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## Current Design Methods for Frame Connections

Méthodes ordinaires de dimensionnement des connexions des cadres

Gewöhnliche Entwurfsmethoden für Rahmenknoten

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### SUMMARY

Current requirements for lateral load design of beam-column joints are either on equilibrium considerations (CEB and New Zealand Codes), or are empirically derived from experimental data (ACI Code). As a result, deformations associated with the design limit states are not considered in evaluating the performance of connections. An alternative approach, satisfying both equilibrium and compatibility requirements is discussed in this paper. The proposed model incorporates the effects of axial load, reduction of concrete compressive strength resulting from diagonal tension, and the influence of indeterminacy which arises in statically redundant structures. Design limits for joint shear stress obtained from the model are compared with those adopted by Design Codes.

### RÉSUMÉ

Les codes couramment utilisés pour le dimensionnement sous charge latérale des joints poutres-colonnes sont soit basés sur des principes d'équilibre (CEB et normes de Nouvelle-Zélande), soit dérivés de valeurs expérimentales (Code ACI). Les déformations associées aux états limites de dimensionnement ne sont pas considérées comme véritable résultat lors de l'évaluation de performance des connexions. Une approche alternative satisfaisant à la fois l'équilibre et les conditions de compatibilité est discutée dans cette étude. Le modèle proposé tient compte des effets d'une charge axiale, de la diminution de la résistance à la compression du béton sous l'effet de tensions diagonales, ainsi que de l'indétermination caractérisant les structures hyperstatiques. Les limites de dimensionnement caractérisant les joints soumis à l'effort tranchant obtenues par ce modèle sont comparées à celles adoptées par les codes officiels.

### ZUSAMMENFASSUNG

Die derzeit gültigen Anforderungen für die Bemessung von Knoten in seitlich belasteten Rahmen sind entweder auf Gleichgewichtsbetrachtungen aufgebaut (CEB und Neuseeland Vorschriften) oder sie wurden empirisch hergeleitet (ACI-Vorschriften). Daraus folgt, dass die mit den Bemessungsgrenzwerten verbundenen Verformungen bei der Beurteilung des Verhaltens der Knoten nicht berücksichtigt werden. Eine alternative Methode, die Gleichgewichts- und Verträglichkeitsbedingungen einschließt, wird in diesem Beitrag besprochen. Das vorgestellte Modell berücksichtigt den Einfluss der Normalkraft, die Verringerung der Druckfestigkeit des Betons infolge Querkraft und den Einfluss der statischen Unbestimmtheit von Tragwerken.



## 1. INTRODUCTION

Requirements for lateral-load design of reinforced concrete (RC) beam-column joints currently implemented in design codes worldwide [1, 2, 3] are based on extensive experimental studies of the inelastic behavior of individual RC frame connections. Because of the complexities associated with controlling tests of statically indeterminate systems, most of the experiments included in the data bases of the various codes have been carried out on highly idealized statically determinate assemblies modelling beam-column connections of frame structures [4, 5, 6, 7].

Forces considered for joint design are illustrated in Fig. 1a. In most contemporary design codes, the magnitudes of these forces are associated with a beam flexural hinging mechanism, implying that beams and columns are dimensioned first. Because a large portion of the forces loading the joint are introduced by bond stresses that develop between concrete and reinforcement, codes require that the magnitude of bond stresses be regulated by controlling the size of longitudinal bar diameter with respect to the available development length (column or beam depth). In the following discussion, it will be assumed that the development length requirements are satisfied *a priori*, and that bond deterioration is not significant (referring both to the derivations and the experimental data discussed in this paper).

