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Serviceability Analysis of Reinforced Concrete Slabs

Analyse à l'état de service de dalles en béton armé

Berechnungsverfahren für die Gebrauchsfähigkeit von Stahlbetonplatten

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SUMMARY

Proposed analysis of reinforced concrete slabs in the serviceability limit states is based on the finite element method and the effective stiffness model of the plate cross-section. The nonlinear effects of concrete cracking, tension stiffening, and simplified creep behaviour are included. Examples of the analysis of real structures are described.

RÉSUMÉ

La méthode proposée pour l'analyse de dalles en béton armé à l'état de limite de service est basée sur la méthode des éléments finis, ainsi que sur un modèle de rigidité effective de la section transversale de la dalle. Les effets non-linéaires de la fissuration du béton, ainsi qu'un modèle simplifié pour la fissure sont également inclus. Des exemples d'analyse de structures réelles illustrent ces propos.

ZUSAMMENFASSUNG

Das vorgeschlagene Berechnungsverfahren für Stahlbetonplatten im Gebrauchszustand beruht auf der Finite Elemente Methode und dem Modell effektiver Querschnittssteifigkeiten. Es werden die nichtlinearen Effekte der Rissbildung, der tension-stiffening-Effekt und, auf einfache Weise, das Kriechen des Betons berücksichtigt. Es werden Beispiele für die Berechnung realistischer Bauwerke gegeben.

INTRODUCTION

The authors have developed a method for the computer analysis of reinforced concrete plates under serviceability conditions. The purpose of this work was to provide an efficient tool for practical design which would enhance current engineering practice. It is known that deflection of reinforced concrete structures is greatly affected by concrete cracking and by the long-term effect of creep. However, such effects are non-linear and they can complicate analysis to the extent that it becomes impractical. Therefore, it was decided to simplify the material and numerical model as much as possible and to focus on efficiency at the expense of unnecessary accuracy.

In view of the above considerations the following assumptions are made:

1. Thin-plate theory based on the Kirchhoff hypothesis is used for plate mechanics. This theory is based on the action of bending moments only. Normal forces are not considered and shear forces must be derived from moments.

2. An effective stiffness model is used in which the material behavior is defined for the whole cross-section of the plate. The constitutive law is defined for moments and curvatures.

The work presented here is based on the first author's Ph.D. dissertation [1].

CONSTITUTIVE MODEL

A smeared approach is used to model the material non-homogeneity, i.e., the material properties are constant within a considered element volume but can vary between the elements. The moment-curvature relation for the plate bending element is as follows:

$$\mathbf{m} = \mathbf{D}.\mathbf{k} \tag{1}$$

where $\mathbf{m} = \{m_x, m_y, m_{xy}\}$ is the vector of internal moments per unit width and $\mathbf{k} = \{\kappa_x, \kappa_y, \kappa_{xy}\}$ is the vector of corresponding curvatures. The constitutive matrix $\mathbf{D}$ for uncracked concrete has the form of the elastic matrix for isotropic plates. The cracking criterion is given by the cracking moment $m_c = h^2 R_t/6$, where R_t is the tensile strength of concrete. When the principal moment reaches the cracking moment, the constitutive matrix takes the form of the orthortopic elastic plate matrix. The axes of orthotropy are aligned with the crack direction, Fig.1:

$$\mathbf{D} = \begin{bmatrix} d_{nn} & d_{nt} & 0 \\ d_{tn} & d_{tt} & 0 \\ 0 & 0 & d_{qq} \end{bmatrix} \tag{2}$$

The coefficients of the constitutive matrix are calculated according to Jofriet et al. [2] :

$$d_{nn} = \frac{B_n}{1-\nu^2} \qquad d_{nt} = d_{tn} = \nu\sqrt{d_{nn}d_{tt}}$$
$$d_{tt} = \frac{B_t}{1-\nu^2} \qquad d_{qq} = \tfrac{1-\nu}{2}\sqrt{d_{nn}d_{tt}} \tag{3}$$

where B_n, B_t are the flexural stiffnesses normal and parallel to the cracks, respectively, and ν is the Poisson's ratio. The flexural stiffness of the cracked cross section is modeled according to Hajek [3] as:

$$B_r = \frac{h_o z}{\frac{\psi_s}{E_s A_{st}} + \frac{2\psi_b}{E_c A_{ci}}} \qquad r = n, t \tag{4}$$

in which n, t are indices for the directions normal and parallel to cracks. E_s and E_c are the elastic moduli of the steel and concrete, respectively; A_{st} and A_{ci} are the areas of steel and

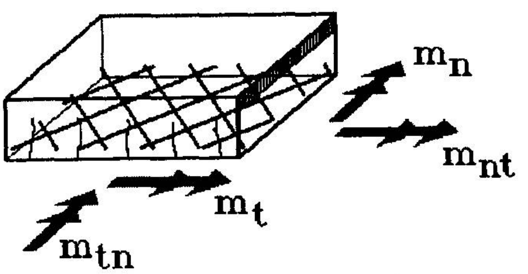

Fig.1 Moments on the cracked plate element.

