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A Consistent Shear Design Model

Modèle cohérent de dimensionnement à l'effort tranchant

Ein konsistentes Schubbemessungsverfahren

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SUMMARY

A shear model is presented which takes into account residual tensile stresses in cracked concrete. The model treats both prestressed and non-prestressed members and accounts for the influence of amount of longitudinal reinforcement, magnitude of moment, axial force and member size.

RÉSUMÉ

Un modèle de dimensionnement à l'effort tranchant qui tient compte des contraintes de traction résiduelles dans le béton fissuré est présenté dans cet article. Il traite des éléments de structure précontraints et non-précontraints et tient compte de l'influence de la quantité d'armature longitudinale, de l'intensité du moment, de la force axiale et de la taille de l'élément-même de structure.

ZUSAMMENFASSUNG

Ein Schubbemessungsverfahren wird beschrieben, das Zugeigenspannungen in gerissenem Beton berücksichtigt. Das Verfahren behandelt sowohl vorgespannte wie auch nicht vorgespannte Elemente und berücksichtigt solche Grössen wie Längsbewehrung, Biegemomente, Axiallasten und Bauteilgrössen.

1. INTRODUCTION

In 1973 the ACI-ASCE Shear Committee [1] concluded the introduction to its state-of-the-art report with the words:

> During the next decade it is hoped that the design regulations for shear strength can be integrated, simplified and given a physical significance ...

The shear provisions of the 1984 Canadian Concrete Code [2, 3], which were based on the compression field model, introduced both strain compatibility and the stress-strain characteristics of diagonally cracked concrete, enabling some of the objectives stated above to be achieved. However, because this model neglected the residual tensile stresses in diagonally cracked concrete, it was restricted to members with shear reinforcement.

The modified compression field model [4] considers the influence of residual tensile stresses in the cracked concrete and hence provides the basis for a consistent shear design model. In this paper a design approach based on the modified compression field theory is presented.

2. RESIDUAL TENSILE STRESSES IN CRACKED CONCRETE

Tests of reinforced concrete panels subjected to pure shear [4] demonstrated that even after cracking, tensile stresses exist in the concrete between the cracks and that these stresses can significantly increase the ability of reinforced concrete to resist shear stresses.

Cracked reinforced concrete transmits load in a relatively complex manner involving opening or closing of pre-existing cracks, formation of new cracks, interface shear transfer at rough crack surfaces, and significant variation of the stresses in reinforcing bars due to bond, with the highest steel stresses occurring at crack locations. The modified compression field model attempts to capture the essence of this behaviour without considering all of the details. The crack pattern is idealized as a series of parallel cracks all occurring at angle θ to the longitudinal direction. In lieu of following the complex stress variations in the cracked concrete, only the average stress state and the stress state at a crack are considered. As these two states of stress are statically equivalent, the loss of tensile stresses in the concrete at the crack must be replaced by increased steel stresses or, after yielding of the reinforcement at the crack, by shear stresses on the crack interface. The shear stress that can be transmitted across the crack will be a function of the crack width. Note that shear stress on the crack implies that the direction of principal stresses in the concrete changes at the crack location.

The average principal tensile strain, ϵ_1, in the cracked concrete is used as a "damage indicator" which controls the average tensile stress, f_1, in the cracked concrete, the ability of the diagonally cracked concrete to carry compressive stresses, f_2, and the shear stress, v_{ci}, that can be transmitted across a crack.

3. SHEAR DESIGN OF BEAMS

In applying the modified compression field theory to the design of beams it is appropriate to make a number of simplifying assumptions. As illustrated in Fig. 1, the shear stresses are assumed to be uniform over the effective shear area, $b_w jd$. The highest longitudinal strain, ϵ_x, within the effective shear area is used to calculate the principal tensile strain, ϵ_1.

The longitudinal strain, ϵ_x, can be determined from a plane sections analysis (see computer program "RESPONSE" [5]) which accounts for the influence of axial load, moment, and shear. For design, ϵ_x can be approximated as the strain in the "bottom chord" of a truss as

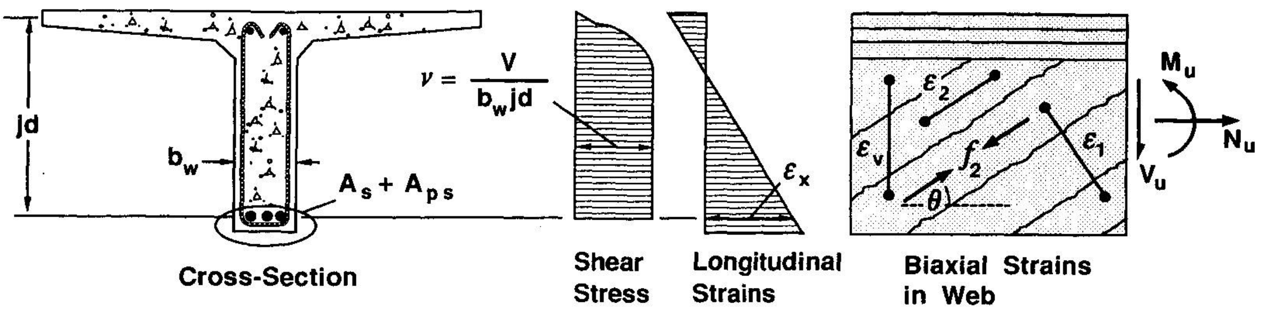

Fig. 1 Beam subjected to shear, moment, and axial load.

$$\epsilon_x = \frac{(M_u/jd) + 0.5N_u + 0.5V_u \cot\theta - A_{ps}f_{se}}{E_s A_s + E_p A_{ps}} \geq 0 \tag{1}$$

where A_s and A_{ps} are the areas of non-prestressed and prestressed longitudinal reinforcement on the flexural tension side of the member.

From strain compatibility, the principal tensile strain, ϵ_1, can be related to the longitudinal compressive stress and the magnitude of the principal compressive strain, ϵ_2, in the following manner:

$$\epsilon_1 = \epsilon_x + (\epsilon_x - \epsilon_2)\cot^2\theta \tag{2}$$

Hence as the longitudinal strain, ϵ_x, becomes larger and the inclination,θ, of the principal compressive stresses becomes smaller, the "damage indicator", ϵ_1, becomes larger.

For design purposes the shear strength, V_u, of a member can be expressed as

$$\begin{aligned} V_u &= V_c + V_s + V_p \qquad (3) \\ &= \beta\sqrt{f_c'}b_w jd + \frac{A_v f_y}{s} jd\cot\theta + V_p \end{aligned}$$

where V_c = shear strength provided by residual tensile stresses in the cracked concrete
V_s = shear strength provided by tensile stresses in the stirrups
V_p = vertical component of force in the prestressing tendons.

The values of θ and β, determined by the modified compression field model are given in Table 1 for members with web reinforcement and in Table 2 for members without web reinforcement.

The tabulated values of the residual tensile stress factor, β, are based on the following expressions:

$$\beta = \frac{0.18}{0.3 + \dfrac{24w}{a+16}} \tag{4}$$

but

$$\beta \leq \frac{0.33\cot\theta}{1+\sqrt{500\epsilon_1}} \tag{5}$$

Equation (4) is based on the shear stress that can be transmitted across diagonal cracks and hence is a function of the crack width, w, and the maximum aggregate size, a. The crack width is assumed

v/f'_c		Longitudinal Strain $\epsilon_x \times 1000$				
		0	0.5	1.0	1.5	2.0
≤ 0.05	β	0.437	0.251	0.194	0.163	0.144
	θ	28°	34°	38°	41°	43°
0.10	β	0.226	0.193	0.174	0.144	0.116
	θ	22°	30°	36°	38°	38°
0.15	β	0.211	0.189	0.144	0.109	0.087
	θ	25°	32°	34°	34°	34°
0.20	β	0.180	0.174	0.127	0.090	0.093
	θ	27°	33°	34°	34°	37°
0.25	β	0.189	0.156	0.121	0.114	0.110
	θ	30°	34°	36°	39°	42°

Table 1 Values of β and θ for members with web reinforcement.

z mm		Longitudinal Strain $\epsilon_x \times 1000$				
		0	0.5	1.0	1.5	2.0
125	β	0.406	0.263	0.214	0.183	0.161
	θ	27°	32°	34°	36°	38°
250	β	0.384	0.235	0.183	0.156	0.138
	θ	30°	37°	41°	43°	45°
500	β	0.359	0.201	0.153	0.127	0.108
	θ	34°	43°	48°	51°	54°
1000	β	0.335	0.163	0.118	0.095	0.080
	θ	37°	51°	56°	60°	63°
2000	β	0.306	0.126	0.084	0.064	0.052
	θ	41°	59°	66°	69°	72°

Table 2 Values of β and θ for members without web reinforcement.

to equal $\epsilon_1 s_{m\theta}$ where $s_{m\theta}$ is the average spacing of the diagonal cracks. Equation (5) is based on the average residual tensile stress in cracked concrete that has a cracking stress of $0.33\sqrt{f'_c}$. See Reference 5 for more details.

In determining the values in Tables 1 and 2 it was assumed that the crack spacing, $s_{m\theta}$, equalled about 300 mm for members containing web reinforcement while, for members without web reinforcement, the spacing of diagonal cracks was assumed to be $s_{mx}/\sin\theta$ where s_{mx} is given in Fig. 2.

To avoid yielding of the longitudinal reinforcement

$$A_s f_y + A_{ps} f_{ps} \geq \frac{M_u}{jd} + 0.5N_u + (V_u - 0.5V_s - V_p)\cot\theta \tag{6}$$

4. INFLUENCE OF MEMBER SIZE

It has been shown [6] that the modified compression field theory can predict the shear capacity of members containing web reinforcement with reasonable accuracy (coefficients of variation about 10%). The influence of axial tension on the shear capacity of members not containing web reinforcement is also predicted accurately (COV 11%) [7].

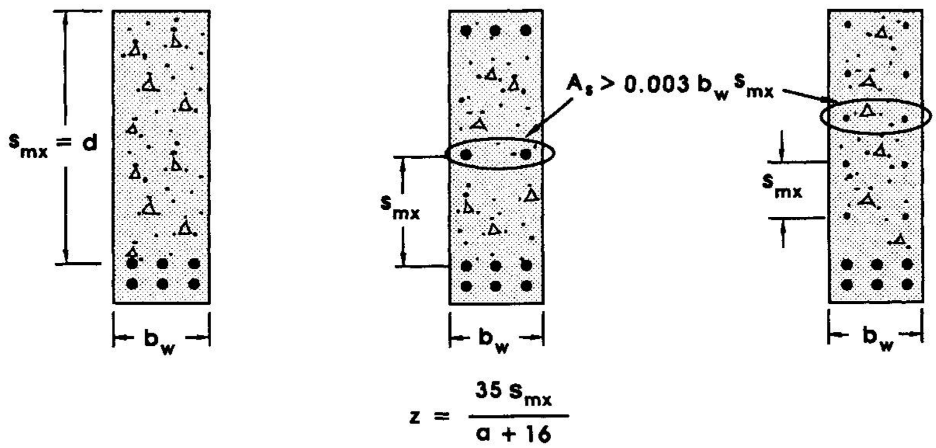

Fig. 2 Crack spacing parameter z.

For the purpose of this colloquium it is of particular interest to discuss the influence of member size upon the shear strength of members not containing web reinforcement. For members not containing crack control reinforcement (Fig. 2), as member size increases the crack spacing, $s_{m\theta}$, will increase and hence, for a given value of strain, ϵ_1, the crack width will increase. An increase in crack width reduces the shear stress that can be transmitted across the crack and hence reduces the shear strength of the member. It can be seen from Table 2 that members containing large amounts of longitudinal reinforcement or prestressed concrete members (i.e., members with low values of ϵ_x) will be less sensitive to member size than lightly reinforced members or members subjected to high moments (i.e., members with high values of ϵ_x). Thus if ϵ_x equals 0 the shear stress at failure increases by a factor of 1.33 as the size decreases by a factor of 16, while if ϵ_x equals 0.002 the shear stress increases by a factor of 3.10.

Figure 3 compares the observed shear stresses at failure for a series of lightly reinforced beams with depths ranging from 200 mm to 3000 mm [8]. Also shown are the shear stresses at failure predicted from the β values in Table 2. It can be seen that the theory predicts the strength of the larger beams very well, but is somewhat conservative for the smallest beam.

5. CONCLUDING REMARKS

The amount of stirrups required to resist a given shear, V_u, can be determined from

$$\frac{A_v f_y}{s} \cdot jd \geq \left(V_u - \beta\sqrt{f_c'} b_w jd - V_p\right) \tan\theta \tag{7}$$

where both β and θ depend on the longitudinal strain parameter, ϵ_x, which accounts for the influence of moment, axial load, prestressing, and longitudinal reinforcement ratios. In addition, for members without web reinforcement, β and θ are strongly dependent on member size.

The sectional design model summarized above is appropriate for those regions of structures where it is reasonable to assume that plane sections remain plane. In regions where there are substantial static or geometric discontinuities, it is more appropriate to use strut-and-tie design models (see Reference 5).

