Computer Programme for Consistent Design of Surface Structures

Programme d'ordinateur pour dimensionnement de structures planes

Programm zur konsistenten Bemessung von Flächentragwerken

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SUMMARY

A computer program for the design of the reinforcement of surface structures has been developed. For given thickness, concrete strength, and location of the reinforcement layers, the required reinforcement areas are calculated for any desired combination of in-plane forces and bending moments. The program can be used in a stand-alone version, for example to determine the load carrying capacity of critical points in reinforced concrete panel, plate, or shell structures. It may also be used as a post-processor for finite element programs determining the required reinforcement at each integration point.

RÉSUMÉ

Un programme d'ordinateur qui calcule l'armature des structures planes a été développé. L'épaisseur lui étant donnée de même que la résistance du béton et l'emplacement des nappes d'armatures, il calcule les sections requises d'acier pour toute combinaison de forces dans le plan de la dalle, ainsi que de moments de flexion. Le programme peut aussi être utilisé par exemple pour déterminer la capacité à supporter une charge de points critiques de structures en béton armé de type panneau, dalle ou coque. Il peut aussi être employé comme post-processeur de programmes d'éléments finis capables de déterminer l'armature requise en chaque point d'intégration.

ZUSAMMENFASSUNG

Ein Computerprogramm für die Bemessung der Bewehrung in Flächentragwerken wurde entwickelt. Für vorgegebene Dicke, Betonfestigkeit und Lage der Bewehrungsscharen werden die erforderlichen Bewehrungsflächen für jede beliebige Kombination von Scheiben- und Plattenbeanspruchungen ermittelt. Das Programm kann zur Ermittlung der Tragfähigkeit von einzelnen kritischen Punkten eines Stahlbetonflächentragwerks benützt werden. Es kann aber auch als Nachlaufprogramm eines Finite Elemente Programms zur Bestimmung der Bewehrungsflächen in jedem Integrationspunkt verwendet werden.

1. INTRODUCTION

Many equations for the determination of the reinforcement in panels for deviating principal stress and reinforcement directions have been published. Baumann [1] has presented a thorough treatment of the subject and also an equation for the design of the reinforcement of panels. The subject of dimensioning the reinforcement in panels subjected to general in-plane loading and in particular Baumann's equation are also discussed in Marti's report [2].

The equations for the reinforcement of panels can be applied to plate elements if the lever arm of the internal forces and the thickness of the load carrying covers is known, as is shown in section 4.4 of [2]. This approach leads to satisfactory results if a realistic guess for the internal lever arm and the depth of the concrete compressive zone can be made. If moments and in-plane compressive forces are acting on an element a realistic estimation of the internal lever arm may become difficult, as will be shown for an example below.

In the remainder of the paper a computer program for the design of reinforced concrete surface structures will be described, which is based on rational mechanic assumptions. The height of the internal lever arm and the depth of the concrete compressive zone need not to be guessed but are the result of the design process for an element subjected to arbitrary in plane forces and moments. Although the application of the computer program requires considerably more computations than programs based on Baumann's equation, this is no serious drawback considering the computing power already available in the design offices today.

2. ALGORITHM FOR THE DESIGN OF SHELL ELEMENTS

2.1 Layered shell element

In accordance with [2] the term shell element is used for an element which may be subjected to in-plane forces and moments. A shell element with unit dimensions in the x- and y-directions and the thickness t is shown in Fig. 1. The element is divided into concrete layers and reinforcement layers. The strain and stress state within each layer is uniform. Assuming a linear variation of the strains through the thickness the strain state in each layer can be calculated for known strains and curvatures of the middle surface.

2.2 Concrete layer

Under loading three states - uncracked, cracks in one direction, cracks in two directions - are possible in each concrete layer. A layer with cracks in one direction is shown in Fig. 2, where axes 1 and 2 denote the principal tensile and principal compressive strain directions, respectively.