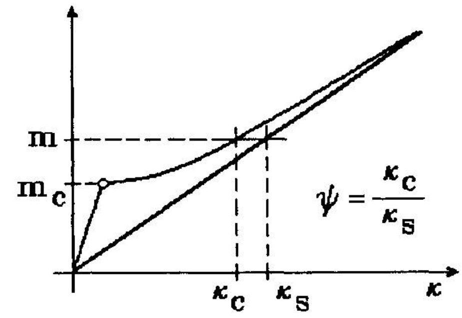

Fig.2 Moment-curvature diagram. Meanig of tension stiffening coefficient ψ.

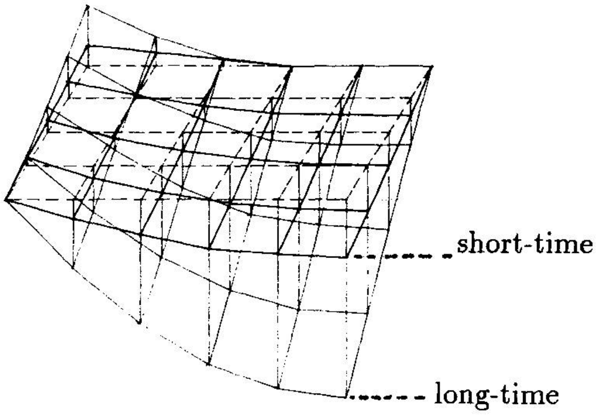

Fig.3 Long- and short-time deflection shapes in Hajek's experiments [3].

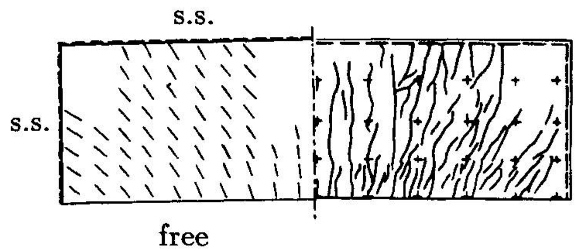

Fig.4 Comparison of experimental and analytical crack patterns.
Legend: free - free edge
s.s. - simply supported edge

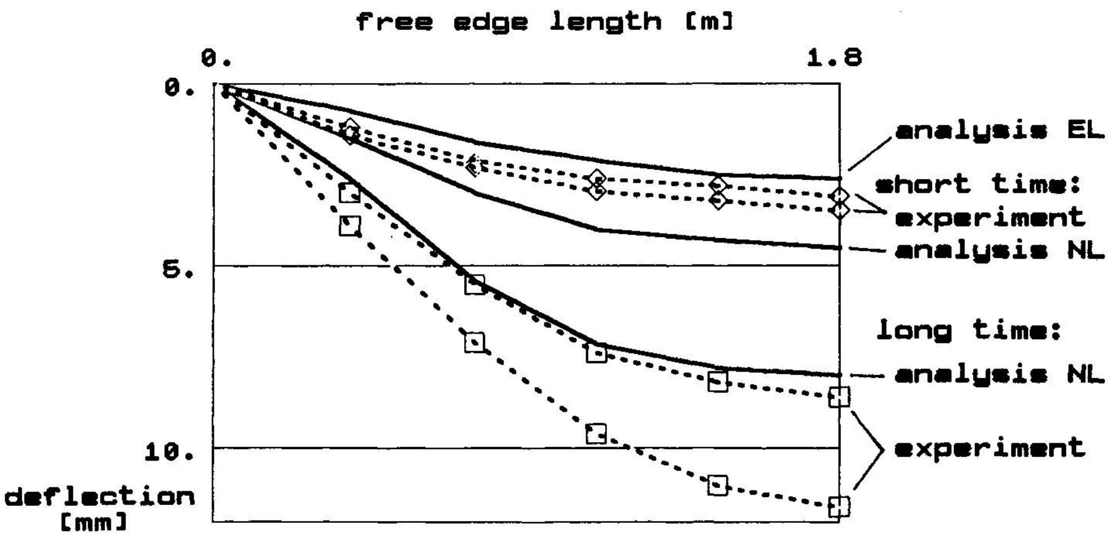

Fig.5 Comparison of analytical and experimental deflections of the free edge for Hajek's slab [3].

