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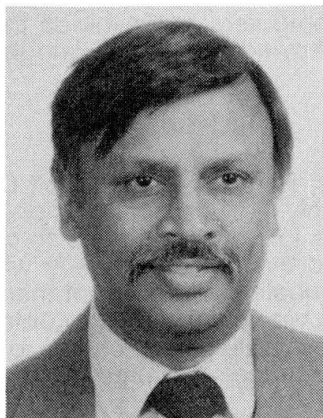
Finite Element Modelling for Analysis of Highly Skewed Bridges

Analyse par éléments finis de ponts à dalles fortement biaisées

Finite-Element-Analyse von stark gekrümmten Brücken

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SUMMARY

Background, reasons for use, and some results of the finite-element method of analysis are presented by the author. The main features of the study are skew bending effects; torsional moments, shears and support reactions; and large reactions caused by post-tensioning forces.

RÉSUMÉ

L'auteur présente la documentation, la justification ainsi que les résultats de la méthode par éléments finis appliquée; les principales caractéristiques de l'étude sont les effets des flexions biaisées, les moments de torsion, cisaillement et réactions d'appui, ainsi que les réactions significatives causées par la précontrainte.

ZUSAMMENFASSUNG

Der Hintergrund und die Ursachen und Ergebnisse der Finiten-Elemente-Methode werden präsentiert. Die wichtigsten Punkte der Untersuchung sind: schiefe Biegung, Torsionsmomente, Querkkräfte und Auflagerkräfte, sowie grosse Auflagerkräfte infolge Vorspannung.



1. INTRODUCTION

1.1 Background

The behavior of skewed bridges has been a pre-occupation of structural engineers for decades. Traditionally skewed bridges were analyzed and designed considering the longest length between the supports as the actual span length. Conventional wisdom assumed that such a conservative approach would result in a very safe design. The means or methods to do a more exact analysis did not exist until the advent of computers. The computers gave the engineers the opportunity to use more refined methods of analysis like the grid and space frame analysis to study the behavior of skewed bridges. The more exact methods of analysis like the "finite element method of analysis" were time consuming and needed large computers. The arrival of the newer, faster and larger computers has avoided this limitation and finally given engineers the means to analyze highly skewed box girder bridges within a reasonable budget and time frame.

1.2 Purpose and Objectives

During the last couple of years, Parsons Brinckerhoff Quade & Douglas, Inc. (PBQ&D) had to analyze and design several highly skewed post-tensioned box girder bridges. The skew angles on these bridges varied from a low of 43 degrees to a high of 70 degrees. The unavoidable high skew angles forced us to evaluate and verify whether the results of the finite element method of analysis would be substantially different than conventional methods or approximate methods. The objective of this whole effort was to obtain realistic behavior of these structures and to examine the local as well as global effects of external and internal loads on these bridges and assure the integrity of the structures.

2. SKEW BENDING

2.1 Cause of Skew Bending

In any bridge, the principal bending occurs along the shortest axis between the supports, which happens to be perpendicular to the axis of supports. In a non-skewed bridge the supports are along the transverse axis which is perpendicular to the longitudinal axis of the bridge. The principal bending of the girders and the whole structure occurs about the transverse axis parallel to the supports, along the longitudinal axis. This behavior has always dictated the direction of the girders in a concrete box girder bridge to be parallel to the longitudinal axis.

In a skewed bridge, the supports are not along the transverse axis. They are located along an axis skewed to the transverse axis of the bridge. The longitudinal axis of the bridge remains the same. The principal bending occurs about an axis parallel to supports along a new longitudinal bending axis which is perpendicular to the supports. This new longitudinal bending axis is skewed to the longitudinal axis of the bridge and is almost parallel to the shortest distance between the supports. The principal bending of the structure along this new longitudinal bending axis is called skew bending and its effects caused in a skewed bridge are referred to as skew bending effects. These effects must be carried by the girders or webs, by resolving them along the longitudinal and transverse axis of the structure.

2.2 Effects of Skew Bending

The principal bending moment along the new longitudinal bending axis is resolved to a bending moment along the longitudinal axis of the structure and a torsional moment about the longitudinal axis. The torsional moments caused by skew bending effects, in turn, cause an uneven distribution of horizontal and vertical shears at a section normal to the longitudinal axis of the bridge. This uneven distribution of horizontal and vertical shears, affects the location of maximum moments, produces large variations in support reactions and the post-tensioning forces which are normally negligible in a non-skewed bridge result in uplifts at certain supports.

