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Strut-Crack-and-Tie Model in Structural Concrete

Le modèle tirant-bielle-fissure en béton armé

Das Risswerkmodell – materialgerechtes Modell für Stahlbeton

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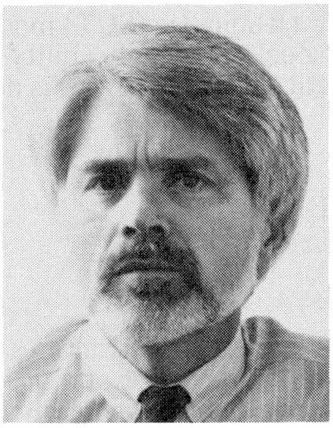

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SUMMARY

The strut-crack-and-tie model, a synthesis of section-by-section design and strut-and-tie model, is presented. The geometry of the cracks and correspondingly that of the struts between cracks follow the linear elastic trajectories and can be predicted explicity. The material parameter of the model is the effective steel strength. It depends on the angle between a crack and the reinforcing bar crossing this crack. The concrete struts are assumed to be stressed biaxially. Applicability of the model to design and detailing is presented in an example.

RÉSUMÉ

Le modèle tirant-bielle-fisure est en fait une synthèse du calcul de résistance section par section et du calcul utilisant l'analogie du treillis. La géométrie des fissures, et donc des bielles en béton délimitées par ces fissures, suit les trajectoires linéaires élastiques et peut ainsi être prévue explicitement. Le paramètre de matériau du modèle est la résistance effective de l'acier, qui dépend de l'analyse formé par la fissure et la barre d'armature traversant la fissure. Les bielles de béton subissent une contrainte biaxiale. Un exemple illustre l'application de ce modèle dans le dimensionnement et les détails de construction.

ZUSAMMENFASSUNG

Das Risswerkmodell, eine Synthese der Querschnittsbemessung und des Stabwerkmodells, wird vorgestellt. Die Rissgeometrie und damit auch die Form und Richtung der dazwischenliegenden Druckstreben wird aufgrund des linear-elastischen Trajektorienbildes angenommen. Die wirksame Stahlfestigkeit, die vom Winkel zwischen Riss und Bewehrungsstab abhängt, dient als Materialparameter. Druckgurt und Druckstreben sind als zweiachsig beansprucht betrachtet. Die Anwendbarkeit des Modells zur Bemessung und zum Bewehren wird an einem Beispiel gezeigt.

1. INTRODUCTION

The section–by–section design for flexure and axial load in B regions is simple, rational and general but generally does not work in D regions. Attempts to adapt the classical 45–degree truss model to the results of shear and torsion tests made it complex, empirical and restricted.

Besides the diagonal compression field theory [1] and methods of dimensioning based on equilibrium solutions from the theory of plasticity [2] the strut–and–tie model [3] has been proposed as a unified design concept which is physical and consistent for all types of structures. Its development is not yet finished.

The paper presents the strut–crack–and–tie (SCT) model: a synthesis of the section–by–section design and the strut–and–tie model. The applicability of the SCT model at dimensioning and detailing of geometrical and statical discontinuities is demonstrated on a dapped end.

2. STRUT–CRACK–AND–TIE MODEL

2.1 Basic considerations

During loading of structural concrete (s.c.) structures first the concrete will crack. As a rule, the cracks are not straight (plain) and consist of a flexural crack and a shear crack. The loads can be increased as long as a) the reinforcement which crosses the cracks is able to carry the tensile forces which are necessary for the equilibrium in the cracked sections or b) the compressive strength of the concrete is reached in the compressive zone or in the web. S.c. structures or parts of them, where their failure is announced by yielding of the reinforcement are called normal–reinforced. Those where the reinforcement remains elastic at failure are called over–reinforced.

80–90 per cent of s.c. structures are normal reinforced. They will fail along one of their cracked sections. This section consists of a flexural–shear crack and a sliding (failure) surface across the compression zone. In case of pure bending only a flexural crack and a failure zone in the compression zone develop, their direction is perpendicular to the member's axis. In all other cases the failing sections are curved: in regions with flexure and shear they are cylindrical, while in those with torsion they are distorted. All these failing sections will be called further *sections.*

It was concluded that the load bearing capacity of normal–reinforced s.c. structures cannot be characterized neither with the concrete compressive strength nor with any reduction of this strength. The adjustment of a calculated ultimate bending moment to a test result, which was influenced by anything, will seldom function by the help of any reduction of the effective compressive strength, as it is well known, that the flexural capacity of a normal–reinforced section is quite insensitive to scatters in the concrete strength.