ACKNOWLEDGEMENT

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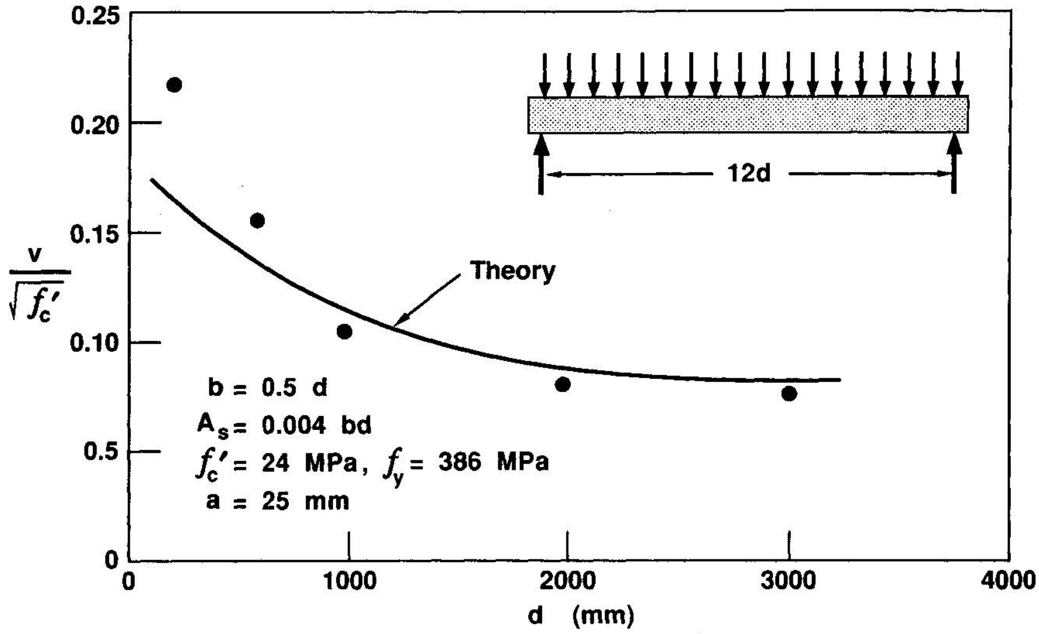

Fig. 3 Comparison of predicted and observed shear stress at failure on section distance *d* from the support.

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Strut-Tie Approach in Higher Strength Concrete Members

Analogie du treillis pour éléments de structures en béton à très haute résistance

Bemessung von Bauteilen aus hochfestem Beton mit Stabwerkmodellen

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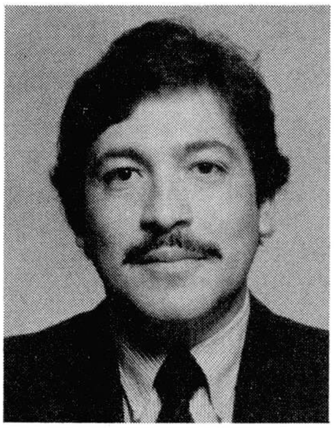

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SUMMARY
The different engineering properties observed in higher-strength concretes clearly indicate the need for further basic information on the behaviour of higher-strength structural concrete. This paper makes an immediate contribution with regard to both, performance of higher-strength structural concrete and the use of strut-tie models.

RÉSUMÉ
Les différentes caractéristiques techniques que l'on observe dans les bétons à très haute résistance indiquent clairement la nécessité d'obtenir de plus amples renseignements sur le comportement de structures réalisées de la sorte. Cet article apporte donc une contribution immédiate en ce qui concerne à la fois les performances du béton à très haute résistance et l'application de modèles d'analogie du treillis.

ZUSAMMENFASSUNG
Die besonderen Materialeigenschaften von hochfesten Betonen zeigen die Notwendigkeit grundlegender Untersuchungen über das Tragverhalten von hochfestem Konstruktionsbeton. Es wird vorgeschlagen, die Methode der Stabwerkmodelle, die im Stahl- und Spannbeton eingesetzt wird, auch für die Erforschung von hochfestem Konstruktionsbeton einzusetzen.

STRUT-TIE MODEL

Strut-tie models can be formulated from experimental observations using failure crack patterns, recorded strains in the concrete and the reinforcement, together with actual specimen detailing, loading and support conditions. In design much of this information is not readily available. However, for simple everyday designs an experienced engineer is generally capable of developing strut-tie models based on common engineering sense and knowledge of the behavior of structural concrete. In the more complex design situations, this practical knowledge is often not enough to develop safe and efficient strut-tie models. In such cases, Schlaich et. al [1] suggest that the load-path method can be aided using the principal stress trajectories based on a linear elastic analysis of the structure. The principal compressive stress trajectories can be used to select the orientation of the strut members of the model. The strut-tie model can then be completed by placing the tie members so as to furnish a stable load-carrying structure.

The strut-tie approach for higher-strength concrete members is illustrated with the analysis of a pretensioned beam, specimen I-4A. This specimen was tested to failure using a point load system [2]. The detailing of the specimen is shown in Figure 1, and the material properties are given in Table 1. The strut-tie model for specimen Type I-4A shown in Figure 2(a), is developed first by placing the strut members in the direction of the principal compressive stress trajectories. Next, the vertical ties of the model are placed at the stirrup locations and the horizontal tie is located at the centroid of the strand pattern. In deep beams the usual assumption of linear distribution of strains over the depth of the section is not adequate. The capacity of this type of member in either flexure or shear depends heavily on the detailing of loading and support. This component of the load carrying mechanism, in the form of an inclined strut going from the support to the point load, is clearly shown in the failure crack pattern in Fig. 1.

The development of a strut-tie model is an iterative process because the widths of the struts and the size of the nodes depend on the forces in the struts and ties. A computer program has been developed at Purdue University to help carry out this process [3]. Initially, the truss model is laid out using the centerline dimensions of the strut and tie members. The effects of the prestressing are represented by the equivalent horizontal loads of 310.3^k (1380.2 kN) and 74.6^k (331.8 kN) shown in Figure 2(a). For the failure load analysis the maximum load of 323^k (1436.7 kN) is applied to the strut-tie model and a preliminary analysis is conducted to determine the internal forces in the individual members of the model. Next, the strut members are dimensioned using allowable compressive stresses checking that the resultant dimensions are compatible with the actual geometric constraints of the specimen. The resultant strut-tie model with finite dimensions for the strut members and nodal zones (Fig. 2b) is then analyzed for the applied load and the external forces representing the effects of prestressing. The equivalent prestressing load applied to the tension chord of the strut-tie model is updated by adding to it the force in the tie member next to the support resulting from a first analysis of the model with finite width for the initial prestress force and ultimate load. Finally, the forces in the horizontal tie member obtained from the analysis of the strut-tie model with finite width members for the applied maximum load and updated equivalent prestressing load must be added to the additional tension force calculated from the initial prestressing in order to obtain the actual tension force in the strands at ultimate load. The resultant analysis forces in the vertical ties of the model can be used directly to calculate the required tension force in the transverse reinforcement (stirrups). Next, the principal stresses in the critical nodal zones are determined using the stresses and the geometry of the individual members framing the nodes. This procedure is illustrated in the following section with the test results of specimen I-4A.

EXPERIMENTAL EVALUATION

In the determination of the strut widths of the model for beam I-4A, the compressive stress levels used were $0.9f'_c$ for the diagonals going from the point load to the support, $0.3f'_c$ for all other strut members, and $0.5f'_c$ for the upper compression members. The selection of the stress levels is an iterative process where the measured forces in the strands at the locations shown in Figure 1 were used to refine the estimates. The geometry of the strut-tie model based on the stress levels mentioned above resulted in the calculated forces shown in Table 2. The calculated strand forces were determined using the procedure outlined in the previous section. As can be seen from Table 2, a tension force of 36 kips must be properly anchored at the support. In specimen I-4A a 2 ft. (609.6 mm) overhang on the N-side and more than 2 ft on the S-side was provided to ensure proper anchorage. At failure, no slippage of the strands was observed. The calculated forces at failure in the vertical tension ties of the strut-tie model indicated yielding of this reinforcement. This was confirmed by the strain measurements in the instrumented stirrups. First diagonal cracking in specimen I-4A occurred at a load of 236^k (1049.7 kN) yielding of the stirrup reinforcement was observed immediately after first cracking; however, no stirrup fracture was noted at failure. Failure in specimen I-4A occurred due to crushing of the concrete under the point load followed by crushing of the web section on S-side as shown by the failure crack pattern in Fig. 1. The size of the critical nodal zone under the point load is controlled by the dimensions of the struts framing into it as well as the dimensions of the bearing plate and the width of the specimen. Once the geometry was determined, a finite element analysis was carried out to determine the principal stresses. After an element grid has been laid out in the nodal zone, the axial forces in the individual struts and applied load are discretized into their components parallel and normal to the boundaries of the nodal zone and applied as concentrated loads at the nodes of the elements bordering each strut and the loading plate. Figure 3 shows the chosen finite element grid and applied boundary forces for the portion of the nodal zone where failure was observed. The results of the analysis indicated a state of biaxial compression for most of the nodal zone and a maximum principal compressive stress of 9.76 ksi (67.3 MPa) at element 90, matching the region where failure occurred.

CONCLUSIONS

The use of increased concrete strengths would produce stronger nodal zones and strut members. If the strut-tie mechanism is properly developed this would result in improved ultimate capacity. Hence, adequate detailing of the member is further emphasized with the use of higher-strength concretes. In deep beams the ultimate capacity in either flexure or shear depends heavily on the strength of the main diagonal strut and the proper detailing of loading and support regions. The loading plate determines one of the dimensions of the nodal zone under the point load and at the support, thus affecting the state of stress at the node. Proper anchorage at the support region of the longitudinal tension reinforcement is also critical for the development of the strut-tie mechanism. Although not so critical for deep beams, in more slender members with low amounts of shear reinforcement the use of higher-strength concrete could jeopardize the formation of an adequate strut-tie mechanism upon first diagonal tension cracking. Because of the increased shear force to be transferred at the onset of first diagonal tension cracking and the possible reduction of aggregate interlock contribution the higher shear force to be transferred upon diagonal cracking could cause the first mobilized stirrups to yield and rupture.

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Table 1 - Type I-4A Information

TYPE I - 4A

Geometry:

Beam Length (ft)	17
Test Span (ft)	10
Shear Span, a (ft)	5
$\frac{a}{d_p}$	2.35

Concrete:

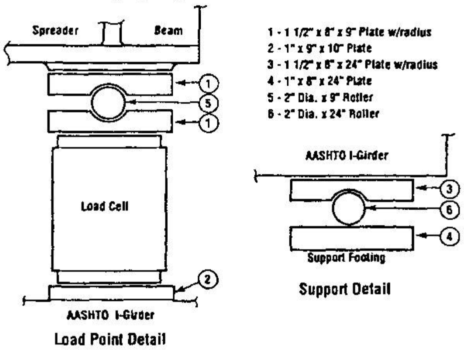
	Transfer	Test
f'_c (psi)	5840	8810
E_c (ksi)	5620	5930
f_r (psi)	920	-

Prestressing Strand:

	Top	Bottom
Grade	270	270
A_{ps} (in^2)	0.1633	0.1633
d'_p, d_p (in)	2.00	26.00
E_{ps} (ksi)	27920	27920
f_{pu} (ksi)	282.0	282.0
f_{si} (ksi)	207.6	193.3
f_{se} (ksi)	199.9	187.8
P_{e1}, P_{e2} (kips)	65.3	245.4

Mild Reinforcement:

	#5 Bar	#4 Bar
Grade	60	40
A'_s, A_v (in^2)	0.31	0.19
d', rf_y (in, psi)	2.00	165
E_s (ksi)	29020	29500
f_y (ksi)	64	52



SI Equivalents

1 in	= 25.4 mm
1 in^2	= 645.2 mm^2
1 lb	= 4.448N
1 psi	= 0.006895 MPa

Table 2. Measured and Predicted Strand Forces (kips) for P = 323 kips

Gage #	14	13	12	11	10
Measured	36	39.03	41.75	42.48	36.7
Predicted	35.37	38.16	40.72	41.32	36.91

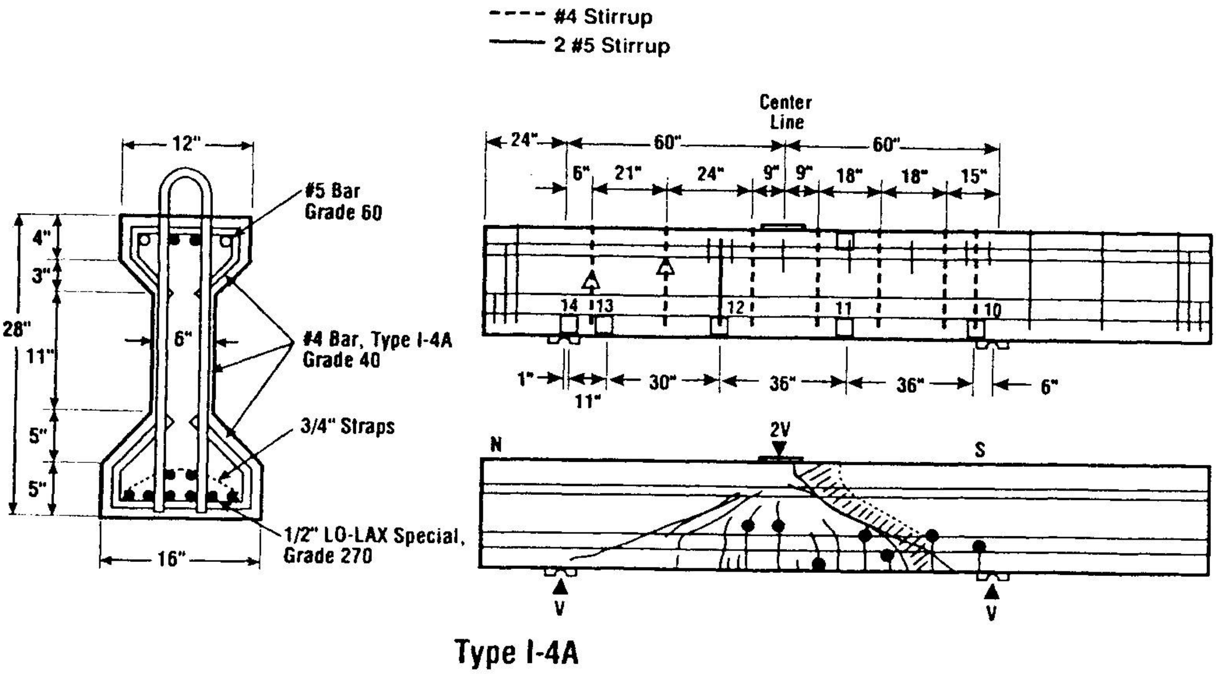


Fig. 1. Detailing and Failure Crack Pattern of I-4A.