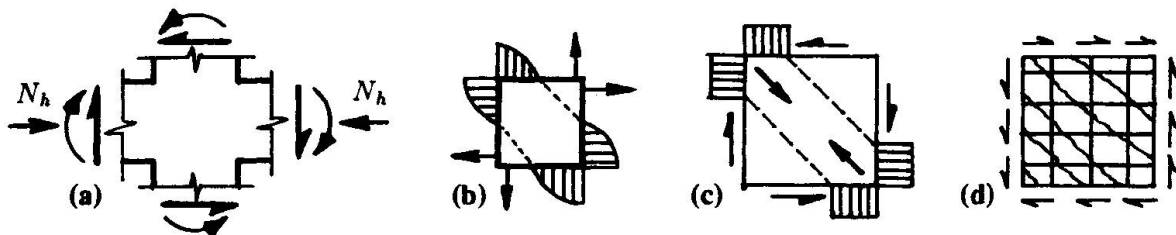


Fig. 1 Shear mechanisms adopted by design codes

Dimensioning and detailing of beam-column joints according to the current design practice is directly linked to evaluation of the so-called joint shear index. This index, which is an estimate of the horizontal and/or vertical joint shear stress, is computed using one of two alternative approaches. The first approach, adopted by the joint ACI-ASCE Committee 352 [1], is characteristic of North American practice. In this approach, the joint-shear index is computed only for a horizontal plane, and is limited by allowable stresses (empirically derived from experiments). The allowable stresses amount to  $0.083 \lambda \sqrt{f'_c}$  (MPa), where  $\lambda$  is 20, 15 and 12 for interior, exterior and corner joints respectively. Furthermore, it has been proposed that development of the concrete strength in the compressed diagonal (Fig. 1b) is facilitated by confining of the concrete core, with closed hoops or other members that frame in to the connection, such as transverse beams and floor slabs [5]. Lateral reinforcement provided in the joint region is the same as that provided in the column critical regions.

The second approach [8], currently adopted by the CEB Model Code [2] and the N. Zealand Code [3], considers both horizontal and vertical joint shear stresses. These stresses, in combination with the normal forces that act at the faces of the joint (Fig. 1a), constitute a loading system for which an admissible state of equilibrium is said to develop from the superposition of a diagonal strut mechanism and a truss mechanism (Fig. 1c, 1d). For the purpose of design, the ultimate joint shear resistance is established by considering the formation of diagonal tension failure inside the joint. For conservatism, the concrete contribution to shear resistance associated with main-strut action (Fig. 1c) is accounted for when the axial force in the column is significant. It has been proposed [8] that axial load improves joint performance by reducing the inclination (from vertical) at which the main-strut mechanism develops. In this model, intermediate horizontal reinforcement is an essential part of the shear-resisting mechanism, comprising the horizontal chords of the idealized truss shown in Fig. 1d. With reference to the role of stirrups or hoops in the overall behavior of beam-column joints, it has been suggested recently that the emphasis which the ACI 352 recommendations place on the confining action of tie reinforcement or other members framing into joint is misleading because the mechanisms of confinement in the critical regions of columns are different from those associated with shear action [9].

The sharp contrast between the two approaches effects different views regarding the definition of acceptable levels of performance, and, to a certain extent, two different interpretations of available experimental evidence as it pertains to the mechanics of joints. Of the two methods, the first is clearly empirical and the latter is based on an admissible equilibrium solution. A consequence of both is that deformations of the joint are not considered in the design process. From the point of view

of structural performance, the amount of deformation required for the joint to develop its resistance is as significant as the magnitude of the resistance. It is therefore desirable that both quantities be reflected in design recommendations.

Although the need for deformation based design criteria for joints has been stated [5], their development has been impeded by the realization that load and deformation demand on the beam-column joints of frames with complex structural configurations is greatly influenced by the three-dimensional effects of the response, and the ability of statically indeterminate structures to redistribute forces. Joint deformation and member expansion are generally unrestrained in statically determinate assemblies, like those used in most experimental studies of joint behavior. Therefore, deformation performance criteria (usually expressed in terms of displacement ductility for the assemblage) quoted by experimentalists in establishing allowable values for joint shear do not directly apply in the case of indeterminate frame structures because of the significant amount of restraint to joint deformation that continuity can cause.