All these circumstances point out that the effective concrete compressive strength cannot be chosen as material parameter in any physical and consistent model for normal–reinforced s.c. structures.

In a second step a possible and reliable reason of the reduced load bearing capacity of D regions with geometrical discontinuity has been found.

Although each member in Figs. 1b–i contains the same bracket shown in Fig.1a, different additional flexural reinforcements are needed in the root sections of the brackets in cases c, d, f, g and i.

Analyzing the crack patterns of these regions it was found that at those members where additional reinforcement was needed, the direction of the flexural crack in the root section (at the geometrical discontinuity) was not perpendicular to that of the usually horizontal flexural reinforcement (cf. Figs. 2a–i). It is well known, that reinforcing bars are less effective if they cross a crack at an angle less than 90°.

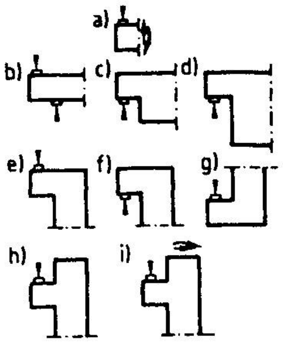


Fig. 1 D regions with the same loading but with partly diffe– rent reinforcement

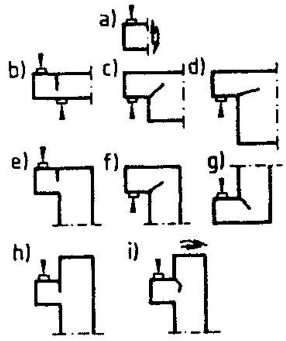


Fig. 2 Direction of flexural cracks in D regions shown in Fig. 1

The idea of the SCT model was born.

2.2 Definition of the SCT Model

The principal characteristic of the SCT model is a system of (cracked) sections. The form of the section depends on the member's geometry and the state of action effects. This form can be simply predicted. The sections are crossed by uniaxially stressed tension ties consisting of reinforcing and/or prestressing steel. The concrete compression struts between the cracks and the compression zone are stressed bi– or triaxially. The tensile strength of the concrete is an integral part of the model, its existence should be guaranteed by both structural and technological measures.

2.3 Selection of Strut–Crack–and–Tie Model

The procedure for laying out SCT models is straightforward omitting any trial and error efforts. Each designer will come with the same model which will reflect the natural load carrying mechanism of s.c. No obscure adaptation of the concrete to the strut–arrangement in the model is demanded. The designer may apply his ingenuity to find the most economic and aesthetic form of the structure.

Starting from a linear elastic analysis of uncracked members and their connections, after applying (if possible) a plastic moment redistribution, the form of the sections can be predicted directly using one and the same model. Instead of looking for the most valid model using minimum strain energy concepts etc. the most convenient reinforcement pattern (e. g. industrialized reinforcement) can be implied directly into the model.

2.4 Interaction of Flexure and Shear

The assumption that flexure and shear can be decoupled and considered separately is assumed to be the source of those troubles which are treated in both invited lectures [4], [5], e.g.:

- shear force carried by concrete
- enhanced shear strength of prestressed members
- shear strength of shear–unreinforced slabs.

If the biaxial state of stress and failure criteria resp. of the compression zone would be rediscovered [6], [7], [8], than all these aforementioned problems and many others could be solved.

In the SCT model the transversal (shear) component of the strength of the compressive zone is fully recognized. The contribution of aggregate interlock along a crack to the shear strength of a member can be easily taken into account at verification of the section's equilibrium.

SCT models indeed allow for consideration of internal forces due to shear, flexure, torsion and axial loads. As when applying this model always the equilibrium of the full member and not a single node is checked, the SCT model provides a real full–member design.