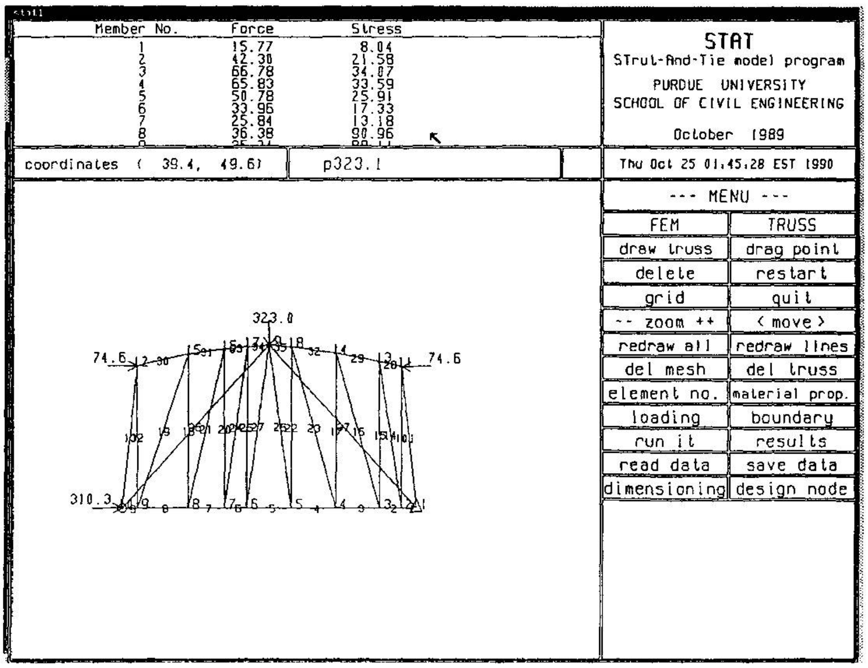


Fig. 2(a). Centerline Dimension Strut-Tie Model

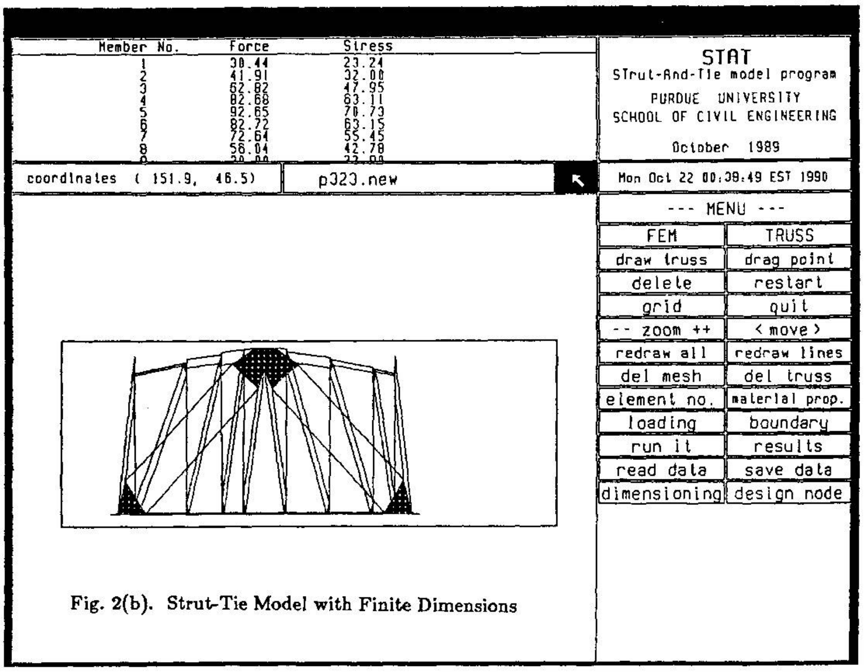


Fig. 2(b). Strut-Tie Model with Finite Dimensions

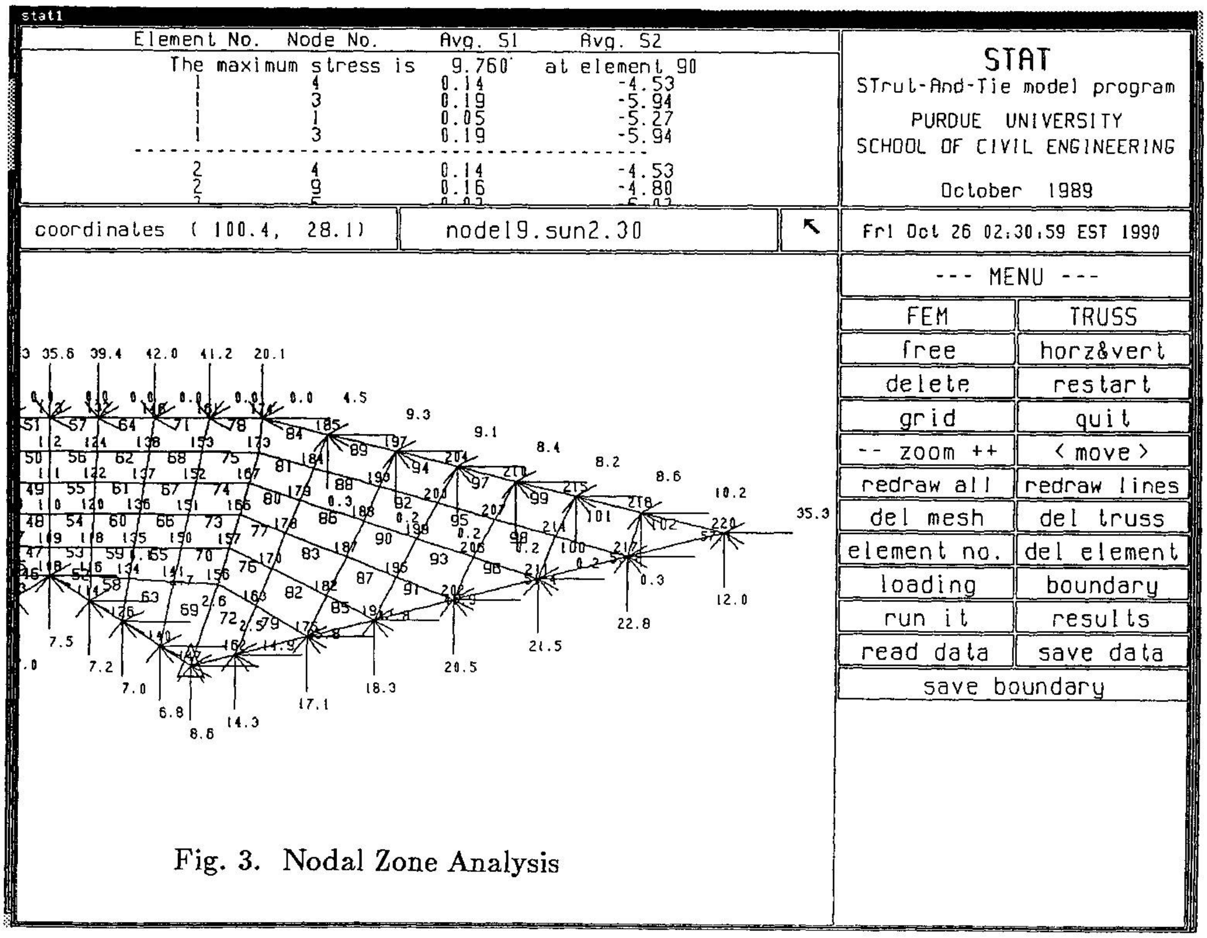


Fig. 3. Nodal Zone Analysis

Design Approaches for Shear Reinforcement in Concrete Beams

Détermination de l'armature à l'effort tranchant

Ermittlung der Schubbewehrung von Betonbalken

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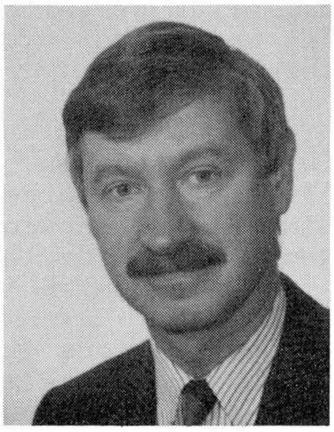

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SUMMARY
Most recent approaches to a rational design of shear reinforcement for reinforced concrete beams are formulated in terms of equilibrium of stress fields and compatibility of corresponding strain fields. The review points out common features and differences of these approaches.

RÉSUMÉ
Plusieurs nouvelles méthodes de détermination rationnelle de l'armature à l'effort tranchant des poutres en béton armé sont dérivées de l'équilibre des champs de contraintes et de la compatibilité du champ de déformations correspondant. Les points communs ou divergents de ces méthodes sont successivement traités.

ZUSAMMENFASSUNG
Die meisten neueren Ansätze zur rationalen Bemessung der Schubbewehrung von Stahlbetonträgern sind auf der Grundlage des Gleichgewichtes von Spannungsfeldern und der Verträglichkeit der zugehörigen Dehnungen abgeleitet. Der Überblick behandelt Gemeinsamkeiten und Unterschiede dieser Ansätze.

1. SCOPE

As pointed out in the Introductory Report by MACGREGOR, there is still no general agreement with respect to design of concrete beams for combined shear, bending moment and axial force. But, there is a clear tendency in recent research work on the subject towards physical models with complete description of equilibrium in the shear zone as a rational basis for concrete design.

The review is concerned with recent research work on the description of the equilibrium system of stresses and the compatibility of strains in the web of reinforced concrete beams containing shear reinforcement.

The review is limited to papers published since 1980. This limiting date was deliberately chosen, because the scientific discussion on design of shear reinforcement was decisively influenced by the COLLINS/MITCHELL paper of 1980 [1]. This influence is clearly demonstrated by the fact that all models treated in this review are formulated in terms of stress fields.

Most theories deal with beam regions in constant shear, but - since they are based on physical models - can be adjusted to situations of varying shear. It is assumed that shear failure is due to yielding of shear reinforcement and/or crushing of web concrete. Thus, bending or bond failure is not taken into account.

2. STRESS-FIELD CONCEPTS IN SHEAR DESIGN

In the webs of slender reinforced concrete beams with shear reinforcement, inclined cracks develop prior to shear failure. It is mostly assumed that these cracks are parallel and straight at approximately constant spacing. This simplification is particularly valid for T- or I-shaped beams.

If there are no bending moments in the concrete struts between inclined cracks, the inclined stresses in the concrete web form a continuous inclined compressive stress field σ_c, the angle α to the beam axis of which does not necessarily coincide with the angle α_{cr} of shear cracks. Furthermore, if the spacing of the shear reinforcement is sufficiently close, stresses in the shear reinforcement can be regarded as a continuous tensile stress field σ_t, the angle β to the horizontal of which is equal to the inclination of shear reinforcement.

The traditional truss model for shear design of concrete beams which dates back to the beginning of the century can also be interpreted as a stress field concept in which the inclination of the concrete compressive stress field is taken as 45°.

If the inclination of the compression stress field σ_c is taken as constant over the height of the beam and given the inclination β of the tension stress field σ_t, the stress intensities follow from equilibrium of vertical forces

$$\sigma_c = \frac{V}{b \cdot z} \cdot \frac{1}{\sin^2\alpha\,(\cot\alpha + \cot\beta)} \qquad (1)$$

$$\sigma_t = \frac{V}{b \cdot z} \cdot \frac{1}{\sin^2\beta\,(\cot\alpha + \cot\beta)} \qquad (2)$$

where V = applied shear force
b = web thickness
z = lever arm of bending stress resultants

Hence, there is no fundamental difference between truss analogy and stress field models if the inclination α of the compression stress field in the web is the same. The main problem is to determine this angle α in a rational manner.

3. LOWER BOUND PLASTIC SOLUTION [2],[3]

In theory of plasticity (also called limit analysis), it is assumed that materials exhibit unlimited plastic (i.e. irreversible) deformations when certain stress combinations (yield condition) are reached. Elastic deformations and workhardening effects are normally neglected.

Given that the function describing the yield condition is convex and that the plastic deformations are normal to the yield surface, upper and lower bounds for the failure load of any structure can be derived. The lower bound theorem of plasticity (limit analysis) states that a lower bound to the true failure load can be found from any stress field in equilibrium which does not violate the yield condition.

With respect to web stresses of concrete beams, it follows from the lower bound theorem of plasticity that the inclination α of the web compression field can be freely chosen as long as yield (limit) conditions are not violated. These conditions are usually taken as the yield strength of shear reinforcement and the effective crushing strength f_c^* of concrete in uniaxial compression. The latter cannot be taken directly from normal specimen tests because of cracking and transverse strains in the web concrete.

A maximum of the lower bounds for the failure load of concrete webs in shear is determined, if the angle α of web compression is chosen in such a way that web crushing and yielding of shear reinforcement occurs simultaneously (web crushing criterion). It should be noted that for low amounts of shear reinforcement this assumption leads to inclinations α of web compression well below the crack inclinations observed in tests.