concrete, respectively; h_o is the effective depth and z is the lever arm of the internal forces. The tension-stiffening coefficient ψ_s represents the reduction of steel strains due to concrete tensile stiffness after cracking. A similar effect in the compression zone is described by ψ_c. These coefficients are defined as follows:

$$\psi_i = 1 - \rho\,(1 - \psi_{ir}), \quad i = s, c, \qquad \rho = \frac{1}{4}\left(5\frac{m_c}{m} - 1\right) \tag{5}$$

$$\psi_{sr} = \frac{6}{0,85} n \frac{A_s\,(2h_o - h)\,z}{bh^3}, \qquad b = 1m \tag{6}$$

$$\psi_{cr} = \frac{6}{1,7} \frac{A_{ci}\,z}{bh^2}, \qquad n = \frac{E_s}{E_c} \tag{7}$$

in which the indices s, b refer to steel and concrete, respectively; ρ is an interpolation parameter for tension stiffening and ψ_{sr}, ψ_{cr} are initial values of ψ_i after cracking. The moment-curvature diagram resulting from this model is schematically shown in Fig.2. In the case of an inclined crack, the effective reinforcement area, A_s, is calculated using appropriate transformation of the two reinforcing directions passing through the cracked cross-section. Three crack modes are recognized: 1. one crack, 2. two orthogonal cracks on the same surface, 3. two orthogonal cracks on opposite surfaces.

The time effect is included as a simplified method based on an extensive experimental investigation by Hajek of real plates [3]. He found an affinity relationship between the short-term and the long-term deflected shapes of plates (Fig.3). The deflection $w\,(t, x, y)$ at the time t is governed by the simple formula

$$w\,(t, x, y) = (1 + \beta)\,.w\,(in, x, y) \tag{8}$$

in which $w\,(in, x, y)$ is the initial deflected surface of the plate and β is the creep coefficient. The creep function is taken from the Czechoslovak National Standard for concrete structures as follows:

$$\beta = \left(0,15 + 0,008e^{-0,015t_1}\right)\left(1 - e^{-0,07\sqrt{t-t_1}}\right)\phi \tag{9}$$

where t is the age of concrete at the considered time, t_1 is the age of concrete at the time of load application, and ϕ is the coefficient of creep.

NUMERICAL SOLUTION

The numerical solution of the plate analysis is performed using the finite element method. The Clough-Felippa [4] quadrilateral finite element is used. The element is composed of four linear-strain subtriangles. It has four external nodes with three degrees of freedom in each node, one deflection and two slopes. This element has piece-wise linear moment distribution and thus gives quite good results even for coarse meshes. The non-linear solution is done by the secant stiffness method. The total loading is applied, then iteration is performed until the constitutive laws and equilibrium are satisfied. Finally, the creep effect is included by scaling the deflected shape using the creep law, Eq.9.

The program can handle irregular geometrical shapes, internal hinges, elastic supports, elastic foundations, concentrated and distributed loadings and other features.

The program is written in FORTAN77 and can be installed on various computer systems (mainframes, workstations, personal computers). It is equipped with efficient pre- and post-processors by means of which numerical results can be presented on grahical terminals, plotters, or printers.

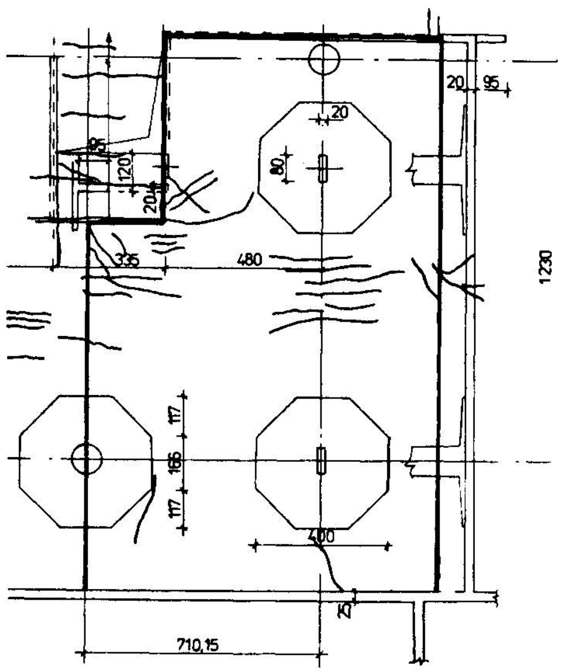

Fig.6 Garage slab. Plan view of the slab section with observed cracks.