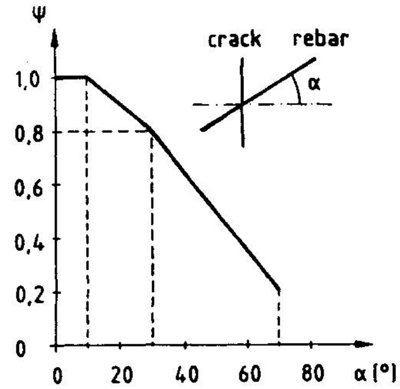
2.5 Material Strength

2.5.1 Steel in Tension

The strength of the steel in tension in ULS is taken as

$$f_{yd} = \Psi \cdot f_{yd}$$

where Ψ is an effectiveness factor, not greater than 1.0. This effectiveness factor accounts for the reduction of the useable strength of a rebar which is not perpendicular to the crack. Factor Ψ, a function of the angle α, shown in Fig. 3, was determined with extended computer calculations using realistic local bond stress – slip relationships, taking into consideration a) the distance between cracks having the characteristic crack width and b) the compatibility requirement that the crack width corresponding to $\alpha = 0$ should remain constant for any α value. The derivation of the effectiveness factor can be found in [9].



For $10° \leq \alpha = 30°$

$\Psi = 1.00 - 0.01\,(\alpha - 10°)$

for $30° \leq \alpha \leq 70°$

$\Psi = 0.80 - 0.015\,(\alpha - 30°)$

Applying the same factors at checking SLS, the proper performance of D regions can be expected, as it was confirmed by control calculations.

Fig. 3 Effectiveness factor Ψ of a skew rebar

2.5.2 Concrete

Compression struts are considered to be stressed biaxially. The components of the strength of a strut are

a) parallel to the strut's axis $D = b \cdot x \cdot f_{cd}$

b) perpendicular to it $\nu \cdot D$.

The assumption a) can be maintained in webs as well until no improved empirical verification of the opposite is available. It is felt, that the strains perpendicular to the strut will not influence very much the effective compressive strength. No evidence of any reduction in the compression zone due to the anchorage of highly stressed stirrups has been found until now. Obviously, reductions at effective web thickness due to ducts for prestressing elements but even to relative thick rebars which are not perpendicular to the strut's axis should be taken into account.

The factor ν in (b) can be deduced from the Mohr–failure–criterion in a similar way as in [6]. It gives a realistic and physical meaning to the V_c term.

No reduction of the concrete strength under loading plates or supports due to anchored tension bars is necessary. Obviously transverse reinforcement is required to "hang up" longitudinal rebars which lie outside the bearing plate, in order to avert a downward splitting of them.

2.6 D Regions

D regions classified in [3] as a) geometrical discontinuities, b) statical discontinuities and c) combination of a) and b) are interpreted in the SCT model as:

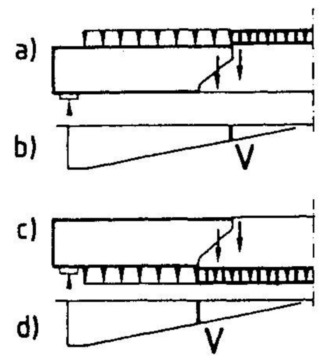
i) the flexural crack at the point of geometrical discontinuity is not perpendicular to any boundary of the region (cf. Fig. 2c, d, f, g and i)

ii) in the neighbourhood of the statical discontinuity only a part of the effective height of the member is really efficient and/or transverse reinforcement (due to bursting and spalling stresses) must be applied when the concrete stress under the loading plate is greater than f_{cd} (cf. deep beams).

Both, variation of the "efficient height", as function of the distance to the concentrated load and inclination of the flexural crack, as function of the linear elastic state of stress at the geometrical discontinuity, can be determined once and for ever and be comprised into quite simple rules.

Knowing the "efficient height" of a given section in a D region, the dimensioning to flexure can be carried out as in a B region: the beam theora can be applied. A similar procedure for deep beams has been suggested in [10].

2.7 Staggering Rule



The form of the sections in both, B and D regions gives a direct interpretation of the required extension of the flexural reinforcement in presence of shear and of the staggering rules as well.

