Design of Disturbed Regions

Conception des zones de discontinuités

Bemessung von Diskontinuitätsbereichen

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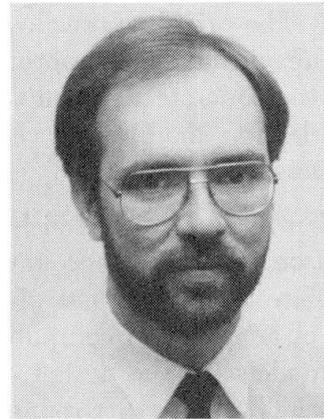Res. Eng.
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SUMMARY

The design method for disturbed regions in the Canadian concrete code is first described and the need to consider strain compatibility is explained. The transition from regions where sectional analysis is applicable to regions where strut-and-tie models apply, is discussed. Examples of the application of strut-and-tie models and non-linear finite element analysis, are presented.

RÉSUMÉ

On décrit la méthode de calcul de zones de discontinuités proposée par le Code Canadien du béton, ainsi que la nécessité de considérer la compatibilité des déformations. La transition entre des zones où l'on peut appliquer l'analyse sectionnelle et celles où l'on prend en considération l'analogie du treillis est discutée. L'application de modèles d'analogie du treillis et d'analyse non-linéaire par éléments finis est illustrée par des exemples.

ZUSAMMENFASSUNG

Das Bemessungsverfahren in der kanadischen Betonnorm für Diskontinuitätsbereiche wird beschrieben und die Notwendigkeit erläutert, die Verträglichkeit der Dehnungen zu berücksichtigen. Es wird der Übergang von den Bereichen, in denen eine Querschnittsbemessung durchgeführt werden kann, zu den Bereichen, in denen Stabwerkmodelle verwendet werden, diskutiert. Beispiele für die Anwendung der Stabwerkmodelle und nichtlinearer Finite Element Methoden werden vorgestellt.

1   INTRODUCTION

The design of a structural concrete member typically involves separating the member into two distinct zones. Regions removed from both geometric and loading discontinuities are usually designed for flexure assuming that plane sections remain plane and designed for shear assuming that the shear stresses are uniform over the nominal shear area of the cross-section. Because the plane-sections analysis for flexure satisfies both equilibrium and compatibility, engineers are able to apply this method to a large variety of loadings and cross-sectional configurations. The 1984 Canadian Concrete Standard (CSA)[1] provides the designer with a sectional design approach for shear based on the compression field model[2], which uses a variable angle for the diagonal compressive stresses in the concrete and satisfies both equilibrium and compatibility. The strain-softening of the diagonally cracked concrete is based on the influence of the principal tensile strain in reducing the compressive load carrying capacity of the concrete as developed by Vecchio and Collins[3]. The principal tensile strain is determined by considering compatibility of strains. However, this current code approach does not consider the beneficial effects of the tension that exists in the concrete between the diagonal cracks. The presence of these tensile stresses has been included in a more recent version of this shear design approach[4] and is based on the modified compression field model[3].

Regions adjacent to discontinuities caused by abrupt cross-sectional changes or concentrated loads or reactions must be designed to account for the resulting disturbed flow of forces. In these regions it is inappropriate to assume that plane sections remain plane and that shear stresses are uniform. In spite of advances in computer analysis techniques, engineers are reverting to simple strut and tie models for the design of disturbed regions, similar to those introduced by Ritter[5] and Mörsch[6] and further refined by Thürlimann et al.[7], Marti[8] and Schlaich et al.[9]. Marti[8] suggested a truss model with a limiting compressive stress in the concrete diagonals of $0.6f_c'$. Schlaich et al.[9] suggested choosing the geometry of the truss model such that the angles of the compressive diagonals are within $\pm15°$ of the angle of the resultant of the compressive stresses obtained from an elastic analysis. These methods, which satisfy equilibrium, do not necessarily satisfy compatibility. The strut and tie model of the 1984 CSA Standard uses compatibility of strains in the determination of the limiting compressive stress in the cracked compressive struts.

