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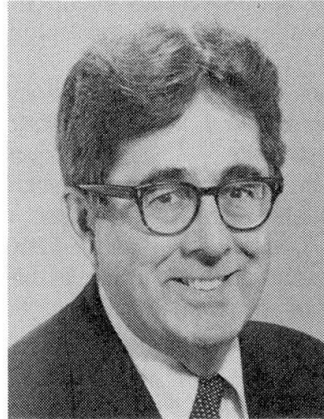
## Analysis of Structural Concrete Systems

Analyse des structures en béton

Berechnungsmethoden von Stahlbetontragwerken

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### SUMMARY

A review of the analysis of structural concrete systems is presented with respect to analytical models, linear elastic analysis, static and dynamic analysis, segmental analysis, and nonlinear analysis. Past history, present status, future trends, and the role of the computer are discussed. Several examples are presented to illustrate points made.

### RÉSUMÉ

Un compte rendu des méthodes d'analyse des structures en béton est présenté où l'on tient compte des modèles analytiques, de l'analyse linéaire élastique, de l'analyse statique et dynamique, de l'analyse concernant des éléments de construction discrétisés ainsi que de l'analyse non-linéaire. L'évolution passée, l'état actuel et les tendances d'avenir vues à travers le rôle de l'ordinateur sont successivement discutés. Plusieurs exemples illustrent les points abordés.

### ZUSAMMENFASSUNG

Diese Arbeit gibt einen Überblick über Berechnungsmethoden von Stahlbetontragwerken. Der Überblick umfasst analytische Modelle, linear elastische sowie statische und dynamische Berechnungsmethoden, Berechnungen von Tragwerken in Segmentbauweise, sowie nichtlineare Berechnungsmethoden. Die Entwicklungsgeschichte, gegenwärtige Fortschritte, zukünftige Tendenzen und der Beitrag des elektronischen Rechners werden vorgestellt. Mehrere Beispiele veranschaulichen die vorgetragenen Argumente.



## 1. INTRODUCTION

A structural concrete system may be defined as any structural system made up of structural concrete in combination with any amounts of reinforcing steel, prestressing steel, and rolled steel sections. Thus, structures constructed of plain concrete, reinforced concrete, prestressed concrete, partially prestressed concrete, composite concrete-steel sections, or any combination thereof, and built cast-in-place, precast, nonsegmentally, or segmentally, are all included in the above definition.

Structural concrete systems have been widely used for buildings, bridges, dams, tanks, cooling towers, nuclear containment vessels, offshore oil platforms, and many other structures. The ultimate goal of the designer of these structures is to produce a structure which satisfies the functional, structural, construction, esthetic, and cost requirements for the completed structure.

The overall design of these structures must insure their safety and serviceability throughout their construction and service life history under normal and abnormal loading and environmental conditions due to dead loads, live loads, construction, winds, earthquakes, floods, temperature variations, support movements, and other effects. In the past the analysis of these structures to determine internal forces, moments, and displacements have been based on simplified analytical models, generally assuming a linear, homogeneous, uncracked material. The designs for the required reinforcement or prestressing to resist these internal forces and moments have then generally been based on empirical procedures prescribed in codes or recommended practices for each particular type of structural concrete system.

Considerable research and effort have been exerted in recent years to develop improved methods of analysis and design of individual members and joints using strut and tie models (Schlaich [1], Marti [2]), truss models (Hsu [3]), truss models and compression field theory (Collins et al. [4,5]), and plasticity theory (Thurlimann [6]). Many other authors have contributed to the above research on individual members and joints. A smaller number of researchers have studied the analysis and design of total structural concrete systems composed of concrete, reinforcing steel, and prestressing tendons. Scordelis [7,8] has presented finite element models, methods of analysis, and computer programs for these systems, built nonsegmentally or segmentally, and loaded through their elastic, cracking, inelastic, and ultimate ranges.

With the increased complexity of structures and the greater computational power and speed available today, there is a need to develop a unified approach for the analysis and design of the entire spectrum of these structural concrete systems which takes advantage of the latest analytical and experimental research, materials, computers, and practical design and construction experience.

The purpose of this introductory report is to present the author's views on some considerations to achieve the above goal primarily with respect to structural analysis. Structural design and structural analysis will be discussed with respect to their past history, present status, future development, and the role of the computer. Several examples will be described to illustrate points to be made. It is hoped that this report will stimulate other authors to present their views on these subjects so as to have a useful discussion at the Stuttgart Colloquium.

## 2. STRUCTURAL DESIGN

The total structural design process involves many aspects, but for purposes of discussion it can be simplified to the following three stages: (1) conceptual design, (2) preliminary design, and (3) final design. Within these three stages the structural system selected must be considered from a global, regional and local analysis, design, and response standpoint to insure its satisfactory performance. The term "satisfactory performance" must be defined with respect to the criteria in existence at the time of the original design and specified for external loads and other actions, as well as displacements, cracking, etc. Changes in these criteria over the years due to any reason must be recognized and may require what has come to be known as "retrofitting" to insure satisfactory performance under the new criteria.



In all aspects of the structural design process, the designer must apply a good knowledge of structural behavior, mechanisms of failure, materials and analysis, and continuously ask himself the question "what if?" to insure that his structure has redundancy and that progressive failure can be avoided. Let us now consider the three design stages and the role of the individual designer, the computer, and education in each of them.

### 2.1 Conceptual Design

The conceptual design stage is perhaps the most important. However, it is not sufficient by itself without the second and third stages. The selection of the correct design criteria and structural system for a particular structure such as a building, bridge, dam, etc. which satisfies functional, esthetic, structural, and construction requirements is certainly dependent on the designer's creative ability, experience, and judgment. The question is: how does he develop these? The author believes that they are developed to a large extent by the detailed analyses and designs that the designer has previously performed and the subsequent observations that he has made of his and other creative designers' structures during construction and after completion over a long period of time.

As in art, music, or in the sciences, the creative ability of structural designers will vary greatly even after a long period of time. Only a few will become truly great designers. One of the biggest challenges to engineering educators is how computers can be brought into the educational process to nurture, rather than destroy, the development of this creative conceptual design ability of structural engineering students. Some educators believe interactive computer graphics will provide the key, where the student will sit in front of his user-friendly-oriented personal computer and communicate with it much as he would with a friendly teacher who has all the answers to all of the questions he has. Other computer advocates believe that expert systems development will provide the link between the expert, knowledgeable engineer, expert system, and the novice user. The author does not personally believe that these can replace the human personal link of a good teacher or contact with a good designer who can provide the enthusiasm and examples for the student to aspire to.

Other necessary components of structural engineering education are personal experience and observation of the behavior of real engineering materials and real structures in the laboratory and in the field, both during and after construction. In the opinion of the author, computer simulations can never replace this experience.

### 2.2 Preliminary Design

Once the conceptual design has been completed and one or more structural systems and materials have been selected, a preliminary analysis and design takes place to determine and compare the general dimensions, proportions, and costs of the structural systems selected. In the past, this was often done using approximate methods on simplified analytical models based on experience and simple calculations using statical control. Most of the great designers of the past and present were and are experts at doing this type of preliminary design by hand without the aid of a computer. In the context of today's availability of personal computers, every engineer with stronger and more powerful interactive computer graphics programs can study simplified or final analytical models of a structural system. The author believes that every designer in the future will use his personal computer as a powerful tool in every phase of the design, including the preliminary design stage.

### 2.3 Final Design

The final stage in the design process is the detailed global, regional, and local analysis and design of the selected structure using accurate analytical models to determine the displacements and internal forces and moments needed to design and proportion the structural elements and joints and to check the strength, stiffness, and stability of the structure as a whole. It is at this stage of final analysis and design where the computer has traditionally played its most dominant role since the beginning of its use in structural engineering. Computer programs which each year have increasing capabilities, speed, versatility, and ease of use are being used for final analysis, design, and even drafting. This trend is bound to continue.



While the computer can play an important role in the final design stage, it is imperative that experienced and qualified structural engineers be involved in the interpretation of the results using their judgment and knowledge of structural behavior to insure that the final design meets all of the design criteria for the particular structure throughout its construction and service load history and under increasing live loads, environmental loads, or other effects throughout its elastic, cracking, inelastic, and ultimate ranges.

Of particular importance in the final design is a final review of the design drawings by an experienced structural designer to insure that the structural system and the concrete dimensions and detailing for reinforcement and prestressing of the members and joints are reasonable and no obvious omissions have been made. It should always be remembered that, finally, only what goes on the drawings is what will be built.

### 3. STRUCTURAL ANALYSIS

#### 3.1 General Remarks

The objective of any structural analysis can be defined in general and simple terms as follows: *Given*: a defined analytical model of some real total structural system or portion thereof, subjected to a prescribed load and environmental history; *find*: the external reactions and displacements and the internal forces and deformations in the system at any location and time. The analytical model may be "global," such as the total structure; "regional," such as a particular rigid frame, beam, or column; or "local," such as a joint or anchorage.

Depending on the design criteria, importance, and complexity of the structure, as well as other factors, a variety of assumptions may be made for the analytical model and the type of analysis to be used. These may involve (1) linear or nonlinear analysis, (2) static or dynamic analysis, (3) time-dependent analysis, and (4) segmental analysis, as well as other analyses.

For many structures considered in present design codes, strength design is based on a linear elastic static analysis of the structural concrete system subjected to factored loads to determine internal forces and moments from which concrete dimensions and reinforcing and prestressing requirements are determined based on ultimate strength design procedures. Past practice often used hand calculations on simplified analytical models, design tables, charts, graphs, and other published aids to carry out this procedure. These have been and are rapidly being replaced by computer aids such as programs, spread sheets, etc. geared to the personal computer which many engineers now possess and utilize. This trend is bound to continue.

While many of the analyses and designs described above can be accomplished by following approximate or empirical procedures prescribed in codes or recommended practices, it is desirable to have refined analytical models and methods of analysis which can trace the structural response, as necessary, throughout the construction, service, and environmental load history and under increasing loads through their elastic, cracking, inelastic, and ultimate ranges.

A unified analytical approach and model is needed which, as necessary, can be used for linear or nonlinear analysis, static or dynamic analysis, time-dependent analysis, segmental analysis, and other analyses. Inherent in such an approach is the use of the computer as a powerful computational tool. Versatility in application should be an important feature of such an approach so that global, regional, or local analyses of a structural concrete systems can be performed as desired. One such unified approach is described below and several examples of its application are given later to stimulate useful discussion at the Stuttgart Colloquium.

#### 3.2 Analytical Models

For the purposes of structural analysis, a structural concrete system can be modelled analytically as a system of joints (nodal points) interconnected by discrete structural elements made up of concrete, reinforcing steel, and prestressing steel. In general, the total structure may be idealized by one or more of the following types of structural systems (Fig. 1):

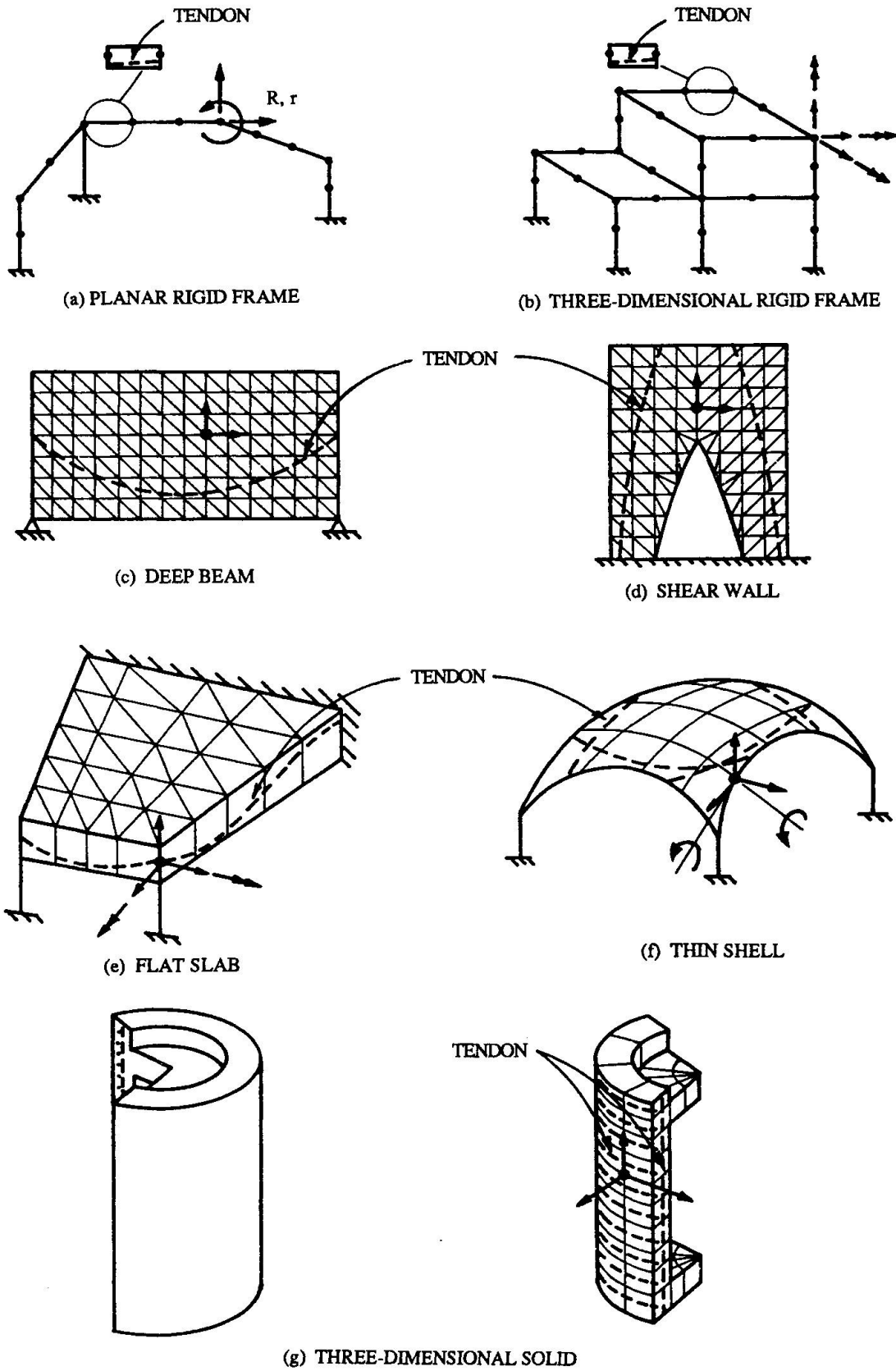


FIG. 1 - FINITE ELEMENT ANALYTICAL MODELS



1. Planar or three-dimensional rigid frames made up of one-dimensional (1D) elements;
2. Panels or slabs made up of two-dimensional (2D) triangular or quadrilateral flat finite elements;
3. Thin shells made up of two-dimensional (2D) flat or curved finite elements or axisymmetric thin shell elements; and
4. Solids made up of three-dimensional (3D) solid finite elements or axisymmetric solid elements.

If desired, a regional or local portion of the total structure also can be modelled as a refined subset of the total global structure, as desired. With such a unified analytical model, various types of analysis can be performed. The selection of an appropriate analytical model to represent an actual structure or portion thereof is a key step in the analysis process and is influenced by many factors. This is illustrated later in Example 1.

### 3.3 Linear Elastic Analysis

Numerous general purpose programs have been developed for personal microcomputers (PC) to perform linear elastic analyses for the above systems. For structural concrete systems the elements usually are assumed to be homogeneous uncracked gross concrete members. The rapid development of the PC during the past few years at decreasing cost has put at the personal command of the individual engineer, at his desk, and at any time, the computational power of the mainframe computers used in the 1970's. The available structural engineering PC software employs computer graphics in color, improved pre- and post-processing, computer-aided analysis and design, database systems, and the use of spreadsheets to simplify and speed up many of the tasks confronting the structural engineer.

A typical example of such a PC computer program available commercially in the U.S. today with which the author is familiar is the SAP90 program developed by Wilson and Habibullah [9]. Many others also exist. The SAP90 computer program consists of five modules which can be linked as desired:

1. Basic statics module for pre-processing and checking of input data, plus the static analysis of 2D and 3D structures made up of 1D frame elements;
2. Finite element module for the analysis of continuum structures made up of plate, membrane, shell, axisolid, or 3D finite elements;
3. Dynamics module for eigenvalue, Ritz, response spectrum, or time history dynamic analysis of structures;
4. Graphics module for plotting and display of undeformed and deformed shapes; mode shapes; axial force, shear, and moment diagrams; and stress contours; and
5. Design module for using the results found in 1. to 4. in post-processor modules to design members to satisfy existing steel or concrete design codes.

It is the author's conclusion that the linear static analysis of any structural system that can be modelled as shown in Fig. 1 can be accomplished today quickly and economically. However, useful and correct results can be obtained only by careful and proper interpretation of the input and output.

### 3.4 Static and Dynamic Analysis

Either or both a static or/and dynamic analysis may be required depending on the nature of the design criteria, loading, and environmental history. Static loadings, analytical procedures, and design criteria are usually well defined and understood. On the contrary, these are more difficult to define and understand for dynamic effects due to earthquakes. For example, the principles used in the development of seismic guideline provisions for bridges in the United States [10] (1981) were:

1. Small-to-moderate earthquakes should be resisted within the elastic range of the structural components without significant damage;
2. Exposure to shaking from large earthquakes should not cause collapse of all or part of the bridge; and
3. Realistic seismic ground motion intensities and forces should be used in the design procedure.

Item 3. is difficult to define for a particular geographical location and is highly dependent on soil conditions. Depending on the complexity and importance of the structure, the analysis may be based on an equivalent static force method, a dynamic response spectrum method, or a dynamic time history method. For structural concrete systems subjected to large earthquakes, a linear elastic dynamic analysis is often used. Dynamic analyses usually indicate internal forces and moments well above the linear elastic range. Concepts of "ductility demand" and "ductility factors" based on engineering judgment and research must be introduced to justify the safety of the structure based on these analytical results. Inherent in their justification is the use of properly detailed reinforcement and confinement of concrete in the joints and members.

Seismic analysis and design of structural concrete systems should be based on the latest knowledge and state of the art in this rapidly changing field of study. Analytical and experimental research, together with field observations of damage during earthquakes during the past thirty years, has resulted in a tremendous change and improvement in rational seismic analysis and design for this complex problem. Computer tools are now available to assist in this process, but the basic methods used in them should be clearly understood by the user. Example 2 illustrates some of the problems associated with seismic analysis and design in an actual structural failure during an earthquake.

### 3.5 Segmental Analysis

Another important consideration in the analysis of certain structural concrete systems such as segmentally erected prestressed concrete bridges, cable-stayed bridges, tall buildings, dams, and other structures is to be able to trace the response of the structure during each stage of its construction. For example, an analysis of a segmentally erected prestressed concrete bridge might use a plane frame model consisting of 1D elements made up of concrete and prestressing tendons. During construction, possible changes in structural configuration or loading can occur at any time and could include restraining and releasing of support conditions, installing and removing frame elements, stressing and removing prestressing tendons and form traveller loadings, and applying or removing external loads. All of these should be considered in a segmental analysis which often is performed considering the structure to be made up of linear elastic uncracked concrete elements and prestressing tendon elements.

Of particular importance in a segmental analysis is its ability to trace the time-dependent response through construction and under service loads, providing a record of stresses in the concrete and the prestressing tendons and displacements of the structure during this history. The time-dependent effects of creep, shrinkage, and aging of the concrete and relaxation of the prestressing steel should be included in the analysis, which is generally based on a step-forward integration in the time domain. Because of its complexity, special purpose computer programs for segmental analysis must be utilized. Such a computer program, SFRAME, has been developed by Ketchum [11]. Others are also available, generally on a proprietary basis. Care must be exercised in selecting the input parameters for creep and shrinkage of the concrete and relaxation of the prestressing steel because of the sensitivity of the final results to this selection. Example 3 demonstrates the implication of some of these factors in the deflection analysis of a 195-meter-span prestressed concrete segmental bridge.

### 3.6 Nonlinear Analysis

The capability for nonlinear material, geometric, and time-dependent analysis of reinforced and prestressed concrete systems of all types has been the goal of research at Berkeley for the past fifteen years. Methods of nonlinear analysis and computer programs, described in some detail in references 7 and 8, have been developed for 2D and 3D frames, slabs, panels, and thin shells

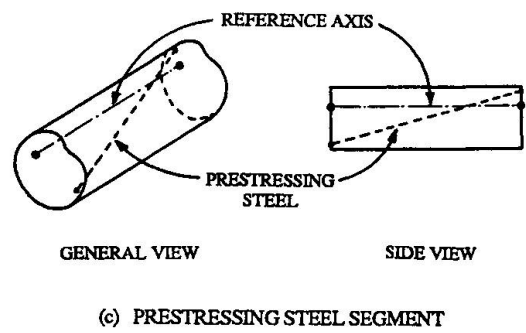
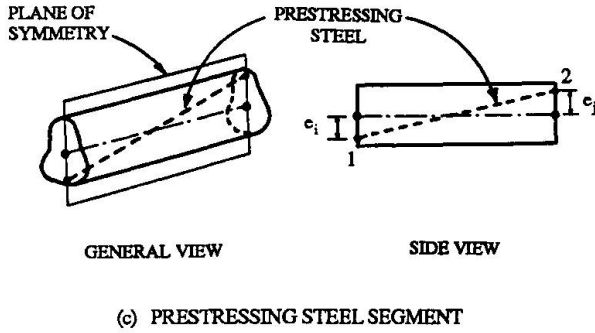
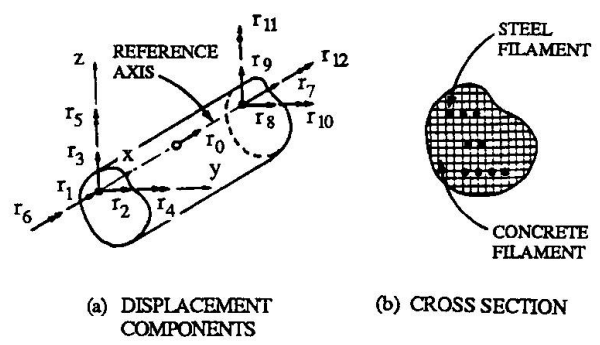
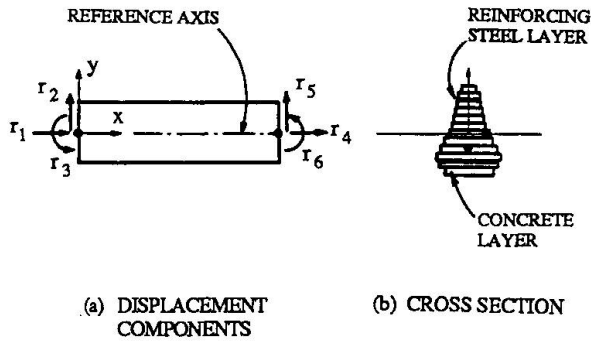


FIG. 2 - ONE DIMENSIONAL ELEMENT FOR PLANAR FRAMES

FIG. 3 - ONE DIMENSIONAL ELEMENT FOR 3D FRAMES

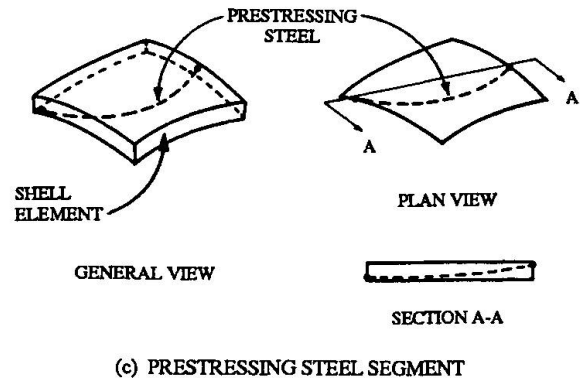
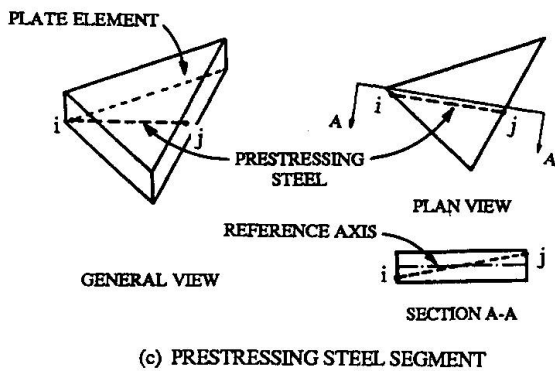
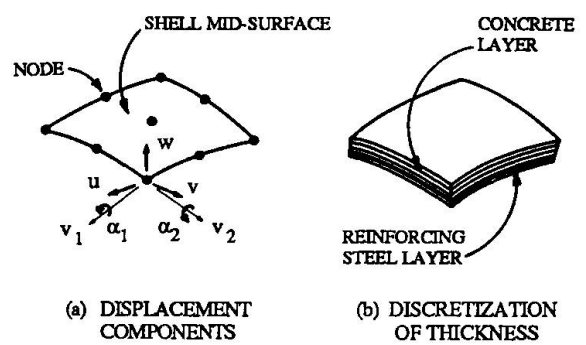
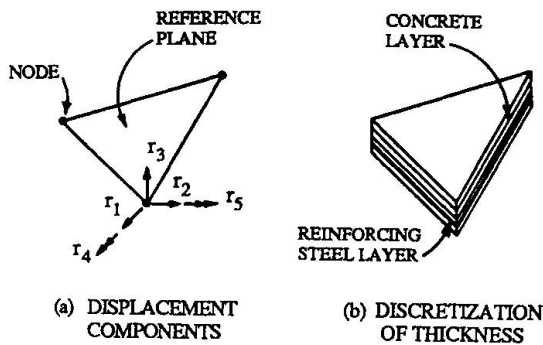


FIG. 4 - TWO DIMENSIONAL ELEMENT FOR PANELS AND SLABS

FIG. 5 - CURVED ISOPARAMETRIC ELEMENT FOR THIN SHELLS

(Fig. 1), as well as for straight and curved box girder bridges. Segmental analysis capabilities have been added recently for planar and three-dimensional frames.

Time-dependent effects due to load history; temperature history; creep, shrinkage, and aging of the concrete; and other prestress losses due to anchor slip, friction, and relaxation are all included in the analysis. Any combination of nonprestressed and prestressed reinforcement may be specified so that partial and fully prestressed cases may be analyzed through their elastic, cracking, inelastic, and ultimate ranges.

For 2D or 3D rigid frames (Figs. 2, 3) a series of 1D longitudinal finite elements with layered or filamented cross-sections of concrete and reinforcing steel are used to model the systems. For slabs, panels, and thin shells (Figs. 4, 5) an assembly of flat or curved 2D layered finite elements are used to model the concrete and multidirectional reinforcement systems. In all of the above, within each finite element the prestressing steel segment is assumed straight or curved and can be defined by vectors in space using its element boundary coordinates.

In the nonlinear analysis of all of the above structural systems, a unified finite element tangent stiffness formulation, coupled with a time-step integration scheme, is used to trace their quasistatic response up to ultimate failure. Nonlinear material stress-strain relationships are used for the concrete, reinforcing steel, and prestressing steel throughout the nonlinear finite element analysis to determine the displacements; crack patterns; and strains and stresses in the concrete and reinforcing and prestressing steel at any time up to failure, under any load or environmental history.

With the increasing computational power and speed of computers every year, these methods of analysis and computer programs can be used for global analysis, where deemed necessary and applicable, in an economical way. Example 4, given later, illustrates one such example involving a nonlinear material, geometric, and time-dependent analysis of a reinforced concrete shell.

While the analytical models for 1D elements accurately model the nonlinear longitudinal stress response due to axial force and bending moments (Figs. 2, 3), additional research needs to be done to represent the nonlinear response due to shear and torsion more accurately without going to 2D or 3D solid elements (Figs. 1c to 1g). These solid elements with reinforcement and prestressing can also be used for the local analysis of joints or portions of members from planar or 3D frames loaded to failure.

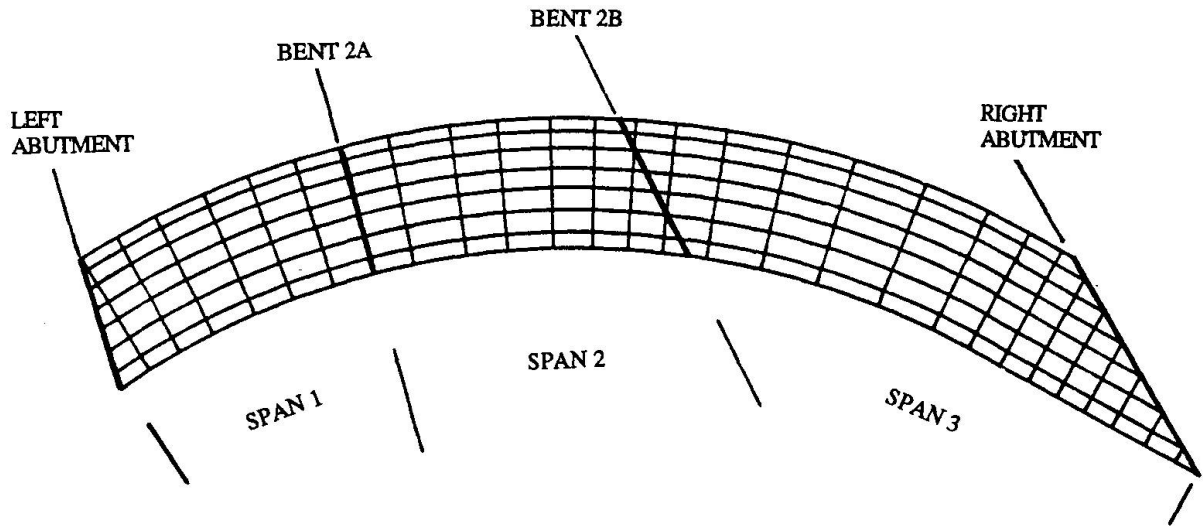
At the basic research level, work must continue on how best to model analytically in detail, especially at the "local level," the nonlinear effects up to failure of the structural concrete system made up of a combination of concrete, reinforcing steel and prestressing steel under combined stress, creep, shrinkage, cracking, aggregate interlock, dowel action, bond, tension stiffening, stress relaxation, cyclic loading, and dynamic loading. These are not simple problems to answer, but the best available analytical tools must be developed and utilized whenever needed and necessary.

#### 4. EXAMPLES

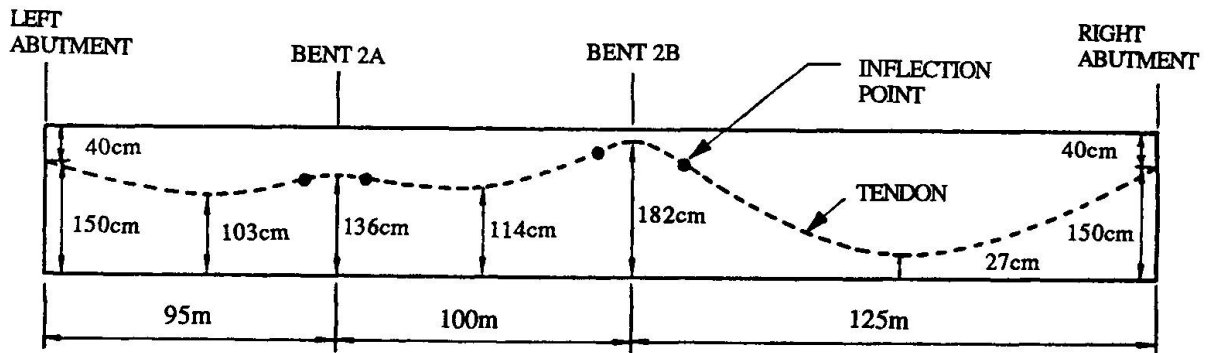
Four examples are presented to illustrate some of the capabilities and problems in the use of the methods of analysis which have been described. The structures considered are: (1) a multicell, multispan, curved, prestressed concrete bridge on skew supports, (2) a multispan double deck viaduct structure, (3) a 195-meter-span prestressed concrete segmental bridge, and (4) a gabled hyperbolic paraboloid (HP) shell roof.

##### 4.1 Example 1 - Highway Bridge, San Juan, Puerto Rico

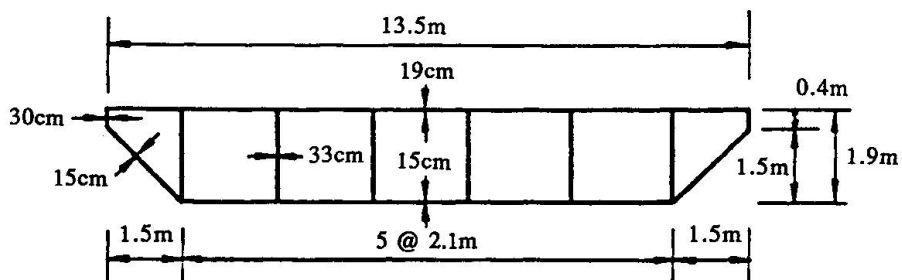
A three-span, multicell, curved, prestressed box girder bridge (Fig. 6), proposed as part of the highway connection to the airport in San Juan is chosen to illustrate the possible selection of analytical models for a linear elastic analysis. Complete details of this study have been presented by Scordelis et al. [12]. The general plan, typical panel tendon profile, and cross-sectional dimensions of the constant depth bridge are shown in Fig. 6.