The beam shown in Fig. 4a is loaded on its top, that in Fig. 4c on its bottom. Each beam is divided in the same manner into two parts by a section. The inner forces along these sections substitute the loading on the parts at the right. The difference in the shear loads corresponding to the two sections is obvious (cf. Figs. 4b and d resp.).

Fig. 4 Staggering rule in beams loaded on their top and bottom resp.

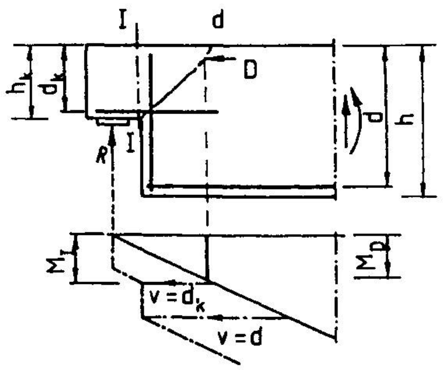
It should be emphasised that SCT models are clearly intended to be behavioural models, thus they remain simple and transparent. The following example should verify it.

3. DAPPED END OF A PREFABRICATED GIRDER

Intensive linear elastic FE calculations showed that the inclination of the flexural crack at the reentrant corner of a dapped end can be taken as

$$\alpha = 90\, h_k / h \quad [°] \qquad (1)$$

Test results [11] confirmed the applicability of (1).



The flexural reinforcement at this corner must be dimensioned for the moment M_D (under the point of application of the compressive inner force on the section d–d shown in Fig. 5a. Applying the horizontal shifts v and v_k resp. due to the effect of the inclined shear crack, approximate the same moment is obtained under the conventional cross section I–I, which is to be dimensioned here actually.

Fig. 5 Dapped end: determination of the design moment for the corner section

At dimensioning the flexural reinforcement in this section the inclined position of the flexural crack relative to the reinforcing bars must be taken into account. If only horizontal reinforcement (A_{sh} in Fig. 6a) will be applied in the dapped end then the flexural equilibrium equation for section d–d in ULS is

$$M_D = M_I = A_{sh} \cdot \Psi_{(90° - \alpha)} \cdot f_{sd} \cdot z_k + A_{sv} \cdot \Psi_{\alpha} \cdot f_{sd} \cdot z_k \cdot \cot \alpha \qquad (2)$$

If inclined reinforcing bars (A_{ss}) will be added (Fig. 6b) then (2) must be extended with

$$A_{ss} \cdot \Psi_{(90° - \alpha - \beta)} \cdot f_{sd} \cdot z_k \cdot (\cos \beta + \sin \beta \cdot \cot \alpha) \qquad (3)$$

The flexural dimensioning of other sections on both sides of the reentrant corner can be performed in the well known manner, as each flexural crack there is perpendicular to the rebars (see Figs. 6c.)

The SCT model gives clear guidance for detailing as well: each rebar may be anchored "behind" that section, where its strength is no more required for maintaining equilibrium in ULS (see Fig. 6d).

Failure loads, calculated with a truss model [11] and SCT model are compared with test results [11] in Fig. 7. The SCT model approximates the test results better.

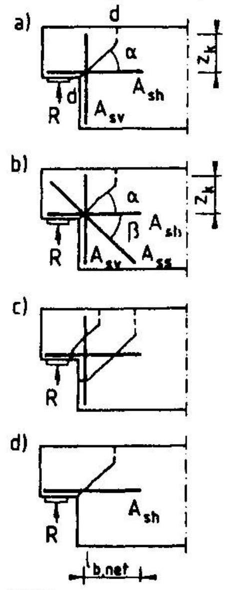


Fig. 6 Dimensioning of a dapped end a) without b) with additional inclined reinforcement at the corner c) at sections on both sides of the corner d) anchorage of the horizontal reinforcement

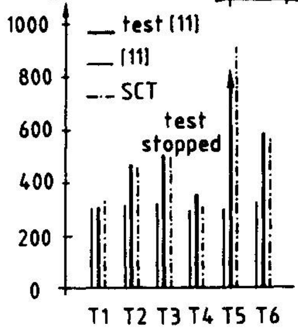


Fig. 7 Comparison of test results [11] with calculated failure loads

SCT models yielded excellent approximations for test results with other D regions as well [12].

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