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## Dimensioning and Detailing

### Dimensionnement et conception des détails

### Bemessung und Bewehrungsführung

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#### **SUMMARY**

Current and proposed Canadian, American and European design rules for dimensioning and detailing concrete structures are reviewed and compared. The majority of the discussion deals with shear in B Regions and D Regions. Major areas needing more experimental evidence and synthesis are identified.

#### **RÉSUMÉ**

Les normes canadiennes, américaines et européennes visant le dimensionnement et la conception des détails constructifs des structures en béton armé actuellement en vigueur ou simplement proposées, sont passées en revue et comparées. La majeure partie de la discussion touche l'effet tranchant dans les régions B et D. Les zones plus larges nécessitant davantage de résultats expérimentaux et de synthèse sont mises en valeur.

#### **ZUSAMMENFASSUNG**

Die derzeit gültigen und die geplanten Regeln kanadischer, amerikanischer und europäischer Normen für die Bemessung und Bewehrungsführung von Betontragwerken werden zusammengestellt und miteinander verglichen. Dabei werden überwiegend die B-Bereiche mit Querkraft und die D-Bereiche behandelt, und die wichtigsten Felder herausgestellt, die eine weitere experimentelle Absicherung und bessere Zusammenfügung erfordern.



## 1. INTRODUCTION

In the past 15 years there has been a major shift in the design methods for reinforced concrete toward methods based on equilibrium solutions from the theory of plasticity and related simple mechanical models. This paper discusses the current state of the Canadian, American and European design rules for regions subjected to combined shear, moment and axial load. Four specific documents are discussed: the 1984 Canadian Standards Association (CSA) code<sup>1</sup>, a 1987 draft of Chapter 11 (Shear and Torsion) of the ACI Code<sup>2</sup> prepared by a subcommittee of ACI Committee 318 but never finally adopted, the First Draft of the Chapter 6 (Verification of the Ultimate Limit States) of the CEB-FIP Model Code 1990<sup>3</sup> and a design method proposed by Collins and Mitchell.<sup>4</sup>

This paper is not intended to be an endorsement of one or other set of design rules, but rather a critique of the state-of-the-art. In spite of the great progress made in recent years, there are major areas needing more experimental evidence and synthesis.

## 2. DEFINITION OF CONCEPTS

### 2.1 Dimensioning and Detailing

*Dimensioning* and *detailing* refer to the process of selecting the dimensions of members and joints within a structure and selecting the amount, the layout, the position and the details of the reinforcement. Traditionally, dimensioning or design has been taken to mean the selection of overall sizes and reinforcement amounts at highly stressed sections and detailing has dealt with selection of the bends, cut-off points, joints and the like. In North America, dimensioning was the job of the engineer, detailing was the job of the reinforcement fabricator. This is an incorrect division since the details of the reinforcement in the discontinuities control the strengths of these regions and hence must be considered by the structural engineer.

### 2.2 D Regions and B Regions

In 1982 Schlaich and Weischede<sup>5,6</sup> introduced the concept of *D regions* and *B regions* where D stands for discontinuity or disturbed and B stands for beam, bending or Bernoulli. D regions, extending a distance equal to the member depth away from a discontinuity such as a change in section, concentrated load or reaction, were assumed to carry load primarily by strut-and-tie action with significant in-plane load components. The regions between D regions were termed B regions. Here beam theory applied as did truss analogies such as those developed at the turn of the century by Ritter and Morsch or the more refined plastic truss analogies<sup>7</sup>. This classification scheme permitted a major step forward in our understanding of the design of concrete members and our ability to write design rules for such members.

### 2.3 Full Member Design and Sectional Design

Truss and strut-and-tie models require consideration of the entire member in the design process. In such methods, a truss which is in equilibrium with the loads is developed. This one model allows consideration of internal forces due to shear,

flexure and axial loads. Such a design procedure will be referred to as *full-member design*. In a full-member design procedure, such things as the extension of flexural reinforcement resulting from the presence of shear forces, are accounted for automatically. A drawback of such methods is that a different set of internal forces and hence member sizes results from each loading case. As a result, multiple load cases must each be considered separately and different loading cases may require different truss layouts. Such an approach is generally too tedious for the design of conventional beams.

The traditional design process for reinforced concrete, particularly in North America, assumes that a beam can be designed section by section for the worst combination of flexure and shear at that section. Generally it is assumed that flexure and shear can be decoupled and considered separately - first designing for flexure using the moment envelope, then designing for shear using the shear envelope. The interaction of shear and flexure is ignored, or dealt with empirically, or considered using equations derived from truss concepts. Such a design procedure is referred to as a *section-by-section design* or *sectional design*. Sectional design procedures generally do not work in D regions.

#### 2.4 Compressive Strength of Cracked Concrete

The concrete in the cracked web of a beam is subjected to diagonal compressive stresses which are parallel or nearly parallel to the inclined cracks. One must know the crushing strength of this concrete to prevent web crushing failures. The strength of this concrete is variously related to (a) the presence or absence of cracks and/or the orientation of these cracks, (b) the tensile *strain* perpendicular to the compressive stress averaged over a width including several cracks, or (c) the transverse tensile *stress*.