(a) PLAN OF BRIDGE AND MESH LAYOUT



(b) CENTERLINE SPAN DIMENSIONS AND PANAL TENDON PROFILE FOR VERTICAL WEBS



(c) CROSS-SECTIONAL DIMENSIONS

FIG. 6 - EXAMPLE 1 - HIGHWAY BRIDGE, SAN JUAN, PUERTO RICO

Several analytical models of increasing complexity were possible:

1. A planar grid, hereinafter termed the "SAP4 model," consisting of a single line of 1D beam elements, with six degrees of freedom at each end, along the longitudinal axis of the total bridge cross-section, rigidly connected and supported by transverse 1D elements at the supports oriented in the direction of the skew end abutments and interior support bents;
2. A planar grid consisting of six lines of 1D beam elements with six degrees of freedom at each end along the six longitudinal axes of the girder webs, supported by transverse 1D elements at the skew end supports and interior support bents, and also connected transversely at specified intervals in each span by 1D elements simulating the transverse bending stiffness of the bridge or diaphragms between longitudinal girders;
3. An orthotropic slab having the plan geometry and support conditions of the actual bridge, with appropriate bending and torsional stiffnesses assigned to the slab in the longitudinal and transverse direction to simulate the actual bridge;
4. A finite element model, hereinafter termed the "CELL4 model," in which the top and bottom slabs, the longitudinal webs, and the transverse diaphragms or supports are modelled with 2D finite elements incorporating both in-plane membrane stiffnesses and plate bending stiffnesses.

Typical of many modern designs today, the selection of the analytical model to use is highly dependent on the computer programs available and their pre- and post-processing and automatic generation capabilities for input of nodal point coordinates, element properties, dead loads, live loads, and prestressing loads, and for output of displacements, internal forces, and moments. Based on these criteria and the availability of a general purpose program for the SAP4 model and a special purpose program for the CELL4 model, only these models were chosen for the analysis. The same mesh layout (Fig. 6a) was used for both models. For the CELL4 model, automatic generation of dead loads and equivalent prestress loads was performed internally in CELL4 from a minimum of necessary of input data. Such a capability was not available for the SAP4 model, and extensive calculations had to be carried out externally for the prestressing load input data into SAP4.

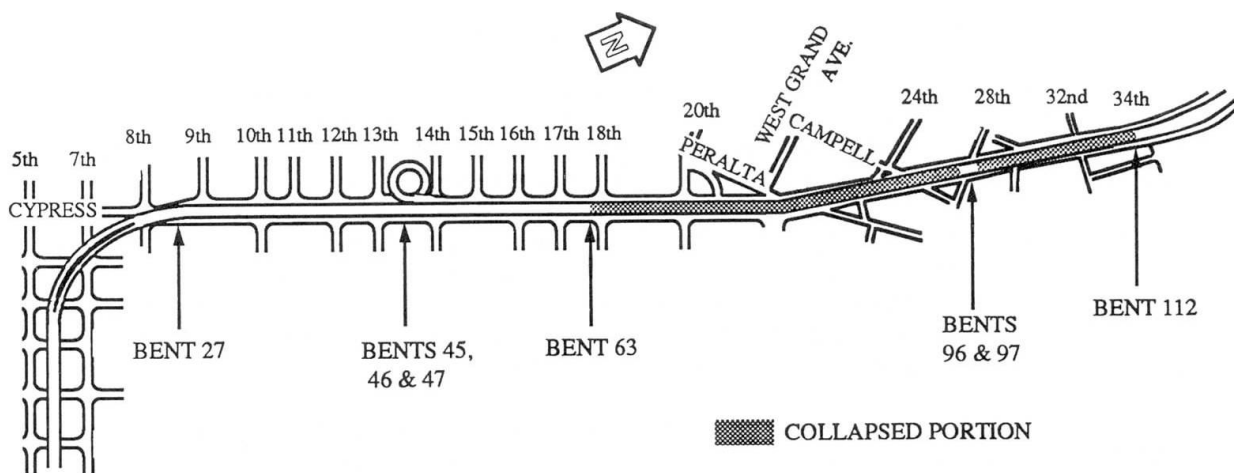
The SAP4 beam model had the advantage of simplicity of output since, as a single longitudinal member, it yielded results for the longitudinal variation of centerline displacements and total sectional forces and moments. However, it had the disadvantage that it did not give any information on the transverse distribution of these quantities. On the contrary, the CELL4 finite element model gave a voluminous output of stresses in all elements and displacements throughout the width and length of the model. For use in interpretation and design, automatic stress integration routines in CELL4 provided results for internal forces and moments in six prescribed individual longitudinal girders and for the total section. Time required for external calculation of these quantities from element stresses would have been prohibitive. Thus, the CELL4 model gave a complete output of the distribution of individual girder forces, moments, and displacements over the entire structure with about the same external effort used for the SAP4 model.

This illustrates the importance of available software and their capabilities in selecting the analytical model to be used.

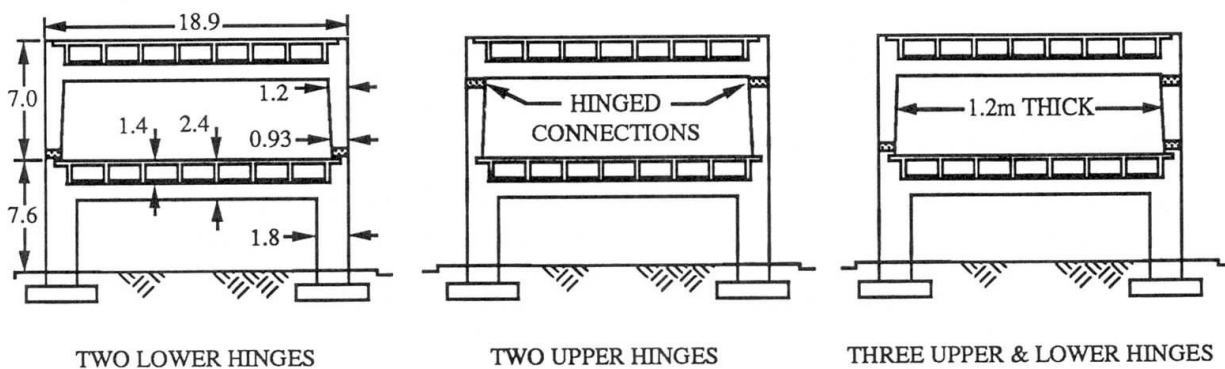
#### 4.2 Example 2 - I880 - Cypress Viaduct, Oakland, California

The Cypress Street Viaduct (Fig. 7) was a 2.5 km long concrete double deck freeway carrying four lanes of traffic on each deck. It consisted of a series of eighty-three two-story bents extending from bent 29 through bent 111 (Fig. 7a), monolithically connected to and supporting seven-cell box girder bridge spans on two levels. Several transition bents also existed at each end. Typical transverse dimensions are shown in Fig. 7b. Typical longitudinal spans varied from 21 to 24 meters. A variety of bent types and hinge combinations were utilized in the upper story columns, with the three most common shown in Fig. 7b.

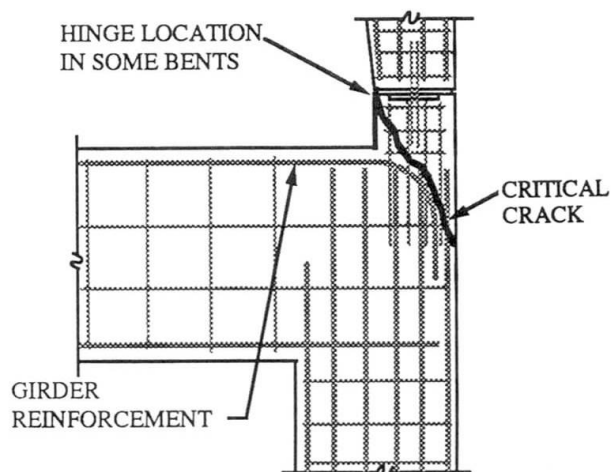
Preliminary design of the structure by CALTRANS (California Department of Transportation) began in 1949 and construction took place between 1955 and 1957. During the magnitude 7.1 Loma Prieta



a) PLAN OF CYPRESS STRUCTURE WITH DAMAGED BENTS



b) TYPES OF BENTS WITH TYPICAL BENT DIMENSIONS (IN METERS)



c) DETAIL OF FAILED CONNECTION



d) PHOTO OF COLLAPSED STRUCTURE

FIG. 7 - EXAMPLE 2 - I 880 - CYPRESS VIADUCT - OAKLAND, CALIFORNIA

earthquake of October 17, 1989, a large portion of the structure collapsed, extending from bent 63 through bent 112, with only bents 96 and 97 remaining standing (Fig. 7). Forty-two persons were killed and 108 injured in the collapse.

The analysis and design of the viaduct between 1949 and 1955 were performed without the benefit of the knowledge gained in seismic design during the past thirty years and without the aid of computers. It is believed that the structural system was designed with many hinges to simplify its analysis and interpretation of behavior, as well as provide for movements due to creep, shrinkage temperature and prestressing, and future construction additions. This, coupled with the minimum seismic design criteria and forces existing in the early 1950's made the structure highly susceptible to damage or collapse under a strong earthquake.

After the 1971 San Fernando earthquake, CALTRANS's seismic design criteria were upgraded for new construction to include more realistic forces, ground motions, and dynamic analyses using computers. However, it was decided as a first priority that the limited available funds for retrofitting only be used to install longitudinal restrainers at transverse expansion joints existing in every third span of the box girder bridge system to prevent failures of the types experienced during the 1971 San Fernando earthquake. Unfortunately, no detailed global, regional, and local analyses of the entire structural system were made, at that time or up until the failure of the viaduct on October 17, 1989, to determine if other weaknesses existed.

The typical failure mode for most of the bents, between bents 63 and 112, is shown in Figs. 7c, d. The heavy lateral shears generated by the earthquake ground motions, which had to be transferred through the hinge joint together with the vertical gravity load of the structure above, caused a critical diagonal crack below the joint and along a plane of weakness above the bent-down girder reinforcement (Fig. 7c). The vertical gravity and lateral seismic forces then pushed the upper column down and away from the joint, causing the collapse of the upper deck onto the lower deck. A lack of sufficient transverse confinement reinforcement in the joint region, as is recommended today, was a key factor in the failure.

Subsequent to the earthquake a number of analytical and experimental investigations have been made which predict the failure as it occurred. These have included static and dynamic analyses at a global, regional, and local level, as well as ambient and forced vibration tests and quasistatic cyclic load tests on an undamaged three-bent (45, 46, 47) portion of the viaduct. A decision to demolish and remove the entire viaduct was made, and this was completed in January, 1990.

This example illustrates the importance of using the latest research findings and the best analytical tools in the design of new structures and the retrofitting of old structures.

#### 4.3 Example 3 - Parrotts Ferry Bridge, California

This example is used to illustrate the importance of basic input parameters and refined analytical models in time-dependent computer analyses of prestressed concrete segmental bridges. The Parrotts Ferry Bridge (Fig. 8) is a segmentally erected, three-span, post-tensioned, single-cell box girder bridge with a center span of 195 meters supported by 68-meter-high bridge piers. The bridge was constructed of cast-in-place lightweight concrete segments using the balanced cantilever method and was completed in 1979.

During design, prior to construction, analyses were performed including the time-dependent effects of creep and shrinkage using simplified analytical models. During and after construction, deflections have been monitored over the past ten years. Both the magnitude and the current rate of the measured midspan deflection have been larger than analytically predicted (Figs. 8c, d). Measured deflections below the predicted closure elevation were 10 cm at closure in March, 1979, 38 cm in September, 1979, and 66 cm in July, 1988. While the bridge is not in any structural distress, its present appearance with a pronounced midspan sag (Figs. 8c, d) is disconcerting to riders and viewers.

Recent detailed segmental analyses of the bridge using the computer program SFRAME [11], with refined analytical models for the structure and for creep, together with analyses and tests of the

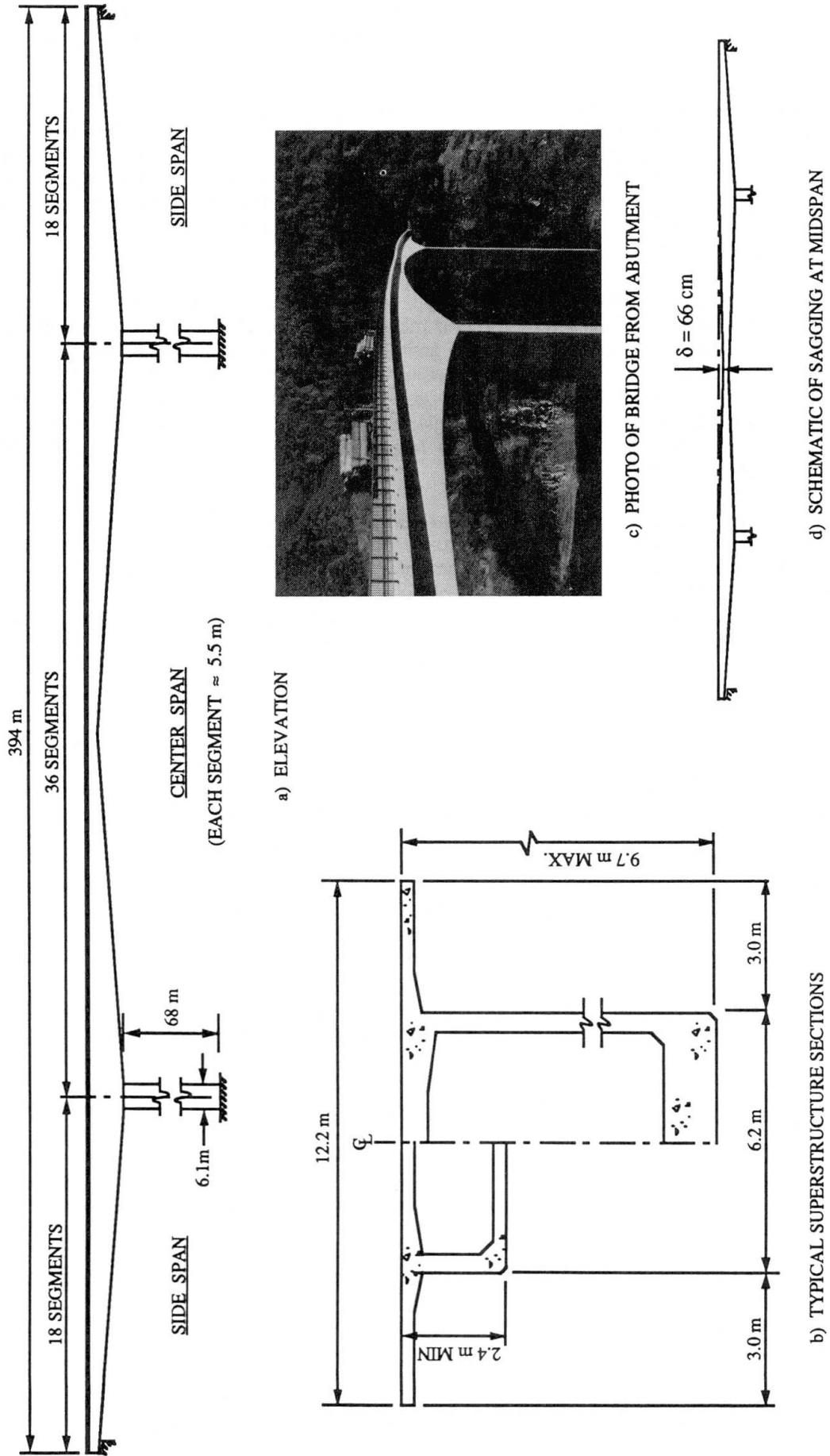
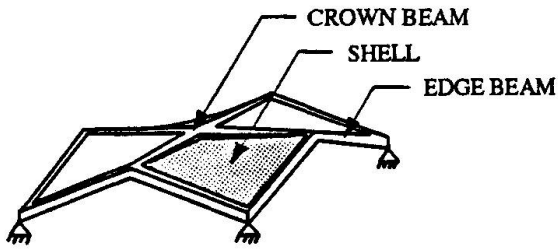
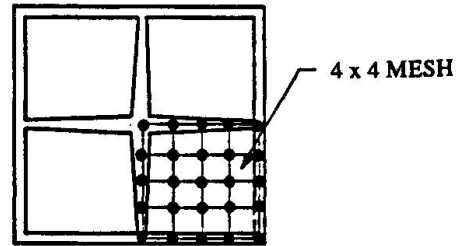


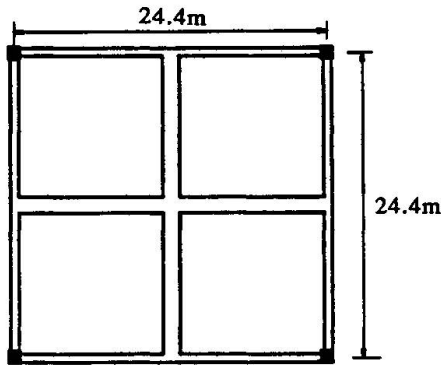
FIG. 8 - EXAMPLE 3 - PARROTTS FERRY BRIDGE, CALIFORNIA



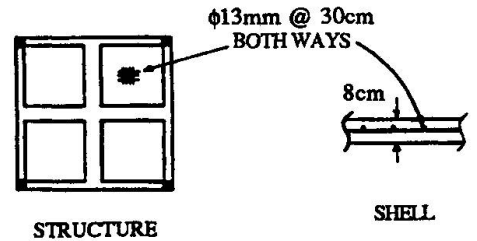
(a) GENERAL VIEW



(b) FINITE ELEMENT MESH

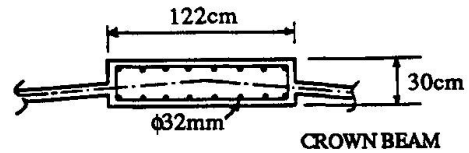


(c) PLAN DIMENSIONS

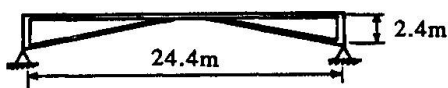


STRUCTURE

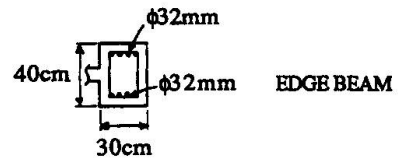
SHELL



CROWN BEAM

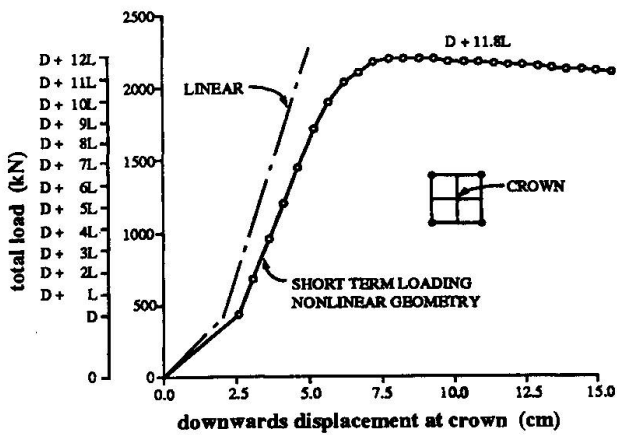


(d) ELEVATION DIMENSIONS

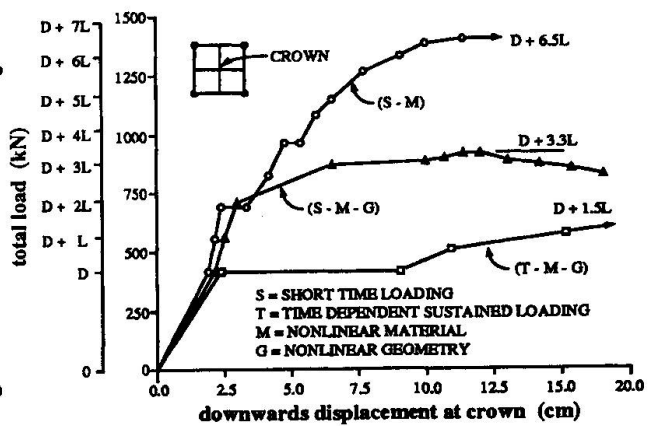


EDGE BEAM

(e) TYPICAL REINFORCEMENT LAYOUT



(f) LOAD VS. DISPLACEMENT FOR UNCRACKED MODEL



(g) LOAD VS. DISPLACEMENT FOR CRACKED RC MODEL

FIG. 9 - EXAMPLE 4 - GABLED HYPERBOLIC PARABOLOID (HP) SHELL ROOF



material properties of the lightweight concrete actually in the bridge, indicate that the probable causes of the higher deflections were that the actual modulus of elasticity is lower than that used in the original design, and the actual creep factor is higher than that used in the original design. This demonstrates that input material properties must be determined carefully and that the sensitivity of final results to variations in input parameters should be considered in analyses of this type to establish upper and lower bounds, if necessary. With the greater computational power and available computer analysis programs, this can be done today efficiently and economically using refined analytical models.

#### 4.4 Example 4 - Gabled Hyperbolic Paraboloid (HP) Shell Roof

Many shell roofs of this type (Fig. 9a) were designed in the 1960's using only simple membrane theory, and several of them suffered large increasing deflections with time. In one such shell a collapse occurred ten years after it had been completed.

In this example selected analytical results are presented from a detailed study by Chan [13] of a gabled HP shell with crown and edge beams, having the dimensions and reinforcement shown in Figs. 9c, d, e, to demonstrate the dramatic decrease in the ultimate load which can occur when nonlinear material, geometric, and time-dependent effects are included in the analysis. The structure was designed, based upon membrane theory, for dead load (D) plus a live load (L) of  $958 \text{ N/m}^2$ .

A number of nonlinear finite element analyses were then carried out using the mesh layout in Fig. 9b and the computer program NASHL [13] in which the reinforced concrete shell and beam elements were modelled as shown in Figs. 1f, 5. All the nonlinear analyses were performed by first applying the total dead load (D) of the shell plus beams and then adding multiples of the  $958 \text{ N/m}^2$  live load (L) until failure occurred. Only the results for a case using an oversized crown beam (Fig. 9e) are presented here in Figs. 9f, g to show the undesirable effects of doing so.

Crown displacement results shown in Fig. 9f from the analysis of a homogeneous uncracked concrete model without (linear) and with nonlinear geometry effects included indicate that the ultimate load in the latter case is  $D+11.8L$ . The response is almost linear up to a load of  $D+10.0L$  after which the continuous change in geometry causes local instability in the vicinity of the crown where the curvatures of the shell are smallest and the axial force is the largest.

Figure 9g shows the results of three analyses for a reinforced concrete analytical model of the HP gable shell which included nonlinear material effects such as cracking, etc. First, for a short time, loading with only nonlinear materials, the ultimate load was  $D+6.5L$ . Second, for a short-time loading with nonlinear material and geometry, the ultimate load drops to  $D+3.3L$ . Finally, for a case in which the dead load is first applied and then left on for five months, during which time creep and shrinkage take place, resulting in increased deflections and redistribution of stresses, and then increments of live load are applied on the structure with nonlinear material and geometry, the ultimate load drops to  $D+1.5L$ . This illustrates the large decrease in the ultimate load that can occur in shells of this type as each nonlinearity is included in the analysis.

## 5. SUMMARY AND CONCLUSIONS

A review of the analysis of structural concrete systems has been presented with respect to analytical models, linear elastic analysis, static and dynamic analysis, segmental analysis, and nonlinear analysis. Past history, present status, and the role of the computer in analysis have been discussed. It can be concluded that tremendous progress has been made during the past thirty years in our analytical capability to predict the response of structural concrete systems. This is due to the development and understanding of more accurate and realistic analytical models and the increasing computational power and speed of computers to perform the necessary analyses accurately, economically, and efficiently. This trend is bound to continue in the future. Of course, the ultimate goal of this improvement in analytical capability is its intelligent implementation in design so as to produce better, safer, and more economical structures, regardless of their complexity. This should be a continuing, unending, and exciting challenge for all structural engineers.

## 6. ACKNOWLEDGEMENTS

The assistance of Jefferey Soulages, Graduate Research Assistant, who reviewed the text and prepared the figures for this paper, is gratefully acknowledged.

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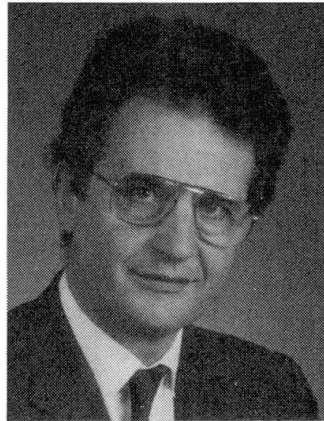
## Concrete Columns

### Colonnes en béton

### Betonstützen

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#### **SUMMARY**

Compression forces and second order effects influence the behaviour of concrete columns and walls in terms of bearing and ductility capacity. An overview is given on the main aspects concerning phenomena, design, analysis and safety concepts.

#### **RÉSUMÉ**

Les efforts de compression et les effets du second ordre affectent le comportement de colonnes et murs en béton, en termes de résistance et de ductilité. Une vue d'ensemble est donnée sur les aspects principaux concernant les phénomènes, le projet, l'analyse et les concepts de sécurité.

#### **ZUSAMMENFASSUNG**

Druckkräfte und die Effekte aus der Theorie Zweiter Ordnung beeinflussen das Verhalten von Betonstützen und -wänden hinsichtlich ihrer Trag- und Verformungsfähigkeit. Es wird ein Überblick über die wichtigsten Gesichtspunkte bezüglich der auftretenden Phänomene, der Bemessung und Berechnung sowie der Sicherheitskonzepte gegeben.



## 1. INTRODUCTION

The main features of columns are that they sustain axial loads and they may be slender.

Therefrom, the capacity of such structures is affected also by additional load eccentricities produced by their deformation, and it is reduced, compared to the capacity of the cross-section alone. The phenomenon is called geometric non linearity, second order effects, stability of deformation, buckling.

Concrete has been viewed in its beginnings as giving bulky structures, not subject to slenderness problems. However, the increase of concrete strength, coupled with the needs of saving material reducing selfweight and gaining free space, have given rise to columns walls and piers that may show sensible second order effects. This because the stiffness does not grow up as much as the strength, unfortunately.

The above problems pertain to "long" columns, thus they are concerned namely with "B-regions" (according to the definition given in [18]) subject to bending and compression. Of course "D-regions" problems may appear in columns, namely at the ends, or in splice-joints of prefabricated units, or in "short" columns that might be acted by high shear and compression. However these are not so typical of columns, and will not be treated here as such.

The observations that follow, on various aspects involved in columns design, reflect the writer's views; thus, reference is made mostly on his previous papers, intended only as a partial justification of the statements.

## 2. COLUMNS CAPACITY

The parameters controlling the deformation of a column, given the loading and the geometry, are the shape of concrete stress-strain curve, its tensile strength, the stiffness of reinforcing steels and possibly the time. The first is governed by the initial  $E_c$  modulus, which may increase with the strength  $f_c$  but not proportionally; the same happens to the second ( $f_{ct}$ ), which seems to have an upper limit and it is not fully reliable; the third ( $E_s$ ) is independent of steel strength, up to yield stress  $f_y$ ; long term strains roughly depend on  $E_c$ , too.

Moreover, the stiffness varies along the member following the local state of stress, thus depending partly on the second order effects themselves. This means that mechanical (material) nonlinearity and geometric nonlinearity interact in modifying the structural behavior.

The service limit states are normally not sensibly affected by second order effects, being the materials still in the linear range over the whole column.

Instead, under ultimate conditions, large deformations appear in critical regions, and the stiffness decreases also along the structure, due to lowering  $E_c$  in compression and cracking in tension. Considerable deformations and second order moments appear. The bearing capacity of the column is reduced. In fact, addition of bending moments is in any case unfavorable for the resistance of a concrete section.

Another relevant structural requirement is the ductility capacity, i.e. the ability of undergoing given "plastic" deformations with constant lateral force response, while dissipating energy. Also ductility also is influenced by the interacting nonlinearities. Normal forces reduce the elongation of tensile reinforcement available before concrete crushing, and second order effects are much increased by plastic rotations, even in columns non slender per se: this results in a reduction of moment redistribution capacity as well as in a rapid drop in the Force-Deflection curve of the column (fig. 1) and of the corresponding energy absorption [8,13].

Therefore, in design of seismic structures, ductility is preferably demanded to non compressed members, such as beams or bracing walls.

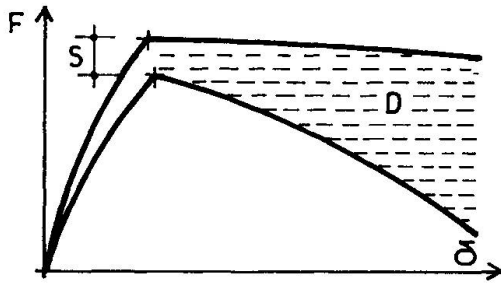


Fig. 1  
Reduction due to 2nd order effects  
S : Strength  
D : Energy dissipation

### 3. PRESTRESSING

Prestressing may be a means of improving the bearing capacity of axially loaded slender columns [9]. In fact, it has an influence on the second order effects by modifying the stiffness of the cross-sections. When applied, prestressing is normally symmetric in the cross section, if bending has not a preferred sign. The influence is twofold: on one hand, the increase of the average compressive stress tends to reduce the stiffness; on the other hand, it tends to improve it by delaying formation and propagation of cracks, as in the example in fig. 2. If the external axial force is very low, centric prestressing may even improve the cross-section strength.

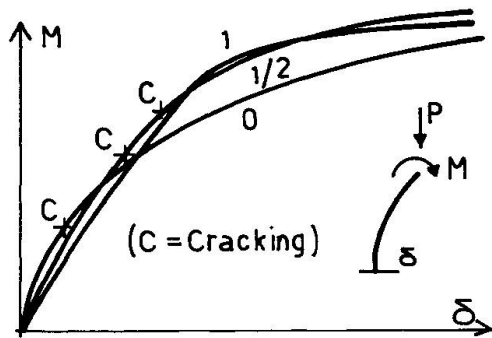


Fig. 2  
Behavior of slender prestressed elements  
1 : fully  
1/2 : partially  
0 : not

But, contrary to an external axial force, internal prestressing does not produce itself second order effects, as it follows the structure's displacements. Also the ductility reduction is less than that of an external axial force [13]. It should be well calibrated to be helpful (the more sensibly the more the column is slender). Partial prestressing is advisable and bonded tendons would also contribute to flexural strength.

Economy governs the "if and how much" to prestress. In the balance, the advantage may come of pretensioning precast columns for resisting transient loading conditions.

Unbonded external tendons are not fitted for the purpose, as they do give second order effects, unless they are multilinked to the structure; the effects are theoretically less than those of a real external force, because eccentric though symmetric tendons strain favorably and oppose ends rotations.

### 4. RESTRAINTS

The effective restraint acted on by the supports of the columns must be known, both in statically determinate and indeterminate cases. In fact, second order effects render all structures statically indeterminate.

The actual stiffness of the restraint, which may vary according to the actual forces applied, determines in particular the "effective length"  $l_0$  between virtual points of contraflexure. Even in the simplest case of an isolated cantilever clamped at the base (as in fig. 1) account should be made in principle both for rotation of foundation, when evaluating  $l_0$ , and for second order moment, when verifying the foundation itself.



Often, columns are part of frames where the beams form the restraint. In principle, restraint parameters should be evaluated at a state of stress corresponding to the u.l.s. of the columns, which comes out automatically in a full nonlinear analysis of the frame, but requires some judgment if the column is taken out of its frame and analyzed as "isolated".

A major point is whether the frame's joints may or may not displace (sway vs. non sway frames). This distinction refers to the presence of a rigid structure bracing the whole frame, i.e. providing a fixed restraint to lateral displacements of all joints (fig. 3).

A frame, whose joints do not displace under the given loading condition, but is not itself positively braced, cannot be considered as non sway if second order effects are expected. Again, either a full nonlinear analysis is performed, including forces exciting lateral displacements, or criteria for evaluating the overall slenderness are needed.

The overall analysis of a sway frame is sufficient if the columns of every storey are regular, i.e. uniform in dimensions and loadings. Otherwise, it must be integrated by the check of the most slender columns in their isolated buckling mode.

Columns much more slender than the nearest become soft elements refusing increments of axial or lateral loadings from the structure, thus altering the force pattern in the beams (fig. 3). It is matter of proper design to avoid such situations. Slender sway frames would not be recommended for buildings anyway.

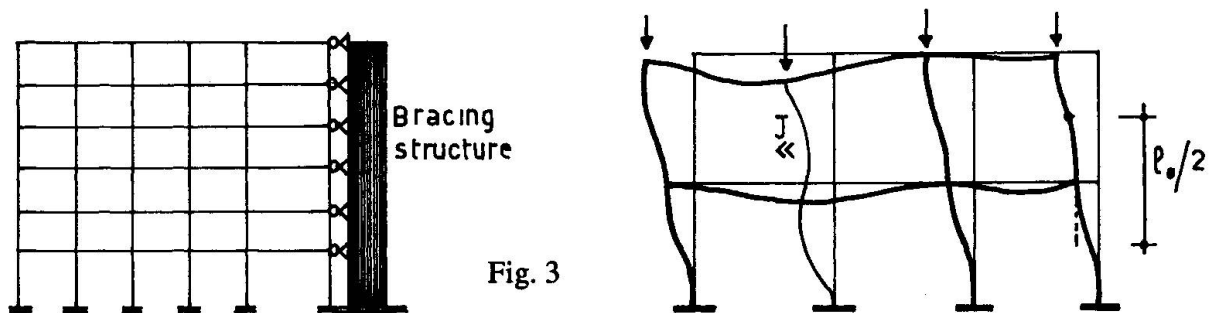


Fig. 3

## 5. SLENDERNESS LIMITS

The analysis of second order effects in a concrete structure implies more or less complex calculations, due to the interaction of geometric and mechanic nonlinearity. Even with the simplified methods and aids, it represents a computational cost. Theoretically, all members subject to external compressive forces show an increase of internal bending moments. Practically, the majority of compressed members may be excluded from that analysis.