To identify the important parameters that control the behavior of joints, it is instructive to review the available experimental evidence. Early tests conducted in the 1960's on isolated connection specimens illustrated that joint reinforcement in the form of horizontal stirrups, in combination with uniformly distributed longitudinal column reinforcement (so as to form a closed cage), significantly enhanced the shear resistance of joints [4]. Since then, a large amount of research has been conducted, with an aim towards establishing relationships between the degree of deterioration of shear resistance under cyclic loads and the amount of lateral reinforcement provided in the joint. Recent tests (as part of a U.S. - Japan - N. Zealand - China cooperative research effort) of connection specimens simulating Japanese, U.S. and N. Zealand practice (of which the first and last pose the lowest and highest requirements of joint reinforcement, respectively), clearly demonstrate that there is an upper limit in the amount of joint reinforcement, beyond which the overall resistance of a connection is not significantly affected [5]. This manifests the obvious fact that, although large amounts of joint reinforcement can increase the available shear resistance of the joint, this additional strength is not likely to be usable because the demand on the joint is eventually limited by the flexural resistance of adjacent members. All reported specimens (which were designed to fail in beam yielding and had very favorable bond conditions) demonstrated similar overall resistance, with the only significant differences being apparent stiffness of the connections and the amount of deformation. Increased amounts of lateral steel delayed the initiation and propagation of cracking in the joint, and reduced the amount of joint distortion that occurred under a given level of joint shear stress. These results suggest that an important consequence of adding joint reinforcement is an increase of joint stiffness.

Based on this experimental evidence, it is possible to idealize joints as two-dimensional (2-D) panels reinforced in two orthogonal directions, and acted upon by in-plane stresses. However, contrary to familiar 2-D panels, the concrete stiffness contribution is not independent of that of the reinforcement. This is supported by experimental evidence obtained from tests of beam-column joints in which the closed stirrups were replaced by longitudinal beam reinforcement that was uniformly distributed along the height and anchored outside of the joint [7]. Although the joints in these specimens were able to resist the shear demanded to develop beam hinging, a rapid deterioration of joint shear resistance was observed with cycling. Evidently, the stiffening action of closed stirrups occurs not only in the direction of the load but in the perpendicular direction as well, making the confining role of stirrups a more transparent phenomenon. Therefore, the experimental evidence suggests that stirrups contribute to the shear resistance of joints directly (by resisting part of the joint shear), and indirectly (by confining the concrete core, thus enhancing its diagonal compressive strength). However, these two functions are not independent or mutually exclusive of each other - a point that fuels the current debate between differing design philosophies.

From tests of interior beam-column joints with transverse beams, it has been established even joints without any stirrup reinforcement can perform satisfactorily within realistic levels of lateral displacement [5]. This suggests that transverse beams at interior connections and closed hoop reinforcement affect the behavior of joints in a similar manner, by restraining volumetric expansion, which eventually leads to deterioration of joint shear resistance. It has been suggested that this is result of insufficient modeling of boundary conditions in the experimental models, since the enhancement of joint performance occurred only when transverse beams were free of load during the tests. Indeed, experiments in which transverse beams were loaded have been carried out, and in these cases transverse beams had negligible confining contribution. Nevertheless, tests conducted on indeterminate specimens have shown that the excessive deformation that would occur in the beam plastic hinge regions if the assembly was statically determinate, is partially restrained by the presence of adjacent



members. This restraint has been measured experimentally as internal axial forces that developed in beams experiencing inelastic deformation near the connection. The internal forces,  $N_3$  and  $N_h$  (Fig. 1a), represent the reactions of adjacent columns to the lateral displacement required to accommodate beam expansion. Because of the presence of these internal actions, the confining effect of transverse beams on the joint is likely to be significant in actual (indeterminate) structures, where each connection is restrained by the presence of adjacent frames.

With respect the overall displacements of beam-column connections, experiments have shown that a satisfactory joint performance is always accompanied by minimal contribution of joint distortion to the overall lateral drift of the structure (joint performance is deemed satisfactory if cracking is controlled and deterioration of resistance does not occur). In statically determinate assemblies of typical proportions, joint distortion accounted for approximately 25% of the total displacement at low levels of lateral drift. At higher displacement levels (corresponding to approximately 2% interstorey drift, which is often considered a design limit), experimental data suggest that the contribution of joint distortion to total drift became less than 15%, when beams developed sustained flexural hinging as a result of sufficiently reinforced joints or joints with low shear stresses (stresses below values associated with cracking of concrete). In contrast, the contribution of joint distortion to the total drift has been observed to increase with increasing magnitude of total displacement, reaching 40% in cases of joints with insufficient hoop reinforcement or excessive levels of joint shear stress (stresses exceeding the empirical limits in the ACI-ASCE 352 Recommendations [1]). This information suggests an opportunity to link connection design to overall structural response. But any such design approach must consider joint deformations as well as internal forces. No current design basis does so explicitly.