2.4.1 Stress Limited as a Function of Presence of Cracks - The 1978 CEB Model Code<sup>8</sup> limited the diagonal compressive stress in the web to  $f_{cd}^* = 0.6 f_{cd}$  where  $f_{cd}$  was the design compressive strength. This value was also used in Reference 7. Schlaich et al.<sup>6</sup> propose

$$\begin{aligned}
 f_{cd}^* &= 1.0 f_{cd} \text{ for uniaxial compression,} \\
 &= 0.8 f_{cd} \text{ if transverse tensile strains cause cracking parallel to the strut} \\
 &\quad \text{with normal crack width,} \\
 &= 0.6 f_{cd} \text{ for skew cracking of normal width or struts crossed by skew} \\
 &\quad \text{reinforcement} \\
 &= 0.4 f_{cd} \text{ for skew cracks of unusual width.}
 \end{aligned}$$

Schäfer, Schelling and Kuchler<sup>9</sup> reported tests of ten specimens and reviewed data from five other test series and reaffirmed the above rules. The First Draft of the CEB-FIP Model Code 1990<sup>3</sup> defines  $f_{cd}^*$  as:

$$f_{cd}^* = \alpha \left[ 0.85 \left( 1 - \frac{f_{ck}}{250} \right) \right] f_{cd} \quad (1)$$





where  $\alpha = 1.0$  for uncracked zones or zones with cracks at angles greater than  $45^\circ$  to the direction of the compressive stresses and  $0.7$  for zones with cracks at less than  $45^\circ$  to the compression. For  $30 \text{ MPa}$  concrete and  $\alpha=0.7$  this works out to  $0.524 f_{cd}$ . Regan<sup>10</sup> has shown this to be a lower bound to the web crushing stress in 31 beams which failed by crushing of the web concrete. Rangan<sup>11</sup> compared Eqs. 1 and 2 to 16 tests of prestressed beams which failed by web crushing and reported equally good agreement by either equation.

**2.4.2 Stress Limited as a Function of Transverse Strain** - In 1978 Collins<sup>12</sup> suggested that the compressive strength of cracked concrete was a function of the strain perpendicular to the direction of the principal compressive stress. The strain used was an average strain based on a gauge length that included several cracks. Collins and Mitchell<sup>13</sup> incorporated these concepts in their Compression Field Theory design method for shear and torsion.

From tests of reinforced concrete panels subjected to in-plane normal and shear stresses Vecchio and Collins<sup>14</sup> derived a relationship between  $f_{cd}^*$  and the transverse principal tensile strain,  $\epsilon_1$ . The 1984 Canadian code has incorporated the following version of this relationship:

$$f_{cd}^* = \frac{\lambda f_{cd}}{(0.8 + 170 \epsilon_1)} \quad (2)$$

where  $\lambda$  ranges from  $1.0$  for normal weight concrete to  $0.75$  for concrete made with lightweight sand and gravel.

**2.4.3. Stress Limited as a Function of Transverse Tensile Stress** - Kollegger and Mehlhorn<sup>15</sup> tested 55 panels under in-plane compression and transverse tension and reviewed data from several other test series and report a maximum reduction of compressive strength of about 20 percent for panels which failed by crushing of the concrete prior to yield of the reinforcement. They concluded that the effective compressive strength was more accurately described as a function of the transverse tensile stress than the transverse tensile strain.

Agreement must be reached as to the best way of defining the compressive strength of cracked webs. Preferably this should be done at two levels: (a) A theoretically correct definition, and (b) a definition which can be easily applied in the design of beams. For design the author favors the values of  $f_{cd}^*$  given in Section 2.4.1.

### 3. GENERAL REQUIREMENTS OF STRUCTURAL DESIGN CODE CLAUSES

Four general rules should be followed in formulating structural design codes:

1. Wherever possible, code provisions should be based on clearly understood mechanical models.

We have a clear physical model for pure flexure - a beam is a compression force and a tension force which form a couple in equilibrium with the applied moment.

Strain compatibility is invoked. These principles are clearly stated in all modern codes. In the Introductory Report for Sub Theme 2.2, Professor Schlaich has pointed out that the mechanical models needed in design should be "just enough" and not "as much as possible". Too complex models obscure the understanding.

Frequently, concepts derived from full-member models such as strut-and-tie models or truss models are applied in a sectional design procedure. Problems have arisen when the code drafting body has given inadequate attention to the fundamental differences between the two procedures.

2. If it is necessary to introduce empirical constants or expressions or simplifying assumptions, the end result should be as simple as possible.

The derivation of simple rules may take considerable effort on the part of the code writers. The rectangular stress block for flexure<sup>16,17</sup> is a simple approximation to the true stress blocks. This simplicity is based on extensive and thoughtful research, based in turn on a professional understanding of the degree of complexity which could be tolerated in a design office. The existence of computers in design offices does not in itself justify complex empirical expressions.

Generally speaking, shear design equations which involve the longitudinal steel ratio,  $\rho$ , or the shear span to depth ( $M/Vd$ ) ratio are tedious to apply in practice since these quantities change from point to point along the beam and change for different load cases. Questions also arise as to where a reinforcing bar is sufficiently well anchored to be counted.

3. When design shifts from one range to another the design models and/or approximate design expressions should meet at a common point unless there is a mechanical reason why they should not.
4. Ductile modes of failures are preferable to brittle failures. The margin of safety should be greater for brittle failures than for ductile failures.