### 5.1 Uniaxial bending

The parameter used in past codes, as for instance in CEB-FIP Model Code 78 [1], is the slenderness ratio  $\lambda$  in the plane of bending.

This ratio is merely geometric and derives from the elastic theory. However, even for the elastic checks of steel structures, the lower bound of  $\lambda$  (combined with the well known coefficient  $\omega$ ) varies according to the different steel grades, thus revealing itself insufficient as such for being an absolute value.

In concrete structures, where the interaction of normal force  $N$  and bending  $M$  is more complex, and  $\omega$  is not used, the slenderness ratio alone is a very poor parameter, not able to represent a significant boundary for the importance of second order effects. In fact, in former CEB Recommendations 1970, the adopted value was 50; because columns with  $\lambda < 50$  may show important second order moments, subsequent MC 78 adopted the value 25, but the same drawback applies. Present draft of MC 90 [2] uses a different approach, as well as the future Eurocode 2.

A better parameter has been worked out [12] incorporating the specific normal force applied ( $v$ ), which is the other essential factor. The new parameter, formulated as  $\lambda\sqrt{v}$ , showed to detect with much greater precision the bound where second order effects rise sensibly. It has been checked in numerical tests [12, 17] against the ratio  $\mu$  of total to first order moment increasing by 10%; the useful limit appeared situated around the value  $\lambda\sqrt{v} \cong 20$ .

That parameter may be used also for defining an upper limit, above which the column would be too slender and is to be avoided : increase of moment  $\cong 100\%$ ;  $\lambda\sqrt{v} \cong 70$ .

Of course the above values are averaging most cases, and do not give the exact solution, which needs the analysis. The use of  $\lambda\sqrt{v}$  as indicative limits represents a substantial improvement with respect to  $\lambda$ , though not more complex (fig. 4).

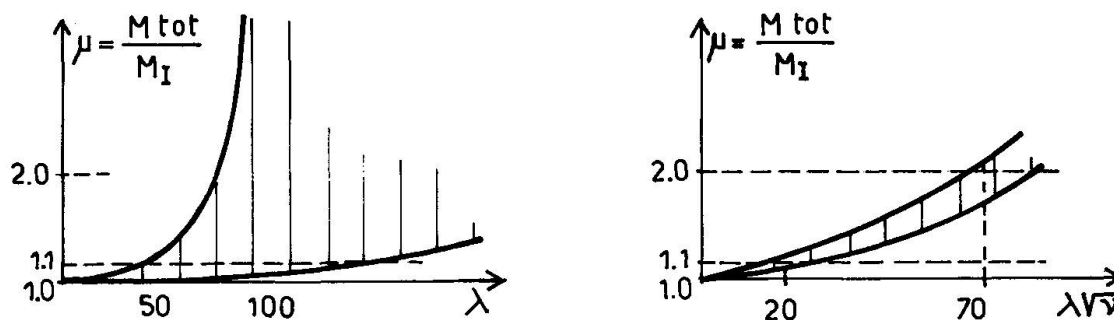


Fig. 4 - Compaction of second order limit estimation using  $\lambda\sqrt{v}$  instead of  $\lambda$

Limits of slenderness based on  $\lambda$  corrected with the ratio of load eccentricities at the ends of the column,  $\lambda / (2 - e_{01} / e_{02}) = 25$ , as adopted by some codes, may be misleading - especially when both eccentricities tend to zero - because they rely upon favorable moment distribution that may not influence really the buckling mode, and allow for  $\lambda$ 's between 50 and 75 to be treated as non slender, independently of the normal force.

## 5.2 Biaxial bending

In the writer's opinion, when the slenderness lower limit in one principal direction is exceeded, biaxial check would be necessary, applying at least the minimum eccentricities provided by the codes.

In fact, the interaction of first order moments about two principal axes of a cross-section is low when the skewed eccentricity is close to one axis. But this may be not true for the second order moments, when slenderness ratios about both axes are much different: the reduced (accounting for second order effects) capacity about the stronger axis has two distinct values, if accounting for side buckling or not, as sketched in the reduced interaction diagram in fig. 7.

A criterion for avoiding biaxial check (when the loading condition lays in a principal plane) should be found, assessing a ratio  $\lambda_x / \lambda_y$  close to 1.

## 5.3 Frames

Slenderness limits for entire non braced frames have been sought. The problem is finding an equivalent  $\lambda$  for the storey, then it may be associated with an average specific normal force to enter in the criterion under § 5.1. If the frame is "regular" an average effective length of columns in lateral buckling mode may be easily estimated and assumed for the scope (fig. 3).

A simplified formula, very conservative, has been proposed [10] for a rough check of the storey slenderness:  $\lambda = \sqrt{12 K A / h}$ , where  $A$  is the sum of concrete areas of columns,  $h$  the storey height and  $K$  a conventionally calculated displacement.



## 6. ANALYSIS

### 6.1 General Methods

It is called "general method" any analytical procedure able to solving with sharp approximation the stress-strain state of the structure.

The great obstacle in these procedures is the computational work: the strain state must be determined all over the structure to work out the deformation, accounting for mechanical and geometrical nonlinearities. It is well known that, by discretization, it is rather simpler to work out the internal forces over a section, given the strains, than vice-versa; while this is needed in the iterative procedures when searching for the deformations under the current tentative action effects along the structure.

To make this search easier, a criterion has been elaborated for building up the correct stiffness matrix of the most generic concrete structure, subject to axial force and biaxial bending, and responding to any given materials  $\sigma$ - $\epsilon$  relationships [5, 6, 15].

The criterion is based on the linearization, in every point of a cross-section, of the local stress-strain path during a given loading step; all the linearized path steps make up a section made of fictitious linear materials, each with a different modulus of elasticity; this can be treated, by a transformation into a common modulus, as a homogeneous elastic section, eventually with the theory of the ellipse of inertia, which figures the relationship between forces and deformations (fig. 5). Any phenomenon, like cracking, tension stiffening, bond, non elasticity, etc., may be incorporated for each material, provided it can be fit into a stress-strain curve.

The above relationships must be found anew at every iteration, as the stresses change and with them the fictitious elastic section. However, they are the most effective in driving the iterations toward convergency.

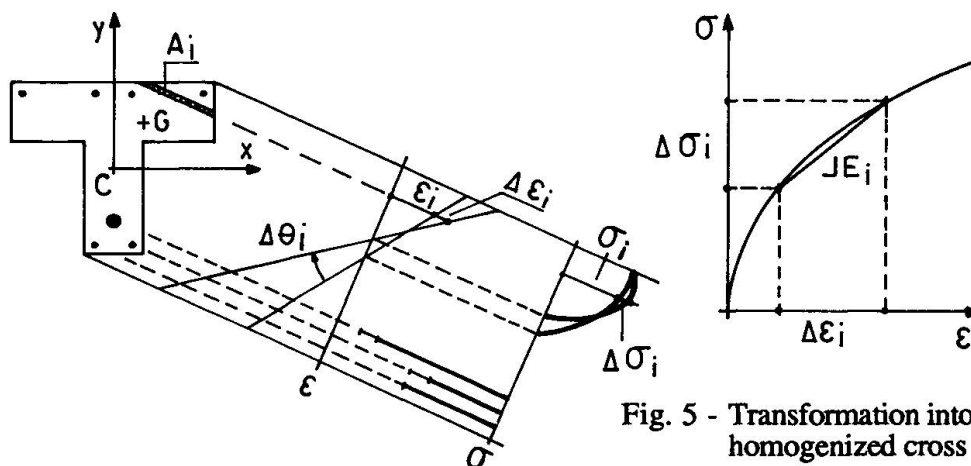


Fig. 5 - Transformation into elastic homogenized cross section

### 6.2 Approximated Methods

#### 6.2.1 Uniaxial bending

For a simple cantilever column, loaded only at the top, the integration of deformations is solved approximately in the "model column" method [4] by assigning the structure a shape function (sinusoidal), whose amplitude is determined by imposing the compatibility, of the curvature of the axis line ( $1/R$ ) and of the deformed cross section ( $\theta$ ), only in the critical section at the base.

That assumption permits to derive easily the second order moment  $M_{II}$  in the critical section:

$$M_{II} = N \cdot \delta = N \cdot \theta^2 l_0^2 / \pi^2 \cong 0.4 (l_0/2)^2 \cdot \theta \cdot N$$

Thus,  $M_{II}$  is represented by a straight line on the Moment-Curvature ( $M$ - $\theta$ ) diagram of the critical

section (for the given  $N$ ).  $M$  being the total moment, a column can be acted only by a first order moment  $M_I$  given by the difference  $M - M_{II}$  (fig. 6).

Therefrom, it is easy to work out the so-called "reduced interaction diagram", by plotting the  $M_{I \max}$  for all values of  $N$ . This diagram becomes referred to the structure, not only to the section,  $M_{I \max}$  being a means of representing the applied forces net of 2nd order effects. Such diagrams are also prepared in nondimensional form as design aids; naturally, they may be built-up using exact methods.

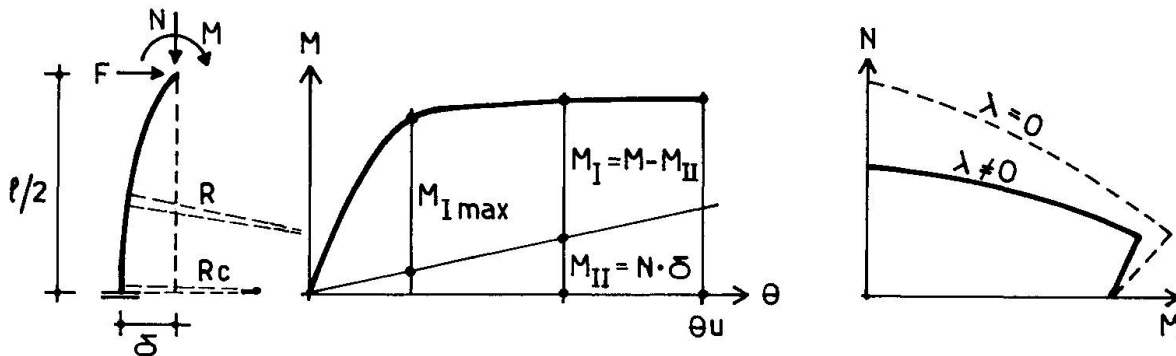


Fig. 6 - Model Column and Reduced Interaction Diagram (R.I.D.)

### 6.2.2 Biaxial bending

The model column may be used for biaxial actions, too. But the approximation is worst, as it disregards that the deflection is not plane (fig. 8) and varies in direction when loads increase; in fact, internal forces vary along the structure and, with them, the angle of skew.

Nevertheless, reduced interaction diagrams may be worked out too, by more refine means. Being the action triaxial ( $N, M_x, M_y$ ) it is worth to plot them in  $M_x, M_y$  planes, for various  $N$  (fig. 7).

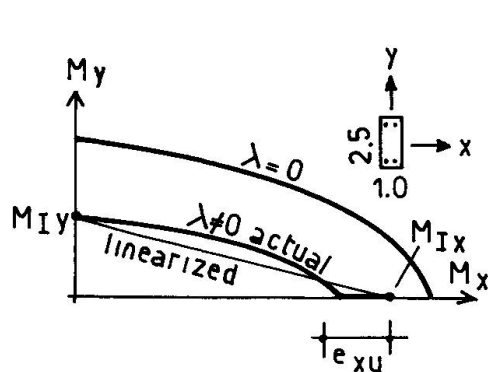


Fig. 7 - R.I.D. of biaxial bending

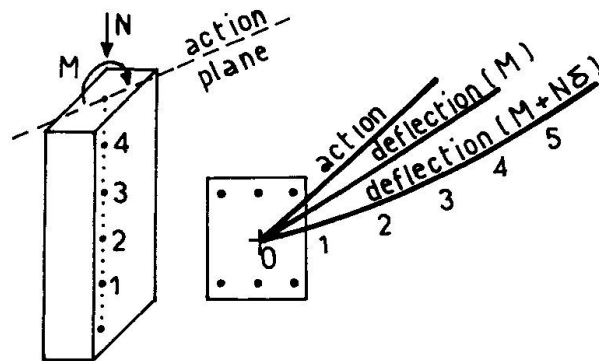


Fig. 8 - Non-plane deflection in biaxial bending

Due to the complexity of biaxial verification with general methods, a rough simplified criterion of check for the most general case was also proposed [7].

Given a column, with any possible kind of section, restraint, loading, it is analyzed in uniaxial bending on both directions  $x, y$  with an appropriate method, accounting for second order effects. The critical multipliers  $\alpha_x^*$  and  $\alpha_y^*$ , applied to all design loads acting in principal planes  $x$  and  $y$  respectively, are found. Finally, an Interaction diagram is drawn with a straight line between both found values, in the plane of the multipliers  $\alpha_x, \alpha_y$ .

The diagram is referred directly to the actions (through the multipliers  $\alpha$ ) and not to the action effects on a section, because in a general case critical sections may be more than one. The check is satisfied if the point ( $\alpha_x = 1, \alpha_y = 1$ ), representing the design load combination considered, is contained in the safe diagram (fig. 9).

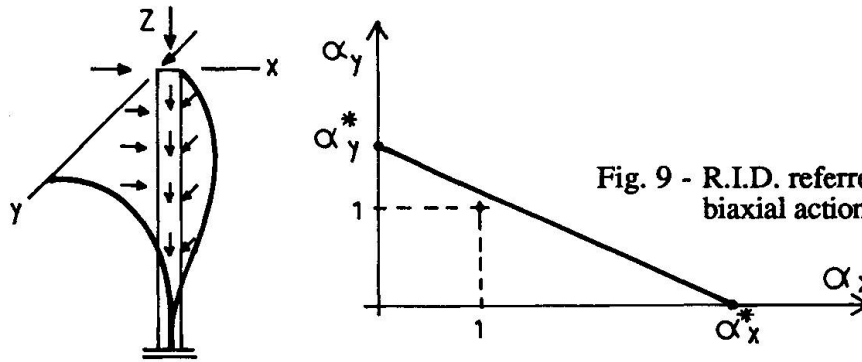


Fig. 9 - R.I.D. referred to generic biaxial actions and structures

The criterion is generally conservative, but in a small regions close to the axis of the greater  $\alpha^*$  in cases of much different slenderness about both planes, as it was mentioned. However, this must be covered by the minimum eccentricity (fig. 7).

### 6.2.3 Frames

A traditional approximated method for overall check of sway frames is the so-called P- $\delta$  method. It consists in an iterative analysis of the frame, acted by lateral forces at the storey levels, "equivalent" to the second order effect ( $F \cdot h = P \cdot \delta$ ),  $F$  being the lateral resultant,  $P$  the vertical one,  $h$  the height and  $\delta$  the deflection, all referred to the storey.

At each iteration the equivalence is checked with the obtained  $\delta$ , until convergency is reached. The difficulty, determining the degree of approximation, is assigning correct stiffness to the elements, consistent with the u.l.s.

### 6.2.4 Walls

Walls are bi-dimensional plane structural elements, subject to complex in-plane and out-plane forces. Concrete walls are usually slender out of their plane. They might be analyzed accurately by means of appropriate non linear methods. However, in building practical design they are treated with approximated criteria, reducing the problem to that of an equivalent one-dimensional element, i.e., a column.

As well as for columns restrained at the ends in a frame, nomograms are given for estimating the reduction of the effective buckling length, as function of the restraints on the four edges and of their respective distance. Codes give guidance for estimating conventional and additional eccentricities. The checks are then performed accordingly, on the critical vertical strips of wall acted by a normal force equivalent to the strip cross-section normal stress resultant, deriving from the overall analysis of the structure.

A particular case is represented by non reinforced concrete walls. For them, the slenderness limits cannot be based on the criterion of 10% increase of the moment due to the second order effect, because the moment capacity depends almost totally on the normal force, and the eccentricity becomes the critical parameter.

Starting from the basic work [3], graphs and coefficients are given for the reduction of capacity of wall strips, accounting for the multiple parameters involved, including safety elements. Some reliance is made both on tension stiffening and on tensile strength of concrete, justified by the redundancy of the bi-dimensional behavior.

Short (horizontally) walls, unrestrained along vertical edges, become "non reinforced columns". Such seldom concrete elements should not rely upon tensile strength and possibly not be affected by second order effects, thanks to geometry or to loadings.

## 7. SAFETY

### 7.1 Code provisions

According to Level I approach, characteristic values of the relevant variables involved in a limit state are to be given partial safety factors, to obtain the "design values": generally, actions are amplified, strengths reduced, possibly the model uncertainty is covered by special factors; the corresponding "state" of the structure must be within the given limit. The u.l.s. of slender structures is affected by the deformation, thus the relevant variables affecting it should be modified, too, for the verification. This is done by the following conventional means:

- i an initial unintentional inclination is introduced, or an equivalent eccentricity;
  - ii the deformabilities of materials are factored;
- furthermore:
- iii creep deformations due to sustained loadings are added;
  - iiii possible deformation of restraints is accounted for.

Some observations are deserved.

The first item (i) accounts for unavoidable geometrical imperfections, that in slender structures may raise or amplify the deflections. Standard specified inclinations are quite severe, between 1/150 and 1/200, thus some reduction are allowed, depending on site controls; it could even be neglected, when many connected columns act in parallel.

Item (ii) implies  $\gamma_c$  factor being applied not only to the strength, but to the whole  $\sigma$ - $\epsilon$  curve, reducing the initial modulus  $E_c$ , too. In fact, it is the same model used for checking the cross-section resistance; but here it assumes the particular meaning of increasing the "design deformation" of the structure for u.l.s. However, the initial part of the curve being relevant, it should be assumed more realistic than the conventional parabola attached at a fixed strain value to the rectangle, in order to better match  $E_c$ .

The question is under discussion whether the same  $\gamma_c$  should factor  $f_c$  and  $E_c$ : in fact, by factoring deformability and loads, the deformation is factored twice. Also the stiffening effect of concrete in tension is to be judged: whereas, for small size isostatic columns, it should be considered naught (consistently with the local effect governing the whole behavior), for large structures like towers, or for highly redundant frames, it would be really too conservative.

The initial deformability of steel is commonly not factored; yielding point is automatically lowered by factoring the strength, thus influencing the ultimate deformability.

Prestressing factors are not mentioned by codes in this context. In fact, it seems reasonable to factor steel curves as for ordinary steel (i.e. only beyond yielding point) and to apply the nominal tensioning strain unfactored for the analysis.

Item (iii), creep deflection, is generally calculated under the unintentional inclination and the quasi-permanent load combination (how factored is also matter of discussion). For sake of simplicity, the creep deflection is calculated separately, then added to the initial one (i); finally the check is performed with all design loads considered as short time.

Item (iiii) covers an obvious extension of the criteria to the connected bodies involved in the deformation. The deformabilities should be referred to the action effects intervening at the u.l.s. of the column.

### 7.2 Safety analyses

A set of checks of MC 78 provisions, applied to 60 different columns, were performed by a Level II method [16], in order to assess the consistency of the rules.

Some indicative conclusions were drawn, so summarized:



- partial factor  $\gamma_F$  on variable axial forces should be increased, compared with that on dead loads;
- the special  $\gamma_F$  on permanent loads, only for calculation of creep deflection (assumed = 1.1), should be increased;
- the minimum reinforcement, assumed  $A_{smin} = 0.8\% A_c$ , should be increased in slender members (as function of concrete grade or of design axial force).

## 8. DETAILING

Few remarks may be set on detailing of columns themselves (apart their D-regions):

- longitudinal reinforcement: minimum value, expressed as % of concrete area as in MC 78 (see § 7.2) should be increased for slender columns as function of design axial force, and in general, for columns with higher concrete grades, to provide bending capacities proportional to the implicit higher axial force capacities;
- confinement: overall confinement of axially loaded slender columns does not increase their bearing capacity, as u.l.s. is governed by bending; placed in the end sections, it increases the ductility capacity.

Walls generally carry lesser average normal stress than columns and may need a lesser minimum vertical reinforcement. Instead, significant horizontal reinforcement is needed, to follow the plate effect. In "non reinforced" walls, minor reinforcements (bars or meshes) are placed for taking over local tensile stresses; care should be taken to prevent the vertical bars from buckling and spalling concrete.

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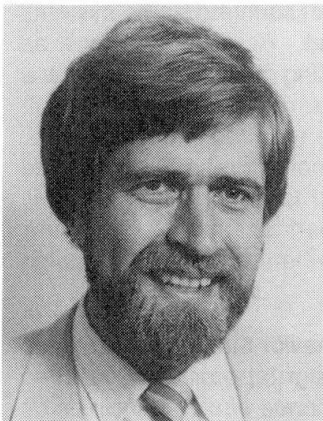
## Performance Assessment of Cap-Column Joints Under Seismic Loading

Evaluation de la résistance de joints pile-pont soumis aux séismes

Begutachtung von Rahmenecken unter Erdbebenbelastung

### Frieder SEIBLE

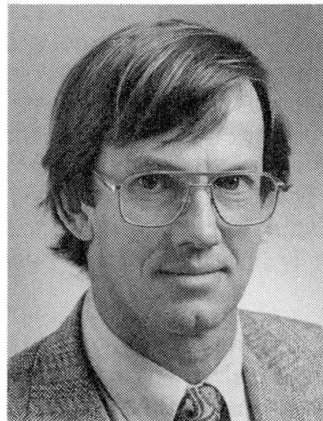
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### SUMMARY

An overview of joint damage in beam-column joints of multi-level bridge bents during the 1989 Loma Prieta earthquake, is presented. Assessment criteria for evaluating seismic performance characteristics and retrofitting schemes for existing concrete bridge structures, are developed. The application of analytical concrete models of various complexities in support of structural concrete design is demonstrated.

### RÉSUMÉ

On présente ici une vue d'ensemble des dommages causés aux angles des cadres soutenant des ponts multi-niveaux, advenus lors du tremblement de terre de Loma Prieta en 1989. Les critères d'évaluation de base concernant l'estimation des caractéristiques de résistance aux séismes et les possibilités de renforcement des ponts existants sont développés; à cet effet, l'application de modèles analytiques de divers niveaux de difficulté servant de support au dimensionnement des structures en béton armé est démontrée.

### ZUSAMMENFASSUNG

Über Schäden in Knotenpunkten von Balken-Stützenverbindungen in mehrstöckigen Brückenrahmen während des Loma Prieta Erdbebens von 1989 wird ein Überblick gegeben. Beurteilungskriterien für die Begutachtung und Einschätzung des Erdbebenverhaltens und der Verstärkung von bestehenden Betonbrücken werden aufgezeigt. Die Anwendung von analytischen Modellen im konstruktiven Betonbau von unterschiedlicher Komplexität wird demonstriert.



## 1. INTRODUCTION

The Loma Prieta (San Francisco) earthquake of October 17, 1989 reemphasized the vulnerability of structural concrete systems to cyclic displacements resulting from seismic attack. The dramatic collapse of a one-mile section of the Cypress Viaduct in Oakland can be traced to inadequate performance of the cap-column joint region in the supporting bents of the elevated bridge structure. Investigation of the joint reinforcement showed inadequate structural detailing of the joint region for the encountered seismic force levels. While the Cypress collapse was well publicized in the press and technical literature, only limited information can be found on similar structural joint damage to other elevated roadways in the San Francisco–Oakland Bay Area, see Fig. 1, which led to the temporary or permanent closure of several major freeway arteries including the Embarcadero Viaduct (I-480), the China Basin/Southern Freeway Viaduct (I-280) and the Central Viaduct (Highway 101) in San Francisco, as well as the Southbound Connector (I-980) in Oakland [1].

While most of these bridge sections were designed and built in the 1950's and 1960's, some of them were completed as late as 1985. This raises questions concerning not only past but current detailing practice for structural concrete joints. Design rules for beam and column members of structural concrete frame systems seem to be widely accepted and standardized in similar form around the world. However, as soon as aspect ratios of structural members approach unity, design guidelines and supporting design models show a wide range of different approaches. Limited detailed design models for these regions exist when full three-dimensional force transfer of axial, flexural and torsional structural action is required simultaneously. Also, most design models focus on single monotonic structural loading and do not address fully reversed cyclic loading patterns. Thus, the question arises whether a unified approach for design and analysis models in support of structural concrete detailing exists or if the state-of-the-art in structural concrete detailing still relies primarily on experience to design and detail complex members for realistic loading conditions.

Both analytical models to study the in-depth mechanism of structural concrete behavior through various limit states and design models developed to unify the structural detailing and design approach have seen comprehensive recent developments. In a direct extension of early structural concrete design principles by Ritter (1899) and Morsch (1909), Schlaich et al. have developed a comprehensive design approach toward structural concrete detailing which ensures internal force transfer through discrete compression and tension (strut and tie) members, satisfying equilibrium by simple truss mechanisms [2]. This approach has become a powerful design tool since it allows a variety of detailing solutions as long as basic anchorage and stress limit states are observed, but most importantly it allows and forces the design engineer to develop a consistent design model resulting in an engineered solution rather than in a design resulting from a recipe application. Problems and limitations arise when design solutions based on inappropriate truss mechanisms are attempted and when the discrete member forces are of magnitudes which cause stress limit and anchorage problems and thus require a distributed or smeared approach. Parallel to the consistent strut and tie model development, Collins et al. developed the juxtaposed position of a smeared or distributed behavior model [3], which is based on homogeneous behavior of structural concrete even in its cracked state and resulting orthogonal principal compression and/or tension fields of the internal forces. Based on mechanical principles of an orthotropic homogeneous material, the orientation of the resulting stress fields is derived from compatibility and equilibrium conditions. The resulting stress fields are subsequently discretized in concrete and reinforcement action which forms the basis for a rational structural concrete design approach. Similar to the discrete strut and tie model, additional considerations for anchorage and local concentrated force transfer are required and limitations exist where either reinforcement is heavily concentrated rather than distributed, and where structural action results in a few large cracks rather than in the ideal distributed (smeared) crack pattern. Thus, while both design models are different in the approach, they are rather complementary in the overall design process, especially when in addition to the force transfer in the joint or member, deformation limit states also need to be considered.

Both of the above models provide comprehensive design approaches to structural concrete detailing but are fully applicable only when simple monotonic loading conditions exist up to design levels with sufficient margin to the ultimate limit state. Where deteriorating bond phenomena along the reinforcement, opening and closing of cracks under reversed cyclic loading, deterioration of concrete contribution in developing local failure mechanisms, and the development of ductile hinges (which incorporate all of the above aspects) are present, the above models may not be adequate and additional considerations to both design

approaches are needed as outlined by Paulay et al. for structural concrete joints under seismic action [4]. It will be shown in the following that, with these additional considerations, the above design models can also be directly applied toward the design of retrofit measures of existing critical structural concrete regions as found in the San Francisco double-deck freeways. Further, it will be shown that complete failure sequences and limit states of these structural systems can be traced using advanced nonlinear analytical structural concrete models.

The critical role of structural concrete joints in beam-column systems and their behavior under seismic loading is evaluated in this paper on the example of joint performance in elevated bridge structures during the Loma Prieta earthquake and the applicability of various design and analysis models to the structural concrete joint problem is demonstrated, both for the assessment of joint performance during the earthquake and subsequent repair and retrofit strategies.

## 2. SEISMIC PERFORMANCE ASSESSMENT

### 2.1 General Assessment Approach

To assess the expected seismic performance of structural concrete beam column joints, it is important that the joint under consideration is evaluated in direct relationship to the actual adjacent member capacities. This requires a state or capacity determination of adjacent beams and columns with consideration of (1) actual material properties at the time of evaluation, i.e., probable concrete strength, not the design strength  $\sqrt{f'_c}$ , actual stress strain behavior for the reinforcement not nominal specified design yield levels, (2) proper consideration of axial load effects, (3) proper consideration of possible confinement effects from transverse reinforcement, (4) reduced concrete shear contribution in areas of large fully reversed cyclic deformation, and (5) realistic bond and anchorage estimates particularly for large diameter reinforcing bars.

A preliminary performance assessment of joints comprises the following steps: *Step I:* Realistic member capacities based on the above considerations are derived for both flexure and shear, and the critical failure mechanism is determined by direct comparison of the shear capacity with the plastic flexural limit state shear  $V_p$  derived from the appropriate flexural plastic hinge failure model of the member. If  $V_p$  is larger than the calculated shear capacity, a potentially brittle shear failure can be expected without the formation of ductile flexural plastic hinge mechanisms. *Step II:* Based on the possible member failure mechanisms, the expected global collapse mechanism for the complete structural system is derived by comparing combined dead load and lateral seismic force action with the derived capacities. *Step III:* From the identified systems collapse mechanism, critical joint forces can now be determined at the collapse state and a direct comparison with most probable joint capacities will indicate if joint distress degrades the capacity of the collapse mechanism or if the joint behaves as ideally assumed within or close to the elastic range. *Step IV:* Finally, equivalent seismic base shear forces are estimated corresponding to the lateral force level which causes collapse based on the above failure mechanism. Excessive joint distress can lead to a reduction of this base shear coefficient, particularly when a large number of cyclic load reversals and the associated joint degradation is considered.

Application of the above preliminary seismic assessment procedure to the San Francisco double-deck bridge bents has shown that the joints did not meet design criteria for earthquake resistant ductile structures summarized by Paulay et al. [4] as: (1) joint strength should exceed the maximum strength of the weakest connecting member, (2) structure capacity should not be jeopardized by strength degradation in the joint, and (3) joint response should be elastic during moderate seismic disturbances.

The preliminary joint behavior assessment outlined above can be supplemented and refined by more detailed analysis and design models as demonstrated in the following for one specific case study performed following the 1989 Loma Prieta earthquake.

### 2.2 The Oakland Southbound Connector, I-980, Bent 38

A single-deck outrigger bent (bent #38) with only a 0.92 m (3 ft) outrigger cap beam extension past the superstructure on I-980 featured heavy joint damage as shown in Fig. 1c,d,e. Built in 1985, the column was well confined with an interlocking spiral, see Fig. 3, however, this spiral did not continue into the joint region



where it was replaced by a 5 gauge wire spiral with  $D = 5 \text{ mm}$  at  $10 \text{ cm}$  ( $\phi 0.2 \text{ in. @ } 4 \text{ in.}$ ). Also, the cap beam reinforcement, aside from the top and bottom bars, see Fig. 3, did not extend into the joint region.

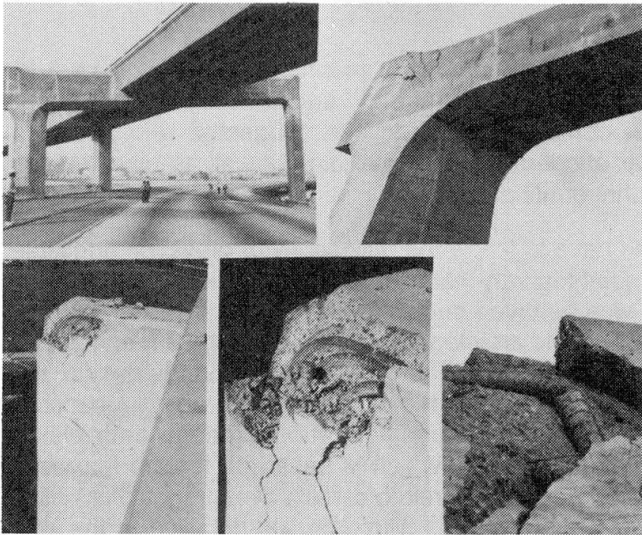
A capacity check on the cap beam and column capacities showed that the cap beam capacity is critical for positive moment due to the insufficient anchorage length of  $1.8 \text{ m}$  ( $72 \text{ in.}$ ) for the  $D = 57 \text{ mm}$  (#18) bars which, based on ACI 318-89, require a basic development length of  $3.0 \text{ m}$  ( $117 \text{ in.}$ ) which is likely to be on the conservative side. In the other loading direction (negative moment in the joint), the column capacity is critical. Joint shear force levels derived from simple stress models, Fig. 4, show joint shear stress levels of  $0.37\sqrt{f'_c}$  MPa ( $4.3\sqrt{f'_c}$  psi) and  $0.5\sqrt{f'_c}$  MPa ( $6.0\sqrt{f'_c}$  psi), under positive and negative moment loading, respectively, which are both above an assumed level of  $0.33\sqrt{f'_c}$  MPa ( $4.0\sqrt{f'_c}$  psi), where diagonal tension cracking in the joint can be expected. Since the shear capacity of the 5 gauge wire spirals does not add significant joint shear capacity, the formation of any flexural hinge mechanism in adjacent members was inhibited. This explains the encountered diagonal joint crack patterns during the Loma Prieta earthquake, see Fig. 1c,d,e.

In addition to the diagonal crack patterns, large areas of cover concrete spalling along the outer cap corner as well as a ruptured  $D = 57 \text{ mm}$  (#18) reinforcement bar which was bent on a  $45 \text{ cm}$  ( $1'-6''$ ) radius were observed, see Fig. 1d,e.