2   STRUT AND TIE MODEL OF THE CSA STANDARD

The steps in design of a disturbed region, such as the deep beam shown in Fig. 1, are:

1. Sketch flow of forces in disturbed region and locate nodal zones which are regions bounded by struts, tension ties or bearing areas.

2. Choose dimensions of loading and reaction areas such that nodal zone stresses stay below permissible limits (i.e., $0.85\phi_c f_c'$ in nodal zones bounded by compressive struts and bearing areas, $0.75\phi_c f_c'$ in nodal zones anchoring only one tension tie and $0.60\phi_c f_c'$ in nodal zones anchoring tension ties in more than one direction).

3. Determine geometry of truss model and determine forces in struts and ties. If truss is statically indeterminate, estimate relative stiffnesses of truss members in order to solve forces in struts and ties.

4. Determine required areas of tension ties ($A_s = T/\phi_s f_y$) and check details of tension reinforcement to ensure adequate anchorage into nodal zones.
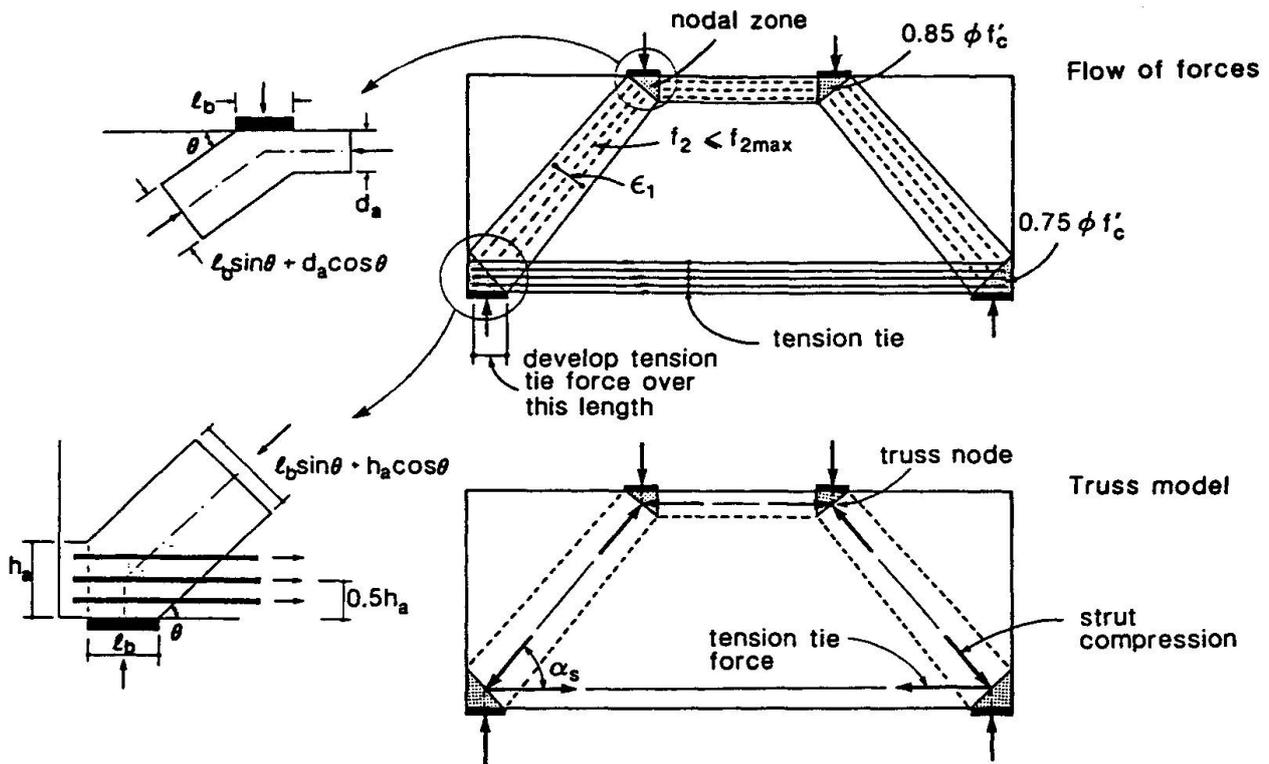
Figure 1: Strut and tie modeling of a deep beam.