#### 4 FLEXURE

At points of maximum moment in B regions, with or without axial load, the internal forces are represented by tensile force in the reinforcement and an assumed compressive stress distribution in the compression zone. A linear strain distribution is assumed at such a location. The compressive stresses are related to the strains by the stress-strain curve of the concrete. For design, the stress-strain curve is simplified to one of the forms shown in Fig. 1. The ACI Code, CSA Code and 1978 CEB-FIP Model Code<sup>16</sup> use variations of the rectangular stress block in Fig. 1b. Here the stress intensity,  $0.85f'_c$ , and the depth of rectangle,  $\beta_1c$ , are chosen so that (i) the

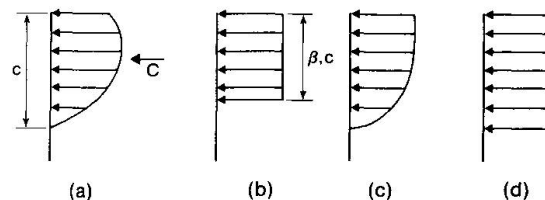


Fig. 1. Compression Stress Blocks



resultant force  $C$ , and (ii) the line of action of this force, are the same as for the "real" stress block in Fig. 1a. The same is true of the parabola-rectangle diagram shown in Fig. 1c. Chapter 6 of the First Draft of the CEB-FIP Model Code 1990<sup>3</sup> permits the use of the parabola-rectangle stress-block or the stress block shown in Fig. 1d where the entire compression zone is assumed to be stressed to  $f_{cd}^*$  given by Eq. 1 with  $\alpha = 1.0$ . While this results in a compression force approximately the same as Fig. 1b and c, the location of the axis of zero strain is closer to the extreme compressive fiber than in Fig. 1b and c. This will affect the strain in the tension steel and whether the steel yields or not in a given beam or column.

Except as noted here the model for flexure is accurate and well established and needs no further comment.

## 5. D REGIONS

D or Discontinuity regions are assumed to extend a distance equal to the member depth away from concentrated loads, reactions, changes in cross section and holes. Such regions can be designed using strut-and-tie models. In their introductory reports for this conference Professor Schlaich and Professor Marti describe such models in detail.

In their most basic form, strut-and-tie models consist of uniaxially stressed tension ties consisting of normal or prestressed reinforcement, uniaxially stressed concrete compression struts, and joint regions referred to as nodal zones at points where three or more struts meet, or where a combination of three or more struts, ties, or external forces meet. In the example shown in Fig. 2 the compression struts are prismatic with perpendicular ends bearing on the nodal zones. Because this does not truly represent the state of stress in the beam, more complex models or bottle-shaped<sup>6</sup> or fan-shaped struts<sup>18</sup> have been suggested.

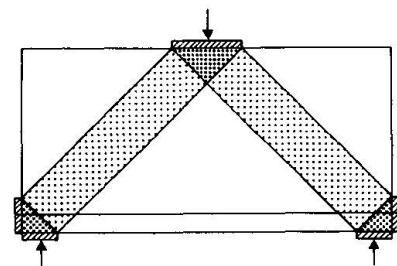


Fig. 2. Strut-and-Tie Models

Although the nodes have finite width, the strut-and-tie model is analyzed as a pin-jointed truss. Walraven and Lehwalter<sup>19</sup> account for the flexural stresses in the struts due to the continuity at the top joint of Figure 2. They also include a size effect.

### 5.1 Selection of Strut-and-Tie Models

The procedure for laying out strut-and-tie models involves a graphical procedure to draw a truss in equilibrium with the loads.<sup>20</sup> The procedure involves trial and error because the widths of the struts and the sizes of the nodes depend on the forces in the struts and ties. Recently computer programs have been developed to carry out this process<sup>21,22</sup>.

Different designers may come up with different strut-and-tie models. Although the models selected are to some extent self fulfilling prophecies, problems may develop if the model selected in the design differs too much from the natural load carrying

mechanism because the concrete may have inadequate ductility to adapt to the strut arrangement in the model.

Schlaich et al.<sup>6</sup> suggest two guides in selecting a workable strut-and-tie model. First, the compatibility of deformations may be approximately considered by orienting the struts and ties within 15 deg. of the force systems obtained from a linear elastic analysis of uncracked members and connections. Second, the most valid model tends to be the one that minimizes the amount of reinforcement since this corresponds to the minimum strain energy solution.

## 5.2 Material Strengths

5.2.1 Tension Ties - Tension ties are designed on the assumption they are steel ties stressed to the design yield strength at points of maximum stress. Adequate anchorage must be provided. The tensile resistance of the concrete is not utilized.

5.2.2 Compression Struts - The strength of the concrete in the compression struts is taken as:

$$f_{cd}^* = v f_{cd} \quad (3)$$

where  $v$  is an effectiveness factor in the order of 0.5 to 1.0. The effectiveness factor accounts for: (a) reduction of the useable concrete strength due to cracking of the struts and/or tensile strains or stresses transverse to the struts, and (b) strain gradients across the width of the struts arising from the fact that the strut-and-tie model is not truly a pin-jointed truss.<sup>19</sup> Several approaches to defining  $v$  were given in Section 2.4. The effective compression strength of the concrete making up the compression struts varies from code draft to code draft.

The 1990 Draft of Chapter 6 of the CEB-FIP Model Code 1990<sup>3</sup> assumes the full width of the strut is stressed to  $f_{cd}^*$  given by Eq. 1 with  $\alpha = 0.7$ .