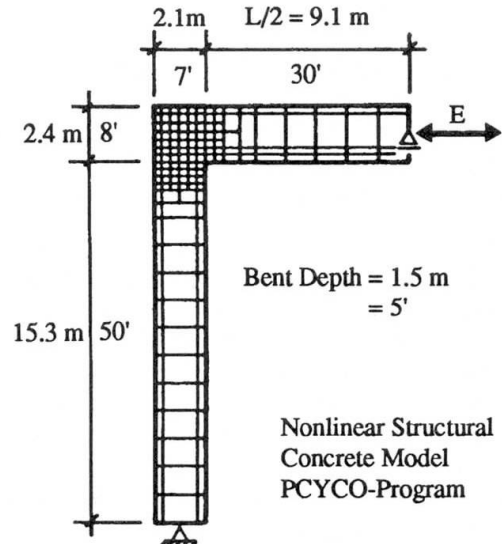
The first phenomenon of cover concrete spalling can be explained with the fully reversed cyclic loading. Under negative moment, flexural cracks open on the cap surface as shown in Fig. 3 and under subsequent positive moment loading, the entire compression force has to be transferred through the negative moment reinforcement until the cracks can close. These high compression forces in the negative moment reinforcement transferred to the concrete by bond, have a tendency to spall off the concrete cover between flexural cracks developed in the previous tensile excursions. The second phenomenon, the ruptured reinforcing bar, see Fig. 1d, points to a potentially critical problem which needs further investigation. Common ultimate strain levels in  $D = 57 \text{ mm}$  (#18)  $f_y = 414 \text{ MPa}$  (Grade 60) bars are in a range from 7 to 15%. Introducing a  $R = 45 \text{ cm}$  ( $18 \text{ in.}$ ) radius bent into a  $D = 57 \text{ mm}$  (#18 or  $\phi = 2.25 \text{ in.}$ ) bar causes strain levels of  $D/(2R) = 225/(2 \times 18) = 6.25\%$ , which is close to the ultimate strain range. The very low strain reserves and possible strain aging effects which raise the notch ductile temperature at which steel will fail in a brittle mode can cause sudden failure in these bent bars at very low additional strain levels.

In addition to the simple stress models of the knee bent joint, detailed nonlinear finite element simulations, see Fig. 2, based on extended compression field principles were performed to determine analytically failure modes and joint deformation contributions to the overall bent deformations. Reinforcement development of straight bars was modeled by assuming a reduced yield level in the anchor zone decreasing linearly from the full yield at the ACI 318-89 basic development location to zero at the bar end. Subsequently derived yield patterns in bar anchorage regions therefore are indicative of bond failure or slip. Superimposed to the dead load case, the half bent, Fig. 2, was subjected to lateral force, and force-deformation envelopes with major event indicators, depicted for positive moment loading in Fig. 5, were obtained. Associated crack, slip/yield and first crushing patterns, see Fig. 5, indicate the failure mechanisms in the joint region. The heavy yield in the joint center, both horizontally and vertically, indicates the deficiency of both vertical and horizontal joint shear reinforcement, particularly under negative moment on the joint. Also, the crushing of the concrete in the outer joint region under the bent negative moment reinforcement under loading to the left or negative moment indicates the high compressive stress state and associated transverse prying forces in this region and the need for sufficient transverse confinement reinforcement, as outlined by Schlaich et al. [2], for strut and tie models for negative moment knee joints.

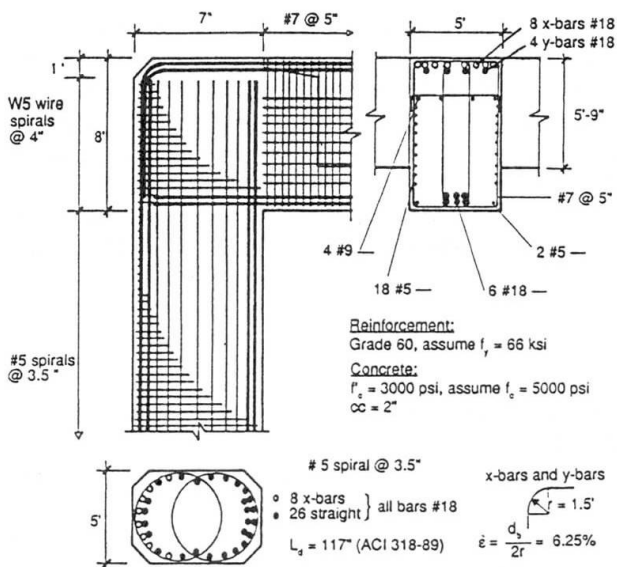
Since joints should be detailed based on capacity considerations such that the major inelastic action occurs in the cap or column, and since they should remain effectively elastic for small seismic disturbances (Paulay et al. [4]), the nonlinear finite element analysis was repeated with a linear elastic joint for a direct comparison of deformation limit states. As can be seen from Fig. 5, while joint deformations did not contribute significantly to the initial overall structural deformations, the failure mode in the positive moment direction shows improved ductile behavior when the failure mechanism is shifted from joint distress to flexural cap beam hinging. It should be noted that the cap beam capacity still may be artificially low due to the reduced yield strength in the bottom  $D = 57 \text{ mm}$  (#18) reinforcing bars based on a reduced development length according to ACI 318-89. Negative moment loading behavior is also improved by forcing the yield mechanism clearly into the column (not shown). Since the bent joint failure is the critical link in the overall behavior, repair/retrofit of these joints is a logical next step.



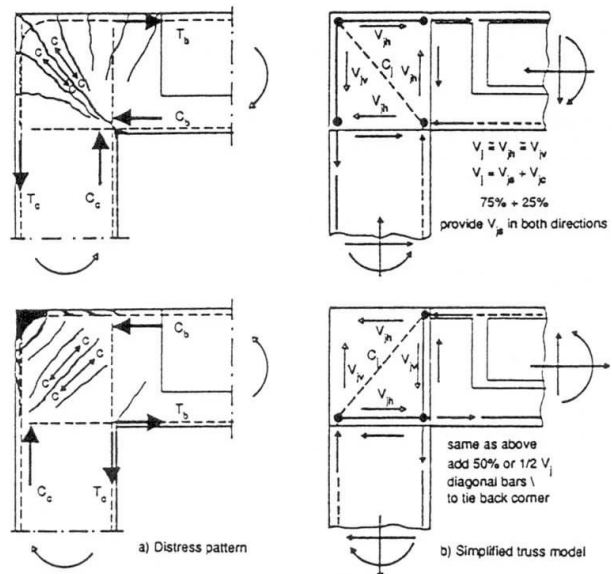
**Fig. 1** Outrigger bent joint damage



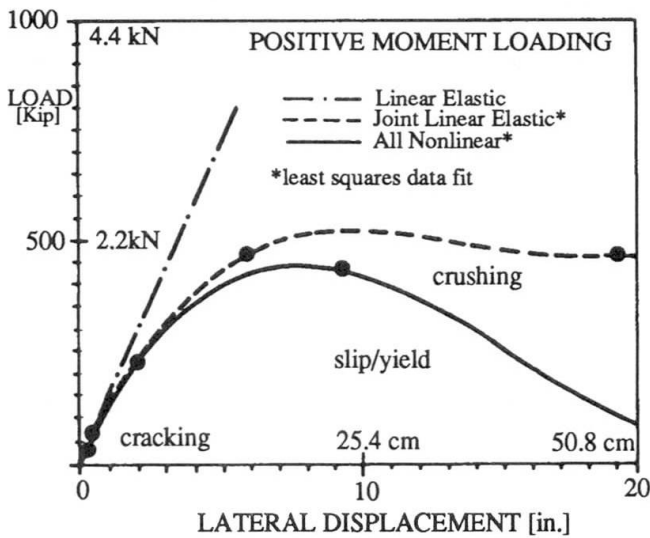
**Fig. 2** Analytical bent model



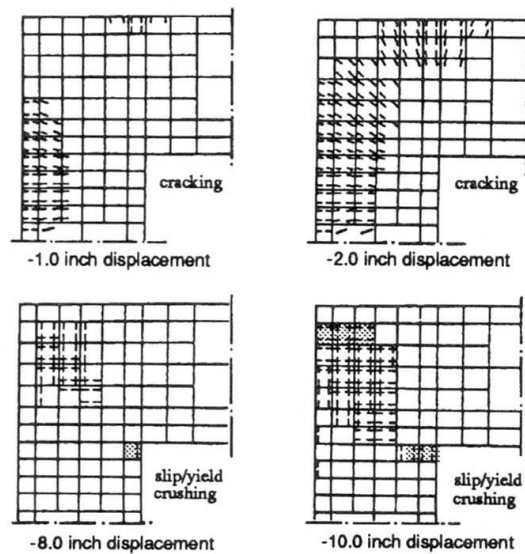
**Fig. 3** I-980, #38, Reinforcement details



**Fig. 4** Joint behavior models



**Fig. 5** Positive moment limit states



**Fig. 6** Negative moment distress patterns



### 3. REPAIR AND RETROFIT

Cap/column joints in elevated roadways are probably the most difficult members in a bridge bent to successfully repair and/or retrofit for seismic loading. Both horizontal and vertical heavy reinforcement patterns from cap beams and columns provide for complex geometric and congested reinforcement patterns in the joint. Also, the critical aspects of bar anchorage within the joint as well as plastic hinge development directly adjacent to the joint complicate the retrofit design.

Criteria for seismic retrofit design of cap/column joints have to follow the philosophy of providing a reliable ductile structural system. Since ductility within the joints is very hard to achieve, the joint retrofitting is geared toward the formation of clearly defined and well behaved ductile plastic hinges in either the cap beam or the column. In many bridge decks, the cap beam is an integral component of the superstructure which would make post-earthquake repair in this member difficult. Thus, frequently in bridge design, the ductile framing system is provided by reliable column hinges. For the retrofit design, again a capacity design approach should be employed which ensures in the case of the joint retrofit predominantly elastic behavior of the joint region. This can be achieved by joint design which is based on factored nominal column design moments, e.g.,  $1.5 \times M_n$ , where the factor accounts for reinforcement overstrength, including strain hardening, confinement effects and concrete strength increase with time. In detailing of the joint repair retrofit, the congested reinforcement layout within the joint as well as high nominal joint shear stress levels typically require an increase in size of the joint region.

Detailing for repair or retrofit of the damaged knee joint on I-980, bent #38, can be derived using either strut and tie models as outlined in Fig. 3 with additional considerations as outlined in [2] for transverse splitting under negative moment and tie back for fully reversed cyclic loading, or directly from the force states derived from the compression field analysis, see Fig. 5. A possible repair measure consists of providing an additional 23 cm (9 in.) concrete jacket around the existing joint with horizontal and vertical distributed joint reinforcement and a diagonal corner tie back based on force and reinforcement quantities derived in Fig. 4. The damaged joint concrete can either be removed and the joint rebuilt completely or the damaged joint can be epoxy injected subsequent to removal of loose concrete, roughening of the interface, and adequate doweling bonding of the added structural concrete jacket.

### 4. CONCLUSIONS

The assessment of beam-column joint performance during the 1989 Loma Prieta earthquake has shown that compression field based analysis and design models are invaluable tools to investigate existing capacities, failure modes, and deformation limit states. Strut and tie models are particularly suited for design of new joints and retrofit and repair measures. However, in capacity assessment, where dependence on distributed concrete tensile stresses is essential and where bond forces cause distributed shear input, the basic strut and tie model is limited. The detailing of structural concrete joints subjected to fully reversed cyclic loading requires additional design detailing considerations which account for opening and closing of large flexural cracks and the associated compression force transfer through the reinforcement. Due to the large column height, the joint deterioration was shown to contribute insignificantly to the initial structural deformation, while non-ductile joint failure resulted in substantially reduced ultimate deformation capacities.

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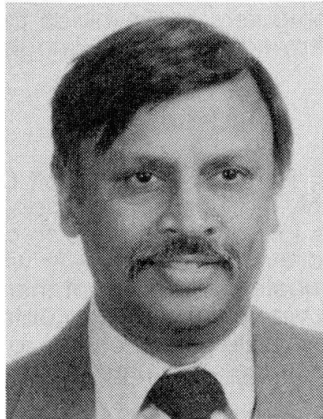
## Finite Element Modelling for Analysis of Highly Skewed Bridges

Analyse par éléments finis de ponts à dalles fortement biaisés

Finite-Element-Analyse von stark gekrümmten Brücken

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### SUMMARY

Background, reasons for use, and some results of the finite-element method of analysis are presented by the author. The main features of the study are skew bending effects; torsional moments, shears and support reactions; and large reactions caused by post-tensioning forces.

### RÉSUMÉ

L'auteur présente la documentation, la justification ainsi que les résultats de la méthode par éléments finis appliquée; les principales caractéristiques de l'étude sont les effets des flexions biaisés, les moments de torsion, cisaillement et réactions d'appui, ainsi que les réactions significatives causées par la précontrainte.

### ZUSAMMENFASSUNG

Der Hintergrund und die Ursachen und Ergebnisse der Finiten-Elemente-Methode werden präsentiert. Die wichtigsten Punkte der Untersuchung sind: schiefe Biegung, Torsionsmomente, Querkräfte und Auflagerkräfte, sowie grosse Auflagerkräfte infolge Vorspannung.



## **1. INTRODUCTION**

### **1.1 Background**

The behavior of skewed bridges has been a pre-occupation of structural engineers for decades. Traditionally skewed bridges were analyzed and designed considering the longest length between the supports as the actual span length. Conventional wisdom assumed that such a conservative approach would result in a very safe design. The means or methods to do a more exact analysis did not exist until the advent of computers. The computers gave the engineers the opportunity to use more refined methods of analysis like the grid and space frame analysis to study the behavior of skewed bridges. The more exact methods of analysis like the "finite element method of analysis" were time consuming and needed large computers. The arrival of the newer, faster and larger computers has avoided this limitation and finally given engineers the means to analyze highly skewed box girder bridges within a reasonable budget and time frame.

### **1.2 Purpose and Objectives**

During the last couple of years, Parsons Brinckerhoff Quade & Douglas, Inc. (PBQ&D) had to analyze and design several highly skewed post-tensioned box girder bridges. The skew angles on these bridges varied from a low of 43 degrees to a high of 70 degrees. The unavoidable high skew angles forced us to evaluate and verify whether the results of the finite element method of analysis would be substantially different than conventional methods or approximate methods. The objective of this whole effort was to obtain realistic behavior of these structures and to examine the local as well as global effects of external and internal loads on these bridges and assure the integrity of the structures.

## **2. SKEW BENDING**

### **2.1 Cause of Skew Bending**

In any bridge, the principal bending occurs along the shortest axis between the supports, which happens to be perpendicular to the axis of supports. In a non-skewed bridge the supports are along the transverse axis which is perpendicular to the longitudinal axis of the bridge. The principal bending of the girders and the whole structure occurs about the transverse axis parallel to the supports, along the longitudinal axis. This behavior has always dictated the direction of the girders in a concrete box girder bridge to be parallel to the longitudinal axis.

In a skewed bridge, the supports are not along the transverse axis. They are located along an axis skewed to the transverse axis of the bridge. The longitudinal axis of the bridge remains the same. The principal bending occurs about an axis parallel to supports along a new longitudinal bending axis which is perpendicular to the supports. This new longitudinal bending axis is skewed to the longitudinal axis of the bridge and is almost parallel to the shortest distance between the supports. The principal bending of the structure along this new longitudinal bending axis is called skew bending and its effects caused in a skewed bridge are referred to as skew bending effects. These effects must be carried by the girders or webs, by resolving them along the longitudinal and transverse axis of the structure.

### **2.2 Effects of Skew Bending**

The principal bending moment along the new longitudinal bending axis is resolved to a bending moment along the longitudinal axis of the structure and a torsional moment about the longitudinal axis. The torsional moments caused by skew bending effects, in turn, cause an uneven distribution of horizontal and vertical shears at a section normal to the longitudinal axis of the bridge. This uneven distribution of horizontal and vertical shears, affects the location of maximum moments, produces large variations in support reactions and the post-tensioning forces which are normally negligible in a non-skewed bridge result in uplifts at certain supports.

Five highly skewed bridges included in the three bridge sites listed below were part of the Aviation Project in Tucson Arizona and are the subject of this paper.

### 3. METHODS OF ANALYSIS AND PROGRAMS EVALUATED

Three dimensional grid, plane frame with charts for increased shears and finite element method of analysis were evaluated for the reliability of results. Programs considered were CALTRAN Bridge Memo (Reference 1) for Designers 15-1, Cell4 Program (Reference 2), MDC STRUDL Program (Reference 3) and the 3-Dimensional Grid Analysis Program (Reference 4).

The CELL4 program was selected for use in view of the savings offered in modelling, computer time, ease of obtaining sectional forces, moments and automatic generation of equivalent loads for post-tensioning forces. In order to ascertain and validate the results of CELL4 program two identical models were tested using CELL4 and STRUDL programs (Figures 1 and 2). The differences between results of the two models though not exactly the same were within reasonable limits of accuracy required for design. The maximum stresses, moments and deflections were within 5% of each other. The STRUDL model in Figure 2 has a thicker pier diaphragm than the CELL4 model, which accounts for some of the differences in the reactions. The only results that have a wide variation are the two uplift reactions at the acute corners.

The closeness of results from both the programs gave us sufficient confidence to use CELL4 on all bridges except the Council/Toole Avenue bridge, which has a variable depth.

### 4. DISCUSSION OF MODELS

#### 4.1 S.P.R.R. Bridge (Figures 3 and 4)

In order to increase the accuracy of the results in the finite element analysis, the aspect ratio of the elements was kept to 1 in critical areas and to 2 in non critical areas. The finite element model for this structure has 1428 joints and 1818 elements.

#### 4.2 Council/Toole Avenue Bridges (Figures 6 and 7)

The finite element used is called "SIPQ", a quadrilateral curved element with four corner nodes and four midside nodes with five degrees of freedom (DOF). The aspect ratio varied from 1 to 2 depending on the geometry and the importance of certain critical areas. This model had a total of 1482 elements and 4119 joints.

#### 4.3 Euclid/Park Structures (Figure 8)

PBQ&D was involved in the review of final design for these two structures. The design was based on a 3-dimensional grid analysis. CELL4 program was used for the design verification. The finite element model consisted of 1,116 joints; and 1,370 elements. The aspect ratio of elements was 1 in critical areas and 2 in less critical areas.

### 5. DISCUSSION OF RESULTS

#### 5.1 DL Reactions (Figures 3, 6 and 8)

The reactions do not follow the normal pattern given by other conventional methods of analysis. The higher reactions seem to occur at supports that lie very close to the principal longitudinal bending axis, illustrating the skew bending effects. At certain locations, supports reactions vary by 100%.

#### 5.2 Reactions Due to Post-Tensioning Force (Figures 5, 6, and 8)

Normally one does not expect any significant reactions due to internal equivalent loads due to post-tensioning force. The results show this is apparently not the case. We attribute this to the heavy torsional moments caused by the skew bending effect.

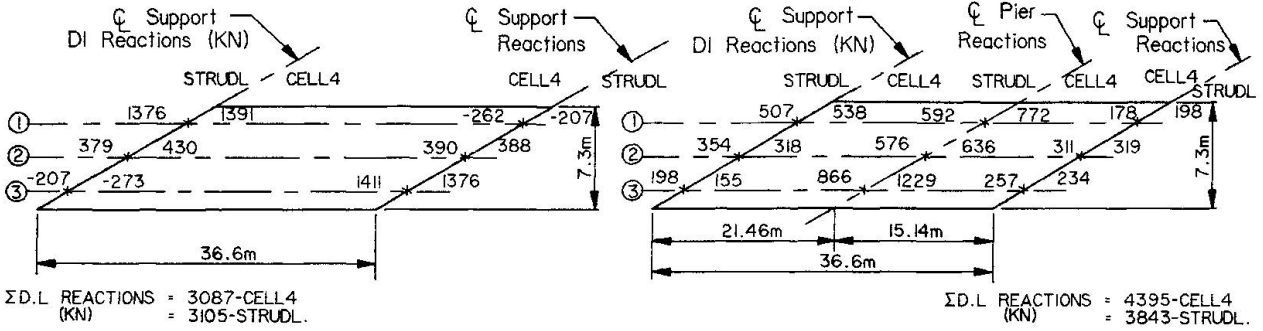


Fig. 1 Single Span TEST Model

Fig. 2 Two Span TEST Model

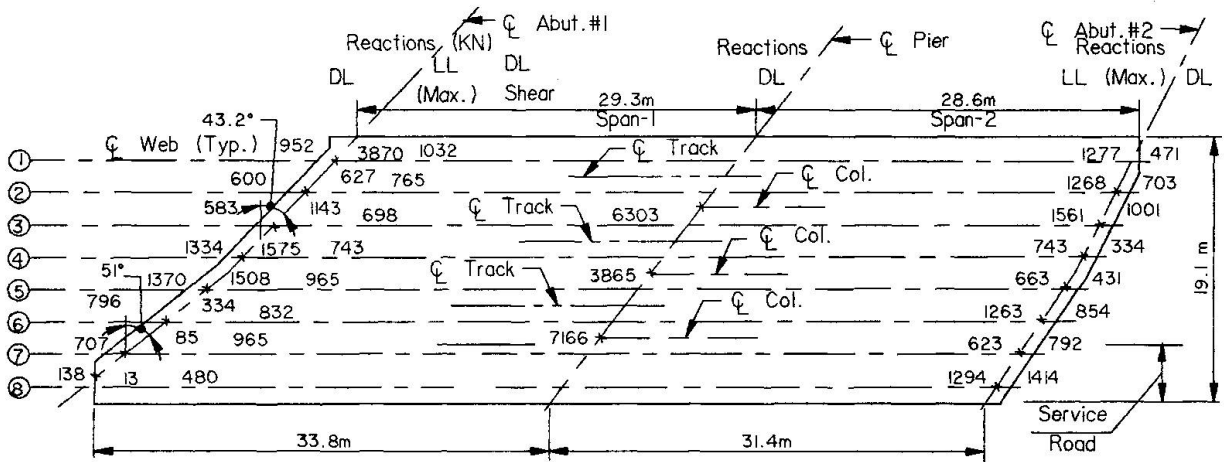


Fig. 3 Sectional Framing Plan With Support Reactions - SPRR Bridge

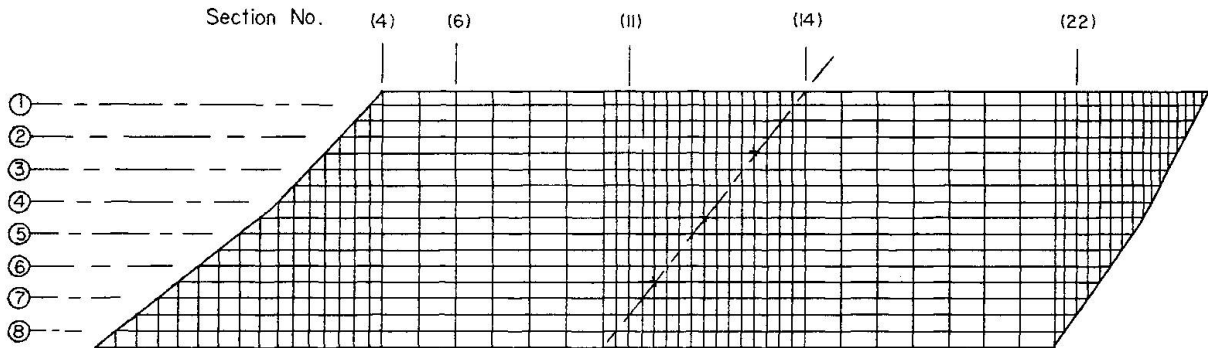


Fig. 4 Finite Element Discretization Mesh - SPRR Bridge

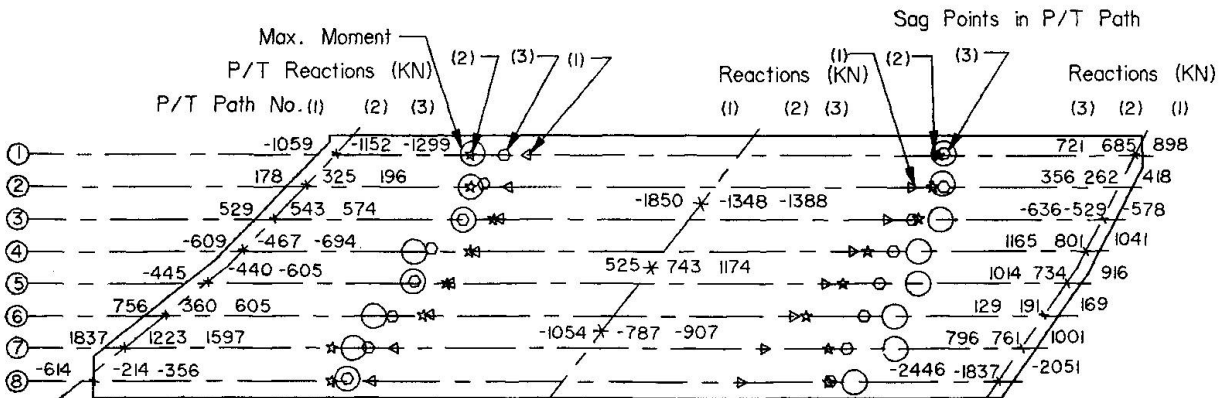


Fig. 5 Post-Tension Reactions (KN), Points of Max. Moments and Sag Points in Tendon Paths - SPRR Bridge

Table 1 Section Torsional Moment and Shear for SPRR Bridge

SECTION	4		6		11		14		22	
	Torsion	Shear	Torsion	Shear	Torsion	Shear	Torsion	Shear	Torsion	Shear
DL	-16592	-1112	-16357	672	319	3287	-12496	-4435	-10527	2393
DL+SDL	-25397	-1841	-23780	747	1588	4839	-20283	-6610	-13049	3363
DL+SDL+LL	-52597	-4537	-42440	-98	5109	7446	-16686	-7135	-9731	2953
P <sub>j</sub> = 17392 Path 1	23495	3443	29787	654	-14261	-7250	34247	9212	16934	-4497
P <sub>j</sub> = 15657 Path 2	30335	3269	35683	-476	-10893	-7602	17848	8558	20269	-4822
P <sub>j</sub> = 15657 Path 3	27086	2896	33311	-267	-13887	-7272	21300	8674	20265	-4492

Units Are KN-m and KN

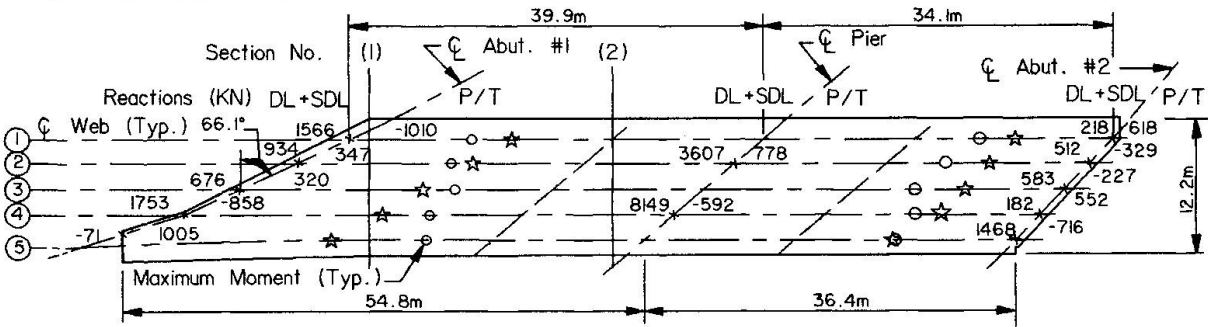


Fig. 6 Sectional Framing Plan for EB Council/Toole Ave. Bridge (DL+SDL) And P/T Reactions.  $\Sigma P/T$  Reaction = -112 (KN)

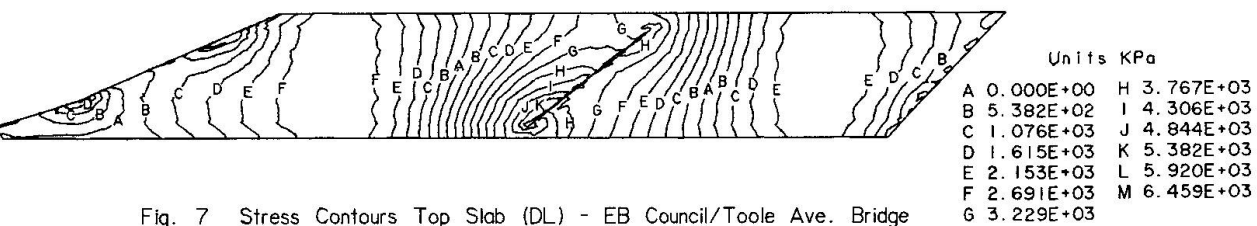


Fig. 7 Stress Contours Top Slab (DL) - EB Council/Toole Ave. Bridge

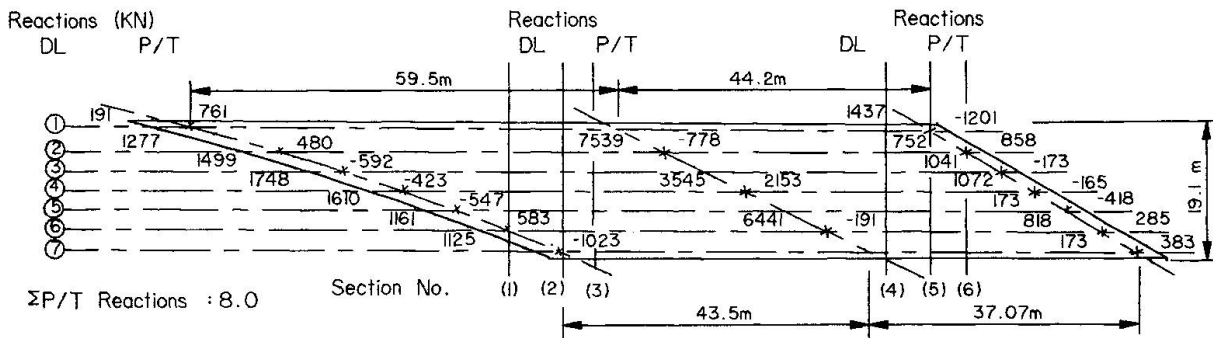


Fig. 8 Sectional Framing Plan With Support Reactions for Euclid/Park Ave. Bridge

Table 2 Torsional Moments for EB Council/Toole Ave. Bridge

SECTION	1	2
DL	-9994	-10285
DL+SDL	-12430	-12794
DL+SDL+LL	-15679	-16127
P/T	8421	6772

Units Are KN-m

Table 3 Section Torsional Moments for WB Euclid/Park Ave. Bridge

SECTION	1	2	3	4	5	6
	Torsion	Torsion	Torsion	Torsion	Torsion	Torsion
DL	14428	29599	29793	22793	22581	9607
DL+SDL	17208	35335	35539	26984	26790	11286
DL+SDL+LL	20877	42977	42893	33090	33082	13999
P/T	-15286	-31720	-23184	-18090	-25717	-11309

Units Are KN-m



### 5.3 Points of Maximum Moment (Figures 5 and 6)

In a two span highway bridge with no skews and simple supports at each end, the points of maximum moment would be located usually at 0.4 of the span from the exterior abutment support. The points of maximum moment in the SPRR bridge vary from 32 percent in girders 1 and 2 to 54 percent in girder 8 in span 1. This is reversed in span 2 and the points of maximum moment vary from 48 percent in girder 1 and 2 to 32 percent in girders 7 and 8.

The fluctuations in the location of points of maximum moments in the Council/Toole Avenue bridges are very large. The distance from the center of abutment to these points varies from 25 percent of span at girder 1 to 55 percent at girder 5. The support reactions are more than 100 percent different at certain locations. Note the heavy reactions at the top left hand corner and the bottom right hand corner supports. The line joining these points, almost coincides with one of pier supports. Though it is not perpendicular to the lines of support, it seems to act like the principal longitudinal axis of the structure. The reaction of this pier is more than twice the reaction at the other pier and greatly affects the design of the cantilever pier diaphragm.

### 5.4 Torsional Moments (Tables 1, 2 and 3)

The Tables 1, 2 and 3 illustrate the magnitude of torsional moments induced due to the skew bending effect. Torsion steel was required for all five bridges as per references (5), (6).

## 6. CONCLUSIONS AND RECOMMENDATIONS

Analysis and design of skewed bridges requires careful evaluation of skew-bending effects. Three dimensional grid analysis does not give a true account of the behavior of the skewed structure. The results are very sensitive to the torsional rigidity of the members assumed in a grid model and the comfort level for dependability of results is low.

Finite element models, with proper aspect ratios, provide the best means to evaluate the behavior of skewed bridges. Intermediate diaphragms do not have any noticeable effect on the behavior or the re-distribution of loads in a skewed box girder.

Support reactions are unpredictable by conventional procedures. Torsional moments induced by the skew need to be addressed in the design. The torsion capacity of the post-tensioned concrete box girders was not adequate to resist the total torsions induced on the structures.

The load balancing procedure does not offset all the skew bending effects in a post-tensioned concrete structure. The post-tensioning force required for load balancing is much higher than what is required for balancing the stresses.

It seems imperative that structures with skews greater than 25 degrees should be analyzed by an exact method of analysis like finite element methodology to assure the structural design integrity.

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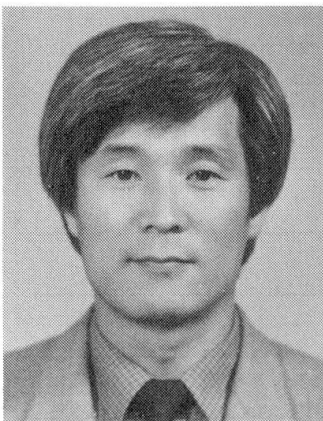
## Behaviour of Prestressed Concrete Bridges Considering Construction Stages

Comportement de ponts précontraints en fonction des étapes de construction

Verhalten von Spannbetonbrücken unter Berücksichtigung des Bauzustandes

### Sung Pil CHANG

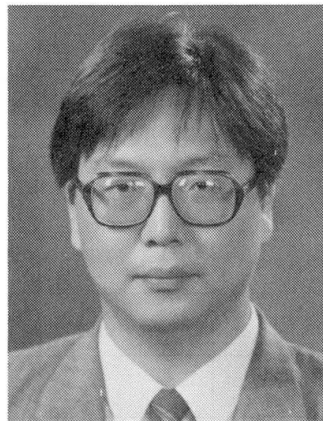
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### SUMMARY

A numerical procedure is presented for 3-dimensional prestressed concrete frame structures with stay cables considering construction stages. A finite element type is developed including the warping and flexural-torsional effects of arbitrary cross sections. The geometric nonlinear analysis during construction and linear dynamic analysis at any construction stage are incorporated. The ultimate load of the structure is not studied. The effects of creep, shrinkage and prestressing are considered by nonconservative equivalent loads.

