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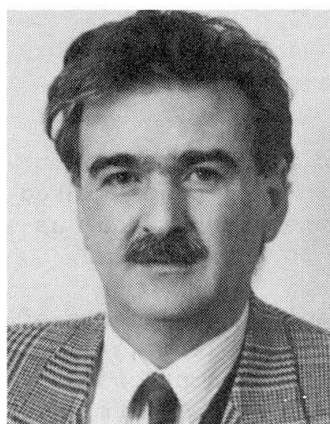
## Plane Elements Analysed Via a Simple Microplane Model

Analyse de structures planes selon un modèle simple par microplans

Analyse von Stahlbetonscheibenelementen  
mit einem einfachen Mikroebenen-Modell

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### SUMMARY

Panels subjected to monotonically increasing loads and deep beams with and without cutouts are analysed assuming a simple microplane model for concrete. The microplane model is incorporated into a finite element program based on an incremental-iterative procedure, which is well suited to the description of the highly non-linear behaviour of reinforced concrete elements. The reinforcement is smeared; bond-induced stiffening effects are included.

### RÉSUMÉ

Des parois en béton armé ainsi que des murs porteurs soumis à des charges continument croissantes sont examinés en recourant pour le béton à un modèle simplifié par microplans; ce dernier fait partie d'un programme par éléments finis basé sur une procédure itérative pas-à-pas. L'armature est disposée sur chaque élément, de même que l'on tient compte de l'adhérence acier-béton et de ses effets raidisseurs. Le modèle de microplans étudié s'adapte de façon satisfaisante aux résultats expérimentaux obtenus.

### ZUSAMMENFASSUNG

Stahlbetonscheiben bei stetig ansteigender Belastung und wandartige Träger mit Öffnungen werden mit Hilfe eines vereinfachten Mikroebenen-Modells für den Beton untersucht. Dieses Modell ist in ein Finite Elemente Programm eingebaut, das auf einem schrittweise wiederholenden Verfahren beruht. Die Bewehrung ist über jedes Element verschmiert und auch der Verbund zwischen Stahl und Beton ist mit seiner Steifigkeit vorhanden.



## 1. INTRODUCTION

The so-called "local models" for the description of the multiaxial behavior of concrete have become very appealing lately due to their intrinsic simplicity and to their strict connection with the micromechanics of aggregate materials. Among the local models, the Microplane Model (Bazant and Oh [1], Gambarova and Floris [2]) has enjoyed special attention, since it seems well suited to the modellization of a variety of loading conditions, monotonic as well as cyclic (Bazant and Prat [3]).

Here an initial and relatively simple version for 2-D problems [2] is introduced into a pre-existing F.E. code and is applied to the analysis of several RC plane structures, such as Collins' panels [4], Cervenka and Gerstle's ribbed panels [5] and Kong's deep beams [6,7], in order to assess the ability of the model to describe the structural behavior (load-displacement response, cracking and path-dependency).

In a previous paper (Donida et al. [8]), Maier and Thürlimann's shear walls were successfully analysed with the proposed model.

## 2. MICROPLANE MODEL FOR CONCRETE SUBJECTED TO PLANE STRESSES

Concrete is considered as a system of randomly oriented planes (the microplanes), in which the elastic and inelastic deformations are concentrated (Figs.1a,b). In the simplified formulation adopted here, three fundamental assumptions are introduced, with a fourth assumption referring to 2-D problems:

- the local strains, acting on each microplane, are the resolved components of the applied strains (macroscopic strain tensor):  $\epsilon_n = n_i n_j \epsilon_{ij}$ , with  $i, j=1,2$ .
- the shear stiffness of the microplanes is neglected: this assumption has a physical explanation [1,2], but has been introduced mostly for its intrinsic simplicity, and may be dropped in a more general approach [3];
- only the normal stiffness of the microplanes is introduced, and the coupling between the normal microstress  $\sigma_n$  and the shear strain  $\gamma_{nt}$  is disregarded. Both the elastic and inelastic behavior of the concrete is described by assuming that  $\sigma_n$  is a function of  $\epsilon_n$ :  $\sigma_n = F(\epsilon_n) \epsilon_n$ ;
- in 2-D problems only the microplanes at right angles to the reference plane are considered (Fig.1b).