The 1984 Canadian code<sup>1</sup> and the draft code by Collins and Mitchell<sup>4</sup> base the strength of the compression struts,  $f_{cd}^*$ , on Eq. 2 where  $\epsilon_1$  is the *average* strain perpendicular to the strut as illustrated in Fig. 3. This average reflects the restraint that the adjoining concrete gives to the highly stressed concrete in the strut. In a varying strain field the value of the average depends on the gauge length used to define the average. In many D regions there is no rational way of estimating  $\epsilon_1$  in a cracked web. In such cases the Explanatory Notes for the Canadian code<sup>23</sup> suggest

$$\epsilon_1 = \epsilon_s + (\epsilon_s + 0.002) / \tan^2 \alpha_s \quad (4)$$

where  $\epsilon_s$  is the average tensile strain in reinforcement crossing the strut at an angle  $\alpha_s$  to the axis of the strut. Again there is the problem of how to define  $\epsilon_s$ . The

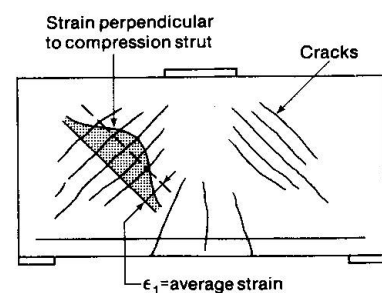


Fig. 3. Definition of  $\epsilon_1$  for D Region



Explanatory Notes suggest  $\epsilon_s$  can be taken as  $f_y/E_s$ . For 400 MPa steel at angles of 30, 45 and 60 degrees to the strut, Eqs. 3 and 4 and  $\epsilon_s = f_y/E_s$  give  $f_{cd}^*$  equal to  $0.315\lambda f_{cd}$ ,  $0.55\lambda f_{cd}$  and  $0.732\lambda f_{cd}$ . Eqn. 4 was derived by Collins and Mitchell<sup>13</sup> using a Mohr's circle for strain.

The 1987 draft ACI Code Chapter 11<sup>2</sup> expresses  $f_{cd}^*$  in a cracked beam web as

$$f_{cd}^* = \frac{\phi f'_c (\theta_{st} - 10)}{(50 + f_y/2000)} \quad (5)$$

for reinforced concrete where  $\phi$  is a resistance factor equal to 0.85,  $f'_c$  is the cylinder compressive strength,  $\theta_{st}$  is the angle between the tension tie and the compression strut in degrees and  $f_y$  is the yield strength in psi. Equation 5 was derived from Eqs. 3 and 4 to avoid the need to compute the strain  $\epsilon_1$ .

Since there is no simple procedure for obtaining  $\epsilon_1$  the writer prefers to define  $f_{cd}^*$  using expressions similar to Eqs. 1 or 2.

More data is required on the effects of compression steel and confining reinforcement in struts. Analytical solutions must be checked experimentally.

5.2.3 Nodal Zones - The strength of concrete in the nodal zones depends on: (a) the confinement of the zones by reactions, compression struts, prestress anchorage plates, reinforcement from the adjoining members and hoop reinforcement; (b) the effects of strain discontinuities within the nodal zone when ties strained in tension are anchored in, or cross, a compressed nodal zone; (c) the splitting stresses and hook bearing stresses resulting from the anchorage of the reinforcing bars of a tension tie in or immediately behind a nodal zone; (d) the weakening effects of grouted or ungrouted prestressing ducts which frequently extend through a nodal zone.

Chapter 6 of the First Draft of the CEB-FIP Model Code 1990<sup>3</sup> requires that the forces in the struts and ties be anchored and balanced in the nodal regions. The Commentary on this section of the draft Model Code states that compressive stresses within the nodes normally only need to be checked at nodes where concentrated loads are applied to the surface of the structural element by means of bearing plates, anchor plates or supports. They also may need to be checked at discontinuities such as holes or corners. The bearing stress is limited to

$$f_{b1} = \alpha \beta f_{cd} \quad (6)$$

where  $\alpha = 1.0$  at nodes where only compression struts meet and 0.8 for nodes at which main tension bars are anchored, and  $\beta$  allows up to a four times increase in the bearing stress if the member is wider than the bearing plate. Transverse reinforcement is required in cases where  $\beta$  is greater than 1.0. Prestressing ducts

crossing the nodal zone are assumed to weaken the nodal zone. No guidance is given for compressive stresses in nodal zones which are not bearing areas.

Except where special confining reinforcement is provided, the 1984 Canadian code<sup>1</sup> and the new draft by Collins and Mitchell<sup>4</sup> limit the concrete compressive stresses in nodal zones to:

- 0.85 $f_{cd}$  in nodal zones bounded by compression struts or bearing areas;
- 0.75 $f_{cd}$  in nodal zones anchoring only one tension tie,
- 0.60 $f_{cd}$  in nodal zones anchoring tension ties in more than one direction.

These values were selected empirically to reflect items (a) to (c) above.

The author does not know of any published experimental study of the strength of nodal zones. This is a major drawback in the development of strut-and-tie models for D regions. Another area needing study is the strength of nodal zones in members comprised of precast and cast-in-place concrete.

The strengths chosen for concrete in nodal zones must be compatible with other similar situations in structures such as the transmission of column load through building floors where, for example, the ACI and CSA codes allow a nodal zone stress of  $\phi 1.4f'_c$  where  $\phi$  is a resistance (safety) factor.