### RÉSUMÉ

Une procédure numérique est présentée dans le cas de cadres tri-dimensionnels à haubans avec prise en compte des étapes de construction. Une procédure basée sur les éléments finis est développée; elle tient compte des effets de gauchissement et de torsion-flexion de certaines sections-types. L'analyse géométrique non-linéaire effectuée durant la construction, ainsi que l'analyse dynamique linéaire y sont également incluses. Le charge ultime de la structure n'est pas étudiée. Les effets du fluage, du retrait et de la précontrainte sont pris en considération sous forme de charges équivalentes non-conservatives.

### ZUSAMMENFASSUNG

Ein Berechnungsverfahren für die dreidimensionale Rahmenstruktur aus Spannbeton mit Schrägkabeln unter Berücksichtigung des Bauzustandes wird vorgestellt. Ein finites Element wird entwickelt, welches die Effekte der Verwölbung und Biegedrillung eines beliebigen Querschnittes enthält. Die Traglast des Tragwerkes wird nicht untersucht. Die geometrisch nicht-linearen, statischen Analysen während des Bauzustandes und die linearen dynamischen Berechnungen für jeden Zeitpunkt der Konstruktion sind inbegriffen. Die Effekte des Kriechens, des Schwindens und der Vorspannung werden durch nicht-konservative Gleichgewichtskräfte berücksichtigt.



## 1. INTRODUCTION

In general, prestressed concrete bridges with long spans are constructed by carefully selected construction methods. Most of these construction methods consist of a number of construction steps which normally mean a number of different structural systems during construction. Moreover creep and shrinkage of concrete cause stress redistribution which makes the behaviour of structures more complicated. During the past two decades, a series of efforts to simulate complicated construction methods were made as shown in Table 1, where this study is listed together and compared. [1,2,3,4,5]

The purpose of this paper is to describe the geometric nonlinear analysis procedure of prestressed concrete bridges with flexural-torsion and warping effects implemented in the program SNUBR and present several examples for the validity and applicability.

ANALY -SIS PRO -GRAM	Used Element		Additional degree of freedom		Shape of section	Segmental Creep & Shrinkage Analysis	Geometric Material nonlinear
	Frame d.o.f	Stay Cable	warping	distorsn			
R M	6	Used				*	
B C	3	Used				*	
SEGAN	8		*	*	1 cellbox	*	
SFRAME	3					*	
SPCFRAME	3					*	G + M
present SNUBR	7	Used	*		arbitrary	*	G

Table 1 Computer Programs for Prestressed Concrete Bridge

## 2. FINITE ELEMENT ANALYSIS

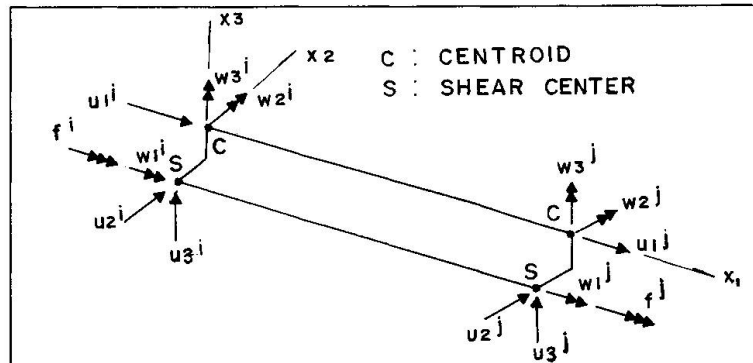
### 2.1 General Remarks

A numerical procedure for the geometric nonlinear analysis considering feasible construction stages and linear dynamic analysis at any construction time of three dimensional prestressed concrete frames is presented. This study includes not only the time dependent effects due to load history, creep, shrinkage and aging of concrete and relaxation of prestressing tendons and stay cables but also the flexural-torsional effects including warping in structures with nonsymmetric sections. In the present study tangential stiffness matrices of the elements are obtained using updated Lagrangian formulation based on finite element method. The following assumptions are based.

- (1) All materials are linear-elastic.
- (2) It is assumed that the shape of a cross section varies only with warping.
- (3) Nonlinear terms in shear strains of a frame element are neglected.
- (4) Large displacements and rotations and small strains are assumed.

**2.2 Stiffness and Mass Matrix**

The frame element has two nodes and each node has 7 degrees of freedom including warping and flexural-torsional effects as shown in Fig.1. The cross section of a frame element has an arbitrary shape which is represented by 7 sectional constants and shear center position. The 14 x 14 tangential stiffness matrix is derived from the virtual work theorem and 14x14 mass matrix based on consistent mass is derived from the energy principle and variational method. Stay cable elements are included in the present nonlinear analysis as truss elements using Ernst's equivalent elastic modulus. Stiffness and mass matrix are also derived by a similar method with frame elements.



**Fig.1** Displacements of a frame element

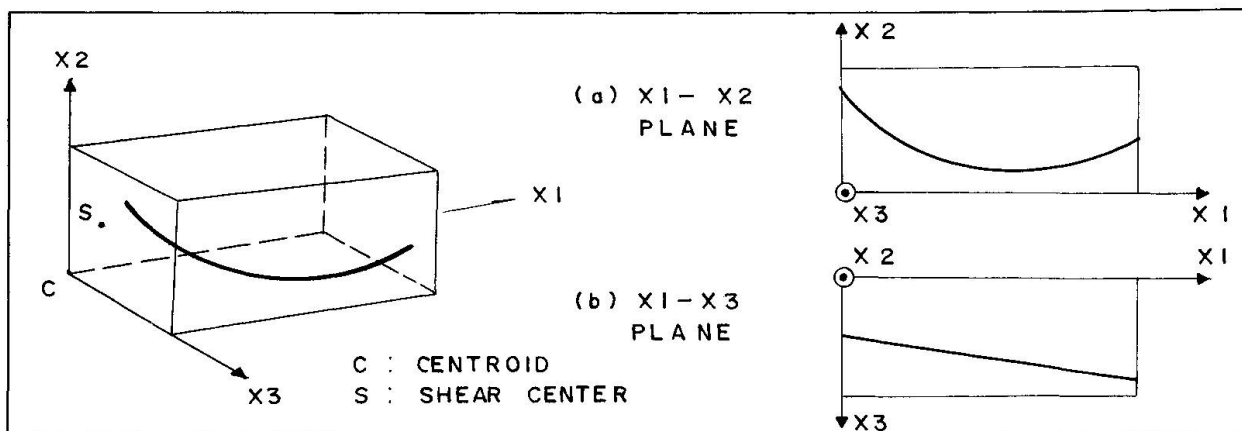
**2.3 Time Dependent Effects in Concrete Members**

The typical time dependent characteristics of concrete, i.e. creep, shrinkage and aging are considered in the present study. It is assumed that creep and shrinkage influence axial deformation, bending deformation, pure torsion deformation and warping deformation at same time. As creep and shrinkage models CEB/FIP and ACI model are used which define their own strain behaviour with time due to loading-unloading stress condition.

The time dependent analysis is based on the superposition principle. The mathematical forms of the strain and curvature increments are obtained using DIRICHLET series using the previous time interval. So equivalent load vector at each time interval is obtained from the general equilibrium equation of initial strains.

**2.4 Prestressing Force of a Tendon**

The geometry of a tendon within an element is defined by two profiles, i.e. a cubic curve in the vertical projection plane and a straight line in the horizontal projection plane as shown in Fig.2.



**Fig.2** Profiles of a prestressing tendon

The effects of prestressing are considered by the form of equivalent loads. At the jacking stage, the loads are derived from self-equilibrium equations. The equivalent loads by the change of the tendon force due to deformation after



jacking are calculated based on the strains of the prestressed members.

The geometric stiffness of the prestressing tendons are considered in the geometric nonlinear analysis. The geometric stiffness of tendons are derived from the deformation and shape functions of main concrete members because the grouted tendons do not have independent shape functions of deformations but are subject to concrete members. After tendons are stressed, the geometric stiffness of tendons are added to the geometric stiffness of a current structure. And then the geometric stiffness of the total structure is not continuous immediately after stressing of tendons because geometric stiffness of a member is embodied from the total sectional resultants.

### 2.5 Conservative and Nonconservative Loads

In our presentation, the effect of creep & shrinkage and prestressing tendon forces are considered in the form of equivalent loads. The total external loads acting on a structure which are resulted from the prescribed equivalent loads do not keep consistent in the global coordinate system, because the loading directions of these forces would vary according to the relocation of structural members followed by the resulting deformations. Therefore, these forces can be said to be non-conservative. On the other hand, the self-weight of a structure and ordinary loads can be defined as conservative loads because the total external load of them would always keep consistent in spite of structural deformation.

In the case of nonlinear analysis, the unbalanced loads which result in the incremental displacement are found from the difference between the total external loads acting on a structure and the total internal resisting load upto current time-step. Therefore, for the non-conservative forces the total external loads are revised in accordance with the corresponding displacement.

### 3. DESCRIPTION OF COMPUTER PROGRAM

Program named 'SNUBR' was developed in order to analyze the three dimensional prestressed concrete frame structures according to the presented manner. For the simulation of construction steps, the following "Construction Commands" are defined and directly used for user's input data for the help of a free field interpreter adopted in this program. [6]

- |                          |                           |
|--------------------------|---------------------------|
| (1) Time                 | (2) Erect                 |
| (3) Stress/Remove Tendon | (4) Stress/Remove Cable   |
| (5) Load                 | (6) Move/Remove Load      |
| (7) Support              | (8) Change/Remove Support |

Static analysis is carried out according to the successive construction steps simulated by several "Construction Commands" which represent unit construction works and time in days. Each construction command defines structural system and/or produces unbalanced loads at any time. Tangential equilibrium equations are solved by the combined method, i.e. the tangent stiffness method with the iterative method.

In the case of dynamic analysis at any construction time, a linear analysis is carried out based on the structural geometry and tangential stiffnesses transferred from the current stage. Then the assembled mass matrix of frames and stay cables are calculated. Frequency analysis and/or forced vibration analysis due to a moving concentrated load and general time function loads are carried out. For free vibration analysis, Subspace iteration method is used and for forced vibration analysis, Mode superposition method and Newmark direct integration method are used.

4. NUMERICAL EXAMPLES

4.1 Natural Frequencies of Thin Walled Beams

To verify the validity of the present stiffness matrix and mass matrix, ten natural frequencies of a thin walled beam with a nonsymmetric section are calculated. The results are compared with the analysis of P.O.FRIBERG who calculated the natural frequencies of thin walled cantilever as shown in Fig.3 including warping derived from VLASOV's differential equations. The difference is about 1.0% as shown in Table 2. [7]

Compression P= 0		Mode No	Compression P=1790 N	
By FRIBERG	Present		By FRIBERG	Present
31.80	31.80	1	25.01	25.01
63.76	63.77	2	61.28	61.31
137.5	137.8	3	136.0	136.3
199.0	199.1	4	192.4	192.4
278.2	278.6	5	274.9	275.3
483.9	484.6	6	478.5	479.3
556.3	556.4	7	550.7	550.9
657.3	658.3	8	654.8	655.9
767.5	769.2	9	760.8	762.6
1075.	1078.	10	1067.	1070.

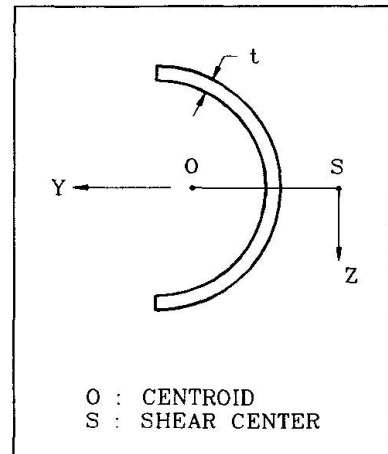


Table 2 Natural Frequencies(Hz) of Thin Walled Cantilever

Fig.3 Half Circle Cross Section

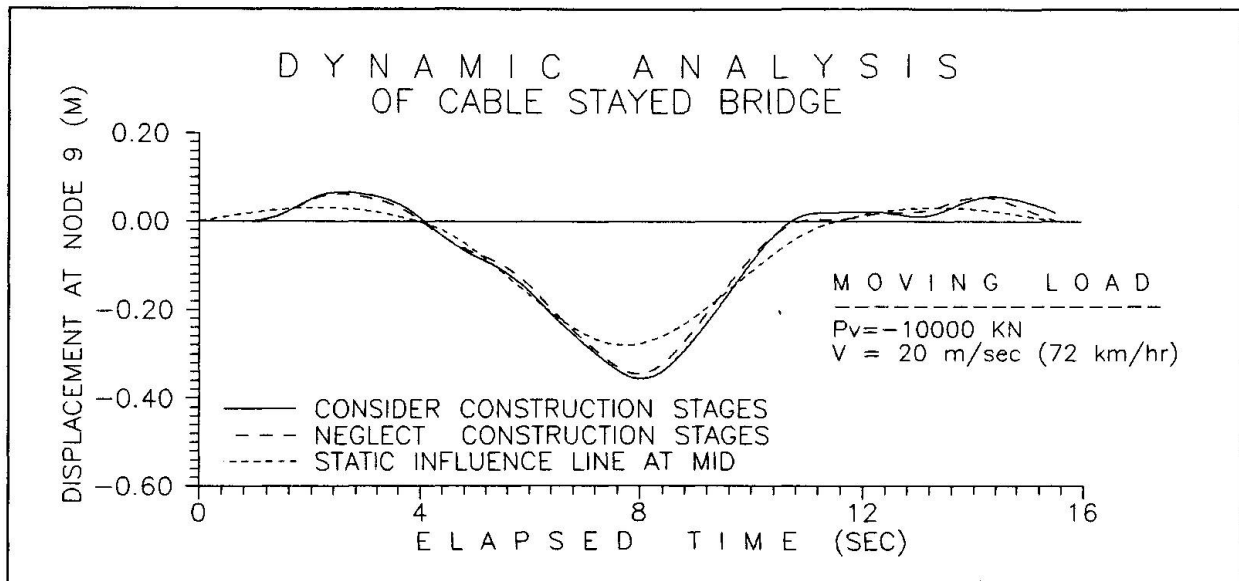
4.2 Dynamic analysis of P.C. Cable Stayed Bridge

In this example, a cable stayed bridge 80 + 150 + 80 = 310 M long is analysed considering construction steps. Pylon is 40m high above deck and 20m long below deck. For construction method, free cantilever method is used. And then the influence on natural frequencies and forced vibration are investigated. The change of natural frequencies as time passes are shown in Table 3, where it is found that 1st natural frequency is increased 4 % as time passes and 6 % error can happen in case of neglecting construction steps.

Construction steps	day	NATURAL FREQUENCIES in Hz				Remarks
		1st	2nd	3rd	4th	
Considered	200	0.2414	0.3205	0.5697	0.6624	4%small
	6000	0.2512	0.3251	0.5866	0.6784	1.000
Neglected	200	0.2560	0.3248	0.5818	0.6730	6 % err
	6000	0.2581	0.3275	0.5860	0.6810	3 % err

Table 3 Natural Frequencies of Cable Stayed Bridge (Hz)

For the effect on forced vibration, a moving concentrated load with the speed of 20m/sec (72km/hr) is applied. As shown in Fig.4, dynamic load factor is increased about 3.5% at the center of the bridge.



**Fig.4** Effect of Moving Load on Cable Stayed Bridge

## 5. CONCLUSION

A numerical procedure and a computer program SNUBR for the geometric nonlinear analysis considering feasible construction stages and linear dynamic analysis at any construction time of three dimensional prestressed concrete frames are presented. This study includes not only the time dependent effects due to load history, creep, shrinkage and aging of concrete and relaxation of prestressing tendons and stay cables but also the flexural-torsional effects including warping in structures with nonsymmetric sections. Capabilities of the program has been demonstrated by several examples. It can be concluded that the program SNUBR can be useful tool for the analysis and design of segmentally erected prestressed concrete bridges with stay cables.

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## Lateral Load Behaviour of Large Panel Precast Buildings

Comportement sous charge latérale de bâtiments réalisés  
avec de grands panneaux préfabriqués

Verhalten von aus Stahlbetonscheiben vorgefertigter Hochbauten  
unter horizontaler Erdbebenbelastung

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### SUMMARY

This paper describes the observed behaviour of a large panel concrete precast building under simulated seismic loads and its mathematical modeling. This mathematical model, calibrated by a series of static tests, was used to study the dynamic response of the structure for different types of earthquake motions.

### RÉSUMÉ

Cet article décrit le comportement d'un édifice réalisé avec de grands panneaux préfabriqués soumis à un séisme simulé, ainsi qu'à sa modélisation mathématique; celle-ci étalonnée par une série d'essais statiques, a été exploitée pour étudier la réponse dynamique de la structure pour divers types de mouvements pouvant être provoqués par un tremblement de terre.

### ZUSAMMENFASSUNG

Dieser Bericht beschreibt das Verhalten von grossen Stahlbetonscheiben vorgefertigten Hochbauten unter der Einwirkung von seitlichen Erdbebenlasten und das sich daraus ergebende mathematische Modell, das aus einem 1:1-Versuch hervorging. Dieses Modell wurde erweitert, um auch Antwortspektren verschiedener Erdbeben abzudecken.



## 1. INTRODUCTION

An insight into the behavior of large panel precast reinforced concrete buildings under lateral loading and its mathematical modeling has been obtained through a series of full scale lateral load tests on a five story large panel precast building. The static lateral loads simulated the distribution the inertial forces during an earthquake. Mathematical models were developed before the tests, which were subsequently calibrated using the actual test results. These models were extended to cover the behavior under seismic actions, which led to definition of behavior patterns under different types of earthquakes [1]. The building tested was very lightly reinforced and represented the type of precast construction used in the Republic of Colombia before the enactment of the Colombian Code [2]. In addition the building was not connected with tension reinforcement to the foundation.

## 2. EXPERIMENTAL PROGRAM

### 2.1 Precast Building Tested

The full scale precast building tested was a five-story low-rise apartment building with an area per floor of 157,64 m<sup>2</sup> as shown in Figure 1. The clear height of the precast panels was 2,20 m and the floor slabs had a thickness of 250 mm. The total height of the building was 12,25 m (Figure 2). The total mass of the building was 460 metric tons. All continuity reinforcement was located and welded in the gaps between precast elements. These gaps were filled later with grout. All the precast panels were 80 mm thick. The internal reinforcement of the panel did not extend from the borders of the panel. Each panel had horizontal bars that extended into the vertical connection. One 9,5-mm diameter vertical reinforcing bar was located in all vertical joints between panels. All 63 vertical joints between precast panels had vertical reinforcement; but only in 24 of them it was continuous from floor to floor. No vertical reinforcement was anchored to the foundation. The building was, therefore, very lightly reinforced having only 24 bars with a 9,5 mm diameter from top to bottom without any connection to the foundation. All floors, including the roof, were made up of plant precast steam cured elements. The edges the floor elements had connecting reinforcing bars that were welded to similar bars in the neighboring elements, achieving the continuity needed to obtain a floor diaphragm effect.

### 2.2 Test Setup and Procedure

Lateral loads were applied from the reaction frame to the precast building using high-strength steel cables attached to hydraulic rams. Two rams per floor were used. Applied loads were measured with twenty load cells, one at each ram. Lateral deflection measurements were obtained using extension bars that recorded the drift (relative lateral deflection) between floors. Each extension bar was provided with a 0.001-in mechanical dial gage. Horizontal and vertical base movements were obtained with additional mechanical dial gages. Ambient vibration records were made, before and after each series of lateral load applications.

### 2.3 Earthquake Simulator Friction Tests

A series of static and dynamic tests [3], with the dynamic tests including two different types of earthquakes, were performed as part of the research project on the earthquake simulator of the Católica University in Lima, Peru. The prototypes were designed to have normal contact stresses in the same range of those found in the horizontal joints of the precast building. The observed static friction coefficient ranged from 0,65 to 0,68. The dynamic friction coefficient observed in the harmonic tests varied from 0,71 to 0,73. In the seismic tests a value of 0,78 was obtained. Based on this, for the evaluation of the experimental tests performed on the precast building a value of 0,66 was used.

### 3. EXPERIMENTAL RESULTS

#### 3.1 Base Shear

Seven lateral load tests were performed on the structure from July to September of 1989. These tests are named 0, A, B, C, D, E and F. Table 1 gives a summary of the direction of loading of each test, the maximum static base shear reached,  $V_{max}$ , in kN and the value of the seismic coefficient,  $C_s$ , defined as  $C_s = V_{max}/W$ , where  $W$  is the active lateral load mass ( $W = 437$  metric tons). Test 0 corresponds to first cracking of the building in the E-W direction. Test B corresponds to the first time the lateral load strength of the building was reached in the E-W direction, obtaining a seismic coefficient of 37,9% of the mass of the building, value that was surprisingly high for a building so lightly reinforced and disconnected from the foundation. Test C correspond to the only test that was performed in the N-S direction and it did not reach the strength of the building although a base shear of 37,3% of  $W$  was measured. With Test E the lateral strength of the building was reached in the opposite direction (W-E) of Test B. A value of  $C_s$  of 41% of  $W$  was measured. Test F was performed to obtain the reduction in stiffness of the building after having reached twice the lateral load strength.

#### 3.2 Measured Deformations

A large decrease of stiffness was observed for the two tests that reached the strength level in the E-W direction (Test B) and then in the W-E direction (Test E) as the roof deflection reached in Test B was 16,5 mm for a base shear of 37,9% of the mass of the building, compared with 90,6 mm for a base shear of 41% of the mass of the building. It is note worthy that only for Test E the story drift in two of the floors was greater than 1% of the story clear height. For Test E the graph of story shear vs. story drift is presented in Figure 3. From these graphs it is evident that the structure went well into the inelastic range.

#### 3.3 Observed Cracking

During Tests 0 and A, no cracking due to the applied lateral loads was observed. During Test B the first cracks in the vertical joints between precast panels were observed. These cracks extended as the lateral load was increased. Then the first cracks in the top horizontal joint of the panels appeared in the north and south exterior walls of the western sector of the building. These horizontal cracks had extended to all the building before the strength level was reached. No internal cracks in the precast panels were observed, with the exception of panels with window openings. During Test C, the only one in the N-S direction, no cracking was observed, different form that caused from the previous tests. The same is true for Test D. For Test E there was a notorious increment of the width of the vertical cracks in the vertical joint of the precast panels. There were interior cracks in some of the panels. Notorious work in the horizontal joints with vertical continuous reinforcement caused by sliding of the joint was observed.

### 4. INTERPRETATION OF OBSERVED BEHAVIOR

The cracking sequence was: (a) Fine cracking was observed in the vertical joints between panels ( $C_s=25,6\%$ ). (b) The cracks extended and increased in width as more lateral load was applied. (c) The first cracks at the top horizontal joint of the panels appeared at the third floor of the West side of the precast building at a  $C_s$  of 32,1% of  $W$ . (d) The cracks at the top joint of the panel had appeared in all of them when the  $C_s$  reached 35,7% of  $W$ . The load was applied from East to West and the first horizontal cracks appeared in the West zone of the building, where the overturning moment introduces compression. These means that the cracks were not caused by flexural induced tension; they were caused by roll-over (overturning) of the precast panels. This interpretation was confirmed by the mathematical modeling.



## 5. ANALYTICAL MODEL

The analytical evaluation of a large panel structure has been performed traditionally assuming that the structure is monolithic. The internal stresses are then obtained from a linear elastic analysis and strength for the elements and their connections are provided for these stresses. A "box effect" is present due to the non planar connection between precast elements thus forming a three dimensional structure. Large differences between the theoretical behavior and the one encountered in tests due to the use of planar analytical models.

An analytical model appropriate for describing the behavior observed during the experimental tests of the building was developed before the tests were carried out and was subsequently calibrated using the experimental results. The model takes into account the non-linear response of the structure by modifying the stiffness of the different elements as they are stressed by the lateral load. A procedure of successive linear stages was adopted to describe the general non linear behavior. The analysis is initiated with a monolithic structure. With the results of this analysis the possible modes of failure of the elements and their connections are evaluated and the one that occurs at a lower lateral load level is taken as critical. The properties of the critical element or connection involved are then modified. The level of lateral load and deflection obtained are recorded and a new linear elastic analysis is performed where a new evaluation is performed, giving a new critical element or connection. This procedure is repeated until loss of equilibrium is detected. The modes of failure that are verified correspond to: (a) sliding of the horizontal connection, (b) overturning of the panel, (c) cracking and sliding of the vertical connection between panels, (d) internal shear failure of the panel, and (e) yielding of the vertical continuity reinforcement. The results obtained with this mathematical model correspond to the first lateral loading of the building. Figure 4 shows, for the E-W direction, the results obtained for the third floor as compared with the measured experimental results of the same floor for Tests 0 and B. The agreement of the results is significant and the same is true for all the other floors.

## 6. DYNAMICAL RESPONSE IN THE INELASTIC RANGE

Using the program presented in [4] and linear idealizations of the relations between story shear and story drift obtained from the experimental lateral load tests of the precast building, the response of the building to base excitation representing strong ground motion corresponding to different soil conditions was obtained. A parametric study of the dynamic response was performed for cases that included: (a) variations of the hysteretic characteristics, (b) different levels of lateral stiffness of the structure, (c) earthquake records with different types of soil conditions, (d) peak ground acceleration of the earthquake records, (e) effects of damage accumulation caused by the structure being subjected to several successive earthquakes, and (f) an earthquake acting in a direction at an angle with the principal plan directions. Records obtained in El Centro (1940 Imperial Valley Earthquake), Castaic (1971 San Fernando Earthquake), Wilshire (1971 San Fernando Earthquake), Viña del Mar (1985 Chile Earthquake), Miyagi (1978 Japan), and Mexico City (1985 Mexico Earthquake) were used.

From the observed damage during the static tests of the precast building, bounds for the response were established. These bounds were a story drift of 4 mm for minimal damage and a story drift of 24 mm for the threshold of life threatening damage.

The effect of the peak ground acceleration is shown in Figure 5, where the maximum story drifts are plotted against peak ground acceleration of the earthquake records. Only those cases where the stiffness of the structure was reduced to one half and one quarter of the initial stiffness have story drift beyond the 24 mm limit. With the soft soil records (Mexico and Miyagi) the maximum story drift obtained was 1,2 mm, well into the minimal damage zone. For damage accumulation, three successive application of the full El Centro record, showed for the third application a maximum story drift of 24,3 mm.





## 7. CONCLUSIONS

A full scale very lightly reinforced large panel precast building was subjected to seven lateral load tests obtaining base shear strengths of the order of 3/8 of the mass of the building and load deflection responses that exhibited a non fragile capacity. Even though the building was not connected to its foundation this did not affect the static lateral load strength of the building. It was possible to explain the failure mechanisms of the building and to establish analytical bounds of response under lateral loads. The static and dynamic friction tests of concrete surfaces cast at different moments gave an insight of the friction phenomena and permitted the development of an analytical model for the response of this type of structures to base excitations representing strong ground motion. Using the dynamic analytical model it was possible to confirm that ground motions rich in short vibration periods tended to affect the building more than motions rich in long periods as encountered in earthquake records from soft soil sites.

The observed behavior in the tests and the results obtained from the analytical models could serve as a basis for future performance requirements [5] and guidelines for analysis methodologies [6] for seismic loading for large-panel precast buildings.

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**TABLE 1**  
**SUMMARY OF THE STATIC EXPERIMENTAL LATERAL LOAD TEST**

TEST	-O-	-A-	-B-	-C-	-D-	-E-	-F-
Direct.	E-W	E-W	E-W	N-S	W-E	W-E	W-E
$C_B$	0,209	0,151	0,379	0,373	0,210	0,410	0,284
$V_{max}^*$	913	660	1655	1630	918	1793	1241

\*  $V_{max}$  in kN

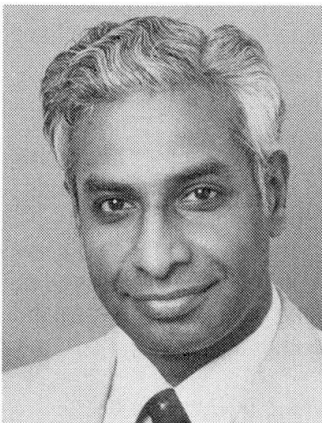
## Urban Interchanges on Elevated Structures

Echangeurs routiers en milieu urbain

Neuartige Kreuzungen innerstädtischer Hochstrassen

### Vijay CHANDRA

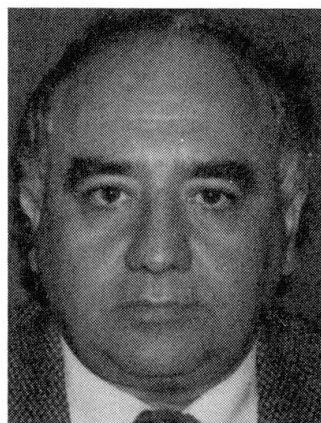
Vice-President  
Parsons Brinckerhoff  
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Vijay Chandra, P.E., vice-president and senior professional associate at Parsons Brinckerhoff, a leading US transportation design firm, has directed the structural design of several award-winning bridges, and is a member of the PCI Bridge Committee.

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### SUMMARY

This paper illustrates the design and construction in Phoenix, Arizona, of a novel type of urban interchange on elevated structures. This approach can often be implemented at a lower cost and on smaller sites than traditional multilevel interchanges.

### RÉSUMÉ

Cet article illustre la conception et la construction à Phénix, Arizona, d'un nouveau type d'échangeur routier en milieu urbain, situé au-dessus de structures élevées préexistantes. Cette approche peut souvent s'appliquer de façon plus économique et sur des surfaces plus restreintes que les solutions traditionnelles d'échangeurs à plusieurs niveaux.

### ZUSAMMENFASSUNG

Dieser Vortrag behandelt den Entwurf und die Konstruktion neuartiger Kreuzungen innerstädtischer Hochstrassen. Diese Lösung kann oft mit geringeren Kosten und mit geringerem Landverbrauch als bei den üblichen mehrstöckigen Kreuzungen erreicht werden.

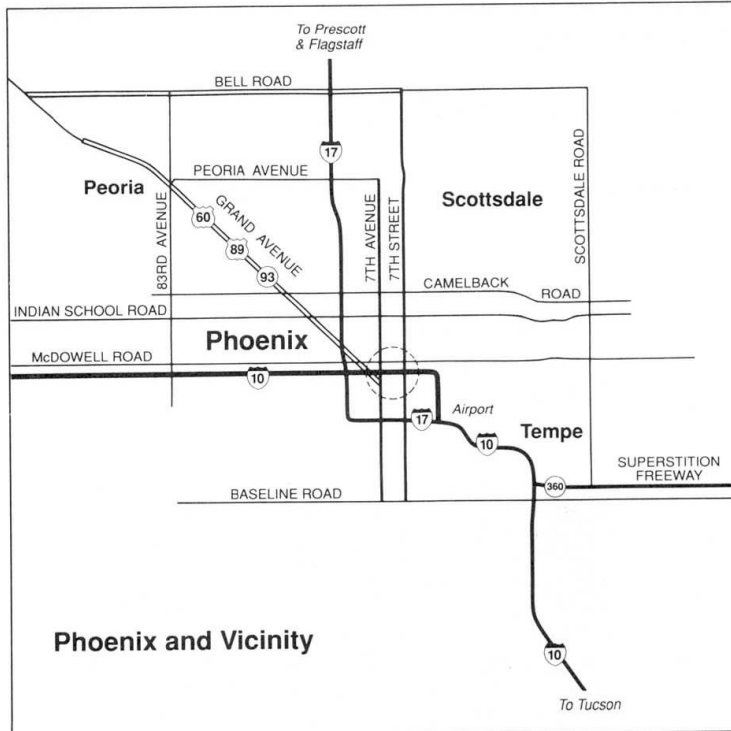


Figure 1. Site Location



Figure 2. Bird's eye view (Seventh Avenue)

## INTRODUCTION

In Phoenix, Arizona, two prestressed concrete interchanges of complex geometry have proved an appropriate structural solution for compressing highway structures into the cramped urban landscape. The elevated structures, linking the north-south arterials Seventh Street and Seventh Avenue to the new depressed highway I-10, are neither full multilevel elevated interchanges with on/off ramps on independent structures, nor completely ground level interchanges. They are in essence compact urban interchanges that have been partly elevated, with certain ramps compressed right into the main structure to reduce the ordinary four levels of an elevated interchange to two.

This paper examines the choices of analytical approach and of construction methods that made rapid (one-year) design and construction possible despite the complexity: finite element analysis of the structures, and postprocessing of results into formats directly useful to engineers.

At these sites, where I-10 was to be connected through the heart of metropolitan Phoenix (Fig. 1), right-of-way could be costly, and lengthy construction disruptive. The Arizona Department of Transportation therefore favored an urban interchange over a multilevel interchange. It was found, however, that the scarcity of land would force the ends of certain ramps to fall *on structure*, with motorists executing their turns on the structure itself (Fig. 2).

## MAKING THE INTERCHANGE CONCEPT WORK

Fusing into the main structure four of the usual eight on/off ramps achieved compactness, but at the cost of generating ramp-structure interactions that challenged analysis. The ramps rest on curving projections of the main spans, making a flared shape, wider at the abutments to accommodate the ramps (Fig. 2). Each structure would look from above like an hourglass, wide at the abutments and narrow at the central pier—the waist of the hourglass. The concept was simple, but the design and construction of such unusually shaped structures would not be.