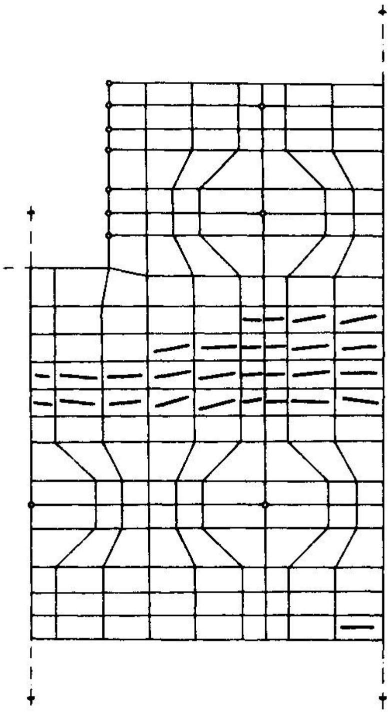

Fig.7 Garage slab. Finite element model with calculated crack pattern.

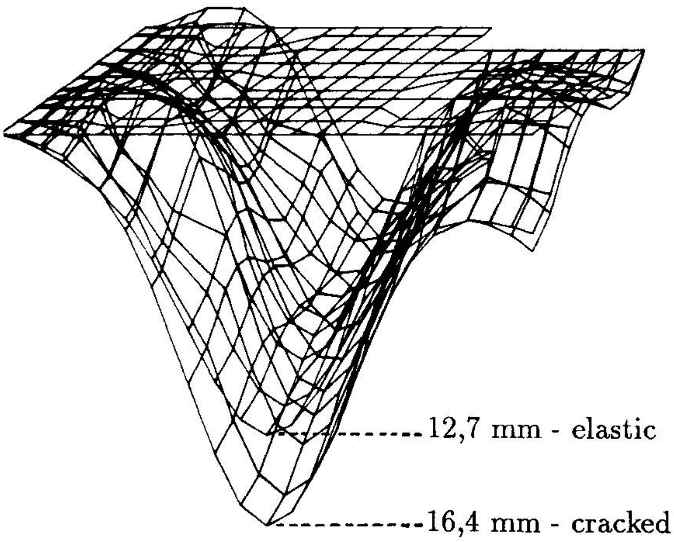

Fig.8 Calculated deflection surfaces.

COMPARISON WITH HAJEK'S EXPERIMENTS

The experimental slabs in ref.[3] were supported on three sides. The slab dimensions were 3,6 x 1,2 x 0,12 m. They were reinforced by mesh with reinforcing ratios in both directions equal to 0,0112. The concrete quality was 30 MPa. The slabs were loaded 28 days after casting by a uniform load of 14,7 kN/m^2. The load was applied by plastic bags under air pressure, and was kept constant during the entire loading history. The slabs were kept under controlled laboratory conditions where deflections under sustained load were monitored during a period exceeding one year. An example of measured deformed shapes is shown in Fig.3. Long term deformations are considered at 400 days.

The slab was analyzed by the above described program. The deflections of the free edge are compared in Fig.5 with experimental results for short-term and long-term effects. The results from two nominally identical experimental slabs A, B are shown to indicate the scatter of the experimental results. The experimental and analytical crack patterns are compared in Fig.4. Good agreement of the analysis with the experiment is found.

EXAMPLE OF A SLAB IN PARKING GARAGE

The program was successfully used for checking deflections of several slab structures in design practice. One example is shown here. It concerns an existing floor structure of a parking garage which did not performed satisfactorily after the first few years of service life. It was required to check the deflections of the slab. The project also included the design of repair provisions. Here we are showing only partial resuts for illustration. Fig.6 shows a section of the floor structure comprised of an irregular slab, supported by columns. The slab, 20 cm thick, is cast-in-place and the octagonal precast shear heads are 35 cm thick. The existing cracks are also recorded. Fig.7 shows the finite element model and crack pattern as calculated by analysis. The calculated deflected shapes are shown in Fig.8. The elastic and cracked shapes are compared. The analysis was used to check the serviceabilty performance of the slab before and after the designed reconstruction.

CONCLUSION

The proposed analysis of reinforced concrete plates based on the finite element method and the effective stiffness of the slab cross section gives a fair approximation of the real performance of slabs under serviceability conditions. It can model complex geometrical shapes and important effects of cracking, tension stiffening, and creep.

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