4. THE COMPRESSION FIELD APPROACH OF COLLINS/MITCHELL [1],[4],[5]

Provided that shear crack openings are "smeared" over the web of the beams, compatibility of average strains is governed by Mohr's circle. The compatibility relation between average strain ε_ℓ in the direction of the beam axis, ε_t perpendicular to this axis and the principal inclined compressive strain ε_d can be derived from Mohr's circle as

$$\tan^2 \alpha = \frac{\varepsilon_\ell + \varepsilon_d}{\varepsilon_t + \varepsilon_d} \tag{3}$$

From eq.(3), the angle α can be determined, if the values of the average strains ε_ℓ, ε_t and ε_d are known.

With the assumption that the directions of the principal compressive strain ε_d and of the inclined web compression field σ_c coincide, this angle α can be used to determine the web stresses from the equilibrium equations (1) and (2). Using this assumption, there is no need to consider the question whether this angle α is equal to the direction α_{cr} of inclined cracks or not.

For stirrups perpendicular to the beam axis (β = 90), ε_t is equal to the average stirrup strain. It can be determined from the stress-strain relationship of stirrup steel, if the stiffening effect of concrete between cracks and the anchorage slip of stirrups are ignored.

The concrete strain in the direction of the inclined compression field, however, cannot be taken from uniaxial load tests, because the large transverse tensile strains exert a softening effect on the stress-strain relationship of web concrete. VECCHIO/COLLINS [4] propose the following relationship between principal compressive stress σ_c and principal compressive strain ε_d which depends also on the magnitude ε_1 of the principal tensile strain.

$$\sigma_c = \sigma_{c,max} \left[2\left(\frac{\varepsilon_d}{\varepsilon_c'}\right) - \left(\frac{\varepsilon_d}{\varepsilon_c'}\right)^2 \right] \tag{4}$$

where $\sigma_{c,max} = \dfrac{f_c'}{0.8 - 0.34\,\varepsilon_1/\varepsilon_c'}$

ε_c' = -0.002

f_c' = concrete cylinder strength

The strength f_{du} of the web concrete in inclined compression is also influenced by the coexisting transverse strain. COLLINS/MITCHELL [1] propose the following relationship

$$f_{du} = \frac{5.5\, f_c'}{4 + \gamma_m / \varepsilon_d} \tag{5}$$

where $\gamma_m = 2\varepsilon_d + \varepsilon_\ell + \varepsilon_t$

It must be noted that in the normal case of combined bending and shear the longitudinal strains in the concrete web vary over the beam height due to bending. The compatibility equation (3) in this case predicts an angle α which also varies over the beam height as a function of ε_ℓ. For normal design situations, it is recommended by the authors to consider the longitudinal strain at middepth of the beam and take the corresponding angle α as constant over the web height.

5. STRAIN COMPATIBILITY AND AGGREGARTE INTERLOCK [6],[7],[8],[9]

If it is assumed that the angle α of the inclined compression field in the web does not coincide with the angle α_{cr} of inclined cracks, forces must be transferred across the cracks by aggregate interlock. These forces depend on the displacements v and w of the crack faces tangential and normal to the crack direction.

The compatibility of strains can be considered independently for the strains in the concrete struts between cracks and for the average web strains (including "smeared" crack openings). The differences between both strain fields can be summed up to determine the crack displacements. For this, the crack spacing must be estimated from bond considerations.

The forces which are transferred across the inclined cracks by aggregate interlock depend on the crack displacements v and w. KUPFER and coworkers [6],[8] use relationships for aggregate interlock stresses determined by WALRAVEN, while DEI POLI et al. [7] consider equations derived by GAMBAROVA.

Taking into account aggregate interlock forces and strain compatibility, the angle α of the compression field can be determined by an iterative procedure.

Again, strain compatibility is dependent on longitudinal strains which normally vary across the web height. As a consequence, crack displacements, aggregate interlock stresses and the angle of the compressive stress field vary accordingly. To simplify calculations, it is again recommended to consider the strains at middepth and treat all related variables as constant over the web height.

REINECK/HARDJASAPUTRA [9] use a kinematic condition to determine the angle α of the inclined compression field. Following considerations of deformations of truss models, they assume that the resulting crack opening is always perpendicular to α. From this assumption, the angle α can be determined by an iterative procedure. Aggregate forces are taken into account by the model allthough their magnitude is not explicitely considered.

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Modelling the Transverse Reinforcement in Reinforced Concrete Structures

Modélisation des armatures transversales dans les structures en béton armé

Modellierung von Querbewehrungen in Stahlbetontragwerken

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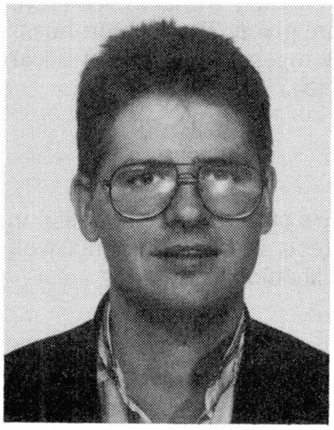

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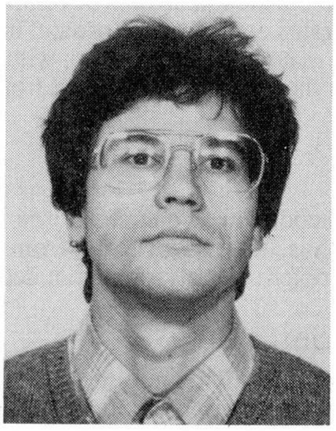

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SUMMARY

The authors present a simplified method to model the influence of transverse reinforcement on the behaviour of reinforced concrete elements. First, a numerical method to compute the three-dimensional stress and strain fields around heterogeneities like transverse reinforcement (stirrups in beams or hoops in columns) embedded in an elastic matrix is developed. Then, a non-linear homogeneous equivalent material is introduced, averaging the local heterogeneities. This unified method allows to treat both the shear reinforcement in beams and the confinement reinforcement in columns.

RÉSUMÉ

Les auteurs proposent une méthode simplifiée pour modéliser l'influence des armatures sur le comportement non linéaire d'éléments de béton armé. Dans un premier temps, une méthode numérique permettant d'obtenir les champs tridimensionnels de déformations et de contraintes localisés autour d'hétérogénéités de type armatures transversales (cadres dans les poutres ou frettes dans les colonnes) incluses dans une matrice élastique est développée. Ensuite, un matériau homogène équivalent rendant compte de façon moyenne des informations fines précédemment obtenues est construit. La méthode permet de traiter de façon unifiée le cas du frettage et celui du cisaillement dans les poutres.

ZUSAMMENFASSUNG

Eine vereinfachte Methode zur Modellierung des Einflusses von den Querbewehrungen auf das nichtlineare Verhalten von Stahlbeton-Konstruktionen wird hier vorgestellt. Zuerst wird eine numerische Methode zur Berechnung der 3D-Verformung- und Spannungsfelder zur Erfassung der Ungleichartigkeiten wie Querbewehrung (Umschnürungsbügel in Stützen oder Bügel in Träger) in einer elastischen Matrix vorgeschlagen. Darauf wird ein gleichartiger Äquivalentstoff mit verschmierten Ungleichartigkeiten formuliert. Die Probleme mit dem Umschnürungsbügel in Stützen und dem Bügel in Träger wird einheitlich behandelt.

1. INTRODUCTION

The behavior of reinforced concrete members including transverse reinforcement (shear reinforcement in beams or confinement reinforcement in columns) is not well understood until now and empirical design methods used in codes and specifications are very different around the world [1]. The concepts that underline current design practice are based partly on rational analysis, partly on test evidence, and partly on successful long-term experience with satisfactory structural performance. A theoretical treatment on the subject is needed.

The purpose of the paper is to model this behavior in the context of a general two-level approach [2] which consists in using for the structures global simplified methods issued from detailed local analysis:

- detailed analyses are first developed to compute the 3D stress and strain fields, warping of cross sections and other local informations in beams and columns taking into account the exact geometry of such elements (spatial distribution of reinforcement for instance).

- then global methods are built using global variables, such as generalized forces on the cross section, and taking into account the previous refined informations. These methods are simpler to use and quicker. The complex local behavior is integrated but not seen by the user who needs only a global response.

Following this general two-level approach, a numerical method to compute three-dimensional stress and strain fields localized around heterogeneities like transverse reinforcement (stirrups in beams or hoops in columns) embedded in an elastical matrix is developed. This method is based on Eshelby works [3, 4] where analytical stresses and strains around an ellipsoidal inclusion in an infinite body are given. Then, a non-linear homogeneous equivalent material is constructed, with an averaging of the local heterogeneities. The non-linear constitutive equations for the concrete are derived from continuous damage theory [5].

2. LOCAL ANALYSIS

A reinforcement bar embedded in a concrete matrix is a local perturbation (there is only a percent of steel in most R/C buildings). Thus, the use of tools from the "micromecanics of defects in solids" seems well adaptated to solve this problem. Equations are not detailed here, interested readers should refer to [4].

2.1 Ellipsoidal heterogeneity

The most popular author in this domain of the mechanics is Eshelby. He obtained in 1959, in an elegant way, the distribution of perturbations around an elipsoidal heterogeneity embedded in an uniformly loaded infinite elastic media [3] (Fig.1).
It can be expressed with:

$$\varepsilon(x) = S(x)\,[(K^*-K)S(x)+K]^{-1}\,(K-K^*)\,\varepsilon^o = H(x)\,\varepsilon^o \qquad (1)$$

where $\varepsilon(x)$ is the strain perturbation at point M with coordinates x, ε^0 the uniform strain at infinity, K^* and K the Hooke tensors of the heterogeneity and the matrix respectively and S is the Eshelby tensor which can be expressed from the two elliptic integrals:

$$\Psi(x) = \int_\Omega |x-x'|\,dx' \quad \text{et} \quad \Phi(x) = \int_\Omega \frac{dx'}{|x-x'|} \qquad (2)$$

These integrals are analytical for specials ellipsoids like sphere or infinite cylinders for instance.

2.1 Transverse reinforcement in concrete

In the case of transverse reinforcement studied here (Fig.2), the assumptions must be revised. These hypothesis are treated and discussed in [6] and will be the object of further publications. Only their list is given here.

2.2.1 Assumptions on geometry

- A stirrup or a hoop is not an ellipsoid: the tensor H of eq.1 must be computed with special technics (numerically).
- Reinforcement bars are not unique: interactions are neglected and perturbations are added.
- They are embedded in a finite body: the approximation of infinite media is still acceptable.

2.2.2 Assumption on loading

- In the case of a beam, the loading is not uniform: the beam is discretized in layers where the gradient of loading is supposed small.

2.2.3 Assumption on materials

- The behavior of concrete is not elastic: the continuous damage theory coupled with an homogeneisation procedure is used.

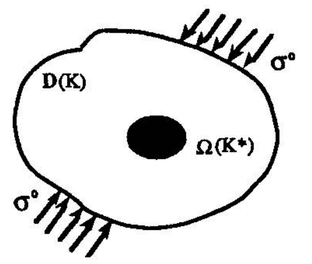


Fig.1 Ellipsoid in an infinite media

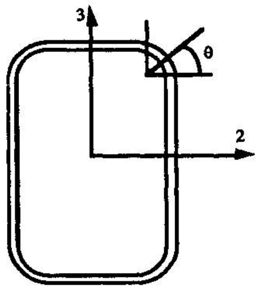


Fig.2 General shape for hoops and stirrups

3. GLOBAL ANALYSIS

3.1 Homogeneous equivalent material

The beam or the column is divided into elementary cells whose height depends of the number of layers choosen (depending on the sollicitation), whose width is the width of the beam and whose length depends on longitudinal distribution of transverse reinforcement (Fig.3,4).

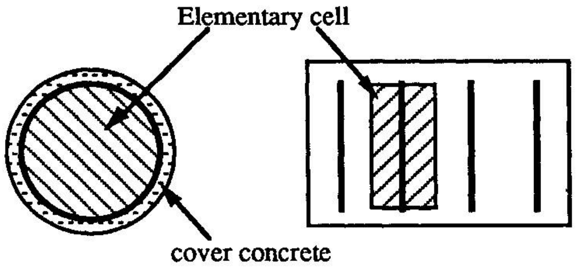


Fig.3 Hoop confined specimens

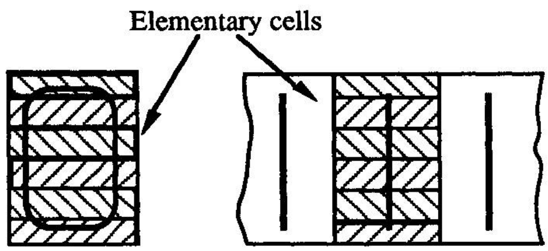


Fig.4 Beams subjected to shear

In relation with the choosen constitutive equations for concrete [5], the homogeneisation variable k is an average on the cell of the leading scalar variable of the damage model, which is calculated at each point within the cell from the positive principal strains $<\varepsilon_i>_+$:

$$\tilde{\varepsilon} = \sqrt{\sum <\varepsilon_i>_+{}^2} \tag{3}$$

$$k = \frac{1}{V}\int_{V_t} \frac{\tilde{\varepsilon}}{\tilde{\varepsilon}^o}\, dv \tag{4}$$

where V is the volume of the cell and $\tilde{\varepsilon}^o$ the value of $\tilde{\varepsilon}$ in the concrete alone (if there were no stirrup or hoop).