5. Check strut compressive stresses from $f_2 = C/\phi_c A_c$ where A_c is effective area of strut as determined by end anchorage conditions (see Fig. 1). Check that f_2 does not exceed crushing strength, f_{2max}, of cracked concrete where:

$$f_{2max} = \frac{\lambda \phi_c f_c'}{0.8 + 170\epsilon_1} \leq 0.85\phi_c f_c' \qquad (1)$$

where ϵ_1 is the principal tensile strain and λ is a factor varying from 1.0 for normal density to 0.75 for structural low-density concrete.

Compatibility of strains is used to determine the principal tensile strain as:

$$\epsilon_1 = \epsilon_s + (\epsilon_s + 0.002)\cot^2 \alpha_s \qquad (2)$$

where ϵ_s is the required strain in tension tie (usually taken as ϵ_y) and α_s is the angle between the strut and the tie crossing the strut.

Although design using strut and tie models appears simple, it takes considerable experience in choosing an appropriate model. Guidance on the use of strut and tie models is given by Schlaich et al.[9], Marti[8,10], MacGregor[11], Collins and Mitchell[12] and Cook and Mitchell[13].

3 SECTIONAL ANALYSIS VERSUS DISTURBED REGION ANALYSIS

Figure 2 compares the experimentally obtained shear strengths of a series of beams tested by Kani[14] with the predicted capacities from both sectional and strut and tie analyses[4]. In these tests the shear span to depth ratio, a/d, was varied from 1 to 7 and no web reinforcement was provided. At a/d ratios less than about 2.5 the resistance is governed by strut and tie
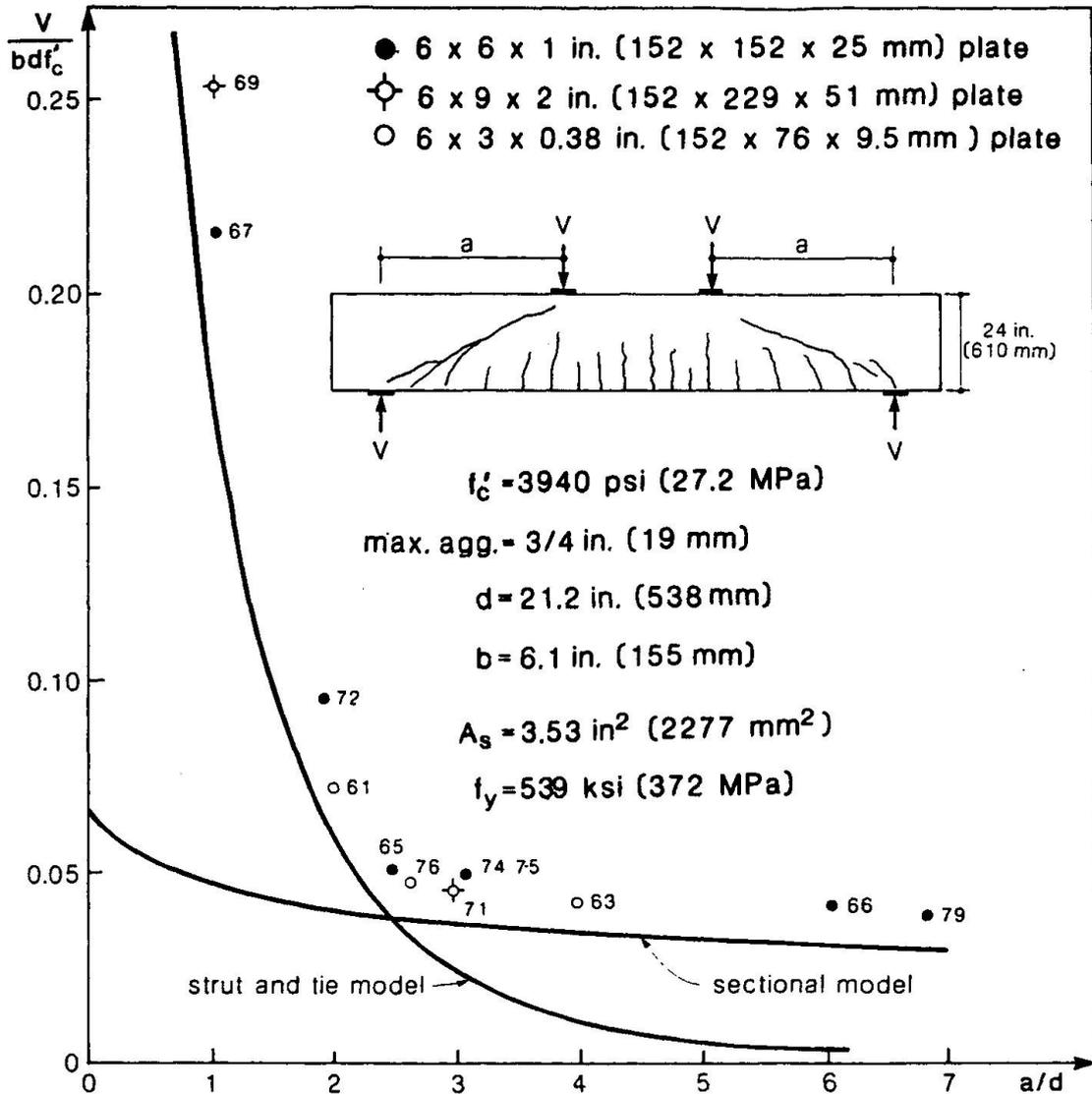
Figure 2: Predictions[4] of shear strength versus a/d ratio for tests reported by Kani[14].