In order to work out the coefficients of concrete stiffness matrix it is necessary to formulate the constitutive law for the microplanes: then, by suitably superimposing the contributions from all microplanes, the stiffness characteristics of the concrete can be obtained [2], as well as the increments of the stresses in the general reference system. Under increasing loads, it suffices to specify the stress-strain relations for loading, unloading and reloading in tension and compression (Fig.1c). The model is -by its very nature- a kind of "rotating crack model" (crack planes coincide with the microplanes exhibiting the maximum normal strain in tension) and is path-dependent (the microplanes are activated independently of each other).

## 3. F.E. PROGRAM IMPLEMENTATION WITH THE MICROPLANE MODEL

A suitable F.E. code based on an incremental-iterative procedure, has been implemented with the microplane model. Quadrangular 4-node elements are used (Fig.3), a fifth inner auxiliary node being provided in order to subdivide each element into four constant-stress triangular elements. The fifth node does not contribute to the degrees of freedom of the structure, because it is removed before the stiffness matrix of the structure is assembled, by means of a condensation process [8]. The introduction of the microplane model, as well as the

evaluation and updating of concrete stiffness matrix, are worked out in a first subroutine dealing with the triangular elements; a system of 12 microplanes suffices for concrete description. The stiffness matrix of the quadrangular elements is evaluated and assembled in a second subroutine.

General properties of the F.E. code are: (a) the stiffness coefficients are formulated by the "direct method"; (b) the solution is based on the Gauss-Doolittle method; (c) the shape functions are of the polynomial type; (d) at the moment only monotonic load histories can be studied; (e) the reinforcement is smeared in two directions at right angles to each other.

Bond-induced tension stiffening effects are introduced by modifying the stress-strain law of the reinforcement: to this purpose, crack orientation, spacing and opening have to be evaluated at each load step and in each triangular element (Fig.2a). Once cracks are formed, their orientation remains fixed. In order to evaluate crack spacing, an "equivalent" steel ratio (Fig.2b) has to be defined [8]: the Young's modulus  $E'_s$  of the steel is a function of the average steel strain according to two different bond-stress situations (Fig.2c,d and Fig.4). Finally, a very simple failure criterion has been introduced for the microplane system: as soon as 2/3 of the microplanes reach a prefixed limit strain in compression and/or in tension (where strain softening automatically diminishes the stiffness), the stiffness of the material is put to zero; subsequently, as soon as the solution process no longer converges, the whole structure fails.

#### 4. FITTING OF TEST DATA

Nine different cases are here examined (Figs.5,6 and 7) and no detailed comments are necessary, since the results are mostly self-explanatory; the principal material properties and the size of test specimens are reported in the figures or in the captions below. For further details see the references, which are easily