## 2. EQUILIBRIUM AND KINEMATICS OF JOINTS

In this section it is assumed that the joint is properly detailed and that reinforcement is present in quantities sufficient to provide adequate crack control; stresses and strains are averaged over the dimensions of the entire joint [10].

### 2.1 Equilibrium

Average stresses in the joint are depicted in Fig. 1. Shear stresses are introduced by direct member action and by the bond that develops between the main reinforcement and the joint core concrete. Shear stress,  $v$ , is assumed to be uniformly distributed over the boundaries of the joint (Eqn. 1, Table 1). Eqns. 2 and 3 establish equilibrium in the vertical  $l$  and horizontal  $t$  directions at the center of the joint.  $\sigma_l$  and  $\sigma_t$  represent the average vertical and horizontal compressive stresses of the concrete.  $\rho_l$ ,  $\rho_t$  are the available amounts of vertical and horizontal reinforcement (where  $\rho_t = \rho_b + \rho_s$ , for which  $\rho_b$  and  $\rho_s$  are the percentages of horizontal beam reinforcement and horizontal stirrups in the joint, respectively). The corresponding average stresses in the reinforcement are  $f_l$  and  $f_t$ . Dimensions of the joint (depth, width and height), are denoted by  $d_w$ ,  $b$  and  $h$ ;  $N_v$  is the column axial force.  $N_h$  represents the beam axial force, which results from partial restraint to beam expansion provided by adjacent columns in indeterminate frames.

$v = \frac{V_h}{bd_w} = \frac{V_v}{bh}$ (1)	$\sigma_t = -\rho_t f_t - \frac{N_h}{bh}$ (3)	$\sigma_1 - \sigma_t = v \tan \theta$	$\sigma_t = -v \tan \theta$ (6)
$\sigma_l = -\rho_l f_l - \frac{N_v}{bd_w}$ (2)	$\sigma = \begin{pmatrix} \sigma_t & v & 0 \\ v & \sigma_l & 0 \\ 0 & 0 & \sigma_3 \end{pmatrix}$ (4)	$\sigma_1 - \sigma_l = \frac{v}{\tan \theta}$ (5)	$\sigma_l = -\frac{v}{\tan \theta}$
			$\sigma_2 = -v(\tan \theta + \frac{1}{\tan \theta})$

Table 1 Joint equilibrium equations

Furthermore, if the joint reinforcement is of the closed-hoop type, the concrete of the joint is subjected to passive confining stress  $\sigma_3 = -\rho_s f_3$ . Here,  $f_3$  represents the hoop stress in direction normal to the plane of action of the applied shear force. Eqn. 4 describes the average stress tensor associated with the joint. The maximum principal stress,  $\sigma_1$ , associated with the stress tensor must not exceed the tensile capacity of concrete. If, for the sake of simplicity, it is assumed that this capacity is negligible then, since in any plane stresses are either negative (compressive) or zero, it is evident that  $\sigma_2 = \sigma_l + \sigma_t$  (conservation of the first invariant, given that  $\sigma_1$  is assumed zero). The elements of the stress tensor in the  $(t, l)$  coordinate system are related to the principal stresses via Eqns. 5 (Table 1); expressions for  $\sigma_t$ ,  $\sigma_l$  and  $\sigma_2$  in terms of the applied shear stress  $v$  are obtained from Eqns. 5 upon substitution of  $\sigma_1 = 0$  (Eqns. 6).



