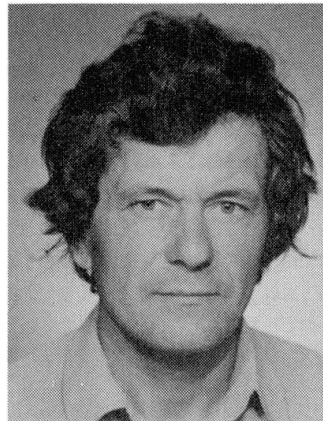# Deformation and Bending-Shear-Torque Failure

Déformation et rupture sous l'action de flexion, cisaillement et torsion

Verformung und Versagen unter Biegung, Schub und Verdrehung

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SUMMARY

It is proposed that nonlinear continuum analysis by the finite element method will become the standard method for the analysis and dimensioning of structural concrete. Potential simplifications of the method are indicated. A simple, unified, continuum model is proposed for beams, slabs and shells.

RÉSUMÉ

Une analyse non linéaire de la structure, utilisant la méthode des éléments finis est proposée afin d'obtenir un modèle unifié de dimensionnement du béton armé Les simplifications potentielles introduites par la méthode sont indiquées; un modèle de continuité unifié est ensuite introduit pour le cas des poutres, dalles et coques.

ZUSAMMENFASSUNG

Die nichtlineare Analyse des Kontinuums mit Hilfe der Methode der Finiten Elemente wird als einheitliche Methode für die Berechnung und Bemessung des Konstruktionsbetons vorgeschlagen. Mögliche Vereinfachungen der Methode werden aufgezeigt. Es wird ein einfaches und einheitliches Kontinuum-Modell für Balken, Platten und Schalen vorgeschlagen.

1.INTRODUCTION

The assessment of the present design practice as well as the objectives presented in the Breen's introductory report reflect actual, real and even acute needs of the building industry and research. The present paper adresses two of the objectives set forth in the report which call for the consideration of the overall structural behaviour and for new transparent methods.

It is common practice that the overall structural behaviour is analyzed by conventional elasticity models. They provide cross-sectional variables for dimensioning but hardly reflect the actual flow of the forces throughout the structure even at service load level not to speak of the ultimate limit load state. This practice entails strictly sectional approach in dimensioning and detailing. Apparently, it also is inconsistent from the purely mechanical point of view in regard to statically indeterminate structures. Actual flexibility of a reinforced concrete structure is far from that assumed in the elastic analysis. If some more global models are used for dimensioning, typically strut-and-tie models (STM), then in effect some of the conventional elasticity analysis might be eliminated. Marti [3], for example, suggests the application of the STM for continuous beams. The sole purpose of the elastic analysis then is to determine the residual forces (reactions at intermediate supports in the above example). Even these moments can be modified (to some extent arbitrarily) owing to redistribution.

Another substantial flaw of this inconsistent mixture of the elastic analysis and limit state dimensioning turns up when the limit state of deformations is considered. Here the results of the elastic analysis are literally useless. Moreover, the rules for the evaluation of deformations of reinforced concrete members are a real maze in the Czechoslovak standards. These considerations suggest that a consistent unified approach to structural concrete should avoid the traditional elasticity methods of analysis.

2.NONLINEAR CONTINUUM APPROACH

At present there is no feasible method that could replace the elasticity methods. It also seems to be too daring an idea to develop what in fact would be 'another nonlinear mechanics', specific for reinforced concrete. In the long perspective, however, taking into account the amazing advance of the computer technology it seems more imaginable. What should feature such a method?

First, it must be nonlinear since reinforced concrete simply is not linear elastic even at service loads. This brings about the problem of variable loads. Various load cases must be analyzed separately in the realm of the nonlinear analysis. But even today there is a trend to reduce the number of load cases and an exact evaluation of the absolute maxima of the cross-sectional forces is not deemed necessary. A compromise appers to be attainable in this respect.

Second, the method should be able to furnish strains, deflections, stresses and forces at various load levels. Single computational model can then be used to assess all kinds of limit states. This need has indirectly been emphasized in the 'Performance requirements' of the Breen's introductory report. STM approaches

cannot meet this requirement.