### 5.3 Serviceability

None of the three documents under consideration adequately treat the Serviceability Limit State for D regions. The major serviceability condition is inclined crack width at service loads. The 1990 draft of Chapter 6 of the CEB-FIP Model Code 1990 suggests that an SLS check can normally be avoided if the secondary and main reinforcement together are oriented at the direction of the linear elastic stress fields.

The CSA Code and the ACI draft do not consider the serviceability of D regions. Means for doing this need to be developed.

## **6. B REGIONS**

B regions are regions of structural members where conventional beam theory or the Ritter, Mörsch or Thürlimann types of truss analogies apply. These regions can be designed by full-member design procedures although most commonly they are designed by sectional design procedures some of which are derived from truss analogies.

### 6.1 Basic Design Models

The Simplified Methods of the 1987 ACI Code draft<sup>2</sup> and the 1984 Canadian code<sup>1</sup> assume that a portion of the factored shear  $V_u$  is resisted by "shear in the concrete",





$V_c$ , and the rest is resisted by stirrups,  $V_s$ .  $V_c$  is constant for all values of  $V_u$  and  $V_s$  is calculated using a 45 deg. truss.

Chapter 6 of the First Draft of the CEB-FIP Model Code 1990<sup>3</sup>, the General Methods of the 1984 Canadian code<sup>1</sup> and the 1987 ACI Code draft<sup>2</sup> and the draft by Collins Mitchell<sup>4</sup> are based on the plastic truss model, developed by Thürlimann and his colleagues<sup>7</sup> and shown in Fig. 4. The beam is modelled by a compression chord, shown dashed, a tension chord and a web. The web is assumed to be cracked at  $\theta$  with the horizontal.

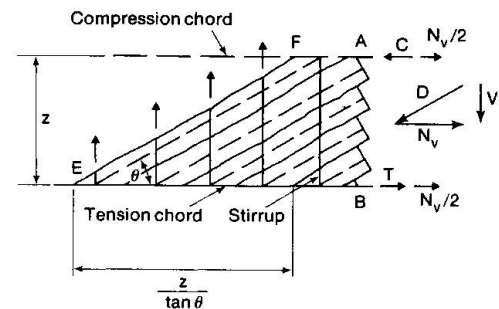


Fig. 4. Plastic Truss Model

The shear,  $V$ , acting on section A-B is assumed to be resisted by a diagonal compression force  $D$  parallel to the cracks and an axial tension,  $N_v$ . The stirrup reinforcement is designed considering the stirrup forces across section E-F in Fig. 4. If  $\theta$  is less than 45 deg, the shear  $V_s$  carried by the stirrups for a given amount and spacing of stirrups exceeds the value used in the Simplified Methods of the ACI and CSA Codes. This partially, but not completely, offsets the lack of a  $V_c$  term in the so called General Methods. Collins and Mitchell's draft includes a  $V_c$  term based on the tension between the cracks.

In design, four modes of failure must be considered: (a) the web must not crush due to the diagonal compression force,  $D$ ; (b) the longitudinal reinforcement must be able to resist  $(T + N_v/2)$ , except that, at points of maximum moment,  $N_v$  goes to zero; (c) the stirrups must be able to resist  $V$ ; and (d) the compression chord must be able to resist a compression of  $C$  at the points of maximum moment and  $(C - N_v/2)$  at other points.

The ambiguity in items (b) and (d), above, results from attempting to express the full-member design concept of a compression fan region at concentrated loads and supports into a sectional design procedure based on a constant angle  $\theta$ .

The General Method of the Canadian Code has not been widely used because designers regard it to be more complex to apply and because it generally requires more stirrups than the Simplified ( $V_c + V_s$ ) method.

## 6.2 Recent Developments in Basic Design Models

The truss model in Fig. 4 assumes that the compression struts are parallel to the direction of cracking and that stresses are transferred across the cracks. More recent theories have made one of two assumptions: (a) Tensile stresses transverse to the axis of the strut exist at points between the inclined cracks but drop to zero at the cracks, Fig. 5a, (b) Shearing stresses are transferred across the inclined cracks by aggregate interlock or friction, Fig. 5b.

Vecchio and Collins<sup>24</sup> show that these two assumptions are the same or, at least very closely related. Two results of these assumptions are: first, the angle of the principal



compression stress in the web is less than the angle of the cracks  $\theta$ , and second, the vertical component of the force along the crack in Fig. 5b is

$$V_c = v_{ci} b_w d \tag{7}$$

assuming  $v_{ci}$  is constant.

In the analysis of a beam using the Modified Compression Field theory of Vecchio and Collins<sup>24</sup> a value of the principal tensile strain  $\epsilon_1$ , perpendicular to the cracks, is estimated. From this, the width of the cracks and the average transverse tensile stress in the struts,  $f_1$ , are calculated. The transverse tensile stresses and resulting  $V_{ci}$  add what amounts to a  $V_c$  term.

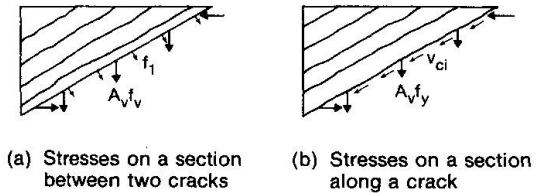


Fig. 5. Modified Compression Field Theory

Dei Poli, Gambarova and Karakoc<sup>25</sup> assume the beam behaves like a plane truss with aggregate interlock shear and compression stresses across the inclined cracks, the latter occurring because the angle of web compression is flatter than the crack angle. This analysis resulted in a  $V_c$  term due to the aggregate interlock. This term was larger for parts of beams with web-shear cracking (tension chord of the truss uncracked) than for parts of beams developing flexure-shear cracking (tension chord cracked).