Thus, during the monodimensionnal calculation described in the following, the variable $\tilde{\varepsilon}_c$ of the cell with transverse reinforcement influence will be calculated from $\tilde{\varepsilon}^o$ (case with no stirrup):

$$\tilde{\varepsilon}_c = k\ \tilde{\varepsilon}^o \tag{5}$$

The homogeneisation variable k varies with the geometry (% and distribution of steel), with the state of the materials (toughness or damage of the concrete matrix) and with the sollicitation of the cell (traction, compression or shear). k is less than one, so it is called the damage delaying indicator. On Fig. 5, one can see the variation of k with the damage variable:

$$D = \frac{E_c - E_{c0}}{E_{c0}} \tag{6}$$

where E_c and E_{c0} are the actual and initial Young modulus of concrete respectively. It can be seen that the more the concrete is damaged, the more the transverse reinforcement effects are great (k<1).

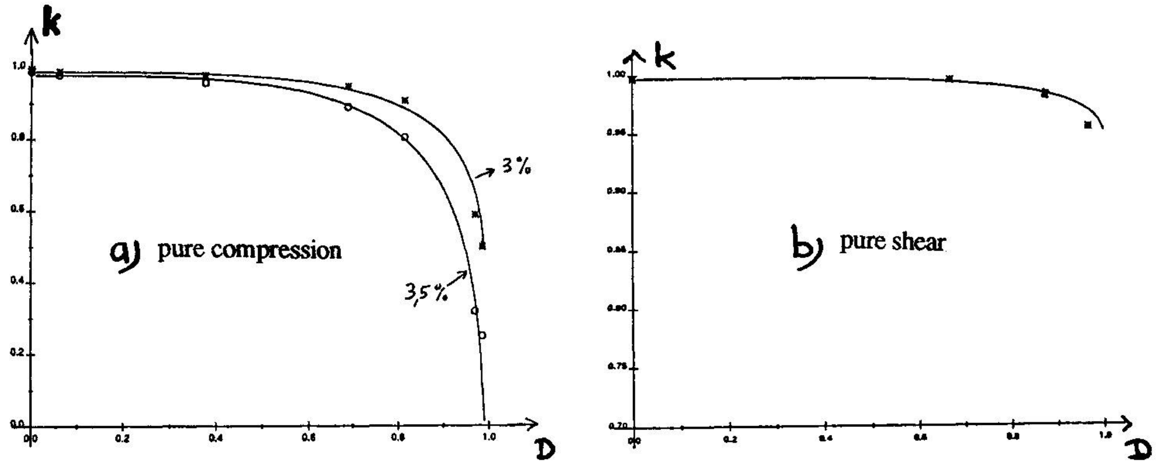


Fig.5 Homogeneisation variable k versus damage $D = \frac{E_c - E_{c0}}{E_{c0}}$

3.2 Global calculation

The heterogeneous elementary cell is replaced by an equivalent homogeneous cell with the same dimensions and whose behavior is affected by the presence of the steel. The transverse steel has no influence on the secant modulus of the cell which is the one of the matrix. But the degradation of the matrix is considerably delayed by the presence of transverse reinforcement.

3.2.1 Hoop confined specimens

The computation of a hoop confined specimen or column is first shown for simplicity:
- First stage: local computation (3D) and building of relations k(D) of Fig.5a.
- Second stage: global computation (1D):

i) The equivalent homogeneous cell is loaded in compression with ε^0, $\tilde{\varepsilon}^0$ is computed with eq.3
ii) The damage D_0 is calculated with the choosen damage law
iii) k is obtained from the curves Fig.5a (numerically stored at the first stage)
iv) The new state of the cell is computed with eq.5: $\tilde{\varepsilon}_c = k\ \tilde{\varepsilon}^o$
v) The damage of the cell D_c is calculated with the choosen damage law
vi) Return to step iii) until convergence (very fast in practice)
vii) Calculation of the stress $\sigma_c = E_{b0}\,(1 - D_c)\,\varepsilon^o$

The first and second stage are completely independant. The (long) three-dimensionnal local computation is done only once. The global monodimensionnal computation is very fast but takes into account the refined information of the first stage.

3.2.2 Beams subjected to flexure and shear

The procedure is about the same but a little more complicated because the cells are loaded with an axial strain ε and a shear strain γ. The corresponding stresses are deduced from:

$\sigma = E_{c0}\,(1\text{-}D_c)\,\varepsilon$ and $\tau = G_{c0}\,(1\text{-}D_c)\,\gamma$ ()

where E_{c0} and G_{c0} are the Young and shear modulus of the sound concrete and D_c the damage of the cell.
The distribution of the strains along the height of a section of the beam are obtained following the iterative method described in [7]: ε is proportional to the rotation ω of the section (supposed to remain plane) and γ is obtained by equilibrium conditions on a layer.

The homogeneisation variable k of a cell subjected to σ and τ is obtained by combination of the curve for pure compression (Fig.5a) and the curve for pure shear (Fig.5b) computed during the first stage.
Interested readers should refer to [6, 7].

4. RESULTS AND COMPARISON TO TEST DATA

4.1 Hoop confined specimens

The method was applied to the hoop confined cylinders described in Fig.6. The parameters of the damage law were identified from the stress-strain curves of the plain concrete. The stress-strain curves of the confined concrete (core concrete) were then calculated by the proposed method. This was done for two different concretes, one with low qualities (f_c = 26 MPa, Fig.7a) and the other with better qualities (f_c = 52 MPa, Fig.7b).
One can see that numericals results are in good agreements with experimental data [8]: the presence of hoops increases just a little the strength of the concrete (pic stress) but a lot its ductility. The better the concrete, the more pronounced these effects.

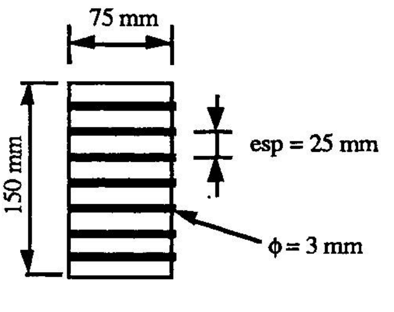


Fig.6 Hoop confined cylinder

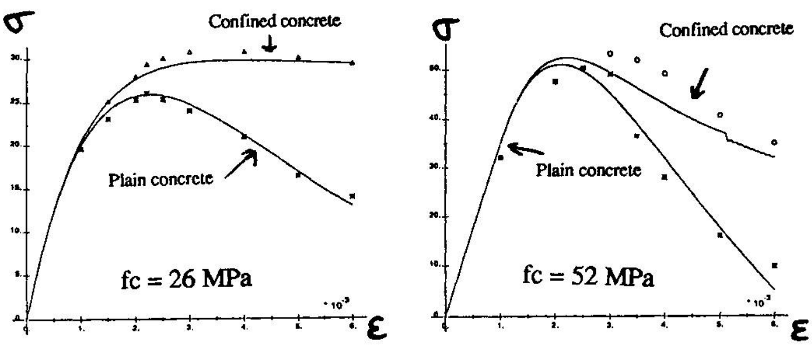
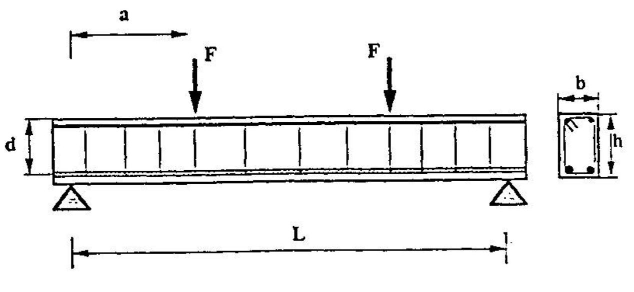


Fig.7 Stress-strain curves

4.2 Beams subjected to flexure and shear

With the proposed method, it is also possible to study beams subjected to shear (Fig.8). In this isostatic case, the sections of the beam differ by the ratio M/Vd where M is the bending moment in the section, V the shear effort and d the effective depth of the beam. The greatest M/Vd ratio is the shear span to depth ratio of the beam a/d (in the section under the applied load), which is a well known influencing parameter in such a problem.



b = 150 mm
h = 300 mm
d = 270 mm
Stirrup: ϕ 6 mm
Longi. bars: top ϕ 10 mm
bottom ϕ 22 mm

Fig.8 Beam subjected to shear

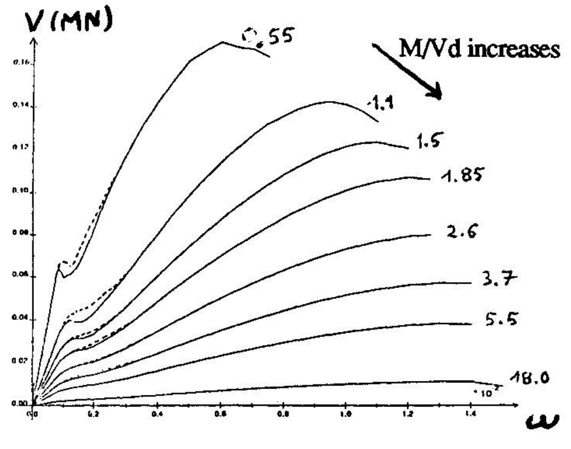


Fig.9 Shear effort V versus rotationof the sections ω

On Fig.9 one can see the simultaneous evolution of the different sections of the beam: the shear effort V is directly related to the applied effort F (V = -F) and loading the beam increases V on the diagramme 9. The sections which have a small M/Vd ratio have two pics. The first pic is due to crushing of concrete under shear in the middle of the section and the second pic correspond to plastification of longitudinals bars or failing of the concrete in compression. The sections with a large M/Vd ratio have only the second pic, the influence of shear being lower.

The ultimate state of the beam is reached when one of the sections reaches a pic. If a/d is large, the beam fails in bending in the sections located in its middle (vertical cracks, Fig.10a). For beams with a small a/d ratio, sections with large M/Vd ratio do not exist and the beam fails in shearing of a section located between the support and the point of application of the load (horizontal cracks, Fig.10b).
The presence of transverse reinforcement has a significant influence only on the first pic in the sections where the M/Vd ratio is small (Fig.10). Thus, the stirrups modify the failure mode of the short beams.

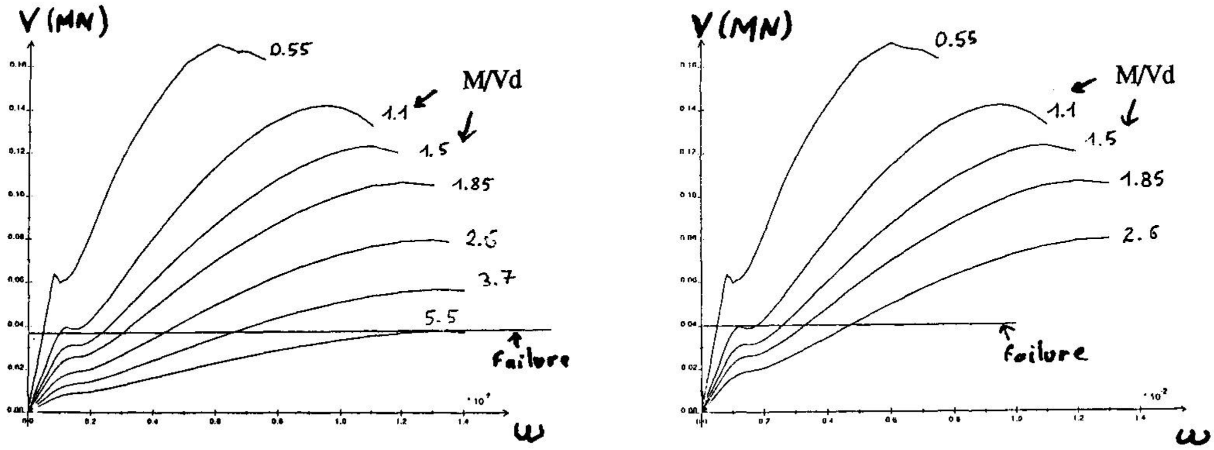


Fig.10 a) Beam with a/d = 5.5 $\frac{Mu}{Mfl} = 1.$ b) Beam with a/d = 2.6 $\frac{Mu}{Mfl} = 0.51$

5. CONCLUSION

Usually, the shear reinforcement in beams and the confinement reinforcement in columns are treated using very different concepts. In the present paper, a unified approach is proposed and the two problems are treated in the same way. This simplified method allows to treat these problems rapidly but with a good accuracy The first results are in good agreement with experimental observations.

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Modelling of Members with Transverse Reinforcement

Modélisation d'un élément en béton pourvu d'armatures transversales

Modellieren von Konstruktionsbauteilen mit Stegbewehrung

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SUMMARY

The biaxial state of stress in the webs of structural concrete members is described, which is presented by a truss model with the combined action of the stirrups and concrete tensile ties. The magnitude of the load carried by concrete in tension is determined by the friction of the crack faces and therefore the state of strain in the web and crack width have to be determined. However, this allows the calculation of the behaviour from cracking until failure. The influence of axial forces and of the prestress on the ultimate resistance can be consistently described as well as the effective concrete strength explained.

RÉSUMÉ

L'état de contrainte biaxial au sein d'éléments en béton est traduit par l'analogie du treillis, qui modélise les efforts de traction apparaissant soit dans les étiers, soit dans le béton. L'amplitude de la charge reprise par le béton en traction est déterminée par le frottement des surfaces des fissures, ce qui impose la détermination de la largeur des fissures ainsi que de l'état de contrainte au sein de l'élément considéré. Or cette démarche permet le calcul du comportement allant de la fissuration à la rupture; par conséquent, on pourra décire l'influence des forces axiales et de la précontrainte sur la résistance ultime aussi bien que la résistance effective du béton de l'élément étudié.