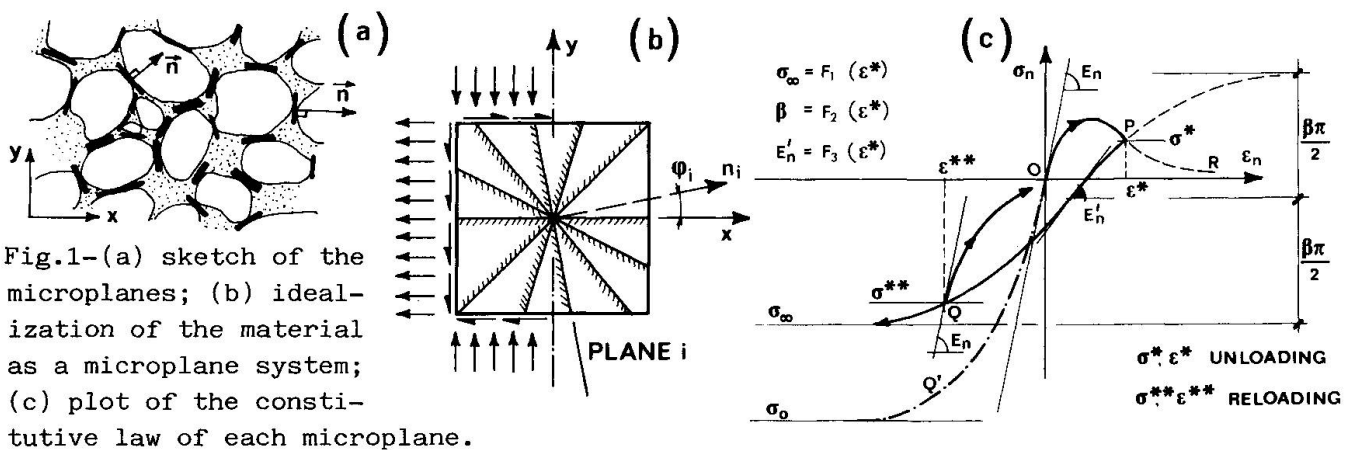


Fig.1-(a) sketch of the microplanes; (b) idealization of the material as a microplane system; (c) plot of the constitutive law of each microplane.

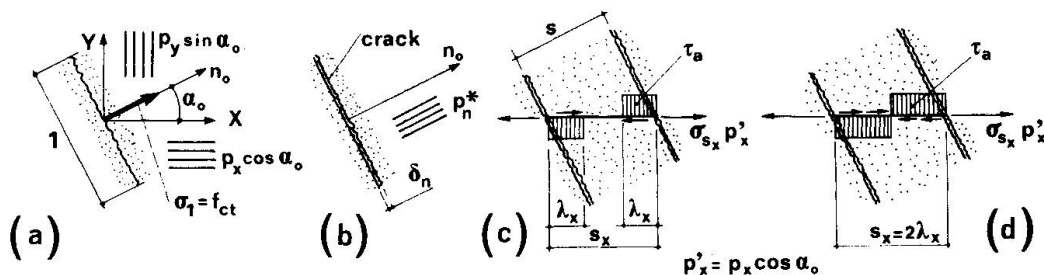


Fig.2 - Formation of a crack (a); equivalent steel ratio (b); bond stresses for mixed bond conditions (chemical adhesion and mechanical interaction) (c); bond stresses after the loss of chemical adhesion (mechanical interaction only) (d).

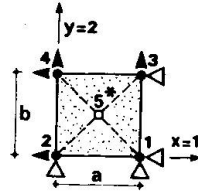


Fig. 3 - Typical 4-node element used in F.E. analysis; (\*) means node condensation.

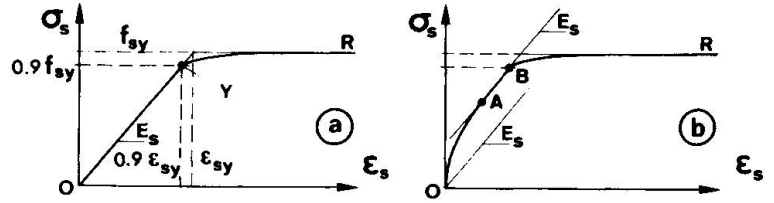


Fig. 4 - Constitutive law of the reinforcement without tension stiffening (no bond) (a) and with tension stiffening (embedded bars) (b).