Five highly skewed bridges included in the three bridge sites listed below were part of the Aviation Project in Tucson Arizona and are the subject of this paper.

3. METHODS OF ANALYSIS AND PROGRAMS EVALUATED

Three dimensional grid, plane frame with charts for increased shears and finite element method of analysis were evaluated for the reliability of results. Programs considered were CALTRAN Bridge Memo (Reference 1) for Designers 15-1, Cell4 Program (Reference 2), MDC STRUDL Program (Reference 3) and the 3-Dimensional Grid Analysis Program (Reference 4).

The CELL4 program was selected for use in view of the savings offered in modelling, computer time, ease of obtaining sectional forces, moments and automatic generation of equivalent loads for post-tensioning forces. In order to ascertain and validate the results of CELL4 program two identical models were tested using CELL4 and STRUDL programs (Figures 1 and 2). The differences between results of the two models though not exactly the same were within reasonable limits of accuracy required for design. The maximum stresses, moments and deflections were within 5% of each other. The STRUDL model in Figure 2 has a thicker pier diaphragm than the CELL4 model, which accounts for some of the differences in the reactions. The only results that have a wide variation are the two uplift reactions at the acute corners.

The closeness of results from both the programs gave us sufficient confidence to use CELL4 on all bridges except the Council/Toole Avenue bridge, which has a variable depth.

4. DISCUSSION OF MODELS

4.1 S.P.R.R. Bridge (Figures 3 and 4)

In order to increase the accuracy of the results in the finite element analysis, the aspect ratio of the elements was kept to 1 in critical areas and to 2 in non critical areas. The finite element model for this structure has 1428 joints and 1818 elements.

4.2 Council/Toole Avenue Bridges (Figures 6 and 7)

The finite element used is called "SIPQ", a quadrilateral curved element with four corner nodes and four midside nodes with five degrees of freedom (DOF). The aspect ratio varied from 1 to 2 depending on the geometry and the importance of certain critical areas. This model had a total of 1482 elements and 4119 joints.

4.3 Euclid/Park Structures (Figure 8)

PBQ&D was involved in the review of final design for these two structures. The design was based on a 3-dimensional grid analysis. CELL4 program was used for the design verification. The finite element model consisted of 1,116 joints; and 1,370 elements. The aspect ratio of elements was 1 in critical areas and 2 in less critical areas.

5. DISCUSSION OF RESULTS

5.1 DL Reactions (Figures 3, 6 and 8)

The reactions do not follow the normal pattern given by other conventional methods of analysis. The higher reactions seem to occur at supports that lie very close to the principal longitudinal bending axis, illustrating the skew bending effects. At certain locations, supports reactions vary by 100%.

5.2 Reactions Due to Post-Tensioning Force (Figures 5, 6, and 8)

Normally one does not expect any significant reactions due to internal equivalent loads due to post-tensioning force. The results show this is apparently not the case. We attribute this to the heavy torsional moments caused by the skew bending effect.