What makes the urban interchange on structure so complex for design and analysis is the interactive load transfer among all these parts that rest on one another. The loads of the four curved and skewed prestressed concrete ramps, for example, would have to be transferred onto the prestressed concrete main span late in construction—when a small error might easily make a ramp rotate from its own weight instead of uniformly sitting on the bearings. The interactive stresses of the entire odd-shaped concrete structure would have to be understood at each stage of construction and load transfer if the structures were to survive the hard-to-predict stresses of their own construction. The final simplicity would be achieved at the price of a major analytical effort, individualized at each of the two sites.

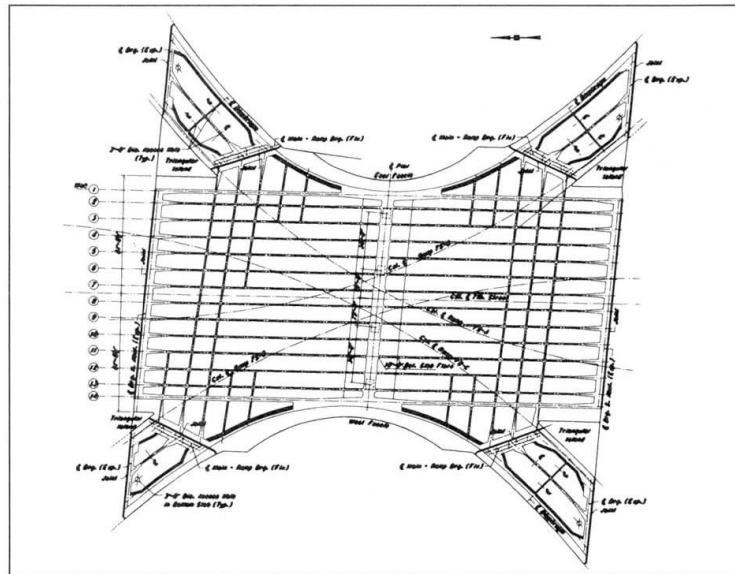
**UNUSUAL STRUCTURES/UNUSUAL ANALYSES**

Unusual structures require unusual analysis techniques. State-of-the-art finite element technique with up to 2,000 elements of various shapes was employed to model the structures, and greatly assisted in understanding the interactive stresses and the true behavior of the ramp structures with their extreme skews.

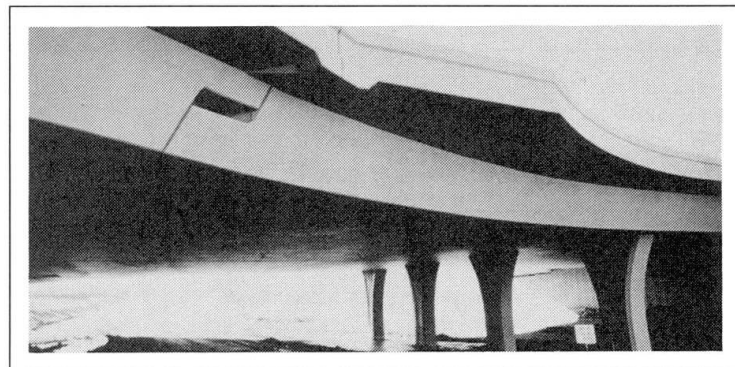
The main part of the structures at both locations is a two-span continuous post-tensioned concrete structure supported on concrete abutments at both ends and supported through the midsection by concrete columns erected in the median of I-10. The curved ramp structures are of concrete box section, each of a different length and different skew to the main structure (Fig. 3). The portion that is on structure rests at one end on a protrusion on the side of the main structure (Fig. 4). The triangular gaps resulting from the peeling off of the ramps from the main structure in front of the abutments also had to be covered—to support landscaping in these visually important spots. Both structures included separate pedestrian walkways, with ramps for disabled persons.

In part, the design was driven by aesthetic needs, and planned to be visually pleasing for users and neighbors alike. Seen from above it flares out like a great concrete hourglass, wide at the ends. Seen in profile by I-10 motorists, its notched curves arc powerfully against the sky. Driving up the ramps, motorists discover landscaping high above ground level, thanks to hidden structural elements that take the weight of earth and plants.

To meet our aesthetic goals, we streamlined the fascia to present a smooth, aesthetically pleasing appearance to the travelling public on I-10, with canopied fences that offered smooth transitions at their ends and at the intersections of the ramps and the main structures. Even landscaping requirements affected structural design: the final landscaping decision to plant shrubs into the triangular areas formed between the ramp structures, main structure, and abutments was not a decision calculated to make life easy for the structural designer. We would need to support a layer of soil at least a foot deep and accommodate a maintenance



**Figure 3.** Typical framing (Seventh Street)

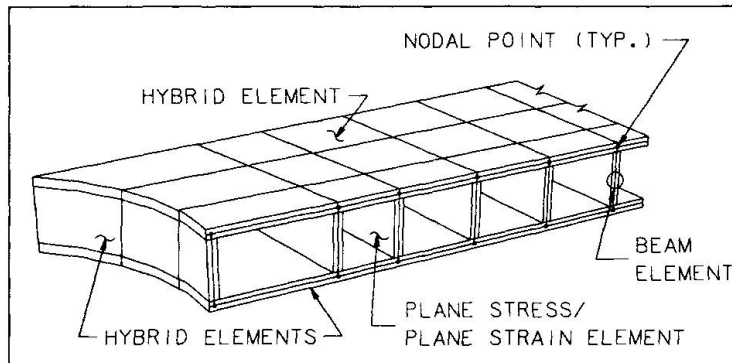


**Figure 4.** Ramp (left) sitting on the main structure (right)

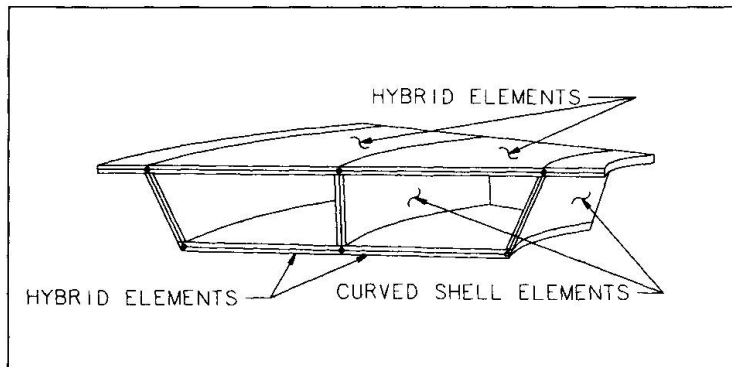
THE STRUCTURES AT A GLANCE		
	Seventh Street Structure	Seventh Avenue Structure
<b>Main Structure</b>		
-Span lengths ft	121' & 121'	141' & 129'
-Width ft	260' & 280' at abutment & 125' at piers	360' & 400' at abutments & 130' at piers
-Type	Cast-in-place post-tensioned concrete with curved fascia	Cast-in-place post-tensioned concrete with curved fascia
<b>Ramp Structures</b>		
-Span lengths ft	Vary, 60'-85'	Vary 110'-170'
-Width ft	32'	32'
-Type	Cast-in-place curved reinforced concrete	Cast-in-place curved post-tensioned concrete
<b>Foundation</b>	Spread footings on S-G-C (sand, gravel, and cobbles) material	Spread footings on S-G-C material
<b>Triangular areas</b>	Cantilever slabs with 29' max. overhang	Cantilever slabs with 27' max. overhang coupled with pretensioned concrete beams



vehicle live load right in the gap between structures. Bridging this gap to support that load threatened to create major structural problems by imposing heavy loads on the ramps. By cantilevering the abutment wall up to about 25 feet, however, much of the triangular area load was relieved off the ramps and transferred directly to the abutments. At Seventh Avenue, the cantilevering alone was insufficient to cover the entire triangle, and the gap beyond the cantilevers had to be covered using precast concrete elements. Before placing the fill for landscaping, we waterproofed the top of the cantilever and precast elements.



**Figure 5.** Finite elements of main structure



**Figure 6.** Finite elements of ramps

### FINITE ELEMENT APPROACH

The Seventh Street and Seventh Avenue structures were similar in concept but would have to be analyzed independently because of their unique shapes, geometric nonsimilarities, ramp-structure variations, span length variations, etc. What mathematical modeling method we chose for all this analysis might well determine what questions we could ask effectively.

Surveying the available methods, we closed in on two finalists. The space grid method simplifies the structure into a Tinkertoy-like skeleton of lines and nodes; the finite element method has similar nodes, but connects them by surfaces instead of lines. Thus a real three-dimensional beam would be represented as either a single line (space grid) or as a set of surfaces, as if boxing-in the beam (finite element).

Before embarking on the most appropriate method, we critically compared the two candidate methods as to their ability to reflect the true behavior of such difficult features as transfer of torsional loads,

transverse stress distribution, temperature gradient, post-tensioning forces, skewed ends, and curved surface areas. The evaluation showed the finite element procedure to be superior.

Before applying the finite element concept to the entire structure, we evaluated discrete structural models for the various element shapes and applied loads. Once a 'level of comfort' was achieved, the full-structure model was pursued.

The main structure required about 1,700 finite elements for the Seventh Avenue structure and 1,400 finite elements for the Seventh Street structure. Each of the ramp structures was individually analyzed and required about 160 elements. The types of elements were quite varied, including trapezoidal and triangular hybrid for slabs, diaphragms, and curved fascia webs of the main structure; curved shell for webs of ramps; and plane-stress/plane-strain for the longitudinal webs of the main structure. Beam elements were introduced between adjacent plane-stress/plane-strain elements (Figs. 5, 6).

Transient loads, such as live loads, were applied on the top slab as surface pressures. After careful evaluation of alternative modeling approaches, post-tensioning loads were applied as equivalent loads using the parabolic curve approach along the webs and concentrated loads at the ends of the elements. The vertical, lateral, and axial components of the post-tensioning forces were input.

In order to fathom the finite element run output in easy-to-understand terms, we developed a postprocessing program that reduced the stresses at any particular nodal point to shear and moment forces. This proved a tremendous help in verifying the behavior of the structure.



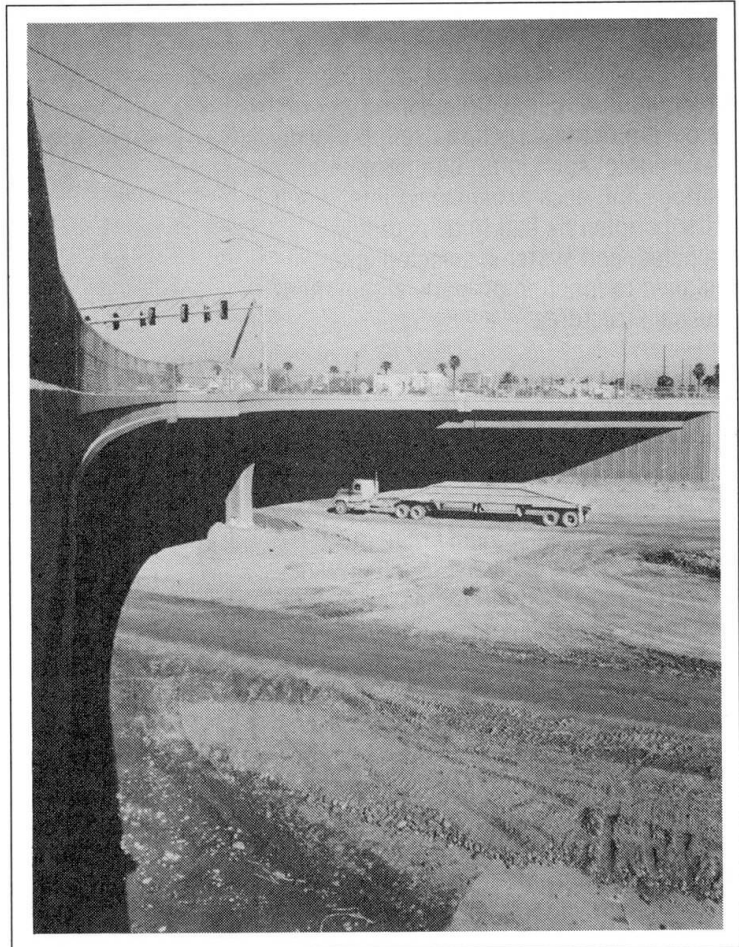


Close coordination between the designer and the contractor is a must: both benefit from the interaction, and this experience benefits their future projects as well. On these two projects, such cooperation was realized to be essential, and with the timely cooperation of the client, problems as they were perceived and encountered were solved to every party's satisfaction.

In Arizona, most concrete box girder superstructures are poured directly on a 'mud slab' as the base form. The mud slab is a thin unreinforced concrete slab poured to grade on a stabilized fill (the 'mud'). Both these projects benefited from the method, which allowed pouring the deck slabs without recourse to the scaffolding commonly used in other regions.

### CONCLUSION

However difficult the analysis and design, the concept of an urban interchange on structure has proved quickly and economically constructible. The \$2.6 million Seventh Street structure (\$62/square foot) and the \$3.6 million Seventh Avenue structure (\$63/square foot) were each built in about a year, and cost considerably less than comparable multilevel interchanges.



**Figure 10.** A driver's view from I-10 (Seventh Avenue)

Phoenix's new urban interchanges on structure, and the design and construction that made them possible, offer a new option for cities seeking to thread highways and interchanges through the already dense urban fabric (Fig. 10).

### CREDITS

#### *Client*

Arizona Department of Transportation

#### *Managing Consultant*

Howard Needles Tammen & Bergendoff

#### *Structural Engineer*

Parsons Brinckerhoff Quade & Douglas, Inc.

#### *Contractors*

Tanner Construction (Seventh Street Structure)

McCarthy Construction (Seventh Avenue Structure)

Portions of this paper appeared, in different form, in *Civil Engineering* (September 1988), published by the American Society of Civil Engineers, and are reused here with their permission. These structures have received engineering awards from the American Concrete Institute (Arizona), Post-Tensioning Institute, and American Consulting Engineers Council (Arizona).

## Computer Models of Concrete Structures

Modèles informatiques pour les structures en béton

Finite-Element-Modelle von Betontragwerken

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### SUMMARY

The application of the nonlinear finite element analysis of concrete structures as a design tool, is discussed. A computer program for structures in plane stress state is described and examples of its application in the research of fastening technique, and in engineering practice, are shown.

### RÉSUMÉ

On discute ici de l'application, en tant qu'outil de dimensionnement, de l'analyse non-linéaire par éléments finis, et ceci dans le cadre des structures en béton armé. Un programme pour des structures en état plan de contrainte est décrit, ainsi que ses applications dans le domaine de la recherche et de la pratique de l'ingénieur.

### ZUSAMMENFASSUNG

Die Anwendung der nichtlinearen Finite Elemente Analyse auf Betontragwerke als Entwurfswerkzeug wird diskutiert. Ein Programm für die Konstruktionen im ebenen Spannungszustand wird beschrieben und Beispiele für die Anwendung in der Forschung der Befestigungstechnik und in der Ingenieurpraxis werden vorgestellt.



## INTRODUCTION

Due to the complexity of concrete behavior under various states of stress, the development of rational design models is a difficult task. This development is treated in detail by many authors at this Colloquium. Schlaich explains why the design process requires several "design models", because more general approaches are too complex for designers and serve as "research models". MacGregor and Marti show the recent development of engineering design models which have evolved from simple equilibrium truss analogy into more consistent models which take into account strain fields and the constitutive laws. Further refinement of these models would require better constitutive laws, better modeling of the multiaxial stress states and better discretization. Then, of course, the simple models turn into the complex ones. This complexity can be handled in a rational manner by the finite element method. FE models of concrete structures have been in development for over 20 years and are now at a stage, that they can be used as design tools, as demonstrated by Scordelis in this Colloquium. The authors believe, that models of all levels of sophistication have their place in design provided that they are rationally based and verified in practice. It is up to the designer to choose the appropriate model under the given circumstances. However, a unified approach for all design models should be accepted which will assure the compatibility between various levels of sophistication. This is also true for safety concepts, which should be extended to the FEM design models. It should be noted, that the CEB has started a significant effort in this respect. It is the purpose of this paper to demonstrate the capabilities of the non-linear finite element method for the analysis of concrete structures and to show its application as an advanced design model. It will be done on the example of the program SBETA, which was developed by the authors. However, we shall first provide a brief summary of the currently used computational models.

## CONSTITUTIVE MODELING

The performance and quality of the non-linear finite element analysis depends on all of its basic components: constitutive model, finite element discretization, solution technique. Of these components, the constitutive model is the most important since it determines the ability of the analysis to model the specific properties of concrete structures. Therefore, we shall make a brief critical overview of some constitutive models which are important for the development of design models.

In the early stages of development two main classes of constitutive models of concrete were used, namely, the models based on the theory of plasticity and the models based on the non-linear elasticity (hypoelasticity). Each of these approaches can well describe some features of concrete behavior, while other features are modeled poorly. The theory of plasticity is suitable for metals but unsuitable for concrete, which is a quasi-brittle material. Hardening plasticity can model the nonlinear behavior of concrete in compression [13], but cannot model cracking and softening behavior. In that sense, the range of application of the plasticity models in concrete structures is restricted to pre-peak compression. Therefore, the plasticity theory was often combined with the brittle-fracturing model for tension [9,2].

Hypoelastic models have been successfully used by many authors [1,15,19]. The orthotropic hypoelastic models have been criticized for their lack of objectivity [3] in the case of rotation of strain fields. In spite of this, they have the ability to cover a wide range of the concrete behavior, i.e. tension and compression, cracking, and softening.

All of the models described above have typically been used within the "smeared material" approach with a local formulation of the stress-strain laws, where stresses are related to strains at a material point. This "local concept" cannot describe the size effect which is evident from experiments. Introduction of the size effect can be done by means of the "non-local concept",

in which the stresses are related to strains in a certain representative volume [4,5].

Cracking has a dominant effect on the non-linear behavior of concrete structures. Therefore, much research is devoted to improve the fracturing model of concrete, as reported recently during a workshop at Torino [7]. The subject is treated in detail in the papers of Hillerborg and König presented at this Colloquium. It is generally accepted that the tensile toughness of concrete is a material property. It is caused by tension softening response after cracking and is characterized by the fracture energy parameter  $G_f$ . In finite element analysis there are two kinds of crack models. In a discrete crack model, a crack is formed by disconnecting the nodes of the finite element mesh and introducing a new boundary. After cracking, re-meshing must be performed in order to adjust the element boundary to the crack path [8,17,18] unless the crack follows a predefined path along the existing element mesh. The softening is modelled by stress-crack-opening law of the crack interface. In a smeared crack model, a band of parallel cracks is formed in the entire element volume under consideration (e.g. volume associated with the integration point). The softening of the crack band is derived from the fracture energy parameter [6,10]. Thus, both approaches have the same theoretical basis and in many cases should give similar results.

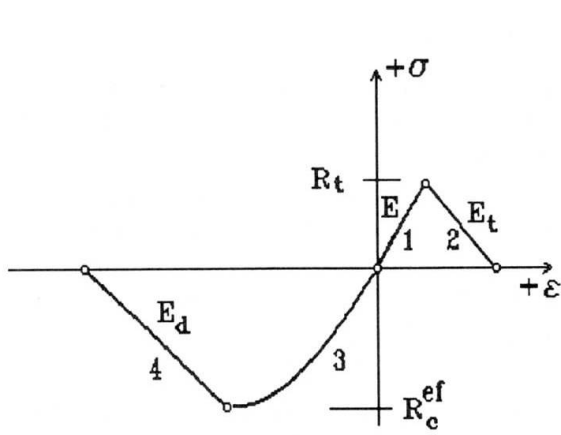
A smeared-crack model based on the orthotropic hypoelastic law can have two basic forms [11]. In a rotated crack model, the axes of principal stress and strain coincide. Rotation of the principal strain axes causes the rotation of material axes (which are coincident with cracks). In a fixed-crack model, the crack direction is determined upon crack initiation, and is kept fixed during the subsequent analysis.

In the fixed-crack model, the crack plane can be subjected to a shear strain and its shear stiffness, representing the aggregate interlock and the dowel action of reinforcement, should therefore be included. This is accomplished by many analysts by means of a shear retention factor, which assigns a constant reduced shear stiffness to the cracked concrete. However, the solution of shear failures is extremely sensitive to the shear retention factor and therefore the use of a constant shear retention factor is not recommended [20]. Improved performance is obtained by decreasing the crack shear stiffness as a function of crack width [11].

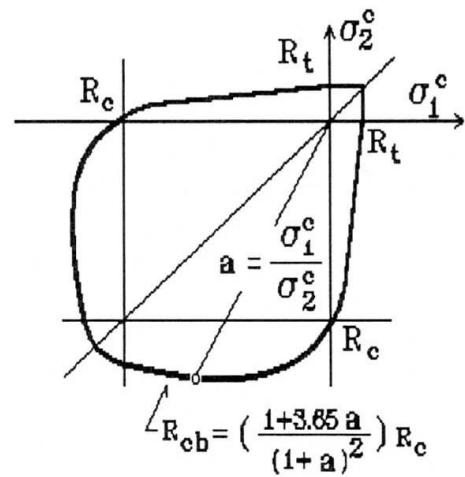
Both discrete and smeared crack models have their own merits. The discrete cracks are appropriate for modeling the fracture of plain concrete with one distinct crack, while smeared cracks are more suitable for reinforced concrete. The advantage of the smeared crack model is that it can cover a variety of crack situations ranging from finely distributed cracks in reinforced concrete to a single discrete crack, without modification of the element mesh, as will be demonstrated in this paper.

All previously described smeared models can be classified as macroscopic models. They directly relate stress and strain components. For general stress states and load path situations, they usually require a large number of material parameters. Further improvement can be expected from a microplane model [5,14]. This is a microstructural model in which the material properties, such as the material stiffness matrix, are integrated from elementar behavior of microplanes. It is a three-dimensional model which is unique for all stress states and a wide range of behavior, including cracking, softening, and dilatancy. It is the most general model developed so far for use in the finite element analysis. It is, however, more demanding on computer capacity because the microplanes introduce another level of discretization. The application of the microplane model is presented in a paper by Eligehausen and Ozbolt at this Colloquium.

The above overview is only a brief outline of the present practice with respect to constitutive modeling of concrete structures. Other aspects of FE modeling, such as the method of finite element discretization and solution techniques, shall not be treated here. The interested reader

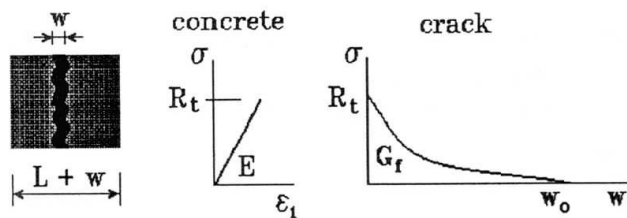


1a. Effective stress-strain law.

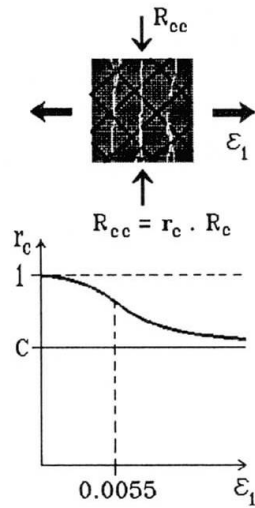


1b. Biaxial failure function.

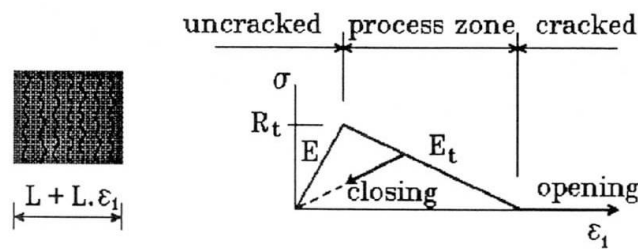
Fictitious Crack:



$$G_f = \int_0^{w_o} \sigma dw$$



Smearred Cracks:

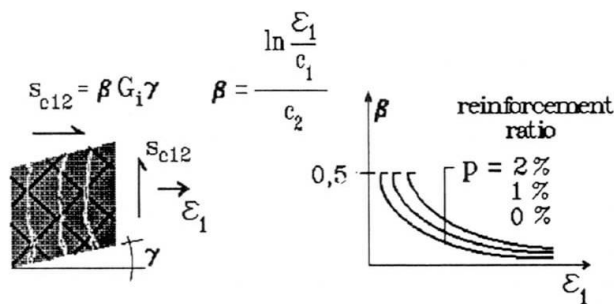


$$\frac{1}{E_t} = \frac{1}{E} \left( 1 - \frac{\lambda}{L} \right)$$

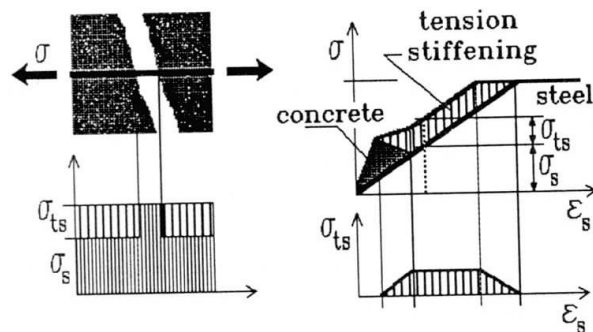
$$\lambda = \frac{2EG_f}{R_t^2} [m]$$

1c. Fracturing in smeared model.

1d. Compressive strength reduction due to cracking.



1e. Shear stiffness of cracked concrete.



1f. Tension stiffening.

Fig.1 Constitutive model in program SBETA.

can find a number of publications on this subject.

## PROGRAM SBETA

Advances in constitutive modeling of concrete and the availability of efficient computers make it possible to produce programs for non-linear finite element analysis which can be used as design tools. Such a program was recently developed by the authors at the Institut für Werkstoffe im Bauwesen at the University of Stuttgart in cooperation with the Building Research Institute of the Czech Technical University in Prague. An overview of the program and examples of its application in research and engineering practice are presented.

The program SBETA [12] is designed for the analysis of reinforced concrete structures in the plane stress state. It can predict the response of complex concrete structures, with or without reinforcement, in all stages of loading, including failure and post-failure. It can serve as a research tool for the simulation of experiments and for the analysis of experimental results. In design practice it can be used to optimize the geometry and reinforcement detailing and to calculate the load-carrying capacity of structures. It can be also used for the diagnosis of the causes of structural damage or failure.

The constitutive model of the program SBETA is summarized in Fig.1. It is based on the smeared material approach using non-linear elasticity and non-linear fracture mechanics. The behavior of concrete is described by a stress-strain diagram, Fig.1a, which is composed of four branches: non-linear loading in compression, linear loading in tension, and linear softening in both tension and compression. The parameters of this diagram are adjusted in the following way: The peak stress  $R_c^{ef}$  is taken from the biaxial failure function of Kupfer, Fig.1b, and the softening modulus in tension  $E_t$  is calculated according to the crack band theory of Bazant [6], Fig.1c.

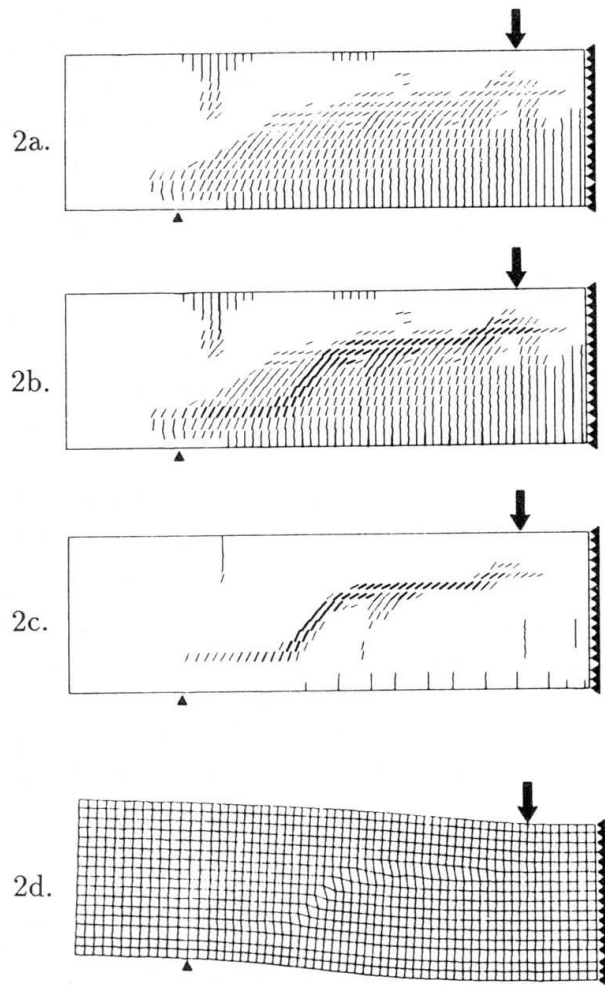


Fig.2 Crack localization in shear.

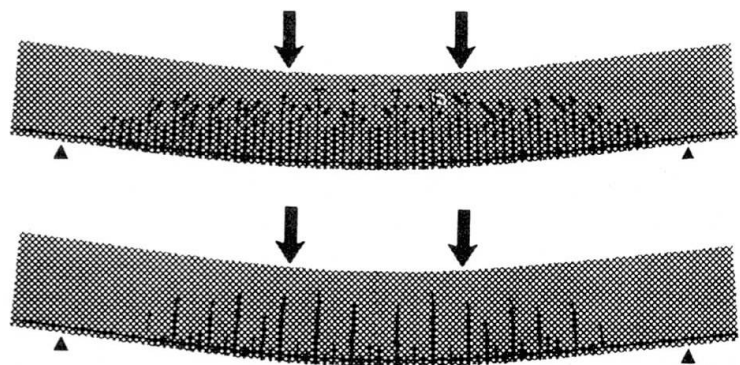


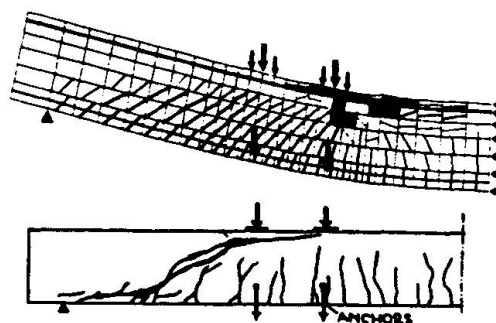
Fig.3 Crack localization in bending.



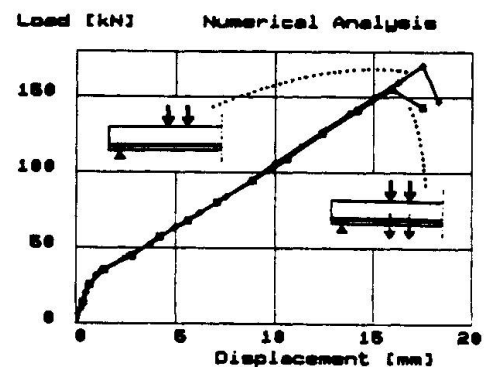
The modelling of cracked reinforced concrete includes the shear resistance of cracks, Fig.1e, reduction of compressive strength in the direction parallel to the cracks, Fig.1.d and the effect of tension stiffening, Fig.1f. Fixed and rotated crack models are implemented. Reinforcement behavior is bi-linear. A monotonic load history is assumed. A four-node quadrilateral finite element is used for the concrete. The reinforcement can be included either smeared, as a part of the concrete element, or discrete, as a bar element passing through the quadrilateral element. The updated Lagrangean formulation is adopted. The non-linear solution is performed by means of step-wise loading and equilibrium iteration within a load step. Newton-Raphson and arc-length methods are the options for the solution strategy.

The program system SBETA includes a pre-processor, FEM solution program, and an efficient post-processor. A graphical, macro-instruction-based pre-processor generates the FE numerical model. The FEM program can be interactively controlled and runs in several levels of real-time graphics. Thus, the solution process can be observed and solution parameters can be adjusted by the user if necessary. A restart option is available. The dialog-oriented post-processor generates the deformed shapes and images of stress, strain and damage fields (cracking, crushing). An efficient data management (generic names, profile files, etc.) enables the generation of animation sequences, which are important for the detection of failure modes.

A special method has been developed to show crack-localization in the smeared material. Due to strain-softening, deformations localize in narrow bands which indicate the main failure cracks. This is demonstrated with the example of a shear failure of a beam without stirrups. Fig.2a shows the entire crack region (only half of the beam was analyzed), Fig.2b indicates strain-localization within the crack zone, Fig.2c shows the location of the failure crack, and Fig.2d the deformed mesh. Another example of crack-localization for bending is shown in Fig.3.



Analytical and experimental failure patterns.



Load-displacement diagrams.

Fig.4 Analysis of the shear resistance of beams with anchors in the tensile zone.

## APPLICATIONS IN FASTENING TECHNOLOGY

Fastening technique is a rapidly developing technology in the concrete industry. The load-carrying capacity of concrete anchors rely entirely on the tensile strength and toughness of concrete. In order to understand the mechanics of anchor failure, the authors have performed a number of numerical studies which simulate experimental investigations. In one such investigation [11] a beam with anchors located in the cracked zone was examined, Fig.4. The computer simulation confirmed the experimentally observed reduction of the shear resistance of beams due to the anchor loads introduced into the bottom of the beam. A similar study was conducted by Eligehausen and Kazic for T-beams, using the material model from SBETA in another program, AXIS (see paper presented at this Colloquium).