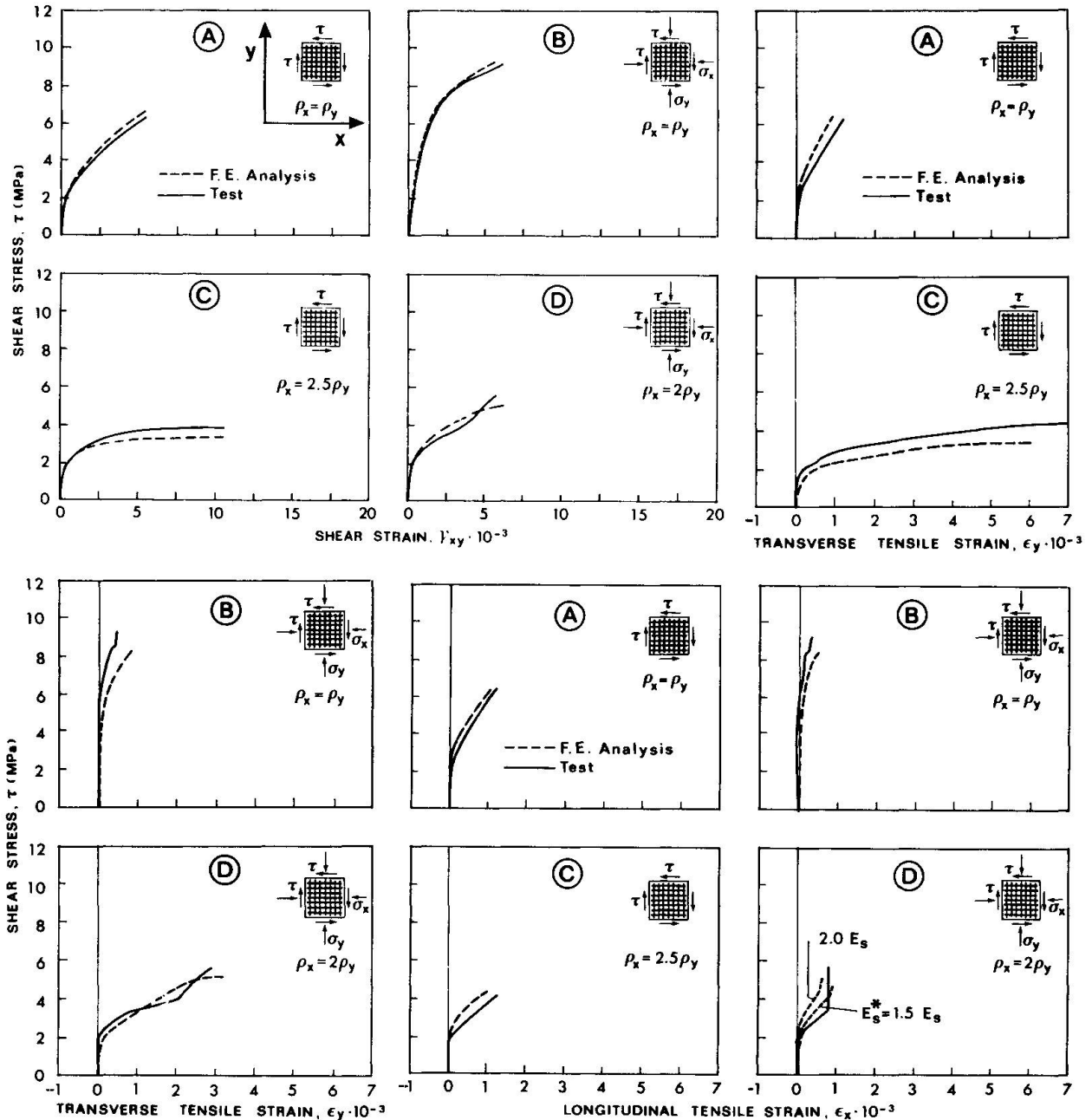


Fig. 5 - Fits of Collins' test results [4]: shear stress  $\tau = 0 - \tau_u$ ;  $\rho_x = 0.01785$ . Panel A:  $f'_c = 20.5$  MPa,  $f_{sy} = 442$  MPa,  $\sigma_x = \sigma_y = 0$ ; Panel B:  $f'_c = 19.3$  MPa,  $f_{sy} = 466$  MPa,  $\sigma_x = \sigma_y = -0.7\tau$ ; Panel C:  $f'_c = 19.0$  MPa,  $f_{sy} = 458$  MPa for x-bars and 299 MPa for y-bars,  $\sigma_x = \sigma_y = 0$ ; Panel D:  $f'_c = 21.7$  MPa,  $f_{sy} = 441$  MPa for x-bars and 324 MPa for y-bars,  $\sigma_x = \sigma_y = 0$  for  $\tau \leq 3.9$  MPa and  $\sigma_x = \sigma_y = -(\tau - 3.9)$  MPa for  $\tau > 3.9$  MPa. Panel size: 890 x 890 x 70 mm. F.E. discretization: 9 square elements.