ZUSAMMENFASSUNG

In Stegen von Konstruktionsbetonteilen herrscht ein zweiachsiger Spannungszustand, der durch ein Fachwerk mit Zugkräften in den Bügeln sowie im Beton modelliert wird. Der vom Beton auf Zug getragene Lastanteil wird aus den durch die Reibung in den Rissen übertragbaren Spannungen bestimmt, und deshalb müssen der Dehnungszustand im Steg und die Rissbreiten bestimmt werden. Dies erlaubt jedoch die Berechnung des Tragverhaltens von der Rissbildung bis zum Bruch. Der Einfluss von Längskräften und der Vorspannung auf die Tragfähigkeit kann konsistent angegeben und die «effektive Betonfestigkeit» erklärt werden.

1. INTRODUCTION

The well-known truss for members with transverse reinforcement is a basic model for structural concrete as shown by MacGregor, Marti and Schlaich in /1/. However, it is also a simple model with only two variables to cover all design cases: the strut inclination Θ and the strength σ_{cw} of the compression struts. This simplicity may lead to contradictions with the real behaviour of members and even to inconsistencies with other models used in the design concept. An important case are the members with moderate shear, which only require light transverse reinforcement: until now most codes provide an empirically derived V_c-term; yet for a truss model unrealistic low values for have to be assumed, which allow almost no staggering of the tension chord reinforcement. A further example is the "effective strength" of the compression fields or struts, where either simply different values depending on the stress situation are proposed or refined strain-considerations are made. Finally, the truss with an uniaxial compression field is an insufficient model for the limit state of serviceability and therefore in codes mostly detailing rules are given (MacGregor /1/).

The aim of this article is to contribute to a clear understanding of the structural behaviour of B-regions with shear forces by presenting a model for the stresses and strains of cracked webs with transverse reinforcement. It will be shown that tensile stresses occur in the web due to the friction of the crack-faces, and that by modelling this the above mentioned inconsistencies are avoided as explained in /2/.

2. EQUILIBRIUM

The equilibrium in the B-region of a r.c.- or p.c.-member is investigated according to the well-known method by Mörsch, to cut the member along the cracks and to deal with the resulting elements in free-body diagrams. For a typical B-region at an end-support the elements are the solid concrete struts between the cracks (Fig.2). For simplicity straight cracks are assumed, whereby their inclinations depend on the degree of prestress and on the magnitude of the axial compressive or tensile force N. Mörsch was satisfied with the capacity of the truss formed by the stirrups and the struts between the cracks (Fig.2a), but now generally the contribution of the friction of the crack faces (Fig.1) is taken into account by assuming flatter strut- than crack-inclinations. (The dowel force of the longitudinal reinforcemnt is neglected in the following for simplicity.) The term "friction" is used here as a general term covering all types of concrete, e.g. also lightweight concrete with no aggregate interlock. Such frictional stresses were so far mostly regarded as components of an uniaxial compression field with an inclination flatter than that of the cracks, like by Kupfer/Mang/Karavesyroglou /3/, Kirmair/Mang /4/, as well as dei Poli/Gambarova/Karakoc /5/. In the following the biaxial state of stress in the web is derived.

From the vertical equilibrium in Fig.1 follows:

$$V = \frac{A_{sw}}{s_w}\,\sigma_{sw}\cdot z\cdot \cot\beta_{cr} + V_f + V_p \tag{1}$$

whereby V_p is the vertical component of the total force in an inclined prestressed reinforcement. Prestressing is here considered internally in the strains, which is here of advantage since later the strains and relative displacements of the crack-faces have to be determined. The strain in the tension-chord follows from the force, which can be calculated from the equilibrium of moments of the end-support region (Fig.1). Similarly the force C and the corresponding strain in the compression chord are determined from the horizontal equilibrium.

The vertical component V_f of the friction forces can be expressed in terms of the friction stresses in the middle of the web

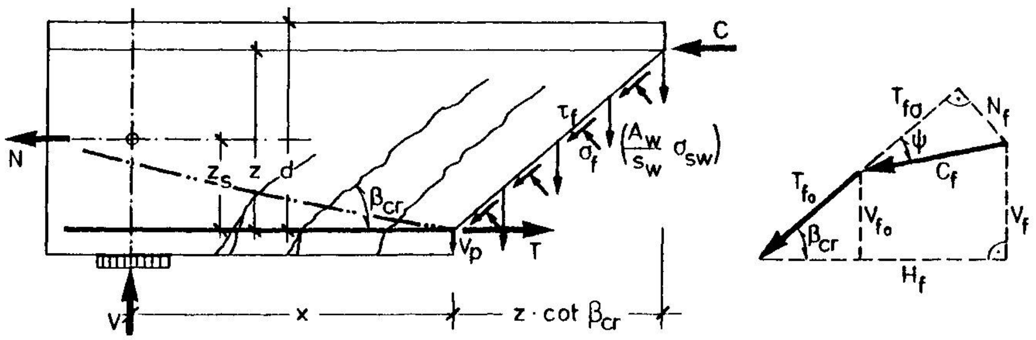
$$V_f = b_w\ z\ [\tau_{f0} + \tau_{f\sigma}\ (1 - \cot\beta_{cr}/\mu_f)\ ] \tag{2}$$

Thereby the well-known relation for friction was assumed

$$\tau_f = \tau_{f0} + \mu_f\cdot\sigma_f = \tau_{f0} + \tau_{f\sigma} \quad \text{with} \quad \mu_f = \cot\psi = 1,7 \tag{3}$$

which, however, contrary to the usual applications depends on the crack displacements. The relatively high value for μ_f was derived from Walraven's constitutive laws for crack widths up to 0,5 mm (see /2, 15/).

Now all the forces and stresses along the crack are defined, and the stress field in the solid concrete strut between the cracks can be determined. In Fig.2 all stresses are given with respect to the crack direction. The truss-action (Fig.2a) is made up by the stirrups and the uniaxial compression field between the cracks. The stress fields due to the friction (Fig.2b) are looked at seperately for a better understanding: the shear stresses τ_f result in a biaxial tension-compression field with an inclination of $\beta_{cr}/2$ of the compression field; the normal stresses σ_f result in principal compressive stresses σ_2 and tensile stresses σ_1 parallel to the crack. Altogether a biaxial stress field exists, whereby the principal stresses and the principal inclination φ_I (Fig.2b) may be determined acc. to the linear elastic theory. This is valid since the principal tensile stresses remain smaller than the concrete tensile strength once the crack pattern has formed.



a) end-support region and forces b) forces due to friction

Fig.1: Free-body diagram for an end-support region of a structural concrete member

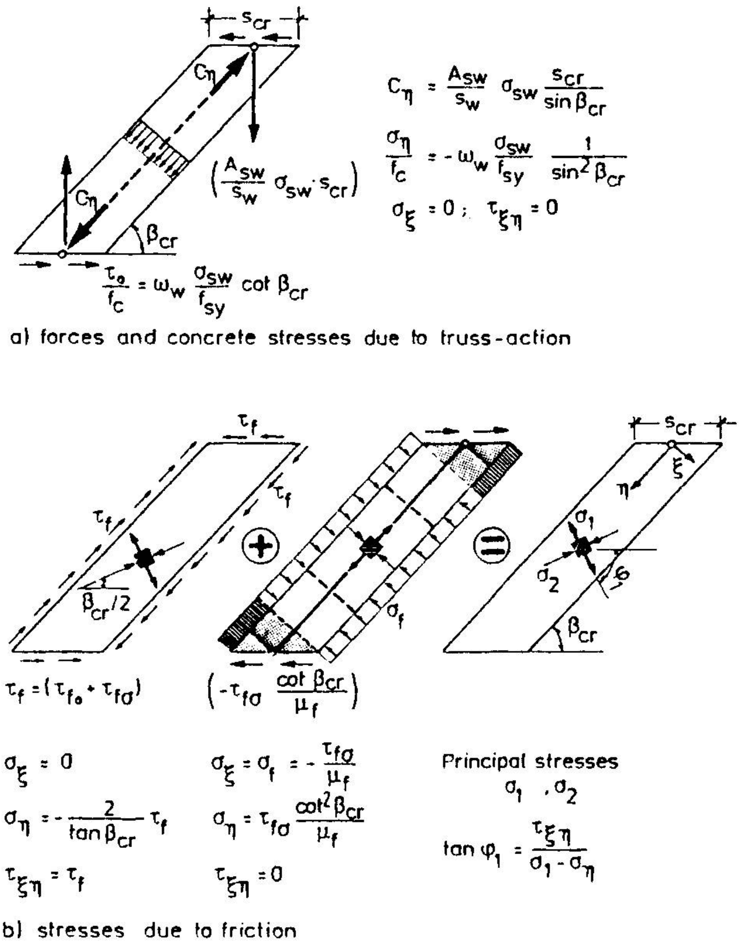
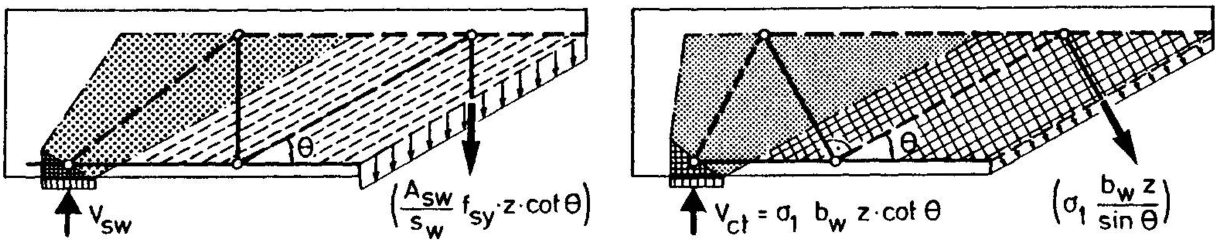


Fig.2: Forces and stresses for the concrete strut between the cracks

The stress field resulting in the web from all the actions in Fig.2 is that of a principal compression σ_2 inclined at the angle Θ and a principal tension(for higher shear also small compression) perpendicular to that. This was already described by Reineck in /6/, and even earlier by Lipski /7/ who, however, gave a different explanation for the tension field. This state of stress results in the two models shown in Fig.3: the well-known truss model formed by an uniaxial compression field and the stirrups (Fig.3a), as well as a truss-model with concrete tensile ties (Fig.3b). These are the two load paths referred to by Schlaich/Schäfer/Jennewein /8/. The models in Fig.3 are statically equivalent to the model shown in Fig.1, and this also means that there is no principal contradiction between the two well-known approaches in the shear design: the "shear-friction theory" leading e.g. to a V_c-term on one side, and the truss-analogy on the other side. In these truss-models the discrete cracks are not modelled but these must be looked at in order to determine the magnitude of the tensile stress σ_1, which only depends on the friction stresses along the crack. However, these trusses visualize the flow of the forces in a member more clearly and simpler. So the overall model (Bruggeling /1/) and the sectional approach are both necessary.



a) stirrups and uniaxial compression field b) biaxial concrete stress field

Fig.3: Truss models following from the principal stresses between the cracks

It must be mentioned that further tensile stresses occur in the struts due to the bond of the stirrups (see /3, 4, 9, 10/). These tensile stresses reduce the stirrup stresses and strains between the cracks (tension stiffening effect), but they do not contribute to the load transfer.

With this model for the biaxial state of stress in the web of structural concrete members with transverse reinforcement a clear transition to the model for members without transverse reinforcement is achieved as also explained in /2, 15/. So apart from being transparent, this model enables a consistent treatment from members with transverse reinforcement to unreinforced members.

3. KINEMATICS AND CONSTITUTIVE RELATIONS

3.1 Kinematics

The strains of an element in the B-region can be calculated from the strains of the truss formed by the stirrups and the solid concrete struts between the cracks (Fig.2a). The stresses due to friction (Fig.2b) are then considered by the deformations of the concrete struts:

$$\epsilon_{cw} = \epsilon_\eta = (\sigma_\eta - 0,2 \cdot \sigma_\xi)/E_c \tag{4}$$

The complete state of strain of a beam-element in a B-region with shear forces is as follows:

- longitudinal strain in the middle of the web: $\epsilon_x = (\epsilon_s - \epsilon_c)\ /2$ (5a)
- curvature: $\bar{\kappa} = \kappa \cdot z = (\epsilon_s + \epsilon_c)\ /2$ (5b)
- vertical strain: $\epsilon_z = \epsilon_{sw}$ (5c)
- shear strain: $\gamma_{xz} = \epsilon_x / \tan\beta_{cr} + \epsilon_{cw} / \sin\beta_{cr} \cdot \cos\beta_{cr} + \epsilon_{sw} \cdot \tan\beta_{cr}$ (5d)

For the vertical strain the beneficial tension stiffening effect between cracks was neglected, because the anchorage slip of the stirrups has a controversial effect; more refined considerations were e.g. made by Kupfer et al. /3, 4/.

With these equations the bending- and shear-stiffnesses of the beam-element are principally given and may be used either for a non-linear analysis or for calculating deformations, since the equations are not limited to the ultimate limit state. The difference to many well-known works, e.g. also by Collins/Mitchell /11/ is, that the crack-inclination is considered and that the direction of the principal strain does not coincide with that of the compression field; further explanations were given by Hardjasaputra /12/ and Reineck/Hardjasaputra /13/.

Since the strains are known also the crack width n and the slip s in the middle of the web can be calculated for a given crack-spacing s_{cr} (measured horizonally):

$$\frac{\Delta n}{s_{cr}} = (\epsilon_x + \epsilon_{sw} + \epsilon_{cw}) \cdot \sin\beta_{cr} + \bar{\kappa} \frac{s_{cr}}{z} \cos\beta_{cr} \tag{6a}$$

$$\frac{\Delta s}{s_{cr}} = -\epsilon_x \cdot \cos\beta_{cr} + (\epsilon_{sw} + \epsilon_{cw}) \sin^2\beta_{cr} \ / \cos\beta_{cr} + \bar{\kappa} \frac{s_{cr}}{z} \sin\beta cr - 2,4 \frac{\tau_f}{E_c} \sin\beta_{cr} \tag{6b}$$

These crack displacements determine the magnitude of the friction transferring the biaxial stress field over the cracks.