## 2.2 Kinematics

We assume that the overall geometry of the joint after deformation is described by the average angle of shear distortion,  $\gamma$ , and by the average longitudinal and transverse strains denoted by  $\epsilon_l$  and  $\epsilon_t$  respectively. Equation 7 (Table 2) describes the tensor of average strains as defined in the  $(t, l)$  system. Some useful relations between the entries of the tensor expressed in various coordinate systems are also given in Table 2. The direction of principal strains, which enters the terms of Eqn. 8 is generally unknown. Considering the behavior before yielding of hoops in the joint (of primary interest from the design point of view), it is assumed that, if the reinforcement has not yielded, the direction of principal strains ( $\alpha$ ) is closely related to that of stresses ( $\theta$ ). If  $\theta = \alpha$  is adopted, then it is possible to express  $\theta$  in terms of the values of stress in Table 1. To do this,  $\tan \theta$  is written in terms of strains (Eqn. 8). Strains are substituted with the ratios  $\sigma_2/E_c$ ,  $f_l/E_s$ ,  $f_t/E_s$ ;  $v$  is replaced by  $(-\sigma_t/\tan \theta) = [\rho_t f_t + (N_h/bh)/\tan \theta]$ . This procedure leads to a quadratic equation for  $\tan \theta$  (Eqn. 9), where  $n = E_s/E_c$ , while the strain ratio  $r = e_h/\epsilon_t$  reflects the amount of lateral restraint to joint growth, which is likely to be significant for indeterminate structures. It is evident from Eqn. 9 that such restraint plays the same role algebraically as horizontal joint reinforcement, which is parallel to experimental observation of improved joint performance when transverse beams were present in tests of interior connections. The quantities  $e_v = N_v/E_c b d_w$  and  $e_h = N_h/E_c h b$  have units of strain, and represent the deformations occurring in the joint under purely axial forces.

$\epsilon = \begin{pmatrix} \epsilon_t & 0.5\gamma \\ 0.5\gamma & \epsilon_l \end{pmatrix} \quad (7)$ $\epsilon_1 + \epsilon_2 = \epsilon_l + \epsilon_t$	$\gamma = \frac{2(\epsilon_1 - \epsilon_t)}{\tan \alpha} = 2(\epsilon_1 - \epsilon_t) \tan \alpha \quad \tan^2 \alpha = \frac{\epsilon_1 - \epsilon_t}{\epsilon_1 - \epsilon_l} = \frac{\epsilon_2 - \epsilon_l}{\epsilon_2 - \epsilon_t} \quad (8)$ $\frac{1 + \frac{1}{n\rho_t} - \frac{r}{n\rho_t(n\rho_t+r)}}{1 + \frac{1}{n\rho_t}} \tan^4 \theta + \frac{e_v/\epsilon_t}{(1 + n\rho_t)(n\rho_t + r)} \tan^2 \theta - 1 = 0 \quad (9)$
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### 2.2.1 Behavior before yielding of joint reinforcement

Before yielding of the horizontal joint reinforcement, the magnitude of joint shear stress is related to hoop strain by Eqn. 10 (Table 3). In a similar manner, expressions for the remaining elements of the strain tensor may be obtained as seen in Table 3. The expression for the principal tensile strain ( $\epsilon_1$ ) indicates that the strain is not only affected by the amount of joint shear stress, but also that it increases with increasing vertical axial load, while the influence of lateral restraint on the joint is the reverse. This effect is significant, because the magnitude of diagonal (principal) compression that can develop in the core concrete decreases with increasing magnitude of the tensile strain in the perpendicular direction.

$\epsilon_t = \frac{1}{\rho_t E_s} \left( v \tan \theta - \frac{N_h}{bh} \right) \quad (10)$ $\epsilon_l = \frac{1}{\rho_l E_s} \left( \frac{v}{\tan \theta} - \frac{N_v}{bd_w} \right)$	$\gamma = \frac{2}{E_s(1 - \tan^2 \theta)} \left[ v \frac{\tan^2 \theta \rho_l - \rho_t}{\rho_l \rho_t} + \left( \frac{N_v}{bd_w \rho_l} - \frac{N_h}{bh \rho_t} \right) \tan \theta \right]$ $\epsilon_1 = \frac{1}{E_s(1 - \tan^2 \theta)} \left[ v \tan \theta \frac{\rho_l - \rho_t}{\rho_l \rho_t} - \frac{N_h}{bh \rho_t} + \frac{N_v \tan^2 \theta}{bd_w \rho_l} \right] \quad (11)$
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Eqn. 11a provides a relationship between average joint shear stress and the amount of associated joint distortion. It is evident from the above that column axial load promotes joint distortion, while restraining horizontal loads reduce the amount of distortion at a given level of shear stress.