Kirmair and Mang<sup>26</sup>, following analyses developed in cooperation with Professor Kupfer found that the  $V_c$  term depended mainly on aggregate interlock along the inclined cracks. The amount of interlock was governed by the relative displacement along the cracks. The  $V_c$  term was affected by the extension of the tension flange, decreasing as the flange strain increased, similar to Vecchio and Collins<sup>24</sup> observation.

Reineck and Hardjasaputra<sup>27</sup> also used a truss model with the angle of compression less than the crack angle. Based on the assumption that the cracks open perpendicular to their axis and an assumed stress-strain curve for concrete in compression they derived expressions for the kinematics of the web deformations. Their results were expressed in a series of design charts. Again, the effective value of  $V_c$  was found to increase as the longitudinal strain at mid-depth decreased as observed in each of the other three papers mentioned in this section.

Reineck<sup>28</sup> presents an explanation of the shear strength of beams without shear reinforcement based on a tooth model. He concludes that the aggregate interlock shear and dowel action shear along the cracks transfer most of the shear in such beams.

A simple means of computing a realistic  $V_c$  term would be a desirable addition to shear design codes. Such a term is important in the design of beams having a factored shear force between one and three times the inclined cracking shear. A large fraction of cast-in-place concrete beams fall in this region. The Collins and Mitchell



draft<sup>4</sup> includes a table of  $V_c$  values and special values of  $V_c$  for beams without stirrups.

### 6.3 Web Crushing Strength for Design

Chapter 6 of the First Draft of the CEB-FIP Model Code 1990 uses Eq. 1 with  $\alpha = 0.7$  to define the web crushing strength in B Regions. The web crushing strength is reduced if prestressing ducts cross the compression field in the web. The draft accounts for this by reducing the effective web width.

The 1984 Canadian code uses Eq. 2 to define the web crushing strength where  $\epsilon_1$  is computed using:

$$\epsilon_1 = \epsilon_x + (\epsilon_x + 0.002) / \tan^2 \theta \quad (8)$$

where  $\epsilon_x$  is the longitudinal strain at mid-depth of the member when the section is subjected to  $M_f$ ,  $N_f$  and the axial force  $N_v$  resulting from the shear, positive when tensile, and  $\theta$  is the angle between the compression struts and the longitudinal axis. Because Eq. 2 was derived from the average strains in panels subjected to a uniform state of stress and strain, the average longitudinal strain (i.e. at mid-depth) is assumed to be the appropriate value here. This has been experimentally checked in several cases.<sup>29</sup>

The 1987 draft of ACI Chapter 11 uses Eq. 5 to compute the crushing strength of the web. As pointed out earlier this was derived from Eqs. 3 and 4. Alternatively, the ACI draft assumes that the web will not crush if the angle  $\theta$  satisfies

$$\theta > 15^\circ + \frac{140 v_n}{f'_c} \quad (9)$$

for reinforced concrete where  $v_n$  is the average shear stress in the web of the beam. This equation was derived in Ref. 13.

The Collins and Mitchell draft<sup>4</sup> presents a table which is entered using  $v_n$  and  $\epsilon_x$ . Values of  $\theta$  which satisfy Eqs. 2 and 8 and values of a  $V_c$  term computed from the tension between the cracks are obtained from the table. This is done at a number of cross-sections along the beam to reflect changes in  $v_n$  and  $\epsilon_x$ .

A major difference between Chapter 6 of the First Draft of the CEB-FIP Model Code 1990 and the other three codes is the dependence of  $f_{cd}^*$  on the angle  $\theta$  in the latter three. This complicates design but may be necessary to properly reflect the true crushing strength of the web concrete. This discrepancy needs to be resolved.

For beams subjected to very high shears the CSA code<sup>1</sup> requires the web width to be reduced to the width inside the stirrups to account for the spalling of the concrete outside the stirrups observed in tests of beams with closely spaced stirrups subjected to very high shears. This concept is not contained in the other two codes.

## 6.4 Allowable Angles

The flattest angle  $\theta$  allowed in Chapter 6 of the First Draft of the CEB-FIP Model Code 1990 is  $18.4^\circ$  ( $\cotan \theta = 3$ ). This is currently under discussion by CEB Commission IV. A more stringent limit based on shear transmitted across the original cracks has been proposed by two members of that Commission.

The 1984 Canadian Code allows any angle  $\theta$  between  $15^\circ$  and  $75^\circ$ . The 1987 ACI draft limited the angle to  $25^\circ$  to  $65^\circ$ . Grob and Thürlimann<sup>6</sup> suggested a limit of  $\cotan \theta = 2$  ( $\theta = 25.6^\circ$ ) to prevent excessive inclined crack widths.

In the Collins and Mitchell draft<sup>4</sup> the angle  $\theta$  is chosen from a table to satisfy Eq. 2 and 8.