3.2 Constitutive Relations

For the concrete and the steel bi-linear stress-strain curves can be used /2/. The strength of the solid concrete struts between the cracks (see also /14/ and section 6) is not lower than:

$$f_{cw} = 0,85 \cdot f_c \quad \text{or} \quad f_{cw} = 0,80 \cdot f_c \tag{7}$$

The constitutive equations for the friction of the crack-faces were already explained in /15/ and so here only the result is given:

$$\frac{\tau_f}{f_c} = \frac{\tau_{f0}}{f_c} \cdot \frac{\Delta s - 0,24\Delta n}{0,096 \cdot \Delta n + 0,01} \quad \text{with } \Delta n, \Delta s \text{ [mm]} \tag{8}$$

The stress σ_f or $\tau_{f\sigma}$ follows then from Eq.(3). The friction stress τ_{f0} is the limiting value without normal stresses σ_f on the crack face and was set to

$$\frac{\tau_{f0}}{f_c} = 0,45 \frac{f_{ct}}{f_c} \left(1 - \frac{\Delta n}{0,9}\right) \quad \text{with } \Delta n \text{ [mm]} \tag{9}$$

This is a much lower value than that given by Vecchio/Collins/Bhide /9, 10/. With these formulations the whole response of the B-region in a member may principally be determined from cracking until failure, and this is demonstrated in the following by two examples.

4. STIRRUP STRAINS AT SERVICE LOAD

For determining the crack width of the inclined cracks in a web all strains must be known acc. to Eq. (6a), but the stirrup strains play of course a dominant role. Presently a semi-empirical approach is used for predicting the stresses and strains for in the stirrups under service load conditions: the stresses are calculated for a truss model with struts at 45° and the actual load V is reduced by a V_c-term. The presented combination of truss models with additional tensile struts in the web can be used for checking the requirements at the serviceability limit state. As an example for this the well-known test series of identically reinforced beams with varying web thicknesses (varying b/b_w-ratio) by Leonhardt/Walther /16/ was calculated, and Fig.4 shows the comparison of the measured and calculated stirrup stresses with increasing load. Contrary to a truss-model, the presented model yields the typical characteristic of the measured curves. It can also be seen that the usually used V_c-term is not equal to the cracking load as often pretended. However, the calculated values are partly very conservative because the tension stiffening effect was not considered for the stirrup strains. If this is improved the model may serve as a relatively simple tool for determining crack widths. Of course, additionally to the strains also the crack spacing must be determined.

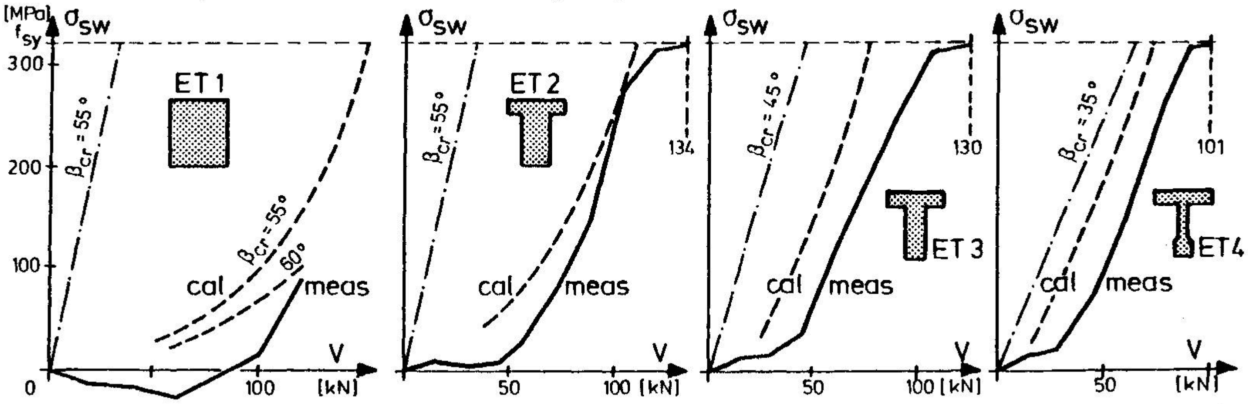


Fig.4: Development of stirrup stresses with the load for test-beams with varying web-thicknesses of Leonhardt/Walther /16/

These results may be interpretated in terms of the load proportions taken by the models with the stirrups or the tensile struts (Fig.3), and such values are given in Fig.5 for the load step 60kN (service load) and the load at yielding of the stirrups. For the two beams with the thick webs the truss with the concrete tensile struts carry quite a considerable part of the load, although the tensile stresses remain low; for the thin-webbed beams the load is almost totally carried by the truss with stirrups. From this it is obvious that the model with the concrete tensile struts is especially relevant for members with moderate shear like in buildings and for foundations.

beam	β_{cr}	Service load $V \sim 60$ kN					Yielding $\varepsilon_{sw} = 1,6$ ‰				
		θ	σ_1/f_c	V_{ct}	σ_{sw}/f_{sy}	V_{sw}	V	θ	σ_1/f_c	V_{ct}	V_{sw}
ET 1	55°	31,7	0,0180	52,6	0,106	7,4	151,9	24,3	0,0140	56,2	95,7
ET 2	55°	26,0	0,0201	37,2	0,256	22,6	113,9	26,0	0,0137	25,5	88,4
ET 3	45°	25,2	0,0014	1,9	0,634	58,2	75,9	27,4	-0,0062	-7,1	83,0
ET 4	35°	30,5	0,0028	1,5	0,794	58,2	72,1	30,8	-0,0002	-0,1	72,2

Fig.5: Calculated values for two load stages of the test-beams in Fig.4

5. DIMENSIONING AT THE ULTIMATE LIMIT STATE

The main results for the ULS are summarized in the well-known dimensioning diagram (Fig.6) which is simplified by assuming a constant shear force component q_f with increasing ultimate shear force. However, this is quite a good approximation as comparisons with similarly derived diagrams by Kupfer et al /3, 4/ and Gambarova /5/ show. Also Hardjasaputra/Reineck /12, 13/ came to similar results based on a simple kinematic condition proposed by Hardjasaputra /12/, which includes the slip in the cracks and thereby the direction of the principal strain deviation from the direction of the principal compression.

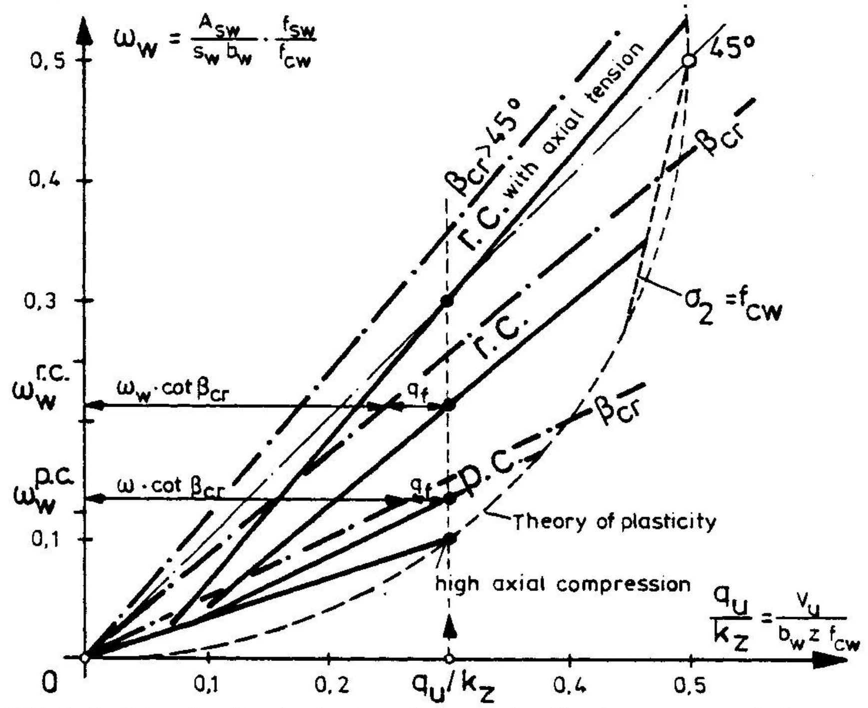


Fig.6: Principal sketch of a dimensioning diagram for the tranverse reinforcement in B-regions and influence of prestress and axial tensile and compressive forces

Since p.c.-beams exhibit a flatter crack inclination than r.c. beams, less stirrups are required although the value for q_f is smaller. For medium values of q_u the friction capacity defines the ultimate load, and only for high values q_u the concrete between the cracks really fails in compression. This occurs at slightly lower shear forces than according to the theory of plasticity because the web is here in biaxial compression (due to high stresses σ_f on the crack face). Of course, in test beams the difference between both "failure types" is not always clearly recognizable, especially not for p.c. beams and webs or panels with small crack spacing.

Axial forces are also constantly considered in the equilibrium equations (see Fig.1), leading e.g. to higher axial strains ε_x as well as steeper crack inclinations for tension flanges of box-girders. It is expecially worth mentioning that for high axial compression very small crack inclinations and therefore also small inclinations Θ for the compression field are possible; therefore a lower limit for Θ is not necessary according to this proposal, since it is taken into account by the limitation of the friction transfer. (Why not 10° for box-columns of bridges with high axial compression? Here the model is that of a very flat strut as explained by Schlaich /1/). However, since structural concrete is capable of some redistribution, also higher strut inclinations than that of the cracks can occur as pointed out by Kupfer/Guckenberger /17/, who tested and clearly explained the structural behaviour of highly compressed structural concrete members.

Finally it may be concluded that the presented model explains the "V_c-term" and that the simple demanded ways of accounting for it (MacGregor /1/) are possible, as demonstrated by a joint proposal from Kupfer and Reineck for the CEB MC 90.

6. EFFECTIVE CONCRETE STRENGTH

When applying the theory of plasticity to structural concrete, an effective concrete strength for the compression field is taken, which is lower than the uniaxial compression strength (Marti and MacGregor /1/). For a compression field crossing cracks it is now obvious from before, that this value is not constant but depends on the strain condition as well as on the direction and spacing of the cracks, if the friction capacity is decisive. If the compression struts between cracks govern the failure, the reduction of the strength of struts between cracks is only up to to 0,85 f_c and 0,8 f_c according to many tests by different researchers as explained in /14/; therefore lower values are unnecessarily restrictive.

If now the ultimate loads according to the diagram (Fig.6), are interpretated by the well-known truss with a uniaxial compression field as shown in Fig.7, then the so-called effective concrete strength ($\nu \cdot f_{cw}$) varies with the ultimate load or the inclination Θ. The value ν now cannot be constant, since the friction characteristics cannot only be formulated in terms of strength values, like normally in "shear-friction theories", but a complete description of the strength as well as relative crack displacements is required. Therefore a crack direction has to be assumed and the crack spacing has to be calculated, in order to evaluate the crack width for a given strain condition.

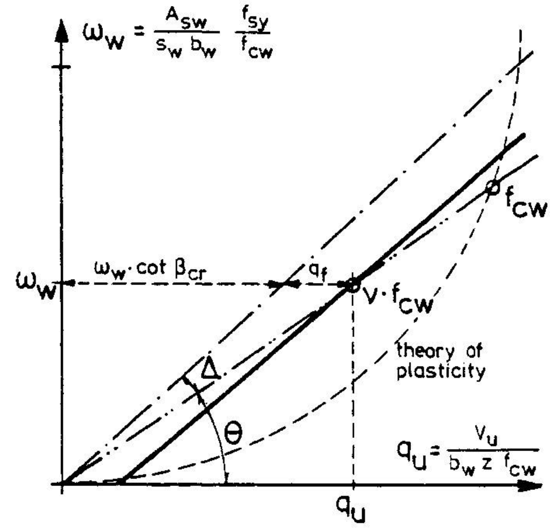


Fig.7: Interpretation of the dimensioning diagram in Fig.6 in terms of an effective concrete strength

All these influencing parameters may simply be described by their effect on the friction component q_f, acc. to Fig.7, and this is shown in Fig.8. The influence of the crack spacing as well as of the crack inclination follows from Fig.8a; thereby the crack spacing is of principal importance because by this parameter a "size-effect" is induced: for larger member depths the crack widths are larger for same rotations or curvatures (as explained in /15/) and also larger bar diameters are used resulting in larger crack widths and smaller values for the friction component q_f. This also follows from the works of Gambarova /5/ and Kupfer et al /4/. The vertical or stirrup strains (Fig.8b) also limit the friction capacity, if large values are reached (unless a second crack field appears and the original cracks close). The longitudinal strains (Fig.8c) reflect mainly the influence of axial forces, which furthermore have an additional effect on the crack inclination. In case of axial compression the crack inclination and also q_f acc. to Fig.8a or 8c decreases, but this is more than made up by the capacity increase of the truss (see Eq.(1) and Fig.2a).