### 2.2.2 Behavior after yielding of joint reinforcement

Upon yielding of the joint hoops, the pattern of deformation in the joint is likely to change noticeably. In terms of stresses, it is evident from Eqn. 3 that  $\sigma_t = -\rho_t f_y - (N_h/bh) = -v \tan \theta$ , which can be solved for the angle of principal stresses  $\tan \theta$ : ( $\tan \theta = [\rho_t f_y + N_h/bh]/v$ ). This result can be used to obtain expressions for the average longitudinal (vertical) stress, the average nonzero principal stress, and the amount of hoop strain,  $\epsilon_t$ , in terms of the joint shear  $v$  (Eqns. 12, 13, Table 4).

$\sigma_l = -\frac{v^2}{\rho_t f_y + N_h/bh} ; \sigma_2 = -\rho_t f_y - \frac{N_h}{bh} - \frac{v^2}{\rho_t f_y + N_h/bh} \quad (12)$ $\epsilon_t = \frac{1 + \frac{1}{n\rho_t}}{E_c [\rho_t f_y + e_h E_c]^3} v^4 - \frac{e_v v^2}{n\rho_t [\rho_t f_y + e_h E_c]^2} - \frac{\rho_t f_y + e_h E_c}{E_c} \quad (13)$	<p>Thus, a dramatic increase occurs in the values of <math>\sigma_l</math>, <math>\sigma_2</math> and <math>\epsilon_t</math>, for small increases in the value of joint shear after yielding of hoops, since all terms (except <math>v</math>) in Eqns. 12 and 13 remain constant thereafter.</p>
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### 3. MECHANISMS CONTROLLING SHEAR RESISTANCE

The shear resistance of a joint is likely to be limited by the occurrence of one of two possible mechanisms: (1) bond failure of the main reinforcement, which is responsible for introducing the shear stresses,  $v$ , to the joint, or (2) yielding of joint reinforcement. Of these, case (1) was excluded in this study by assuming that pertinent development length requirements are satisfied. For case (2), it has been shown that after initiation of yielding of joint hoops, a substantial increase in the values of  $\sigma_1$  and  $\sigma_2$  will occur. Thus, hoop yielding is likely to be succeeded by either a) yielding of the longitudinal column reinforcement, or b) crushing in the principal direction of concrete compressive stresses. Upper limits to the shear capacity associated with these two mechanisms can be established as follows: for case 2(a), the stress in the longitudinal steel reaches the yielding stress,  $f_y$ . Thus,

$$\sigma_1 = -\rho_l f_y - \frac{N_v}{bd_w} = -\frac{v^2}{\rho_l f_y + N_h/bh} \text{ therefore, } v_n = \sqrt{\left\{ \rho_l f_y + \frac{N_h}{bh} \right\} \left\{ \rho_l f_y + \frac{N_v}{bd_w} \right\}} \quad (14)$$

For case 2(b), failure occurs when the principal compressive stress,  $\sigma_2$ , reaches the crushing strength of concrete,  $f_{max}$ . This crushing strength, however, depends upon the amount of restraint to volumetric expansion, which here is represented by stress  $\sigma_3$ . Furthermore,  $f_{max}$  also depends on the amount of tensile deformation in the perpendicular direction, characterized by  $\epsilon_1$  [11]. It is assumed here that the relationship between stress and strain along the principal compressive direction can be described by [11],

$$\sigma_2 = f_{max} \left[ 2 \frac{\epsilon_2}{\epsilon_{max}} - \left( \frac{\epsilon_2}{\epsilon_{max}} \right)^2 \right] \text{ where, } \left\{ \begin{array}{l} f_{max} = \alpha f'_c \\ \epsilon_{max} = \alpha \epsilon_o \end{array} \right\} \text{ and, } \alpha = \frac{K}{0.8 + 0.34 \epsilon_1 / \epsilon_o} \quad (15)$$

where,  $K = 1 + \rho_s(f_y/f'_c)$ . Upon substitution of Eqn. 15 in Eqn. 12, the following alternative expression for the limiting joint shear stress is established:

$$v_n = \sqrt{|(f_{max} + \rho_l f_y + N_h/bh)(\rho_l f_y + N_h/bh)|} \quad (16)$$

### 4. STUDY OF PARAMETERS IN PROPOSED FORMULATION

In this section, the proposed formulation is used to investigate the influence of various connection-design parameters on conditions corresponding to yielding of hoop reinforcement. Equation 10 can be solved for the amount of shear stress, and Eqn. 11 the corresponding shear distortion, that a joint will tolerate before horizontal reinforcement yields. The "design" variables considered for this hypothetical case study are summarized in Table 5. For this example, the ratio of beam reinforcement ( $\rho_s$ ) was set at 0.015 (top and bottom combined).