The concrete cone failure of headed anchor bolts loaded in tension was the subject of an

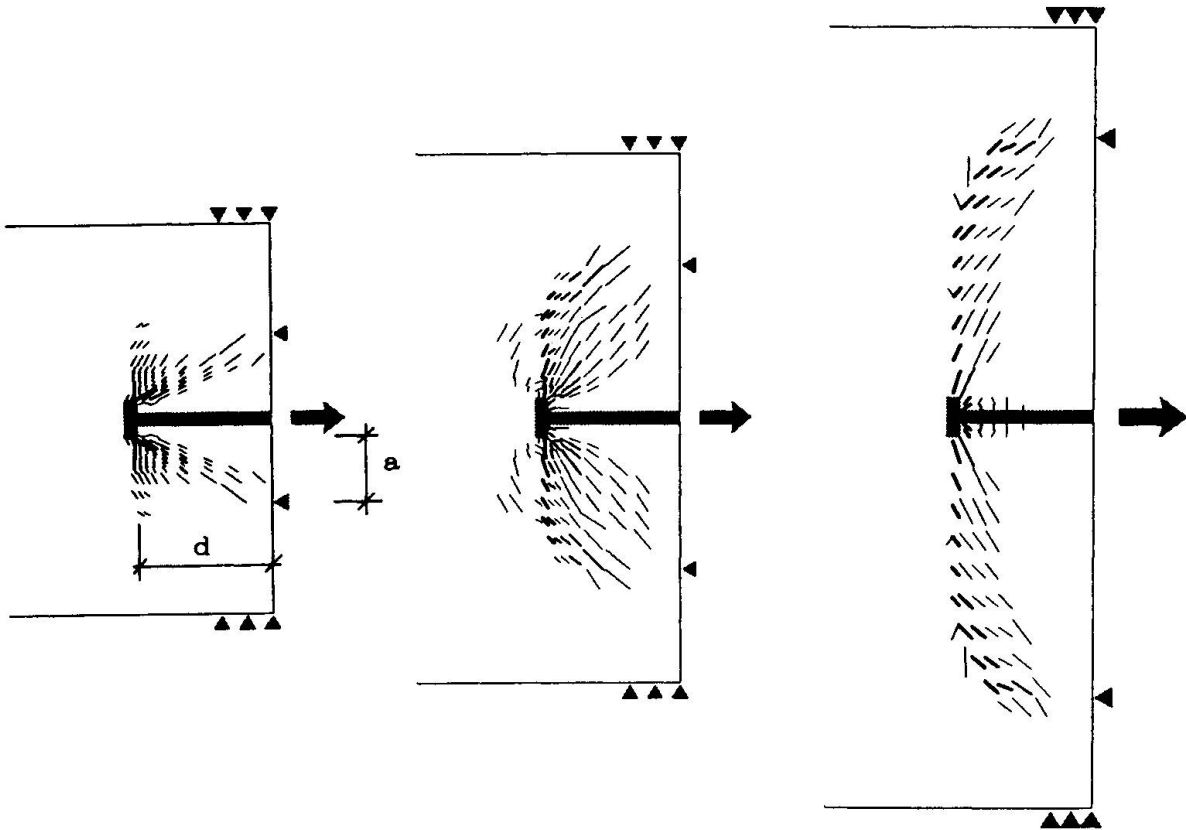


Fig.5 Failure crack patterns of pull-out tests on headed anchors with an embedment depth  $d=150$  mm and three spans  $a=50, 150, 450$  mm.

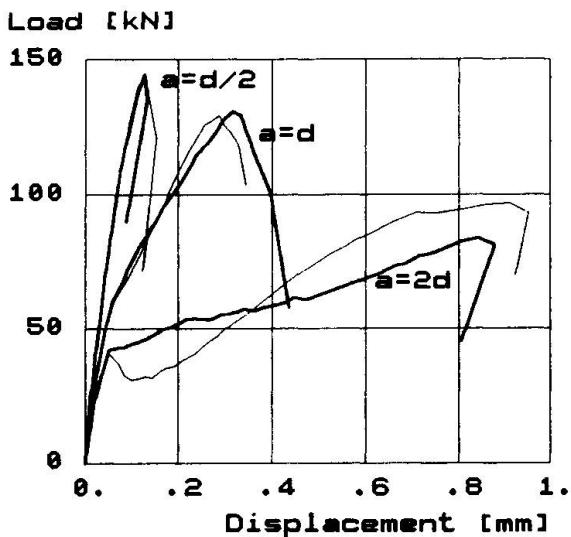


Fig.6 Load-displacement diagrams of pull-out tests for one embedment depth  $d=150$  mm and three spans  $a=50, 150, 450$  mm. Thickness  $b=100$  mm.  
thin line - rotated crack model,  
thick line - fixed crack model

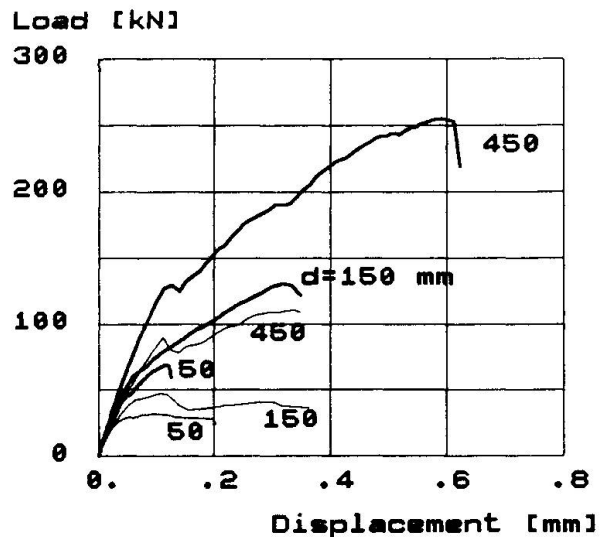
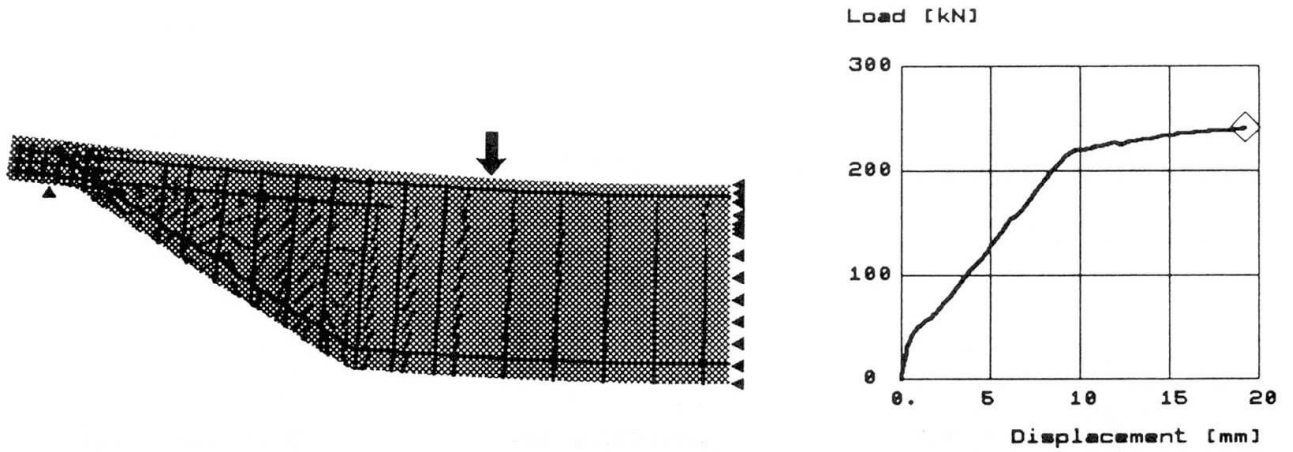


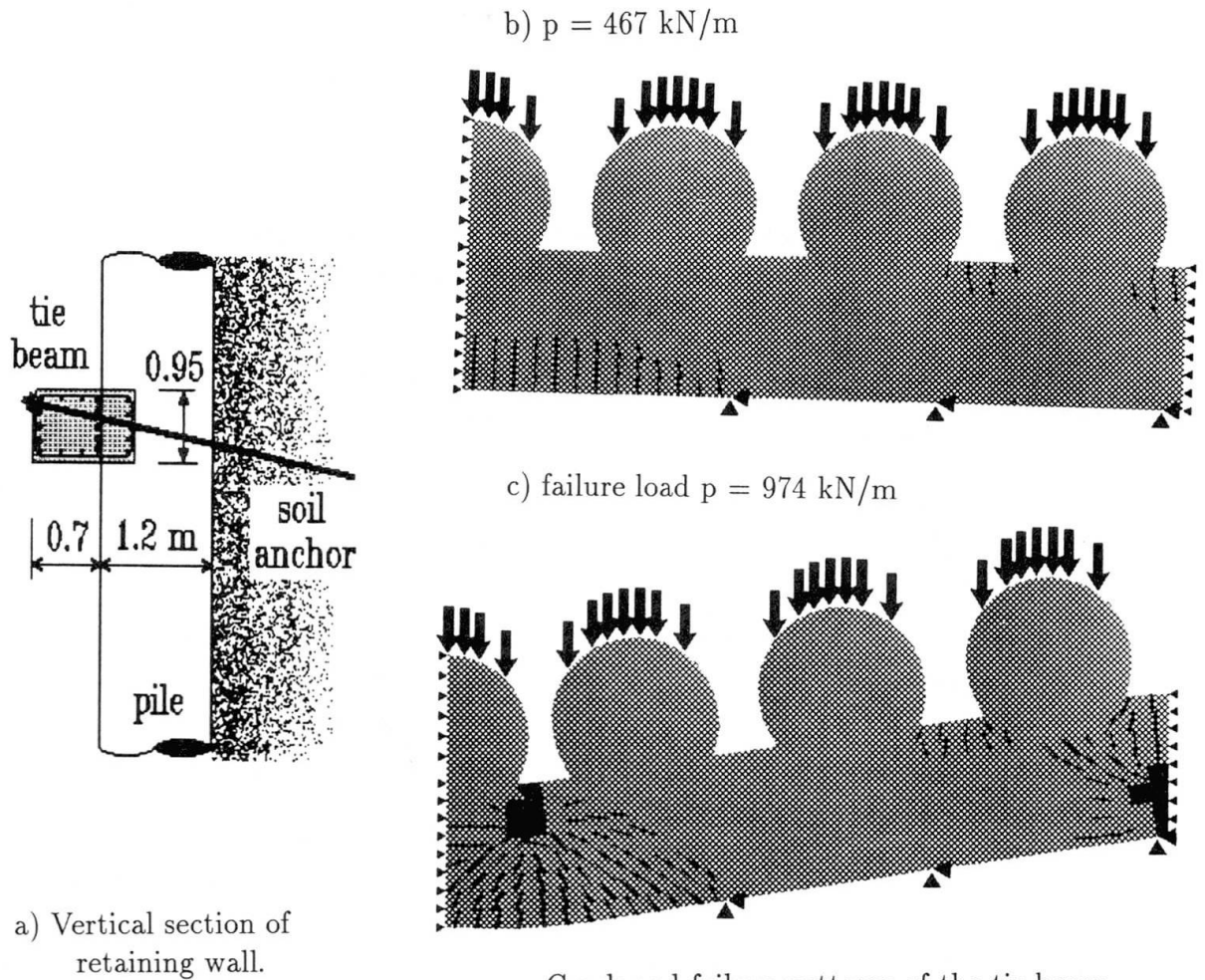
Fig.7 Load-displacement diagrams of pull-out tests for three sizes ( $d=50, 150, 450$  mm) and two lateral constraint conditions. Thickness  $b=100$  mm,  $a/d=1$ .  
thin line - without constraint,  
thick line - with constraint



Deformed shape with crack pattern at failure.

Load-displacement diagram.

Fig.8 Simulation of the ductile failure mode of a tapered beam. Symmetrical half of the beam analyzed.



a) Vertical section of retaining wall.

Crack and failure patterns of the tie beam. Crushed concrete shown by dark shading.

Fig.9 Simulation of the failure of tie beam supported by elastic anchors.

international round-robin analysis organized by the RILEM Committee on Fracture Mechanics. For this round-robin analysis, the authors have made a parameter study on various 2-D pull-out tests [10]. An example from this study concerning a two-dimensional structure in plane stress state is shown here. The embedment depth  $d$  and the shape ratio  $a/d$  ( $a$  is the support span) were varied. Examples of the failure crack patterns for  $d = 150$  mm and three different spans  $a = 50, 150, 450$  mm are shown in Fig.5. The load-displacement diagrams for these cases are shown in Fig.6. Fig.7 shows diagrams for  $a/d = 1$  using three values for the embedment depth  $d = 50, 150, 450$  mm and two assumptions for the lateral constraint (with and without lateral constraint). From these analyses, the influence of the embedment depth (size effect) could be derived. In another application, an SBETA analysis was successfully used to model the behavior of the single anchors and anchor groups subjected to transverse loading [16].

## APPLICATIONS IN ENGINEERING PRACTICE

The program SBETA was used at the Prague University for the solution of several practical problems. Two examples are shown here for illustration. In the first example a precast T-beam was analyzed (Fig.8). The web is tapered and the beam is supported by an overhanging flange. The Building Research Institute of T.U. in Prague has performed experimental and numerical studies in order to optimize the reinforcement detailing. Fig.8 shows the failure state of the final solution with a ductile failure mode due to the yielding of reinforcement.

In the second example, a tie beam of a retaining wall was analyzed. The retaining wall consists of vertical reinforced concrete cast-in-place piles which are supported by a horizontal tie beam, Fig.9. The beam is supported by earth anchors which are located between the piles. It was proposed to investigate the cases when several anchors fail. In such a case the tie beam is subjected to bending, while it is laterally constrained. Elastic supports are used to model the anchors. The maximum soil pressures were obtained for various supporting situations. Fig.9 shows two deformed shapes and crack patterns for two load stages. In the failure stage, concrete crushing is also shown. Yielding of reinforcement was also found by the analysis, but it is not shown here.

## ROLE OF FEM MODELS IN DESIGN OF CONCRETE STRUCTURES

Non-linear FEM is an advanced tool for modeling the behavior of reinforced concrete structures. It's great potential lies in it's ability to work with steadily developing constitutive laws while satisfying the laws of continuum mechanics and fracture mechanics. As with any model, it is an approximation of reality. However, the degree of approximation can be controlled at all levels of the model. As demonstrated here, these models have their application in situations where simple engineering models are not adequate. In practice they have been successfully used for the design of deep beams, reinforcement detailing (D-regions) and for the diagnosis of the causes of structural failure. In research and development, they have been used for the simulation of experiments, prediction of failure modes and for the analysis of experimental results.

It should be emphasized that a non-linear FE analysis must be supported by efficient graphical tools for pre- and post-processing. Just as drawings are indispensable for structural design, graphics is indispensable for the computer analysis of structural behavior.

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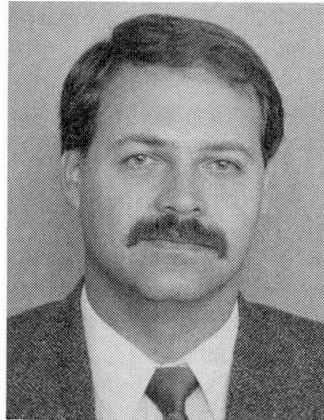
## **Analyses Based on the Modified Compression Field Theory**

**Analyses basées sur la théorie modifiée du champ de contraintes  
en compression**

**Untersuchungen auf Grundlage der modifizierten Druckfeldtheorie**

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### **SUMMARY**

The Modified Compression Field Theory provides a unified, rational approach to the analysis of structural concrete elements under general in-plane stress conditions. Cracked reinforced concrete is treated as a nonlinear elastic orthotropic material based on a smeared, rotating crack assumption. Formulations satisfying equilibrium and compatibility conditions are developed, and new constitutive relations for the component materials are defined. The theory is incorporated into several analytical algorithms. Procedures have been developed for the analysis of membrane structures, beams, plane frames, plates and shells, and three-dimensional solids. Extensive corroborative testing has shown the theory to be able to model accurately the response of structural concrete.

### **RÉSUMÉ**

La théorie du champ de contraintes en compression fournit une approche rationnelle et unifiée à l'analyse des éléments structuraux de béton, sous des conditions de contraintes planes. Le béton armé fissuré est traité comme un matériau orthotropique élastique non-linéaire, se basant sur un concept de fissuration uniforme. Des expressions satisfaisant les conditions d'équilibre ainsi que de comptabilité sont développées, et de nouvelles relations constitutives sont définies pour les matériaux. Des procédures sont développées pour l'analyse des membranes, poutres, cadres plans, plaques, coques et solides tri-dimensionnels. Une série d'essais démontrent que la théorie peut prédire correctement le comportement du béton.

### **ZUSAMMENFASSUNG**

Die modifizierte Druckfeldtheorie stellt eine einheitliche und rationale Methode für die Untersuchung von Konstruktionsbeton-Elementen im ebenen Spannungsfeld dar. Gerissener Stahlbeton wird – basierend auf einem Rissmodell mit verschmierten und rotierenden Rissen – als nicht-lineares, elastisches orthotropisches Material behandelt. Es werden Formulierungen, die Gleichgewichts- und Verträglichkeitsbedingungen genügen, entwickelt und neue konstitutive Beziehungen für die Teil-Materialien definiert. Die Theorie wird in mehrere analytische Algorithmen eingebaut. Für die Untersuchung von Membranwerken, Balken, ebenen Rahmen, Platten, Schalen und drei-dimensionalen Festkörpern werden Berechnungsmethoden entwickelt. Zahlreiche Versuche haben gezeigt, dass die Theorie das Verhalten von Konstruktionsbeton genau beschreiben kann.



## 1. INTRODUCTION

A unified, rational approach to the analysis and design of concrete structures does not currently exist. The difficulty stems from an inability to develop general material behaviour models for structural concrete that are simple, rational, consistent and accurate. This was clearly brought to light in the results of a competition conducted 10 years ago [1].

The international competition was organized to compare analytical methods for predicting the response of reinforced concrete elements subjected to general two-dimensional stress states. Entrants were asked to predict the strength and load-deformation response of four panels tested in a University of Toronto research program. While many of the predictions received were based on analyses conducted using complex nonlinear analysis procedures, a wide scatter in the predictions was evident. Clearly, the ability to accurately model behaviour, using finite element procedures or otherwise, was not very good due to a generally poor state-of-the-art in constitutive modelling.

Much work has been conducted recently at the University of Toronto in an effort to develop improved models. One outcome has been the formulation of the Modified Compression Field Theory.

## 2. MODIFIED COMPRESSION FIELD THEORY

The Compression Field Theory (CFT) was first developed by Collins and Mitchell [2, 3], and co-workers, for the analysis of beams under combined torsion, shear and flexure. The theory provided a conceptual model for the behaviour of cracked reinforced concrete under two-dimensional stress states, following essentially a smeared, rotating crack idealization. Formulations satisfying conditions of equilibrium and compatibility in a continuum were based on average values of stress and strain in the component materials. It was assumed that the directions of the principal stresses coincided with the directions of the principal strains. Concrete in compression was modelled using the Hognestad parabolic curve; concrete in tension was assumed to carry no stress after cracking.

To develop more accurate constitutive relations for cracked reinforced concrete, a new testing facility was developed and utilized in an extensive experimental investigation. The 'Shear Rig' was capable of loading reinforced concrete panels under general conditions of uniform in-plane stress. In particular, it allowed for the first time anywhere, the testing of reinforced concrete under conditions of pure shear, as well as shear combined with biaxial normal stresses. [More recently, at the University of Toronto, the 'Shell Element Tester' facility was developed permitting the testing of larger elements under conditions involving both in-plane and out-of-plane loading]. In the initial series, 30 panels were tested in the years 1979-1981 [4].

Based on the results of these initial tests, the Modified Compression Field Theory (MCFT) was developed [4]. The refinements introduced by the MCFT related to: i) strain softening of concrete in compression, due to the action of transverse tensile strains; ii) tension stiffening effects in cracked concrete in tension, due to the continued presence of tensile stresses in concrete between cracks; iii) the transfer of stresses across cracks (ie. the need to consider local stress conditions at crack surfaces). These effects were embodied in the analytical model by a new set of constitutive relations.

## 3. EXPERIMENTAL CORROBORATION

To corroborate and refine the analytical models, and the finite element formulations developed subsequently, several test programs were undertaken involving membrane elements. Bhide and Collins [5] tested a series of thirty-one uniaxially reinforced concrete panels under various combinations of tension and shear. A series of ten panels, with centre perforations, were tested to study behaviour in situations involving stress disturbances due to structural discontinuities [6]. To examine predicted behaviour under conditions where a uniform system of cracks could not be assumed, panels containing a pre-cracked uniaxially reinforced shear plane were tested [7]. The behaviour of prestressed elements was investigated by Marti and Meyboom, through the testing of three large-scale shell elements. To investigate possible scale effects, panels of varying sizes were tested under similar load conditions using the Shear Rig, the Shell Element Tester and facilities elsewhere. The response of high strength concrete elements was investigated by test programs involving four shell elements and two panel elements. Finally, the influence of cyclic loading on the constitutive response of cracked structural concrete was briefly explored via three shell element tests [8]. Several other experimental programs are currently in progress.

Thus, since the development of the MCFT, several test programs have been undertaken involving a total of over one hundred membrane type specimens. The conditions investigated have encompassed a wide range of specimen construction details and loading conditions. In all cases, the MCFT was able to accurately predict behaviour in terms of crack patterns, deformations, reinforcement stresses, ultimate strengths and failure modes. Detailed comparisons of experimental versus theoretical response, for each of the test series, can be found in the references cited.

#### 4. MEMBRANE STRUCTURES

The MCFT was incorporated into nonlinear finite element analysis algorithms, for membrane structures, using several alternative approaches. Adeghe [9] introduced the model's constitutive relations into the standard program ADINA. Stevens et al [8] developed program FIERCM, a nonlinear algorithm based on a tangent stiffness solution procedure and utilizing high-order quadratic strain elements. Cook and Mitchell [10] developed program FIELDS, also a nonlinear finite element algorithm based on a tangent stiffness scheme. Lastly, a secant-stiffness based algorithm was used by Vecchio [11, 12] in developing program TRIX.

In the secant-stiffness formulation, finite elements were developed such as to completely represent the formulations of the MCFT. By incorporating these elements into a iterative linear elastic procedure, nonlinear analysis capability was achieved. The resulting procedure demonstrated good numerical stability and good convergence characteristics. Extensions to the formulations were later developed [12] which permitted the consideration of prestrain effects in the component materials. Prestressing of the reinforcement, shrinkage or expansion of the concrete, or other types of strain offset effects could then be considered.

The accuracy of the finite element analyses were examined by predicting the response of the various membrane elements tested, as well as by modelling several 'benchmark' tests reported in the literature. In general, aspects of response pertaining to strength, stiffness, cracking patterns, reinforcement stresses, concrete distress regions, and failure modes were all predicted with good accuracy [6, 7, 8, 9, 10, 11, 12].

#### 5. BEAM SECTIONS

The MCFT formulations were adapted to the analysis of structural concrete beams subjected to combined shear, flexure and axial loads by developing a layered section analysis procedure (program SMAL) [13]. In the layered procedure, the only sectional compatibility requirement enforced was that plane sections remain plane. Sectional equilibrium requirements included a balancing of the shear flows as well as of the member end actions. Beyond this, uniform stress conditions were assumed to exist in each concrete layer and each longitudinal rebar element. Conditions of compatibility and equilibrium in the concrete layers were enforced according to the formulations of the MCFT. Thus, given the sectional forces, the two-dimensional stress and strain conditions within each layer of the section could be computed. An iterative solution algorithm was employed.

The formulations were found to provide reasonably accurate predictions of the load-deformation response, ultimate load and failure mode of beam specimens. For example, the thirty-five T-beams tested at the University of Washington, under various conditions of shear and axial load, were modelled analytically and very good agreement was obtained. The ratio of experimental to predicted strength had a mean of 1.01 and a coefficient of variation of 15% [13].

#### 6. PLANE FRAMES

A nonlinear frame analysis procedure (ie. program TEMPEST) was developed to model the response of reinforced concrete plane frames subjected to thermal and mechanical loads [14, 15]. The procedure was primarily based on performing rigorous sectional analyses of frame members at several points along their length, and then enforcing these sectional responses in the overall response of the frame. This was done by various means; for example, by defining effective sectional stiffness factors, unbalanced forces, fixed-end forces, or combinations of these. The sectional analyses were performed in the manner previously described for beam sections, but assuming uniform shear flow distributions through member cross-sections. Thus, the multi-layer section analyses incorporated into the frame procedure considered two-dimensional stress conditions in the manner of the MCFT.



Two large-scale frame models were fabricated and tested to corroborate the analysis program [15, 16]. The one-span, two-storey models had a centre-to-centre span of 3.5m and an overall height of 4.6m. The first specimen was subjected to a concentrated transverse load applied at the midspan of the first-storey beam. The second was subjected to constant axial column loads combined with monotonically increasing lateral load applied at the top storey level. The analysis procedure was found to give reasonably accurate predictions of the complex nonlinear response of the test frames. Shear-related effects contributed significantly to the deformations, and membrane action and geometric nonlinearity significantly affected the load capacities. These effects could only be captured in the theoretical analyses by considering two-dimensional stress conditions within the frame members.

## 7. PLATES AND SHELLS

Analytical procedures based on the MCFT were also developed for modelling the response of reinforced concrete plate and shell elements. Program SEP was initially developed by Kirschner and Collins [17] to predict the response of elements subjected to membrane forces, bending moments and torsional moments. A strain compatibility approach was coupled with a layered element technique. Appropriate assumptions were made regarding the distribution of strains across the thickness of the element. MCFT-derived constitutive relations were then used to calculate stresses within each of the layers. Program SEP was then further developed to account for the influence of out-of-plane shear [17]. This required a consideration of tri-axial stress conditions in each of the layers of the shell element, making the computational algorithm complex, time-consuming, and somewhat unstable. Thus, program SHELL474 was developed by Adebar and Collins [18]. Using the layered element approach, and program SEP as a subroutine, SHELL474 introduced a simplification whereby only the middle layer of the element was analyzed for the three-dimensional stress conditions.

The finite element analysis program APECS was developed to provide global nonlinear analysis capability [19]. The 42-degree-of-freedom quadrilateral shell element formulated allows the modelling of plate or shell structures containing arbitrary in-plane and out-of-plane reinforcement. Also employing a MCFT-based layered-element formulation, the analysis procedure is able to model response to general loading conditions, including a consideration of out-of-plane shear, material prestrains, membrane action, tension stiffening effects, and local stress conditions at crack locations.

Several series of specimens were tested in the 'Shell Element Tester' to corroborate the analytical formulations. Kirschner [17] and Polak [19] tested a total of ten shell elements under various conditions of membrane forces combined with flexure. Adebar [18] tested 9 shell elements and 27 beam elements involving conditions of membrane forces combined with out-of-plane shear. Other test programs are currently underway. In applying the analytical procedures to model the behaviour of the test specimens, good agreement was generally found.

## 8. THREE-DIMENSIONAL SOLIDS

A nonlinear finite element program (SPARCS) was developed for the analysis of reinforced concrete solids [20]. The program was derived using the analytical models previously described for membrane elements, extrapolated to three dimensions. Thus, an iterative linear elastic formulation was used in which secant moduli were defined and progressively refined according to current local stress/strain states. The three-dimensional constitutive relations incorporated into the formulation were ones extrapolated from the two-dimensional models of the MCFT. An eight-noded regular hexahedral element was formulated accordingly.

To obtain an indication of the potential accuracy of the three-dimensional formulation, torsion beams tested by Onsongo [21] were modelled. The hollow, rectangular beams were subjected to varying conditions of torsion and flexure. They represented a stringent test of the analysis procedure because of the complex loading condition, and because the beams were generally over-reinforced and governed by failure of the concrete. The ultimate load and failure mode of the ten beams tested were predicted very well. The ratio of experimental to theoretical strength had a mean of 0.99 and a coefficient of variation of 6.1%. The load-deformation responses and local strain conditions were also modelled well.

## 9. DISCUSSION

Breen [22] has pointed to the need for "developing unified, consistent analysis and design approaches" which can be universally applied to "continuum of structural concrete", but has cautioned that "we must dispel the present preoccupation with complex analysis procedures". Further, he has suggested that "as nonlinear analysis packages develop, it is possible nonlinear finite element analysis may be useful" [in this regard]. Scordelis [23] has echoed these feelings, stating that "there is a need to develop a unified approach for the analysis and design of the entire spectrum of ... structural concrete systems which takes advantage of the latest analytical and experimental research, materials, computers, and practical design and construction experience". He adds that "it is desirable to have refined analytical models and methods of analysis which can trace the structural response ... under increasing loads through their elastic, cracking, inelastic, and ultimate ranges".

The MCFT is a conceptual and mathematical model for structural concrete that is consistent with the perceived needs stated by Breen and by Scordelis. As has been shown, the theory presents a unified, rational analysis approach that can be applied to structural concrete in many of its various forms and applications. The theory's formulations are simple, transparent and easy to implement. Further, emphasis is placed on accurately describing the constitutive behaviour of structural concrete, as oppose to developing complex and sophisticated mathematical solutions. The constitutive models incorporated have been corroborated extensively with experimental data. Thus, while MCFT analysis procedures may not always represent the optimal solution to routine analysis and design applications, they do provide for a consistent approach in performing an accurate 'trace of structural response' in situations where it is deemed necessary.

## 10. CONCLUSIONS

The Modified Compression Field Theory (MCFT) provides a simple, rational framework for modelling the behaviour of cracked reinforced concrete. The theory presents formulations for satisfying conditions of strain compatibility and stress equilibrium in a continuum and, most importantly, defines realistic constitutive relations for the concrete and reinforcement.

The accuracy and range of applicability of the MCFT has been extensively corroborated with additional experimental research since its formulation. The test programs undertaken, conducted on simple panel and shell elements under well controlled conditions, have covered a wide range of structural parameters and loading conditions. The theory, still in essentially its original form, has been found to provide fairly consistent and accurate results in all cases.

The simplicity of the MCFT formulations has allowed them to be easily adopted into various analytical algorithms. Procedures have been developed for the nonlinear analysis of membranes, beams, plane frames, plates and shells, and three-dimensional solids.

In applying the analysis procedures to the modelling of more complex structural systems, generally good correlation was found between predicted and observed responses. The theory was found to provide accurate modelling of crack patterns, deformations, reinforcement stresses, ultimate strengths, and failure modes.

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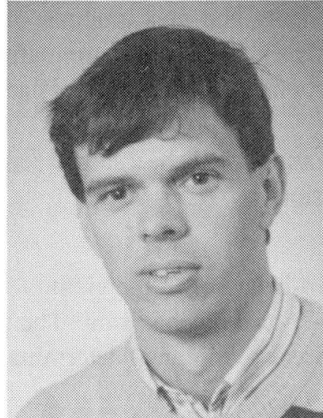
## Computational Bond Models: Three Levels of Accuracy

Modèles de calcul de l'adhérence selon trois niveaux de précision

Rechenmodelle für den Verbund: drei Genauigkeitsstufen

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### SUMMARY

The paper reviews three computational bond models of decreasing degree of precision. Firstly, a detailed resolution of bond-slip is given, which aims at supporting experimental work. Secondly, the bond-slip law is used as input to interface elements with a view to predicting crack spacing and width. Thirdly, embedded reinforcement techniques for global analysis in engineering practice are discussed. Examples demonstrate the capability of nonlinear finite element analysis to simply and correctly reveal «strut-and-tie» systems after stress redistribution.

### RÉSUMÉ

Cet article passe en revue des modèles de calcul de l'adhérence, et ceci, selon 3 niveaux décroissants du degré de précision. Premièrement une résolution détaillée est donnée du problème de glissement-adhérence, qui vise à étayer un travail expérimental. Deuxièmement, la loi de glissement-adhérence est introduite dans le cas des interfaces afin de prédire la largeur et l'espacement des fissures. Troisièmement, les techniques d'enrobage des armatures de la pratique sont analysées globalement. Des exemples démontrent les capacités d'une analyse non-linéaire par éléments finis qui révèlent simplement l'efficacité des systèmes basés sur l'analogie du treillis après redistribution des effets.

### ZUSAMMENFASSUNG

Der Artikel behandelt Rechenmodelle für den Verbund mit den drei folgenden Stufen abnehmender Genauigkeit. Zuerst wird eine ausführliche Lösung der Verbund-Schlupf-Beziehung gegeben, die zur Ergänzung von Versuchsvorhaben dient. Zweitens wird das Verbund-Schlupf-Gesetz als Eingabe für Kontaktflächen-Elemente benutzt, mit der Zielsetzung, Rissabstände und Rissbreiten zu bestimmen. Drittens werden die Verfahren mit in Beton eingebetteten Bewehrungsstäben diskutiert, die zur baupraktischen Berechnung gesamter Systeme dienen. Einige Beispiele demonstrieren die Fähigkeiten nichtlinearer Finite Element Berechnungen, einfach und richtig Stabwerkmodelle nach Spannungsumlagerungen aufzuzeigen.



## 1. INTRODUCTION

Computational simulators for concrete can be applied in three different ways:

- to support experimental research,
- to develop, verify or validate design rules and hand calculation methods,
- to incidentally analyse particular structures.

The former two types of applications are not meant to be made at the desk of a practising engineer. Instead, these are made by researchers, students and post-graduate students who use the computational tools in a manner similar to experimental tools. In this way, computational models are transferred into practice in an indirect manner. The third type of application is of a direct nature.