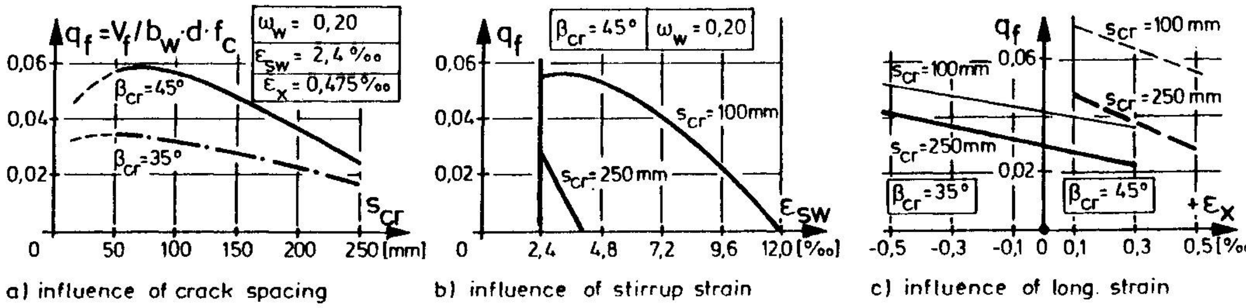


Fig.8: Influence of crack spacing, crack inclination and strains on the shear force component q_f carried by friction

All this shows, that the limited friction transfer also explains the reduction of the effective concrete strength with increasing tranverse strain, which was pointed out by Collins et al /9, 10, 11/; a direct comparison was given in /14/. Furthermore however, the crack spacing and thereby the crack width have an influence, since friction is involved and it is not only a problem of "compression with transverse tension" (as also only in most tests) or of "compression-softening". Consequently it means reversing cause and effect if the "shear transfer strength" is explained by the "softening" of the concrete as done by Hsu /18/. The force transfer over cracks is a discrete problem and must be dealt with as such, similar to the force transfer over joints as discussed by Ruth /19/. Both problems require a model for the structural behaviour of the whole member, but they also have an influence on it, e.g. in terms of a low "effective strength". Finally it must be mentioned that there might be further causes for a strength reduction as shown by Thürlimann /20/.

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Deformation and Bending-Shear-Torque Failure

Déformation et rupture sous l'action de flexion, cisaillement et torsion

Verformung und Versagen unter Biegung, Schub und Verdrehung

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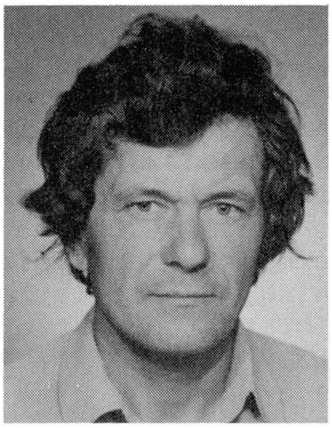

Petr Rericha, born 1944, received his engineering degree from the Faculty of Civil Eng., Technical University Prague. Ever since he has been active in structural mechanics, particularly FEM. He is now back at his alma mater.

SUMMARY
It is proposed that nonlinear continuum analysis by the finite element method will become the standard method for the analysis and dimensioning of structural concrete. Potential simplifications of the method are indicated. A simple, unified, continuum model is proposed for beams, slabs and shells.

RÉSUMÉ
Une analyse non linéaire de la structure, utilisant la méthode des éléments finis est proposée afin d'obtenir un modèle unifié de dimensionnement du béton armé Les simplifications potentielles introduites par la méthode sont indiquées; un modèle de continuité unifié est ensuite introduit pour le cas des poutres, dalles et coques.

ZUSAMMENFASSUNG
Die nichtlineare Analyse des Kontinuums mit Hilfe der Methode der Finiten Elemente wird als einheitliche Methode für die Berechnung und Bemessung des Konstruktionsbetons vorgeschlagen. Mögliche Vereinfachungen der Methode werden aufgezeigt. Es wird ein einfaches und einheitliches Kontinuum-Modell für Balken, Platten und Schalen vorgeschlagen.

1. INTRODUCTION

The assessment of the present design practice as well as the objectives presented in the Breen's introductory report reflect actual, real and even acute needs of the building industry and research. The present paper adresses two of the objectives set forth in the report which call for the consideration of the overall structural behaviour and for new transparent methods.

It is common practice that the overall structural behaviour is analyzed by conventional elasticity models. They provide cross-sectional variables for dimensioning but hardly reflect the actual flow of the forces throughout the structure even at service load level not to speak of the ultimate limit load state. This practice entails strictly sectional approach in dimensioning and detailing. Apparently, it also is inconsistent from the purely mechanical point of view in regard to statically indeterminate structures. Actual flexibility of a reinforced concrete structure is far from that assumed in the elastic analysis. If some more global models are used for dimensioning, typically strut-and-tie models (STM), then in effect some of the conventional elasticity analysis might be eliminated. Marti [3], for example, suggests the application of the STM for continuous beams. The sole purpose of the elastic analysis then is to determine the residual forces (reactions at intermediate supports in the above example). Even these moments can be modified (to some extent arbitrarily) owing to redistribution.

Another substantial flaw of this inconsistent mixture of the elastic analysis and limit state dimensioning turns up when the limit state of deformations is considered. Here the results of the elastic analysis are literally useless. Moreover, the rules for the evaluation of deformations of reinforced concrete members are a real maze in the Czechoslovak standards. These considerations suggest that a consistent unified approach to structural concrete should avoid the traditional elasticity methods of analysis.

2. NONLINEAR CONTINUUM APPROACH

At present there is no feasible method that could replace the elasticity methods. It also seems to be too daring an idea to develop what in fact would be 'another nonlinear mechanics', specific for reinforced concrete. In the long perspective, however, taking into account the amazing advance of the computer technology it seems more imaginable. What should feature such a method?

First, it must be nonlinear since reinforced concrete simply is not linear elastic even at service loads. This brings about the problem of variable loads. Various load cases must be analyzed separately in the realm of the nonlinear analysis. But even today there is a trend to reduce the number of load cases and an exact evaluation of the absolute maxima of the cross-sectional forces is not deemed necessary. A compromise appers to be attainable in this respect.

Second, the method should be able to furnish strains, deflections, stresses and forces at various load levels. Single computational model can then be used to assess all kinds of limit states. This need has indirectly been emphasized in the 'Performance requirements' of the Breen's introductory report. STM approaches cannot meet this requirement.

An incremental nonlinear finite element analysis (FEA) meets this requirements at the expense of prohibitive demands on the computer and analyst parts. There is, however, a great potential for reducing these demands if the material models are simplified to the level required for design purposes. Hardly any FEA package has yet tried to do so. An obvious possibility is to neglect the tension strength of the concrete as most building codes do. The cost of the nonlinear analysis may also be greatly reduced if we realize that severe nonlinearity occurs just in the initial phase of the gradual loading when the crack pattern develops. In this phase, a continuum analogy to the STM builds up. Subsequent loading invokes almost linear behavior. An obvious advantage of the nonlinear continuum analysis is that less engineering judgment or intuition is required in comparison to STM. It should also be recalled that the finite element discretization provides a natural support for the visualization of the structure and its displacements, strains and the flow of internal forces. Current computer graphics software and hardware relies on discretizations identical or similar to those adopted in the FEA. This aspect might become decisive in the future.

3. DEFORMATION PATTERN IN RC BEAMS

As already pointed out earlier, specific simple design oriented material models are necessary in order to make the nonlinear analysis acceptable. The models for B regions of beams, plates and shells should employ appropriate kinematic hypotheses. Present model is based on the assumption of plane cross-sections (Timoshenko or Flugge hypothesis in the scope of plane frame beams). In reinforced concrete beams, this assumption is not sufficient. The deformation pattern must include the transverse strains ε_y , ε_z if stirrups and hoops are to be activated. An obvious possibility is to supplement the required displacement modes into the assumed displacement field. Compatibility of deformations is then maintained but special finite elements are required and in effect fully three-dimensional analysis is entailed. We prefer another option.

The assumed displacement field remains that of the space beam and the strains ε_y and ε_z are evaluated from the transverse equilibrium equations $\sigma_y = \sigma_z = 0$. Compatibility is violated by the terms $\partial v/\partial x$ and $\partial w/\partial x$ where v, w denote the transverse displacements relative to the displaced local reference frame of the cross-section. These terms may be neglected if the displacement field varies only gradually along the x-axis. Analogous assumption on the stress field has been utilized in the derivation of the above transverse equilibrium equations and Marti[3], Nielsen[4] and Harmon[1] also adopted it. The strains ε_y and ε_z must be solved for by an iteration since the material laws of concrete and reinforcement steel are nonlinear.

The discretization in the cross-section plane is performed by dividing it into sufficiently small subregions. In each subregion, homogeneous stress and strain are assumed. This is a direct generalization of the layer concept in the analysis of reinforced concrete plates and shells. Perfect bond is assumed. The same strains thus apply to concrete and reinforcement. Reinforcement bars of any directions are easily included and equally treated. It is important to note that an analogous deformation pattern was developed for reinforced concrete plates and shells. In these structures, there is just one condition of the transverse equilibrium and one corresponding transverse strain component to be solved for. The formulation is simpler. On the contrary, adequate STM are staticaly determinate spatial trusses which may be dificult to construct.

Simple elastic-plastic laws were implemented (principal stresses plasticity condition for concrete) and several sample solutions carried out for the purpose of validation. Most are plane frame beams (no torsion). The T-beam in Fig.1 was tested by Leonhardt[2] for bending-shear failure.

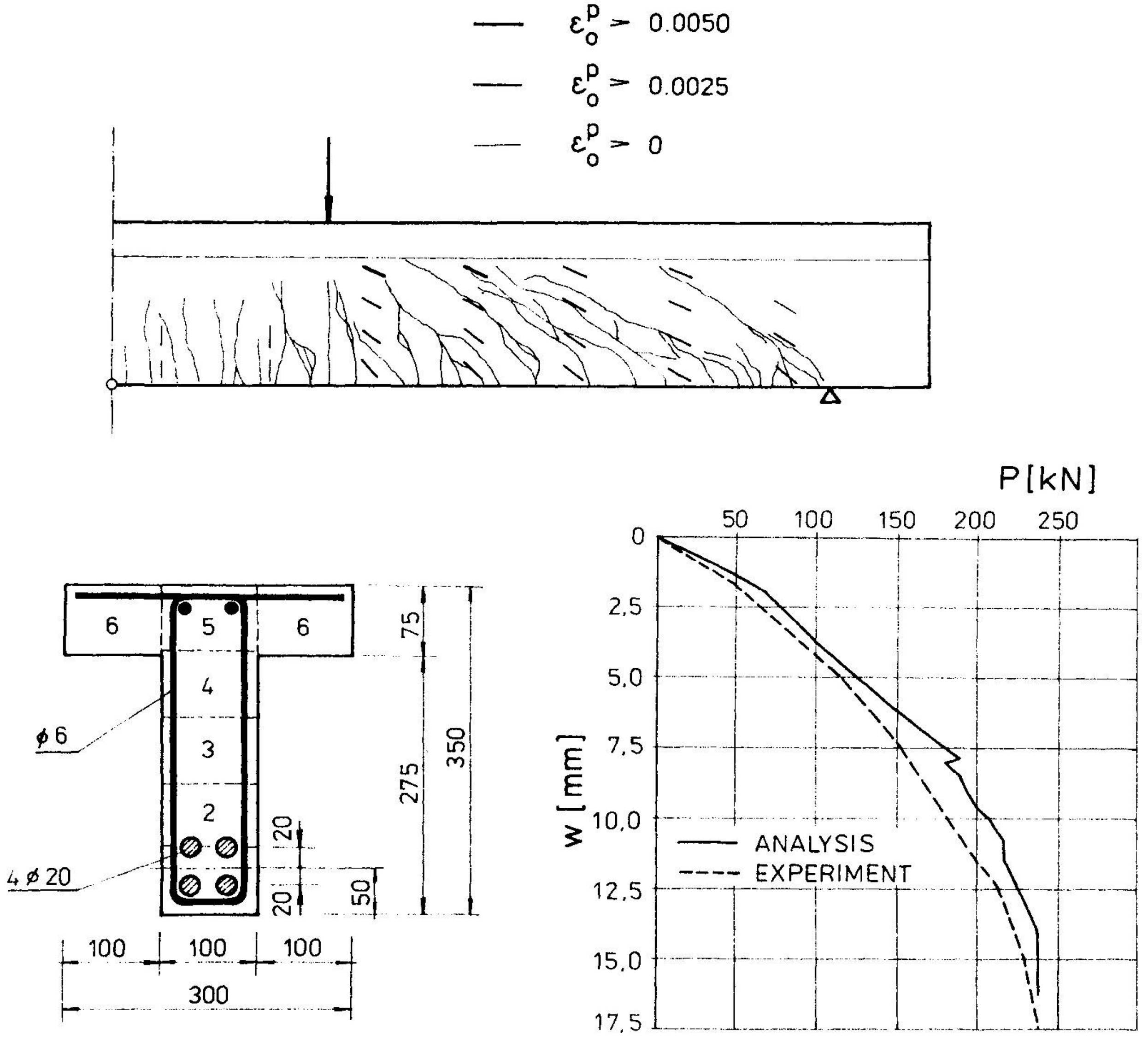

Fig.1 Cross-section, crack pattern (test and analysis) and load deflection curve of a T-beam

The crack pattern at failure compares well to the experimental one and so does the load-deflection curve. The smeared crack width and the direction of the cracks of the analytical (FEA) solution are indicated by abscissas. Actual stiffness and deformation pattern are well modeled. It is worth mentioning that only 7 finite elements were used in this example. The cross-section was divided into 6 concrete subregion (see Fig.1) and 2 reinforcement layers. Tension strain softening was adopted in this example for concrete and the load-deflection graph is therefore curved. If tension-cut-off concrete is adopted the graph is nearly straight.

4.CONCLUSIONS

Nonlinear continuum modeling of structural concrete is proposed to become a unified analytical and dimensioning tool. Possible simplifications of the nonlinear analysis were indicated which would make it more accesible to designers and engineers. A simple model for beams subjected to simultaneous flexure, shear and torsion is described. The model easily includes arbitrary reinforcement. A sample solution demonstrates that the actual crack pattern and corresponding STM action are well approximated by the proposed model.

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