An incremental nonlinear finite element analysis (FEA) meets this requirements at the expense of prohibitive demands on the computer and analyst parts. There is, however, a great potential for reducing these demands if the material models are simplified to the level required for design purposes. Hardly any FEA package has yet tried to do so. An obvious possibility is to neglect the tension strength of the concrete as most building codes do. The cost of the nonlinear analysis may also be greatly reduced if we realize that severe nonlinearity occurs just in the initial phase of the gradual loading when the crack pattern develops. In this phase, a continuum analogy to the STM builds up. Subsequent loading invokes almost linear behavior. An obvious advantage of the nonlinear continuum analysis is that less engineering judgment or intuition is required in comparison to STM. It should also be recalled that the finite element discretization provides a natural support for the visualization of the structure and its displacements, strains and the flow of internal forces. Current computer graphics software and hardware relies on discretizations identical or similar to those adopted in the FEA. This aspect might become decisive in the future.

3.DEFORMATION PATTERN IN RC BEAMS

As already pointed out earlier, specific simple design oriented material models are necessary in order to make the nonlinear analysis acceptable. The models for B regions of beams, plates and shells should employ appropriate kinematic hypotheses. Present model is based on the assumption of plane cross-sections (Timoshenko or Flugge hypothesis in the scope of plane frame beams). In reinforced concrete beams, this assumption is not sufficient. The deformation pattern must include the transverse strains ε_y , ε_z if stirrups and hoops are to be activated. An obvious possibility is to supplement the required displacement modes into the assumed displacement field. Compatibility of deformations is then maintained but special finite elements are required and in effect fully three-dimensional analysis is entailed. We prefer another option.

The assumed displacement field remains that of the space beam and the strains ε_y and ε_z are evaluated from the transverse equilibrium equations $\sigma_y = \sigma_z = 0$. Compatibility is violated by the terms $\partial v/\partial x$ and $\partial w/\partial x$ where v, w denote the transverse displacements relative to the displaced local reference frame of the cross-section. These terms may be neglected if the displacement field varies only gradually along the x-axis. Analogous assumption on the stress field has been utilized in the derivation of the above transverse equilibrium equations and Marti[3], Nielsen[4] and Harmon[1] also adopted it. The strains ε_y and ε_z must be solved for by an iteration since the material laws of concrete and reinforcement steel are nonlinear.

The discretization in the cross-section plane is performed by dividing it into sufficiently small subregions. In each subregion, homogeneous stress and strain are assumed. This is a direct generalization of the layer concept in the analysis of reinforced concrete plates and shells. Perfect bond is assumed. The same

strains thus apply to concrete and reinforcement. Reinforcement bars of any directions are easily included and equally treated. It is important to note that an analogous deformation pattern was developed for reinforced concrete plates and shells. In these structures, there is just one condition of the transverse equilibrium and one corresponding transverse strain component to be solved for. The formulation is simpler. On the contrary, adequate STM are staticaly determinate spatial trusses which may be dificult to construct.