The uniaxial stress-strain diagram of concrete is also shown in Fig. 2. The parabola-rectangle diagram of the German code [3] has been substituted by a fourth order parabola for numeric reasons. Tensile strength of concrete is set to zero and tension stiffening is also neglected since the program is to be used as a design tool. In accordance with the code [3] an increase of the concrete strength under biaxial compression will not be considered. Concrete under biaxial compression is analyzed with an orthotropic material model using

the stress-strain diagram of Fig. 2 in each principal strain direction and a shear modulus based on the tangent moduli in the principal strain directions.

Initially the cracks in a layer will form in a direction normal to the principal stress in concrete as soon as this stress becomes tensile. Upon increased loading the principal strain direction is adjusted according to the principle of the minimum of the internal energy. The crack direction and the direction of the concrete struts in a layer (Fig. 2) will always remain orthogonal to the principal tensile strain direction. Biaxial tension in a concrete layer will lead to the formation of two orthogonal sets of cracks and a complete loss of stiffness of this layer. The programing of this material model with re-orientation of the principal tensile strain direction, including the tension stiffening effect and an effective concrete strength depending on the transverse stress state has been described in [4].
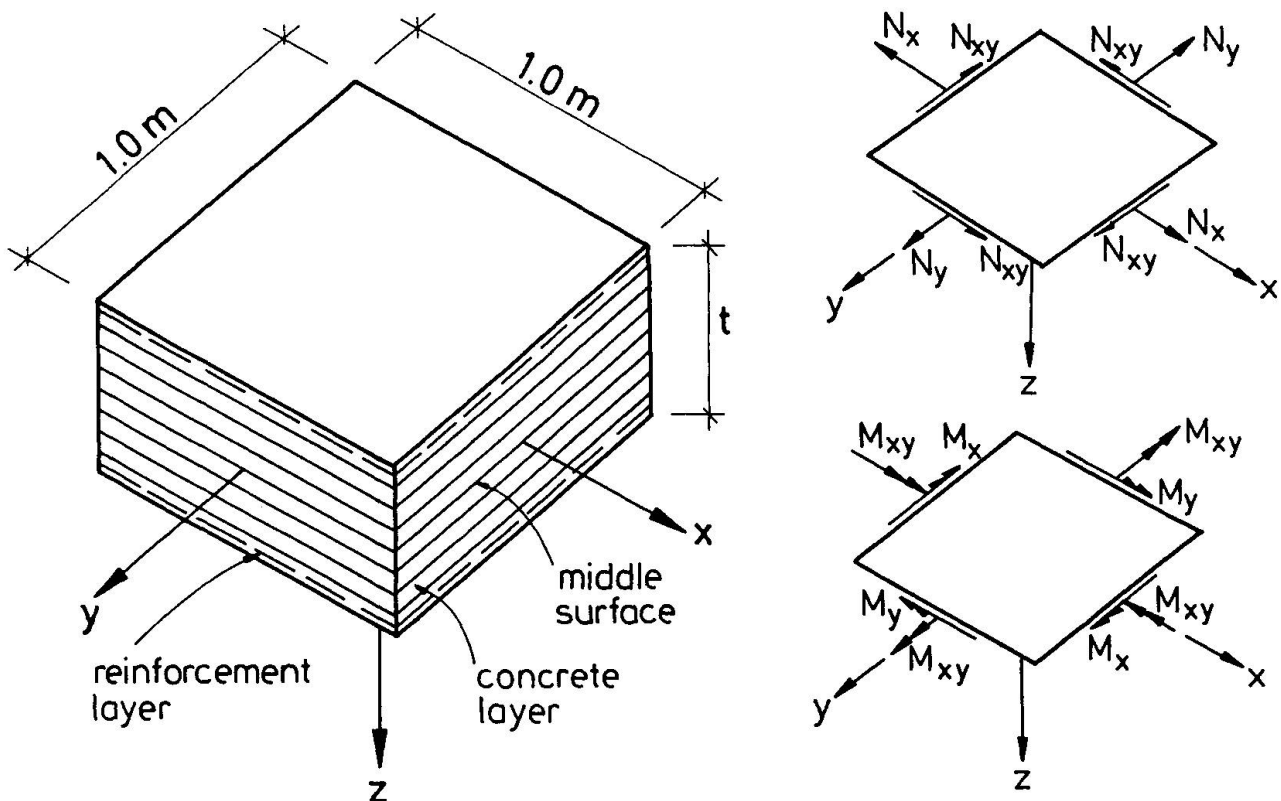