## 6.5 Staggering Rules

When a beam is loaded on its top surface and supported at locations on its lower surface, the stirrups within a distance  $d/\tan \theta$  from the support can be designed for the shear at  $d/\tan \theta$  from the support, those between 1 and 2  $d/\tan \theta$  for the shear at  $2d/\tan \theta$  and so on. This is referred to as the *staggering rule*. For beams loaded and supported in this way this procedure satisfies equilibrium and has been shown experimentally<sup>29</sup> to produce a safe design. On the other hand, the staggering rule does not satisfy equilibrium for beams both loaded and supported on their bottom surfaces or for beams in which dead load is a major fraction of the total load. The ACI Code has allowed the staggering concept to be used within  $d$  ( $=d/\tan 45^\circ$ ) from the support since 1963. The General Method of the 1984 Canadian code allows the staggering rule for uniformly loaded beams loaded on their top surface and supported at locations on its bottom surface. It does not give guidance for other cases. The commentary for Chapter 6 of the First Draft of the CEB-FIP Model Code 1990 allows this procedure in the end  $d/\tan \theta$  but does not appear to make it a general rule.

## 6.6 Prestressed Concrete

One of the most perplexing features in the development of codes for structural concrete is the treatment of shear in prestressed concrete beams. Thus, for example, the CEB-FIP Model Code 1978 presented a Standard Method for design for shear in prestressed concrete which included a  $V_c$  term which was larger for prestressed beams than for reinforced concrete and an Accurate Method which for high shears did not include a  $V_c$  term and did not recognize any favorable effect of prestress. As a result, the Accurate Method required more stirrups than the Standard Method. Similar arguments have prevented the adoption of the 1987 draft of ACI Chapter 11.

Schlaich et al.<sup>6</sup> suggest that the load carrying mechanism of a prestressed beam at ultimate loads consists of an inclined

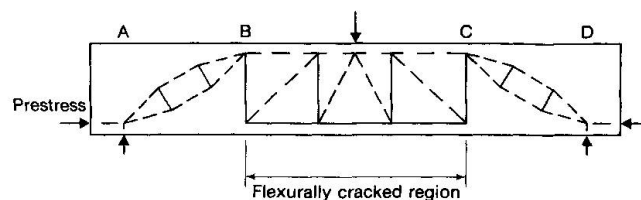


Fig. 6. Truss Model of Prestressed Beam



compression strut in the uncracked regions of the beam, AB and CD in Fig. 6, and a truss in the cracked regions, BC. In ACI terminology, regions AB and CD would be subject to web-shear cracking (inclined cracking prior to flexural cracking) while region BC would be subject to flexure-shear cracking (inclined cracking affected by flexural cracking). Stirrups in regions AB and CD prevent widening of the splitting crack (web-shear crack). The main shear carrying mechanism in this region is strut action. In region B-C the stirrups equilibrate all the shear unless account is taken of the shear transferred across the crack interfaces or the tension transverse to the diagonal compression struts.

For uniformly loaded beams Chapter 6 of the First Draft of the CEB-FIP Model Code 1990 identifies two cases. For the normal case the prestress is assumed to carry a portion,  $\lambda q$ , of the dead and live loads,  $q$ , by the arching action of the compression zone and upward pressure of the tendons as shown in Fig. 7a and b. The remainder of the loads  $(1-\lambda)q$  are carried by truss action as shown in Fig. 7c. As a result, stirrups are only required for  $(1-\lambda)$  times the shear force.

For thin-webbed beams with massive end blocks, the 1990 draft Model Code assumes the prestress force is applied directly to the top and bottom flanges as shown in Fig. 9. In this case the only part of the transverse load supported by the prestress is that supported by the upward pressure of any curved tendons. No quantitative guidance is given as to whether the models in Fig. 7 or 8 should be used.

It should be noted that this portion of Chapter 6 of the 1990 draft received extensive discussion and may be modified in the final Model Code.

The 1984 Canadian code presents a Simplified Method and a General Method for design of prestressed beams for shear. The Simplified Method is based on the  $V_c + V_s$  concept with  $V_s$  calculated using a 45 deg