action, with the resistance dropping off rapidly as a/d increases. The failures in this range were governed by crushing of the compressive struts, which were sensitive to the size of the bearing plates. Using the CSA code approach[1] in determining the strength of a member with $a/d = 2.0$ and having a 152 mm long bearing plate resulted in a value of f_{2max} of 10.2 MPa, or only $0.38f_c'$. This relatively low value of the crushing strength of the strut emphasizes the importance of considering strain compatibility in determining the strain softening effect. For a/d values greater than 2.5 the strength is governed by the conditions away from the disturbances created by the support reactions and the applied loads. Because of the significant amount of longitudinal reinforcement failure is governed by shear and the details of the support and loading bearings have little influence on the shear capacity of the member.
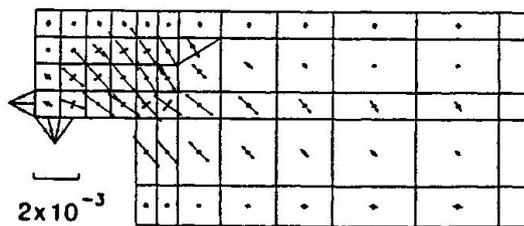
4 DESIGN VERSUS ANALYSIS

Elastic finite element analysis is useful for assessing the serviceability conditions of disturbed regions. It has also been used to approximate the required reinforcement for service and fac-
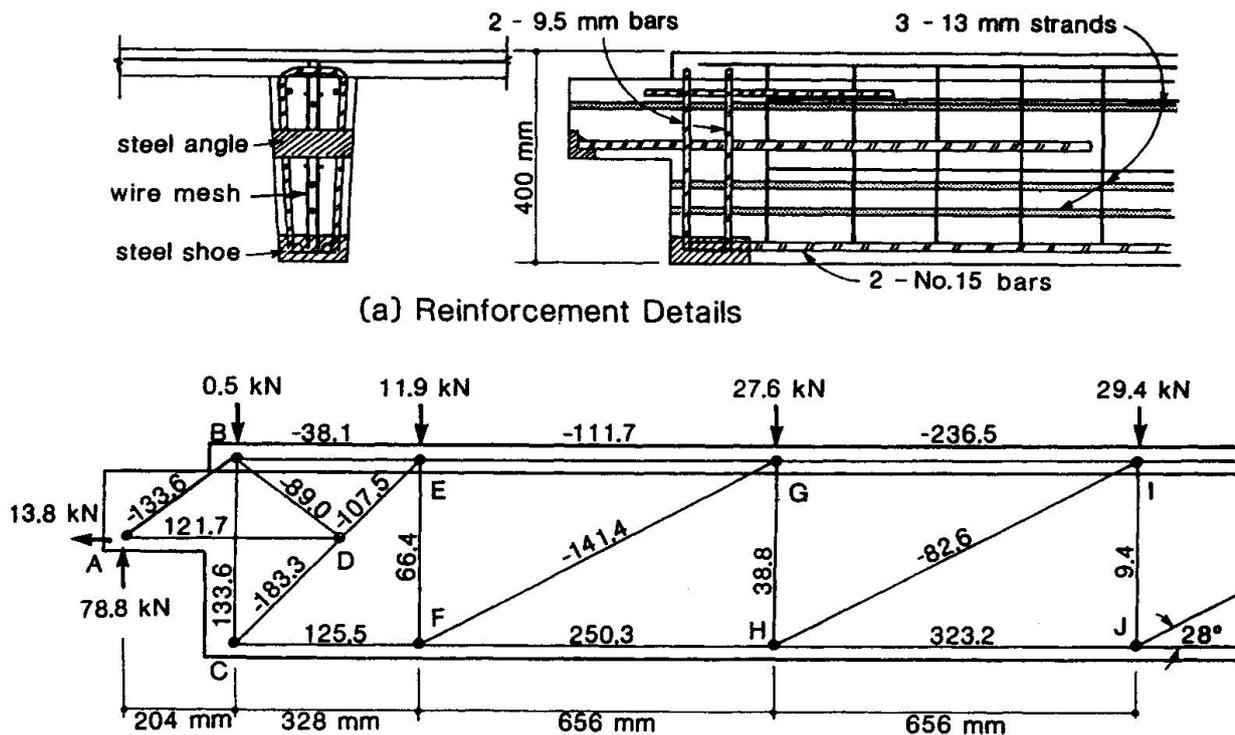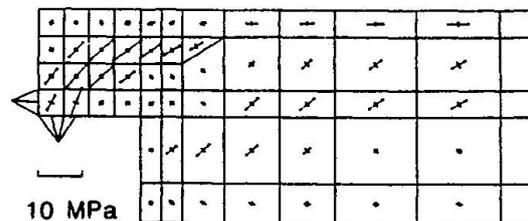
(a) Reinforcement Details

(b) Strut and Tie Model

(c) Principal Concrete Strains   (d) Principal Concrete Stresses

Figure 3: Analyses of thin stemmed precast pretensioned beam with dapped end[16].

tored loading conditions, however, it does not realistically predict the redistribution of stresses that takes place near ultimate. Non-linear finite element analyses[13,15], based on the modified compression field theory, naturally satisfy both equilibrium and