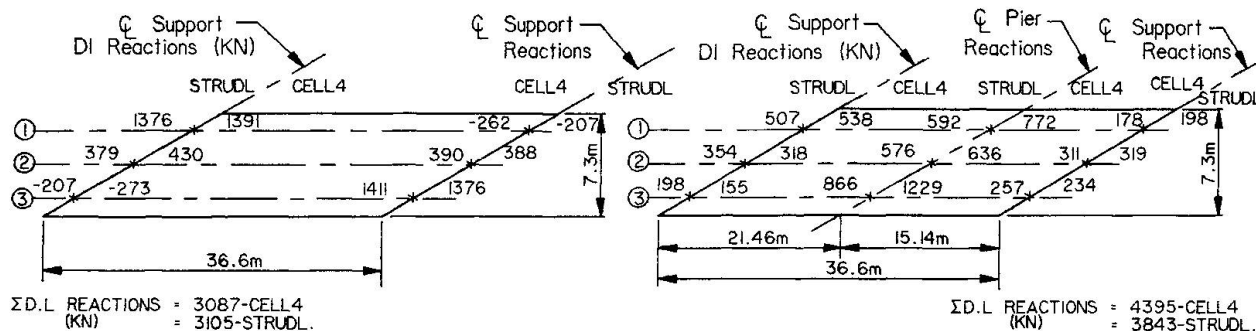


Fig. 1 Single Span TEST Model

Fig. 2 Two Span TEST Model

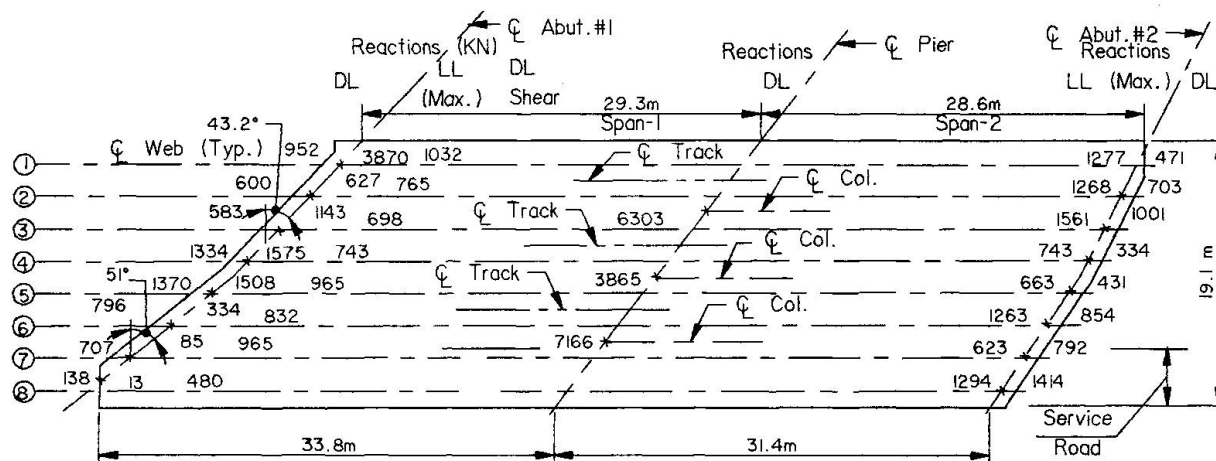


Fig. 3 Sectional Framing Plan With Support Reactions - SPRR Bridge

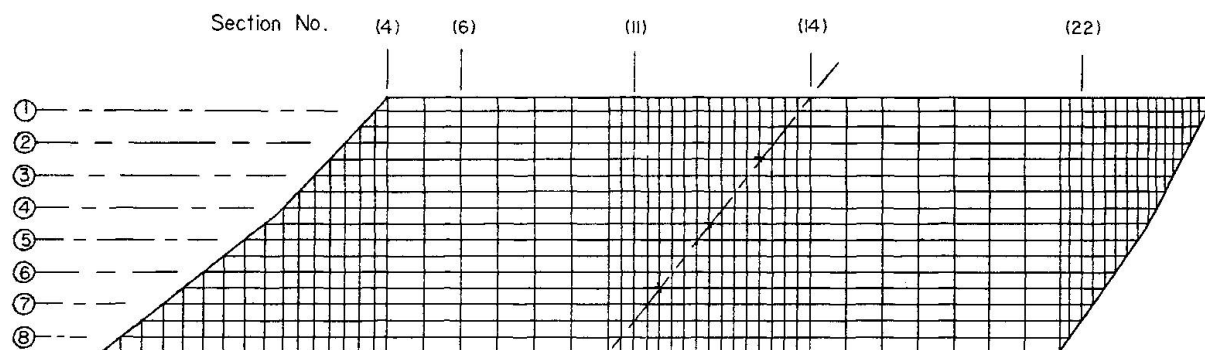


Fig. 4 Finite Element Discretization Mesh - SPRR Bridge

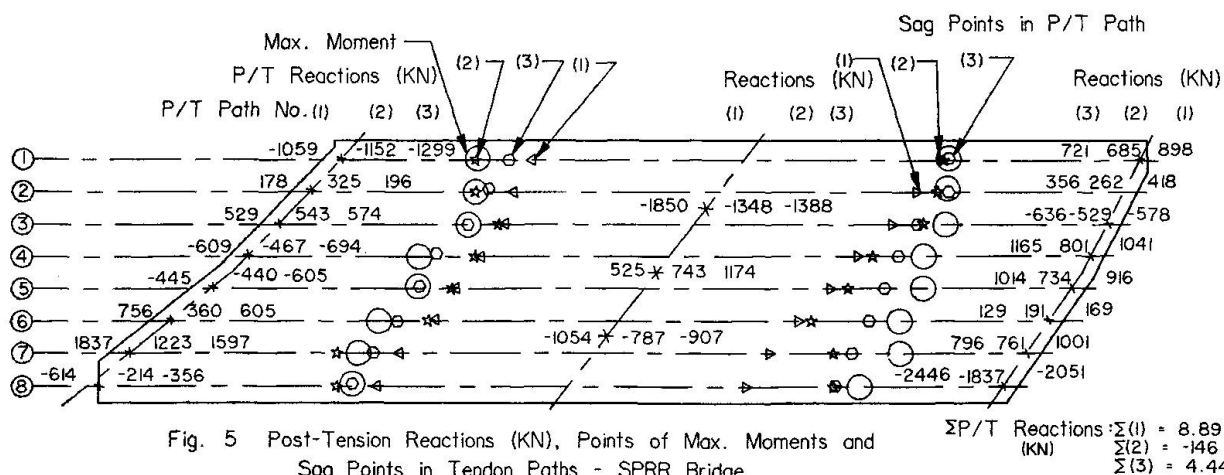
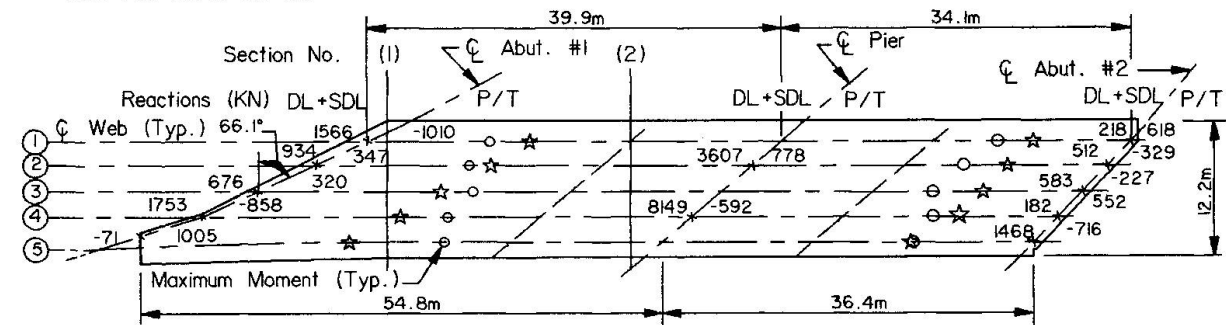


Fig. 5 Post-Tension Reactions (KN), Points of Max. Moments and Sag Points in Tendon Paths - SPRR Bridge

Table 1 Section Torsional Moment and Shear for SPRR Bridge

SECTION	4		6		11		14		22	
	Torsion	Shear	Torsion	Shear	Torsion	Shear	Torsion	Shear	Torsion	Shear