To apply the equations for the purposes of this parameter study, two simplifying assumptions were made:

1. The term  $E_c$  is actually a function of  $\epsilon_2$ , which means that Eqn. 9 should be solved iteratively for the angle  $\theta$ . Instead,  $E_c$  was taken as the secant modulus at the point of peak stress for an assumed parabolic concrete compressive stress-strain relationship (Eqn. 15).
2. The particular response condition examined here corresponds to tensile yield of hoop reinforcement. To account for hoop "pre-strain" that would exist in the presence of vertical axial force,  $N_v$ , Poisson's ratio was taken equal to 0.2 to give the expression  $\epsilon_t(available) = f_{yh}/E_s - 0.2e_v$  for use in Eqns. 9, 10, and 11.

The influence that each of the variables considered had on tolerable shear before hoop yield is summarized in Table 5. Only cases for which failure by crushing or vertical yield did not occur before hoop yield are included in this analysis of parametric influences. It can be observed that increasing hoop yield stress ( $f_{yh}(hoops)$ ), amount of hoop reinforcement ( $\rho_s$ ), and beam axial stress ( $N_h/f'_c bh$ ) had similar effects of increasing both tolerable shear distortion and shear stress (Fig. 2a-2c). Indeed, these were the only parameters that resulted in significant and consistent increase of overall joint capacity as limited by hoop yield. By comparison, the proposed formulation shows (Fig. 2d) that column axial stress ( $N_v/f'_c bd_w$ ) had less of an effect on the shear distortion or stress the joint sustained before hoops yielded.

Design Variable	Nominal Value for Study	Range of Values for Study	Effect of increase of variable	
			$v/\sqrt{f'_c}$ @ hoop yield	$\gamma$ @ hoop yield
$f'_c$	35 MPa	20 - 100 MPa	Nonlinear decrease	Nonlinear decrease
$f_{yt}$ (hoops)	400 MPa	300 - 600 MPa	Strong linear increase	Strong linear increase
$\rho_l$	0.04	0.01 - 0.08	Slight nonlinear increase	Slight nonlinear decrease
$\rho_s$	0.003	0 - 0.010	Linear increase	Linear increase
$N_v/f'_c b d_w$	0.05	0 - 0.25	Slight linear increase	Slight linear decrease
$N_h/f'_c b h$	0.02	0 - 0.25	Linear increase	Linear increase