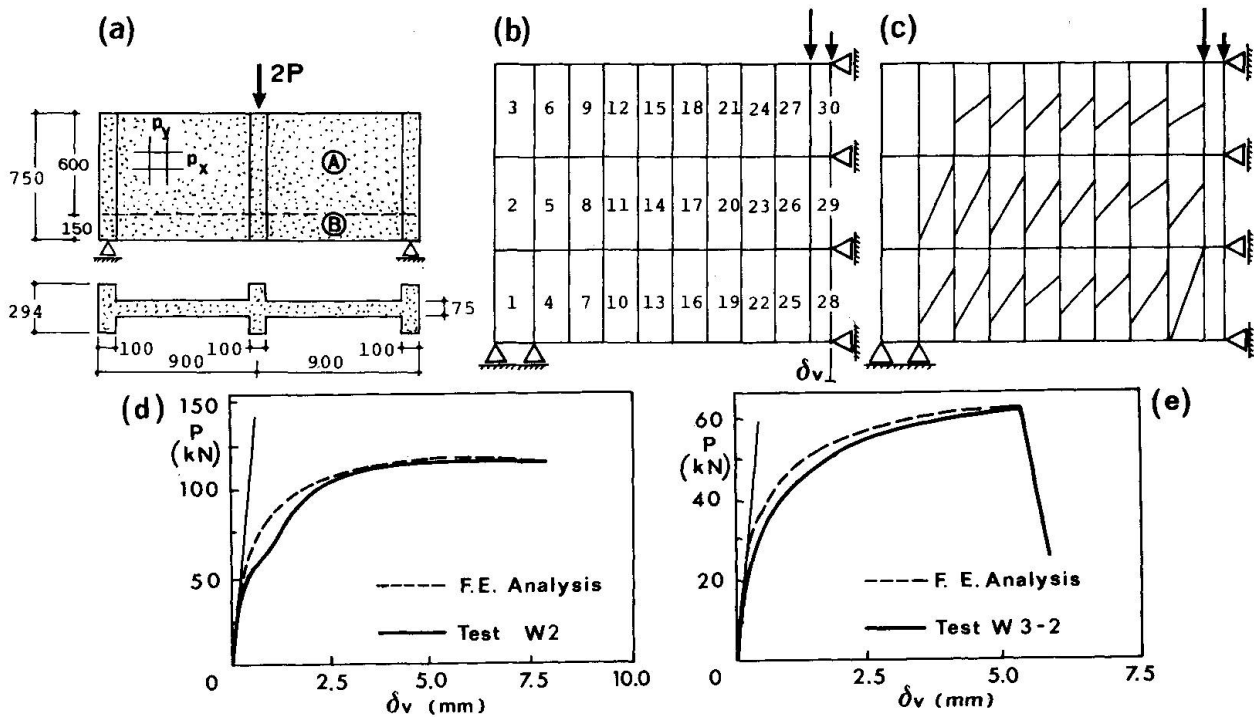


Fig. 6 - Fits of Cervenka and Gerstle's test results [5]: Test W2 (a,c,d) web thickness  $t = 75$  mm, rib thickness  $t' = 294$  mm, Zone A with  $\rho_x = \rho_y = 0.00916$ , Zone B with  $\rho_x = 0.01832$ ,  $\rho_y = 0.00916$ ; Test W3-2 (a,e)  $t = 50$  mm,  $t' = 269$  mm, Zones A and B with  $\rho_x = 0.0123$ ,  $\rho_y = 0$ ; (b) FE mesh, loads and boundary conditions; (c) directions of the principal compressive strain, at collapse; (d,e) load-displacement curves.  $f'_c = 27.4$  MPa,  $f_{sy} = 362$  MPa.