DL	-16592	-1112	-16357	672	319	3287	-12496	-4435	-10527	2393
DL+SDL	-25397	-1841	-23780	747	1588	4839	-20283	-6610	-13049	3363
DL+SDL+LL	-52597	-4537	-42440	-98	5109	7446	-16686	-7135	-9731	2953
P _j =17392 Path 1	23495	3443	29787	654	-14261	-7250	34247	9212	16934	-4497
P _j =15657 Path 2	30335	3269	35683	-476	-10893	-7602	17848	8558	20269	-4822
P _j =15657 Path 3	27086	2896	33311	-267	-13887	-7272	21300	8674	20265	-4492

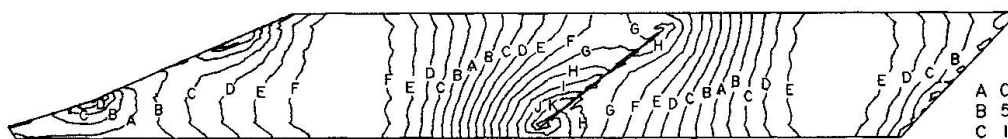
Units Are KN-m and KN



○ Points of Max. Moments

☆ Sag Points in P/T Path

Fig. 6 Sectional Framing Plan for EB Council/Toole Ave. Bridge (DL+SDL) And P/T Reactions.