Table 5 Summary of design parameter study

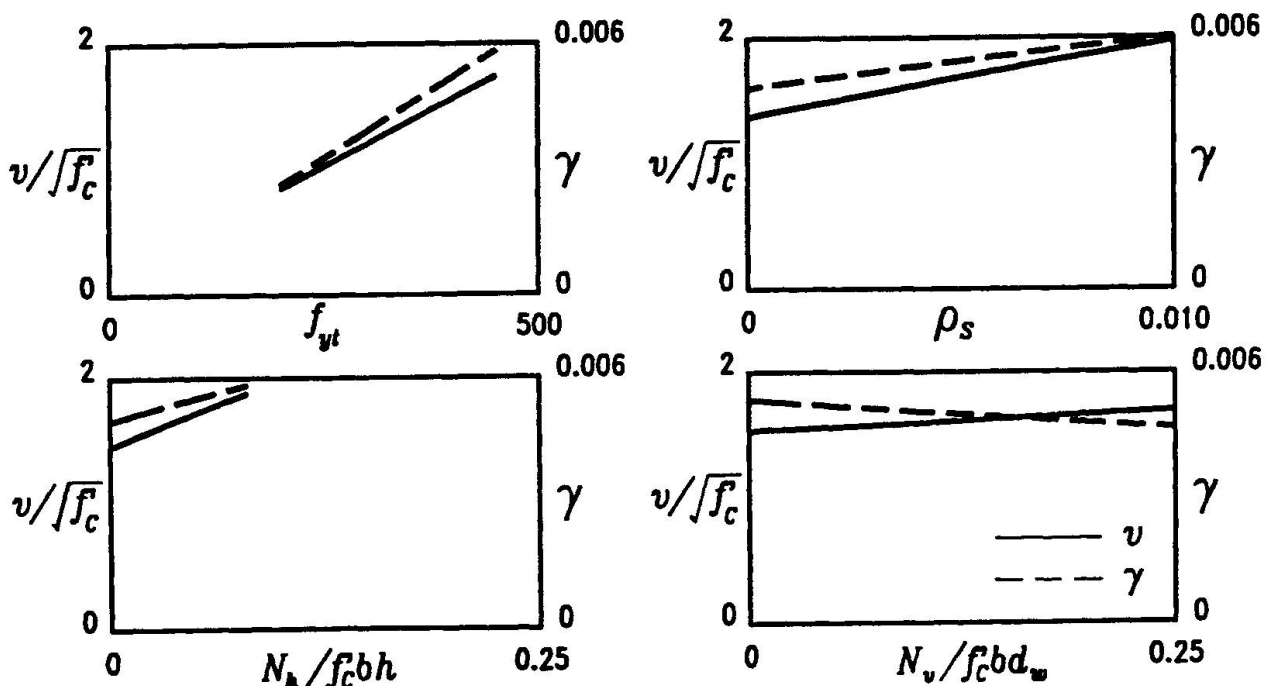


Fig. 2 Results of parametric study

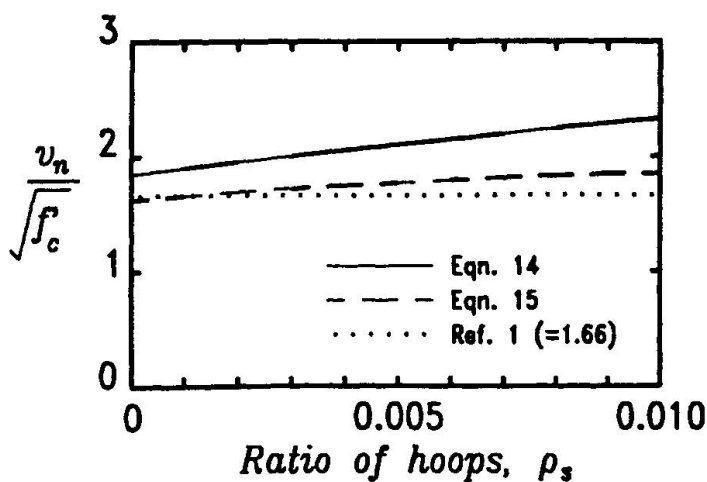


Fig. 3 Shear resistance after hoop yield

Shear capacities associated with connection failure by yield of vertical reinforcement (Eqn. 14) or concrete crushing along the principal diagonal (Eqn. 16) after hoop yield are plotted against the ratio of hoop reinforcement for the sample problem in Fig. 3. For this particular example, it can be observed that capacity will be limited by concrete compression and that, for any ratio of hoops, it is about equal to the value recommended by ACI-ASCE Committee 352 [1]. From the terms of Eqns. 14 and 16, it is apparent that axial stress in the column and beam play an active role, along with the quantity of hoop reinforcement, in determining the shear capacity of connections.





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## 5. CONCLUSIONS

At the root of this Colloquium is the pursuit of generalized approaches for design of structural concrete [12, 13]. Connections in framed structures are a good example of a specific problem in need of unified interpretation— as evidenced by the slowly converging, but still diverse, viewpoints recently presented to the American Concrete Institute by researchers from Japan, New Zealand, and the United States [ACI Fall Convention 1989, San Diego]. In this paper, consideration of both the kinematics and equilibrium of a joint resulted in a comprehensive model that makes it possible to gauge the influence of any design variable at any stage of response and provides design equations for joint shear capacity. While the latter is possible from approaches based strictly on equilibrium and empirical summary, the former can only be achieved by attempting to consider joint deformations. The point is transparent to the particular structural element considered in this paper.

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