Fig. 1 Layered shell element (left), applied forces and moments (right)
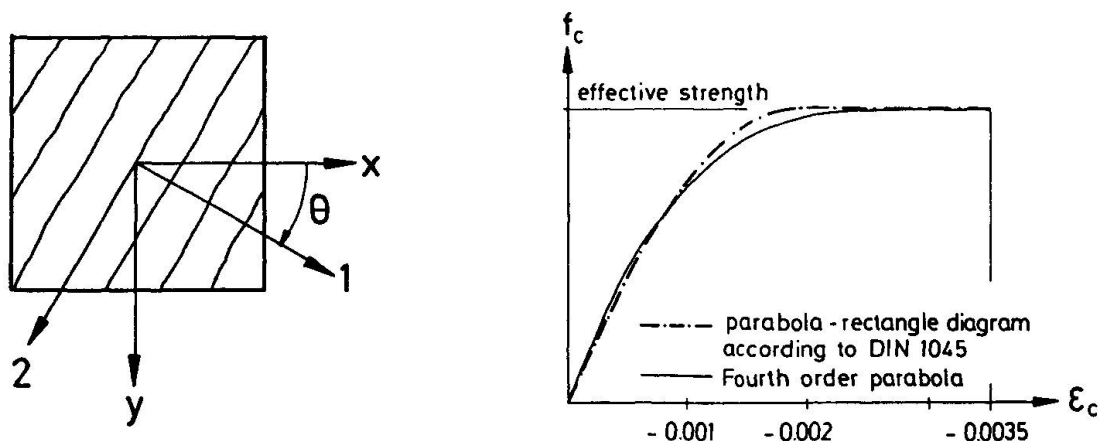

Fig. 2 Concrete layer with cracks in one direction (left) and uniaxial stress-stress strain diagram of concrete (right)

2.3 Reinforcement layer

A reinforcement layer is described by the area of reinforcement, the angle between x-axis and the reinforcing direction, and the distance of the reinforcement layer to the middle surface of the shell element (Fig. 3). An elastic-plastic stress-strain relationship with a hardening modulus is used for tensile and compressive strains as is also shown in Fig. 3. The hardening modulus is set to 1% of the elasitic modulus for numeric reasons. A strain cut-off is assumed for strains larger than 0.005 and smaller than -0.0035 in order to remain compatible with the code [3].
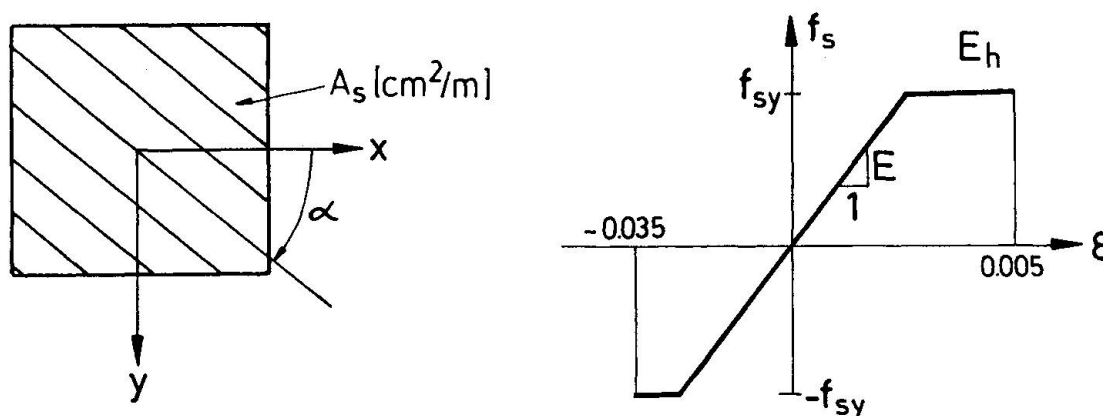