Simple elastic-plastic laws were implemented (principal stresses plasticity condition for concrete) and several sample solutions carried out for the purpose of validation. Most are plane frame beams (no torsion). The T-beam in Fig.1 was tested by Leonhardt[2] for bending-shear failure.
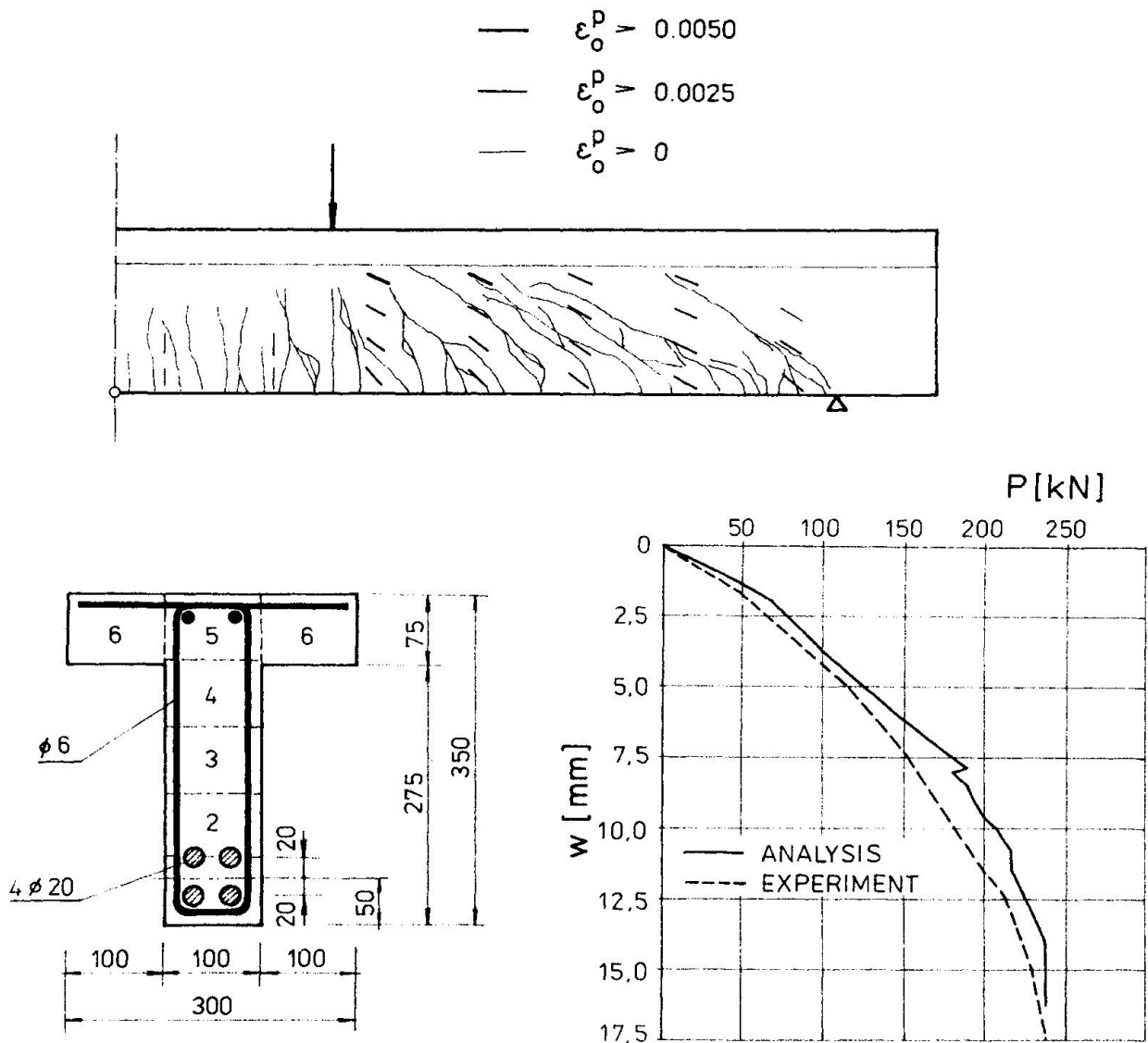

Fig.1 Cross-section, crack pattern (test and analysis) and load deflection curve of a T-beam

The crack pattern at failure compares well to the experimental one

and so does the load-deflection curve. The smeared crack width and the direction of the cracks of the analytical (FEA) solution are indicated by abscissas. Actual stiffness and deformation pattern are well modeled. It is worth mentioning that only 7 finite elements were used in this example. The cross-section was divided into 6 concrete subregion (see Fig.1) and 2 reinforcement layers. Tension strain softening was adopted in this example for concrete and the load-deflection graph is therefore curved. If tension-cut-off concrete is adopted the graph is nearly straight.

4.CONCLUSIONS

Nonlinear continuum modeling of structural concrete is proposed to become a unified analytical and dimensioning tool. Possible simplifications of the nonlinear analysis were indicated which would make it more accesible to designers and engineers. A simple model for beams subjected to simultaneous flexure, shear and torsion is described. The model easily includes arbitrary reinforcement. A sample solution demonstrates that the actual crack pattern and corresponding STM action are well approximated by the proposed model.

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