 $\Sigma P/T$ Reaction = -112 (KN)


Units KPa

A 0.000E+00 H 3.767E+03
B 5.382E+02 I 4.306E+03
C 1.076E+03 J 4.844E+03
D 1.615E+03 K 5.382E+03
E 2.153E+03 L 5.920E+03
F 2.691E+03 M 6.459E+03
G 3.229E+03

Fig. 7 Stress Contours Top Slab (DL) - EB Council/Toole Ave. Bridge

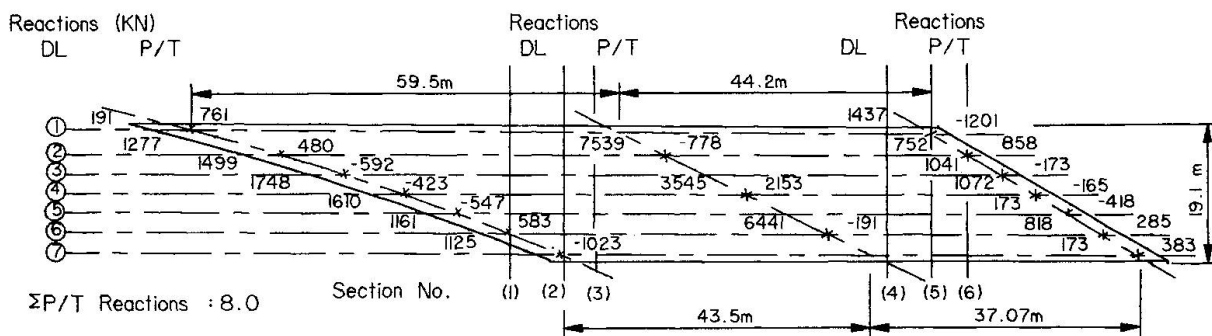


Fig. 8 Sectional Framing Plan With Support Reactions for Euclid/Park Ave. Bridge

Table 2 Torsional Moments for EB Council/Toole Ave. Bridge

SECTION	1	2
DL	-9994	-10285
DL+SDL	-12430	-12794
DL+SDL+LL	-15679	-16127
P/T	8421	6772

Units Are KN-m

Table 3 Section Torsional Moments for WB Euclid/Park Ave. Bridge

SECTION	1	2	3	4	5	6
	Torsion	Torsion	Torsion	Torsion	Torsion	Torsion
DL	14428	29599	29793	22793	22581	9607
DL+SDL	17208	35335	35539	26984	26790	11286
DL+SDL+LL	20877	42977	42893	33090	33082	13999
P/T	-15286	-31720	-23184	-18090	-25717	-11309

Units Are KN-m



5.3 Points of Maximum Moment (Figures 5 and 6)

In a two span highway bridge with no skews and simple supports at each end, the points of maximum moment would be located usually at 0.4 of the span from the exterior abutment support. The points of maximum moment in the SPRR bridge vary from 32 percent in girders 1 and 2 to 54 percent in girder 8 in span 1. This is reversed in span 2 and the points of maximum moment vary from 48 percent in girder 1 and 2 to 32 percent in girders 7 and 8.

The fluctuations in the location of points of maximum moments in the Council/Toole Avenue bridges are very large. The distance from the center of abutment to these points varies from 25 percent of span at girder 1 to 55 percent at girder 5. The support reactions are more than 100 percent different at certain locations. Note the heavy reactions at the top left hand corner and the bottom right hand corner supports. The line joining these points, almost coincides with one of pier supports. Though it is not perpendicular to the lines of support, it seems to act like the principal longitudinal axis of the structure. The reaction of this pier is more than twice the reaction at the other pier and greatly affects the design of the cantilever pier diaphragm.

5.4 Torsional Moments (Tables 1, 2 and 3)

The Tables 1, 2 and 3 illustrate the magnitude of torsional moments induced due to the skew bending effect. Torsion steel was required for all five bridges as per references (5), (6).

6. CONCLUSIONS AND RECOMMENDATIONS

Analysis and design of skewed bridges requires careful evaluation of skew-bending effects. Three dimensional grid analysis does not give a true account of the behavior of the skewed structure. The results are very sensitive to the torsional rigidity of the members assumed in a grid model and the comfort level for dependability of results is low.

Finite element models, with proper aspect ratios, provide the best means to evaluate the behavior of skewed bridges. Intermediate diaphragms do not have any noticeable effect on the behavior or the re-distribution of loads in a skewed box girder.

Support reactions are unpredictable by conventional procedures. Torsional moments induced by the skew need to be addressed in the design. The torsion capacity of the post-tensioned concrete box girders was not adequate to resist the total torsions induced on the structures.

The load balancing procedure does not offset all the skew bending effects in a post-tensioned concrete structure. The post-tensioning force required for load balancing is much higher than what is required for balancing the stresses.

It seems imperative that structures with skews greater than 25 degrees should be analyzed by an exact method of analysis like finite element methodology to assure the structural design integrity.

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