Fig. 3 Reinforcement layer and material model for the reinforcement

2.4 Numerical algorithm to determine the reinforcement of a shell element

The strains in the middle surface and the curvatures of the layered shell element are calculated for any set of applied external forces and moments in an iterative procedure. Starting with linear elastic material properties in the first iteration step, strains and curvatures are calculated. The strains in the individual layers are then determined, and using the material models for the concrete and the reinforcement described above the internal stresses of each layer are calculated. Integrating the internal stresses over the thickness of the element yields the internal forces and moments. Unbalanced forces and moments are calculated as the difference of the externally applied and the internal forces and moments of the element. The unbalanced forces and moments are applied to the layered shell element in the next iteration step. This procedure is repeated until the unbalanced member forces and the differences of the current strains and curvatures with respect to the ones of the last iteration step are smaller than required convergence limits. Then the external member forces of the next load step are applied to the layered shell element and the procedure described above is repeated starting with the element stiffness of the previous load step.

In order to automatically determine the ultimate load of a shell element subjected to proportionally increasing external member forces the following method is used: In the first step the external member forces corresponding to a load multiplier of 1.0 are applied to the shell element. Then the load multiplier is increased in a stepwise manner by 1.0 until no solution can be calculated, i.e. the ultimate load has been overestimated. In the following steps the intervalls are always divided into halves, until the difference between two consecutive load-multipliers remains under a predefined limit. For design examples a value of 0.01 is sufficient for this difference.

In using the layered shell element for the determination of the required reinforcement it is assumed that for each reinforcement layer the reinforcing direction, the distance to the middle surface, and the minimum reinforcement are known. In the first step of the automated design process the load-multiplier for a given set of external forces and moments is calculated with the minimum reinforcement. Then the reinforcement layer with the largest strain is strengthened by adding a reinforcement increment and the load-multiplier is calculated again and compared with the required safety coefficient. The incremental increase of the reinforcement areas of the individual layers and the incremental, iterative determination of the load-multipliers is continued until the required safety coefficient, e.g. 1.75 according to [3], is reached.

The incremental iterative procedure outlined above has been programmed in FORTRAN and implemented on a workstation. The validity of the program has been checked by analyzing numerous experimental tests on panel, plate and shell elements. Comparisons with design examples based on Baumann's theory showed that the same reinforcement areas as determined by Baumann's equation were obtained for panels with in-plane loading and for plates without large reinforcement ratios in the compression zone.