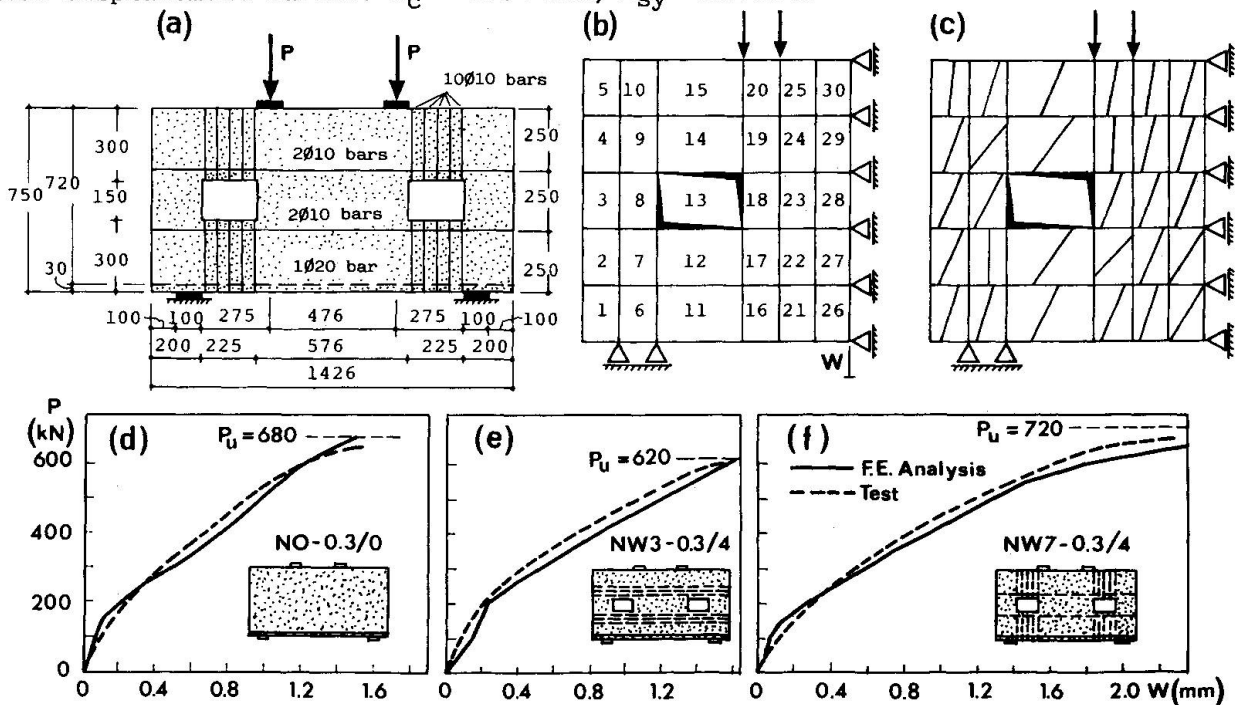


Fig. 7 - Fits of Kong et al. test results [6,7]: Beam NW7-0.3/4 (a) geometry, (b) FE discretization and (c) directions of the principal compressive strain at collapse; (d,e,f) load-displacement curves of Beams NO-0.3/0 ( $f'_c = 44.8$  MPa), NW3-0.3/4 ( $f'_c = 46.2$  MPa) and NW7-0.3/4 ( $f'_c = 42.9$  MPa);  $f'_{ct} = 3.75$  MPa,  $f_{sy} = 430-450$  MPa. Beam thickness = 100 mm.





found in the literature. As a rule, the rebars were smeared in one or two directions over the elements they go through; consequently, the "steel density" is often quite far from the real situation, but this fact does not seem to have a major impact on the results of the analysis. In Panel D tested by Collins ([4] Fig.5) significant unloading occurs after compressive stresses are applied; as a result, the "effective Young's modulus" of the embedded steel during unloading plays a relevant role ( $E_s^* = 1.5-2.0 E_s$ ).

On the whole the fits are more than satisfactory, but a more refined analysis with a better topological description of the steel arrangement is in progress.

## 5. CONCLUDING REMARKS

1. The Microplane Model can be introduced easily into available F.E. codes.
2. The Microplane Model can describe in a relatively simple way a few complex aspects of concrete mechanics, such as multiaxial behavior, path-dependency and cracking.
3. The necessity of storing a few data on the history of each microplane is offset by the limited number of parameters required by the formulation of the microplane constitutive relations.
4. The Microplane Model may be easily improved in order to describe concrete behavior under variable loads (for instance, cyclic and fatigue loads).

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