compatibility and take into account the strain softening of the concrete compressive stress-strain relationship (see Eq. (1)). The complex transfer of forces across crack interfaces, including the influence of surface roughness and the increased tensile stresses in the reinforcement at crack locations, is approximated and the tensile stresses between the cracks (tension stiffening) is accounted for. This analysis provides the complete response through all stages of loading, including redistribution of stresses after cracking and yielding. Non-linear finite element analysis is an analysis tool, rather than a design tool, useful for verifying the behaviour of regions designed using strut and tie models.

Figure 3 shows a thin-stemmed dapped-end precast pretensioned beam tested at McGill University[16]. In order to permit the development of the strut and tie model shown (see Fig. 3b), a steel shoe was provided at the bottom of the thin stem (see Fig. 3a) and horizontal and vertical ties, welded to the shoe, provided the bottom chord and the vertical tension tie forces in the truss. Figure 3c and d show the principal concrete strains and stresses predicted by non-linear

finite element analysis. The test beam failed at an end reaction of 79.8 kN. The strut and tie and finite element predictions were 78.8 and 84.5 kN, respectively.

CONCLUSIONS

Strut and tie models, which appropriately account for compatibility in determining the compressive capacity of the struts, provide a simple design method for disturbed regions. Non-linear finite element analysis provides a more sophisticated tool for checking the design for both service and ultimate conditions.

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