3. ELEMENT SUBJECTED TO AXIAL FORCE AND TWISTING MOMENT

A test program on reinforced concrete plates subjected to torsion was carried out by Marti et al. [5]. Here only plate ML5 will be considered which had a thickness of 20 cm and two reinforcement layers with 20 cm²/m in in the x-direction and two layers with 5 cm²/m in the y-direction. In the analysis an effective concrete strength of 25 MPa was used and the strain cut-off acc. to Fig. 3 was disabled. The envelope of the ultimate loads for different combinations of axial force and twisting moment is shown in Fig. 4. The calculated principal strains and the inclination of principal tensile strain direction are indicated for pure axial force, pure twisting moment, and a combination with Mxy=0.05Nx. While the ultimate load for pure axial force can easily be found by a hand calculation and the core model of [2] will predict the ultimate load for pure twisting moment, the intermediate points of the envelope are most easily determined by a nonlinear analyses with the layered shell element.
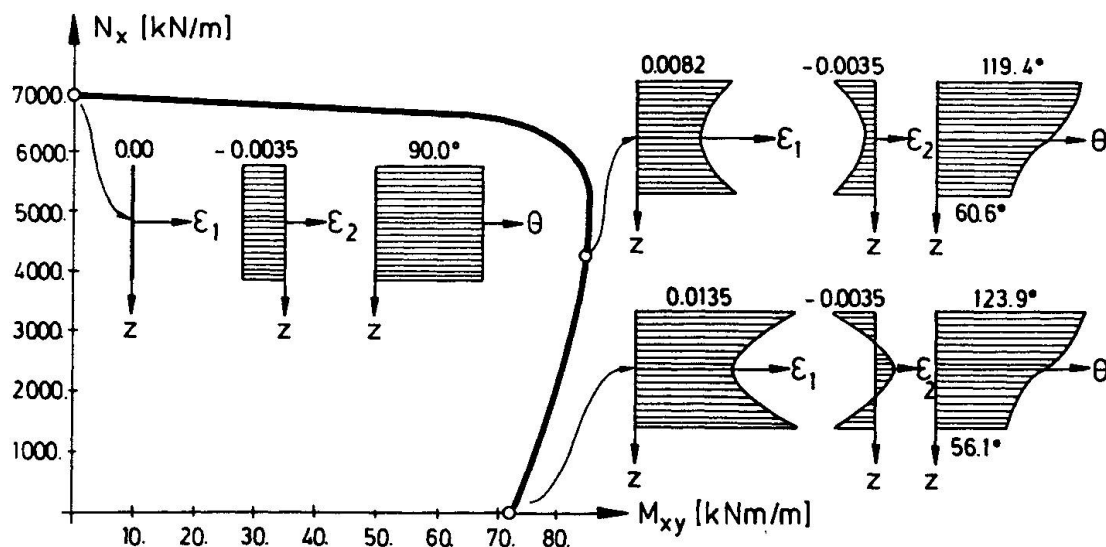

Fig. 4 Calculated ultimate load envelope and selected strain states of plate ML5 [5] subjected to axial force and twisting moment

4. ELEMENT SUBJECTED TO SHEAR FORCE AND BENDING MOMENT

The shell element SE7 was tested at the University of Toronto under a combination of shear force and bending moment [6]. In the following the reinforcement of the shell element will be determined for N_{xy}=1000 kN/m and M_x=113 kNm/m and a required load-multiplier of 1.75. The effective strength of the 28.5 cm thick specimen is taken as 70% of the cylinder crushing strength which was equal to 41.8 MPa. The yield strength of the reinforcement was 492 MPa and the four reinforcement layers had distances of -12.2 cm, -10.0 cm, 12.2 cm, and 10.0 cm, respectively. The reinforcement areas of the test specimen and the calculated strains at failure are shown in Tab. 1. The failure of the test specimen according to the analysis occured at a load-multiplier of 1.754, whereas a load-multilier of 1.81 is reported from the experiment [6].

For the design task the above mentioned properties remained unchanged. Only the reinforcement areas of the four layers were reduced to a minimum reinforcement of 5 cm²/m. The load-multipliers, reinforcement areas, and calculated strains are shown in Tab. 1 for the first step with minimum reinforcement, intermediate steps with stepwise increased reinforcement, and the final step with a load multiplier larger than 1.75. The determination of the required reinforcement areas took only a few seconds on a workstation.

load- multiplier	layer 1 area cm²/m	strain	layer 2 area cm²/m	strain	layer 3 area cm²/m	strain	layer 4 area cm²/m	strain
analysis of ultimate load with reinforcement areas of specimen SE7 [6]								
1.754	41.8	0.0003	13.9	0.0021	41.8	0.0033	13.9	0.0049
incremental, iterative determination of reinforcement areas								
0.283	5.0	0.0004	5.0	0.0011	5.0	0.0049	5.0	0.0020
0.508	5.0	0.0007	5.0	0.0018	11.0	0.0043	5.0	0.0023
1.095	5.0	0.0009	9.0	0.0023	23.0	0.0050	11.0	0.0044
1.539	5.0	0.0015	12.0	0.0038	33.0	0.0046	15.0	0.0043
1.752	5.0	0.0015	14.1	0.0035	37.6	0.0048	16.9	0.0047

Table 1 Analysis of specimen SE7 [6] with reinforcement areas of the test and incremental, iterative determination of reinforcement areas

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