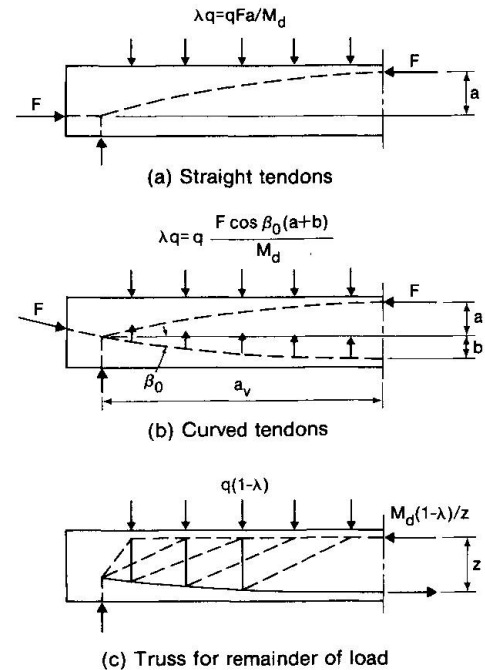


Fig. 7. Prestressed Concrete Draft CEB-FIP Model Code

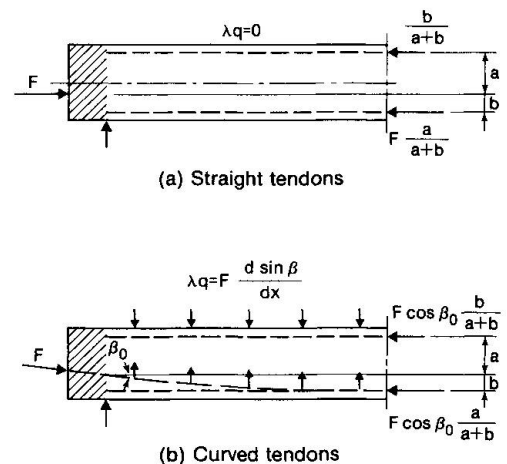


Fig. 8. Prestressed Concrete Beams with Thin Webs and Massive End Blocks

truss. The  $V_c$  term is the smaller of the shear causing web-shear cracking and flexure-shear cracking. The General Method is based on the plastic truss model. The General Method has no  $V_c$  term and hence does not recognize any favorable effect of prestress except its vertical component, unless a strain compatibility solution is used to compute  $\epsilon_x$  in Eq. 13.

The 1987 draft of ACI Chapter 11 follows the same pattern as the 1984 Canadian code although the presentation is simplified to avoid the computation of  $\epsilon_1$ .

The Collins and Mitchell draft<sup>4</sup> includes the effects of prestress in the calculation of the value of  $\epsilon_x$  at each section and thus accounts for the effects of prestress directly.

### 6.5 Serviceability

The 1987 ACI Code draft and the 1990 draft of the CEB-FIP Model Code do not consider the width of shear cracks in B regions except through detailing rules. The Simplified Method of the 1984 Canadian code attempts to limit inclined crack width by limiting the maximum shear stress in a beam web. The upper limit was chosen on the basis of limiting the stirrup stress at service loads to 200 MPa. The General Method of the 1984 Canadian code gives a set of "deemed to satisfy" rules. If these are violated, the designer must limit the strain in the stirrups at service loads to 0.001 for interior exposure. A relatively complex equation is given to estimate this strain.<sup>13</sup>

## 7. MAJOR AREAS NEEDING MORE STUDY

1. Further development of computer programs to lay out and solve strut-and-tie models.
2. A compromise is needed defining the factors which affect the crushing strength of the webs in beams or the struts in strut-and-tie models. The CEB-FIP draft involved concrete strength and a qualitative description of the degree of cracking, the General Method of the CSA code and the Collins and Mitchell draft included concrete type and principal tensile strain, while the ACI code draft included crack angle and reinforcement yield strength as variables.
3. Experimental verification of nodal zone strengths is needed.
4. Simple ways are needed for verifying the serviceability limit states of D regions and B regions. These could take the form of "deemed to satisfy" rules.
5. Recent theories suggest that tension in the concrete between cracks is responsible for  $V_c$ , the shear carried by concrete. Simple ways of accounting for this effect are desirable.
6. A major area requiring more study is the enhanced shear strength of prestressed members.
7. Although not discussed in this paper, the full-member design procedures proposed in the CEB-FIP draft Model Code for T-beam flanges need to be made compatible with sectional design procedures for beam webs and the description of the thin-walled tube used in torsion design needs to be standardized ( $A_o$ , wall thickness).



8. Designers need more guidance about the design for warping torsion.

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## NOTATION

- $c$  = distance from extreme compression fiber to axis of zero strain  
 $C$  = resultant compressive force due to flexure  
 $d$  = effective depth  
 $D$  = diagonal compressive force in the web of a beam  
 $E_s$  = modulus of elasticity of reinforcement  
 $f_{b1}$  = bearing strength  
 $f_{cd}$  = factored or design strength of concrete  
     =  $f_{ck}/\gamma_m$  in CEB code, where  $\gamma_m = 1.5$   
     =  $0.6f'_c$  in CSA code  
 $f_{cd}^*$  = effective compressive strength in compression zones or cracked beam webs  
 $f_{ck}$  = concrete compressive strength from a cylinder test, exceeded by 19 of 20 tests  
 $f'_c$  = concrete compressive strength from a cylinder test, exceeded by 11 of 12 tests  
 $f_y$  = yield strength of reinforcement

- $f_1$  = tension transverse to strut
- $M_f$  = moment due to factored loads
- $N_v$  = axial tensile force due to shear
- $q$  = uniform load
- $T$  = resultant tensile force due to flexure
- $v_{ci}$  = shear stress transferred across an inclined crack
- $v_n$  = nominal shear stress in web of beam
- $V_c$  = component of shear "carried by concrete"
- $V_s$  = shear resisted by stirrups
- $V_u$  = shear due to factored loads (ACI code)
- $\alpha$  = compressive strength factor
- $\alpha_s$  = angle between compression strut and reinforcement crossing it
- $\beta$  = nodal strength factor
- $\beta_1$  = ratio of depth of rectangular stress block to  $c$
- $\epsilon_s$  = strain in reinforcement crossing compression strut
- $\epsilon_x$  = average longitudinal strain in web of beam
- $\epsilon_1$  = average principal tensile strain in cracked web
- $\lambda$  = a factor to account for concrete type ranging from 1.0 for normal-weight concrete to 0.75 for structural concrete with lightweight, coarse and fine aggregate (CSA, ACI)
- = the fraction of the total load resisted by arching action of the prestress force.
- $v$  = effectiveness factor
- $\phi$  = a resistance factor (ACI)
- $\theta$  = inclination of compression struts in cracked beam web
- $\theta_{st}$  = angle between a tension tie and a compression strut

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