In this paper computational bond models will be categorized in the above way. This corresponds to three levels of decreasing degree of precision:

- Resolution of bond-slip.  
This strategy zooms at the micro-behavior in the vicinity of the rebar, where cone-shaped secondary and longitudinal splitting cracks are crucial mechanisms. The method aims at explaining the fundamentals of traction-slip behavior and supports experimental determination of local bond-slip laws.
- Bond-slip interface analysis.  
This approach lumps bond-slip into an interface element, with a view to supporting the derivation of design rules on spacing and width of primary cracks in structural concrete members.
- Embedded reinforcement with tension-stiffening.  
For global analyses even the above approach that zooms at primary cracks becomes too delicate. Instead, the primary cracks are smeared out and techniques of automatic embedment of reinforcing elements and prestressed cables in concrete elements are necessary. In combination with tension-stiffening, these techniques can be directly used in engineering analysis of structural concrete systems.

This paper reviews the three approaches. Most attention will be given to examples of the third category, where results support the 'strut-and-tie' philosophy for structural concrete.

## 2. DETAILED LEVEL: RESOLUTION OF BOND-SLIP

The first and most sophisticated computational approach to bond-slip is to simulate the behavior in the vicinity around the rebar in detail. The bond-slip, i.e. the tangential relative displacement between the deformed rebar and the concrete (measured some distance away from the rebar), is controlled by four mechanisms [12,17]: (a) elastic deformation, (b) internal conical transverse cracking behind the ribs of the rebar, (c) longitudinal cracking in response to tensile ring-stresses, (d) crushing in compressive cones radiating out from the ribs. Fig. 1 shows a result of an elastic-softening simulation which included elastic deformation, transverse cracking and longitudinal cracking. The structure is a simple reinforced tension bar (tension-pull specimen) modelled in axi-symmetry.

Initially, the external force is transferred from the steel into the concrete primarily via axial tensile stresses. On increasing load, cone-shaped transverse secondary cracks nucleate behind the ribs of a rebar. This cracking starts near the end-face of the specimen and gradually moves inwards, which parallels experimental findings [9]. After transverse secondary cracking, the direct bond action is lost and the transfer of bond forces is subsequently furnished by compressive cones that radiate out from the ribs. The radial components of the compressive cones are balanced by rings of tensile stress [17]. When the ring is stressed to rupture a longitudinal crack arises and the balance against the compressive cones is lost, which causes a further break-

down of bond. These longitudinal splitting cracks are plotted as the shaded area in Fig. 1a.

Aside from giving insight in the basic bond mechanisms, simulations provide quantitative information. Fig. 1c shows a local bond traction-slip curve extracted from the analysis. The curve shows a linear-elastic stage, a stage of decreased stiffness and a softening stage. This trilinear idea of bond curves lends support to analytical bond-models that were derived on the line of argument, and agrees with experimental findings [7]. The slip modulus of the second stage of the curves (appr. 300N/mm<sup>3</sup>) falls within experimental scatter [6]. For further results and details the reader is referred to [14,15], where also a numerical contribution is made with regard to the yet unresolved principal issue in bond research, namely the dependence or non-dependence of bond curves on the distance from the primary crack.

In conclusion, a computational resolution of bond-slip contributes to a better understanding of the basic bond mechanisms and works complementary to experimental research in this area. Simulations of this type are not meant to be made at the desk of a practising engineer. Rather, the outcome comes available to practice in an indirect way, via improved bond curves and design rules.

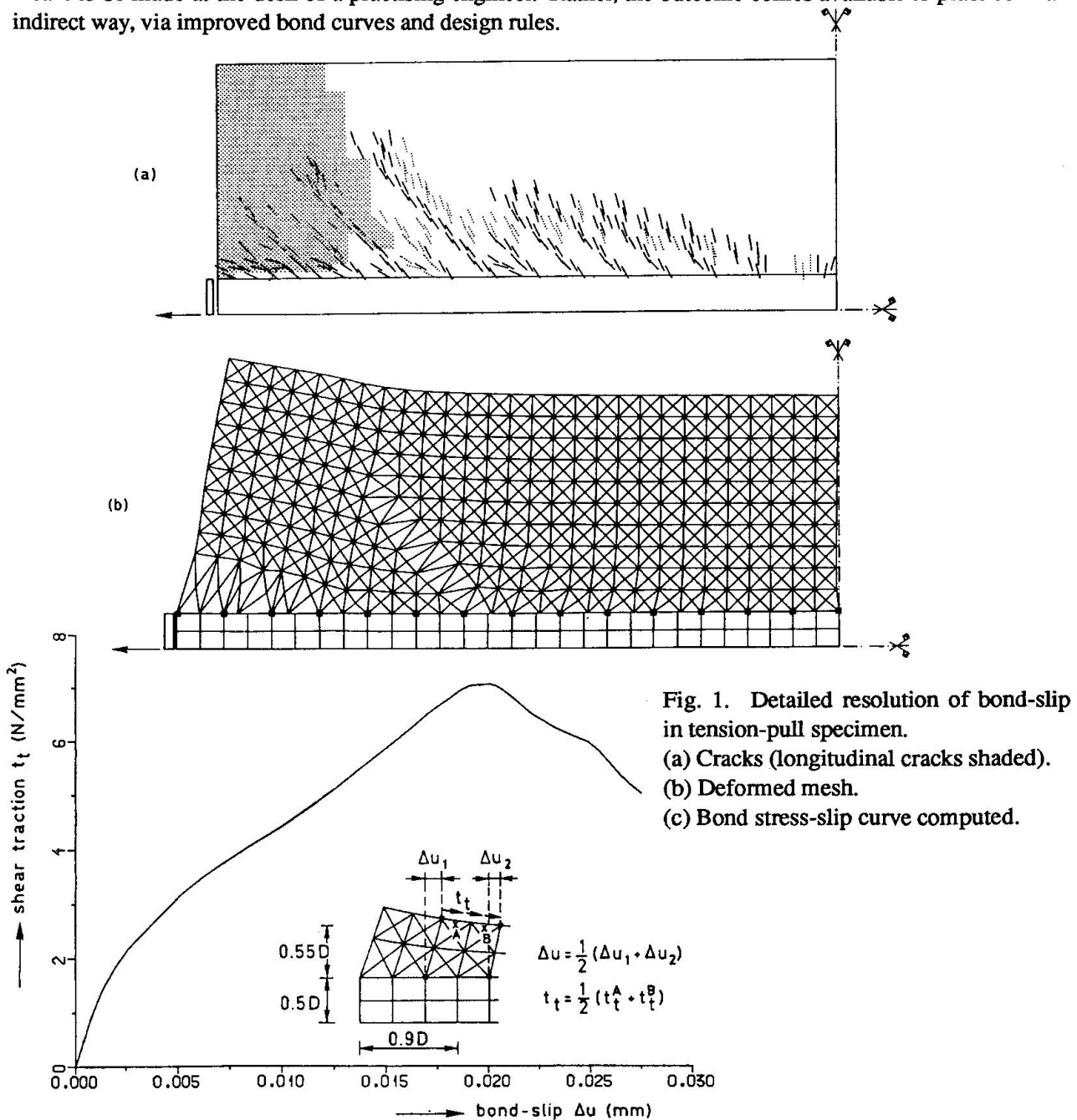


Fig. 1. Detailed resolution of bond-slip in tension-pull specimen.  
 (a) Cracks (longitudinal cracks shaded).  
 (b) Deformed mesh.  
 (c) Bond stress-slip curve computed.



### 3. INTERMEDIATE LEVEL: BOND-SLIP INTERFACE ANALYSIS

A second approach is to lump the bond-slip behavior into a fictitious interface. Consequently, the traction-slip curve, which was output from the previous analysis, is now input. This technique was introduced by Rehm [12] in order to subsequently evolve into a powerful analytical tool for predicting the spacing and width of primary cracks in structural concrete members, e.g. [11,17,3]. In analytical approaches simplifying assumptions have to be made, like the assumption of a simple and unique bond-slip curve or a sudden stress drop for the concrete after cracking. In computational studies, the procedure can be refined since more advanced bond-slip laws can be inserted for the interface elements and gradual softening curves for the cracks.

Fig. 2 shows an example of a long-embedment tension-pull specimen. The steel is modeled by truss elements, the bond-slip layer by interface elements and the surrounded concrete by continuum elements. The bond traction-slip curve for the interface elements was taken according to [6,10]. The tensile strength for the concrete was assigned a Gaussian distribution, which is consistent with the physical process in heterogeneous materials and essential to prevent a wide band of elements under homogeneous stress from cracking simultaneously. The load-elongation response shows four local maxima corresponding to the successive development of four primary cracks. The serrated type of curve is in agreement with experimental results [8] and justifies engineering models [3]. Beyond formation of the fourth primary crack, the crack pattern was fully developed (stable crack spacing) and the solution could be continued up to yielding of the reinforcement without further physical changes. During that stage only the width of the primary cracks increased.

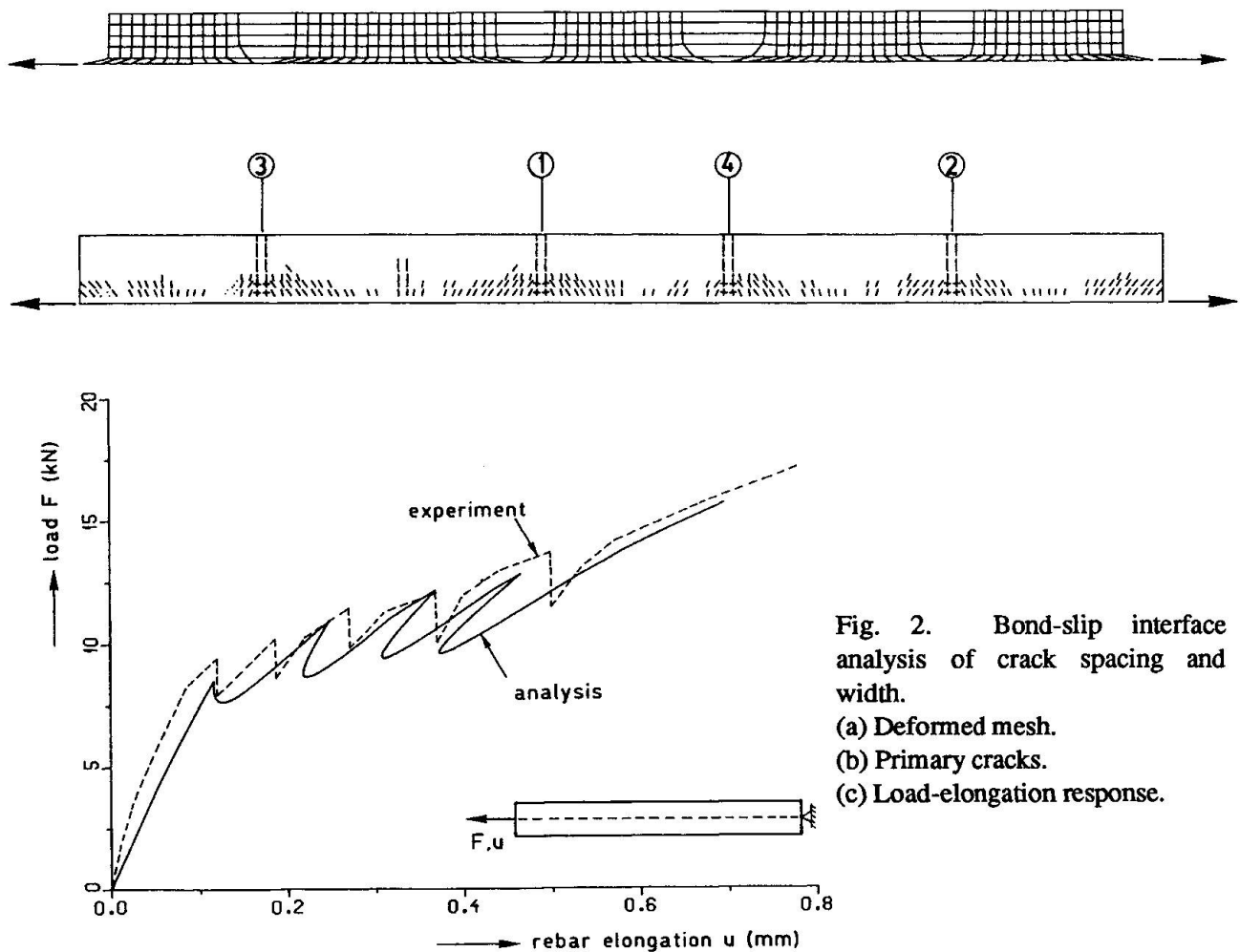


Fig. 2. Bond-slip interface analysis of crack spacing and width.

(a) Deformed mesh.

(b) Primary cracks.

(c) Load-elongation response.

Analyses of this kind apply to cases where information is required regarding crack spacing and/or crack width. This can be the case either in research studies, where adequate design formula for spacing and width are to be developed, or directly in practical analysis of structural details like anchorages, where bond plays a paramount role. An illustrative example of the first category is the combined experimental, computational and analytical study on the control of crack width in deep reinforced concrete beams [2]. Examples of the second category have been given in e.g. [18].

#### 4. GLOBAL LEVEL: EMBEDDED REINFORCEMENT

The third type of approach is full embedment of reinforcements in concrete elements (Fig. 3). With this procedure, the reinforcing elements do not have separate nodes and displacement degrees of freedom, but the strain in the reinforcement is calculated from the nodal displacements of the mother element in which it is embedded. The method generally implies overall perfect bond, i.e. the average strain in the cracked concrete equals the average strain in the reinforcement. For this reason, primary cracks are not treated individually, as though under a magnifying glass, but are smeared out with a view to global analysis of structural concrete systems. The joint action of cracked concrete and reinforcement must then be accounted for via a tension-stiffening model.

The clear advantage is that the lines of the finite element mesh do not need to coincide with the lines of the reinforcement, which is a must when real practical cases with a diffuse set of reinforcing bars, grids and prestressed tendons are analysed.

For practical use of embedded reinforcement techniques in nonlinear finite element analysis, three aspects are essential [4]. Firstly, various embedment combinations must be made available to model the variety of structures and geometries in practice:

- bars in Euler-Bernoulli beam elements  
e.g. 2D and 3D frames
- bars in Mindlin beam elements  
e.g. edge-beams along plates and shells
- bars and grids in plane stress elements  
e.g. panels, deep beams
- bars and grids in axi-symmetric elements  
e.g. storage vessels with radial, tangential and longitudinal reinforcement
- grids in plane-strain elements  
e.g. tunnels
- bars and grids in plate and shell elements  
e.g. slabs, shell roofs, cooling towers
- bars and grids in solid elements  
e.g. massive structures, complicated details

Secondly, options of geometrical preprocessing must be available. It is beyond realism, to let the analyst specify all intersections between reinforcement and element boundaries. Rather, the analyst would like to specify the end-points of the reinforcement, the shape and, depending on the type of shape (straight, parabole, circle etc.) additional information like e.g. a midpoint, see Fig. 3. The preprocessor should then automatically generate all intersections with the lines in the element patch. Once these intersections are known, the reinforcement portions in the elements can be evaluated and the linkage of reinforcement strains to element nodal displacements can be made.

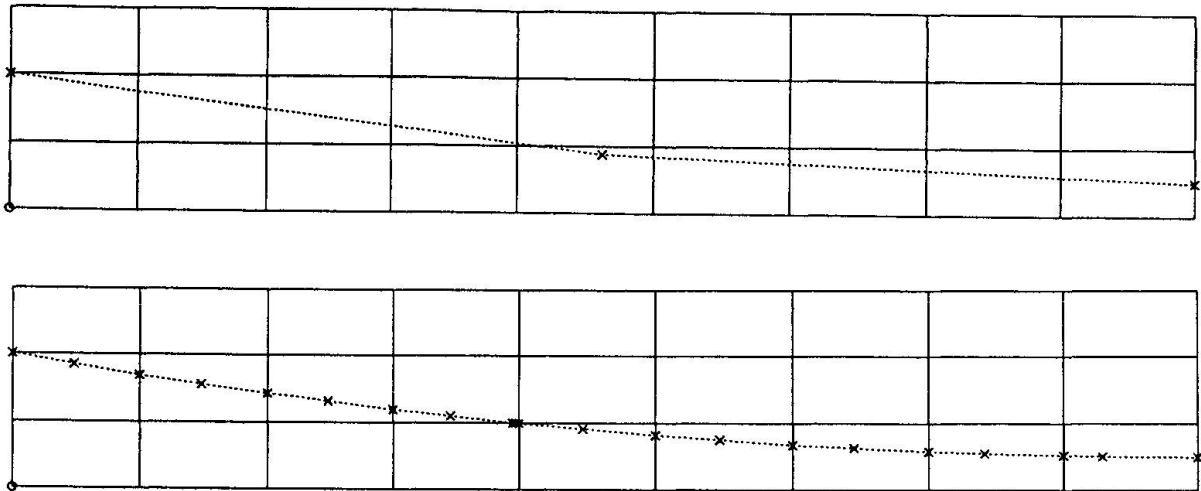


Fig. 3. Embedded reinforcement in finite element analysis.

(a) Simple input of global tendon location, e.g. via three points.

(b) Automatic positioning of reinforcement portions in elements, obtained via geometrical preprocessing routines.

Thirdly, options must be available for prestress, both pre-tensioned and post-tensioned. With pre-tensioning, the reinforcement is assigned an initial stress and the perfect bond assumption is used. With post-tensioning, the reinforcement is given an initial stress distribution (which should guarantee the design value of the jacking force and, simultaneously, account for the losses due to friction, elastic shortening and anchorage slip) and the analysis starts with a no-bond assumption. At this stage, the stiffness of the reinforcement is not included, but only the equivalent nodal element loads due the prestress are placed on the system. After grouting, the no-bond assumption is replaced by perfect bond and the stiffness of the reinforcement is included.

In the latest release of the DIANA finite element program [5] all above listed embedment combinations are available. Furthermore, geometrical preprocessing routines have been included for 2D,  $2\frac{1}{2}$ -D (in-plane and out-of-plane location of curved tendons) and also a limited set for 3D. In the program, the two prestress options are available, whereby it is noted that automatic determination of the initial stress distribution for post-tensioning is currently under development.

## 5. EXAMPLE 1: CONCRETE WALL IN FLAT

A second example concerns a concrete wall in a twelve-storey apartment building which was already under construction up till the seventh floor when the surveyor noticed that the reinforcement in the 'tie' at the first floor was underdimensioned. To decide on the way of repair, the engineering firm in question called for a review analysis at TNO. Fig. 4 shows some key-results thereof.

The ground level was open and two columns of different dimensions supported the wall. In the mesh, only five storeys were taken into account, while the remaining storeys were represented by a load on top. The figure compares the principal stresses in the wall in the linear-elastic stage and in the final stage at load factor 1.7. It clearly reveals a 'strut-and-tie' system whereby in the linear-elastic stage the tie is formed by the concrete and in the final stage by the reinforcement since the concrete has cracked. Note the change in width and inclination of the strut with increasing load. These are definitely factors that should be accounted for in strut-and-tie design models.

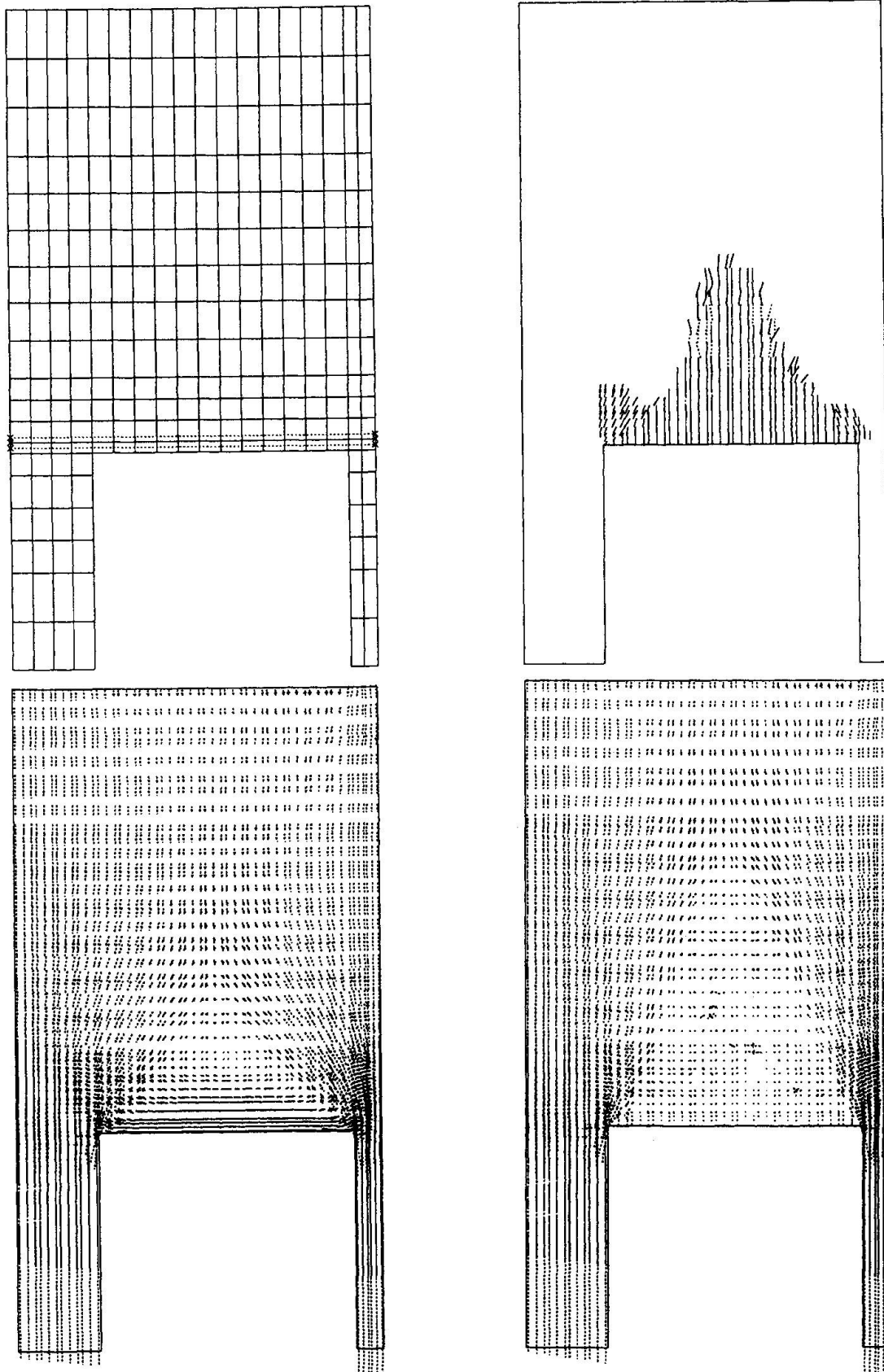


Fig. 4. Review-analysis of concrete wall in apartment building.

(a) Mesh with embedded reinforcement.  
(c) Principal stresses in linear stage.

(b) Crack pattern in final stage.  
(d) Principal stresses in final stage.



Main conclusions from the analysis were:

- The amount of additional reinforcement in the tie, which was accounted for in the analysis, proved to be sufficient.
- The average crack strain at service load 1.0, divided by an assumed crack spacing, led to crack widths that were well within code limits.
- The maximum compressive stresses in the narrow strut remained below the prescribed strength value in the code.

Based on this review analysis, a low-cost repair could be undertaken, consisting of a limited amount of additional horizontal reinforcement in the top layer at the floor close to the wall. Inclusion of additional vertical reinforcement or local thickening of the wall, options that were originally hinted at by the surveying committee, could be circumvented. Even more important was the fact that due to the quick solution of the problem, construction of the building could be continued without interruption, which saved significant costs.

## 6. EXAMPLE 2: CROSS SECTION OF TUNNEL STRUCTURE

The second example relates to a damaged part of an existing tunnel structure, which was submitted to consultancy in the Netherlands. This led to a review analysis of the shear capacity and of the bond, the results of which are described in detail in [13]. Herein, only a brief extract of the results is shown, namely the plots of the principal stress trajectories in the linear-elastic stage and the ultimate-load stage (Fig. 5). In the linear-elastic stage the concrete tensile stresses are found to make a substantial contribution to the transfer of the shear force. At ultimate load these tensile stresses have entirely disappeared because of cracking, and a pronounced thrust arch can be observed which is tied by the the midspan tensile reinforcement. The example strengthens the conclusion in the preceding section as to the capability of nonlinear finite element analysis of predicting 'strut-and-tie' systems in structural concrete after significant stress redistribution.

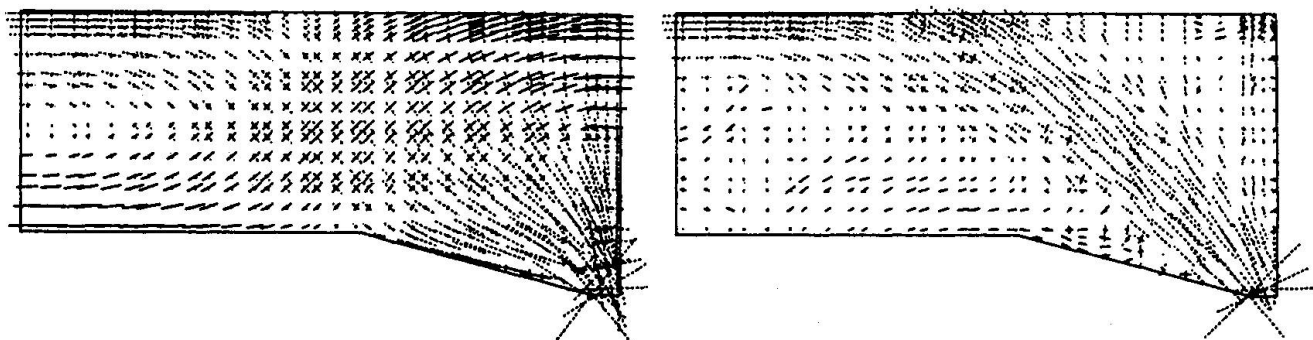


Fig. 5. Transition from linear-elastic behavior to 'strut-and-tie' system in tunnel roof.

- (a) Principal stresses in linear-elastic stage.
- (b) Principal stresses at ultimate-load stage.

## 7. EXAMPLE 3: DEEP BEAM

The third example concerns the reinforced deep beam which is also discussed in the key-note paper by Schlaich [16]. In that paper it was proposed to orientate the geometry of a 'strut-and-tie' model at the elastic stress fields, while this orientation can be adjusted upon approaching failure. It is interesting that a somewhat similar procedure was followed in [14] for nonlinear finite element analysis. The beam was first analysed in a global sense using smeared cracks. Subsequently, based on the crack pattern obtained, a simpler model was

made with a single predefined discrete crack, incorporated via interface elements. All nonlinearity was lumped into this discrete crack, surrounded by elastic elements for the strut and embedded rebar elements for the tie. Fig. 6 shows results of this 'predictor-corrector' approach, which turned out to work also for shear-critical problems [1]. Assuming sets of predefined discrete cracks in essence comes close to the yield line theory in plasticity where one imagines mechanisms of yield lines. This approach possibly has potential not only for review analysis, but also for dimensioning of so-called D-regions, where D stands for discontinuity, disturbance or detail.

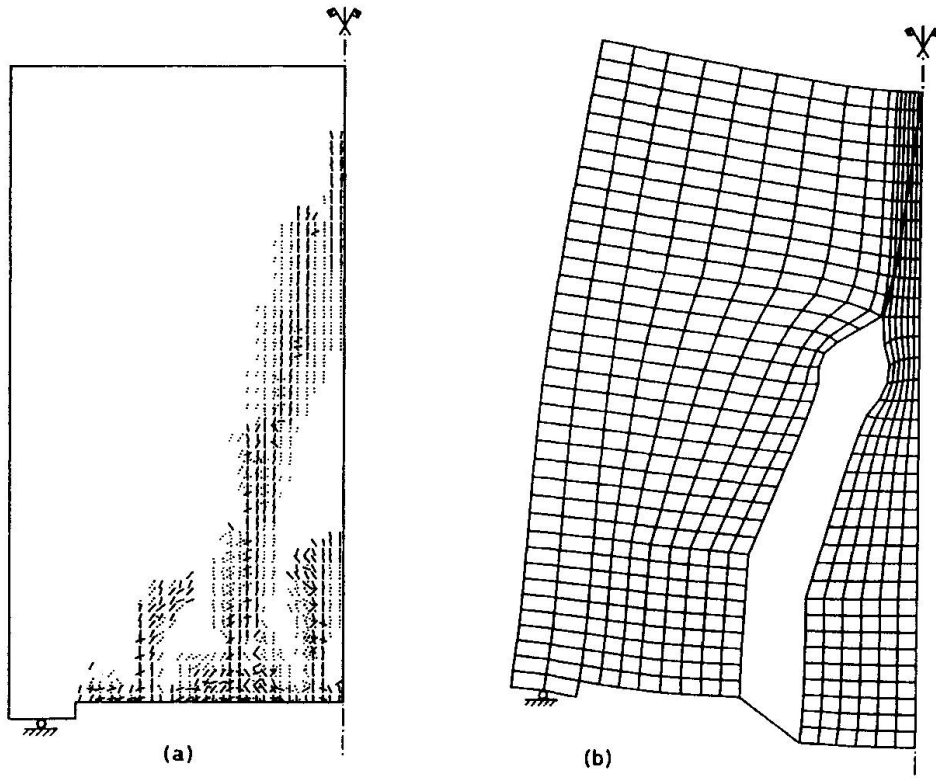


Fig. 6. Deep beam. Two possible strategies in nonlinear finite element analysis.

- (a) Smearred crack analysis for prediction of crack path.
- (b) Discrete crack analysis with predefined mechanism.

#### CONCLUDING REMARKS

It is concluded that the strength of very sophisticated bond models lies more in research than in practice. The utility of such models is of an indirect nature. These models for instance lead to a better understanding of basic bond mechanisms and better design rules for crack spacing and width. Global models with embedded reinforcement techniques are directly applicable in engineering practice. Examples have been presented that simply and correctly reveal 'strut-and-tie' systems after stress redistribution.

#### ACKNOWLEDGEMENTS

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## Safety Considerations for Nonlinear Analysis

Concepts de sécurité dans l'approche non-linéaire du calcul statique

Sicherheitsüberlegungen für nichtlineare Bauwerksberechnungen

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### SUMMARY

It is shown that the recommended safety concepts as proposed in EC 2 and the CEB Model Code are not rational in the case of safety checks for determining the nonlinear bearing capacities of a structure. A new proposal is given.

### RÉSUMÉ

On montre que les concepts de sécurité recommandés dans l'Eurocode 2 et dans le Code Modèle du CEB ne sont pas rationnels pour le contrôle de la sécurité dans le cas des charges critiques non-linéaires. On présente une autre proposition.

### ZUSAMMENFASSUNG

Es wird gezeigt, dass die im EC 2 und im CEB-Model Code empfohlenen Sicherheitskonzepte für nichtlineare Traglastermittlungen nicht sinnvoll sind. Deshalb wird ein neuer, abweichender Vorschlag unterbreitet.



## 1. SAFETY FORMAT AND CURRENT DESIGN

The basis of any rationally founded safety concept must be the probability of failure of a structure. The failure may be defined as a collapse situation, or any other limit state as e.g. the loss of serviceability. In more theoretical terms one asks for the probability of succeeding a limit surface in the space of the different design variable's density functions.

A 'Safety Format' is just one selected method out of an infinite number of simplified methods to guarantee a chosen probability of failure. A special choice is merely justified by reasons of practicability and the unavoidable deviation of the target probability for a whole group of structures. There is no principle advantage of a format using a global safety factor over another with e.g. several splitted partial safety coefficients.

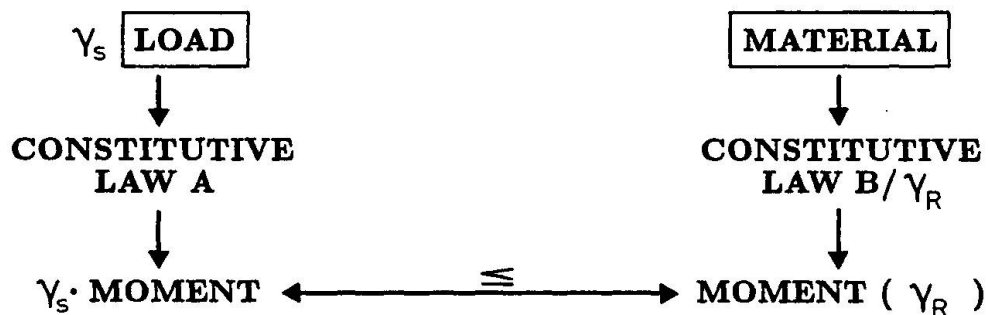


Fig. 1: Linear Analysis / Nonlinear Design

The current design format – called 'Format 1' (Fig.1) – results from the historical development of structural analysis and design. According to this, one first calculates inner forces and moments e.g. by means of the theory of elasticity using a very simple constitutive law A, followed by a so-called cross-section design with a second different set of now nonlinear constitutive laws B for steel and concrete.

Finally on a level of **c r o s s - s e c t i o n a l** quantities one compares e.g. inner moments increased by a safety factor  $\geq 1$  to account for the scatter of acting loads with design moments reduced in a **n o n l i n e a r** manner on the basis of constitutive laws by another safety factor  $\geq 1$  regarding material defects. Depending on the choice of partial safety coefficients a given minimum probability of failure may be secured.

As due to the different constitutive laws A and B cross-section design is independent of the internal moments and forces, no iteration process including the force distribution is necessary at the design stage.

This is a very simple 'Safety Format' with however strong inconsistencies e.g. that the strains calculated at the first process using material behavior A have nothing to do with the ones which result from the second material law B used for design.

## 2. BEARING CAPACITY DESIGN

It is however well known that in general local strength values do not control the safety of a structure at least not in case of indeterminate structures (Fig. 2). So e.g. the governing scatter of resistance within the length L of a yield line drops with L

(Fig. 2a), the local value being not decisive. The same is of course known from continuous girders, where yielding of one cross-section does not cause failure of the structure.

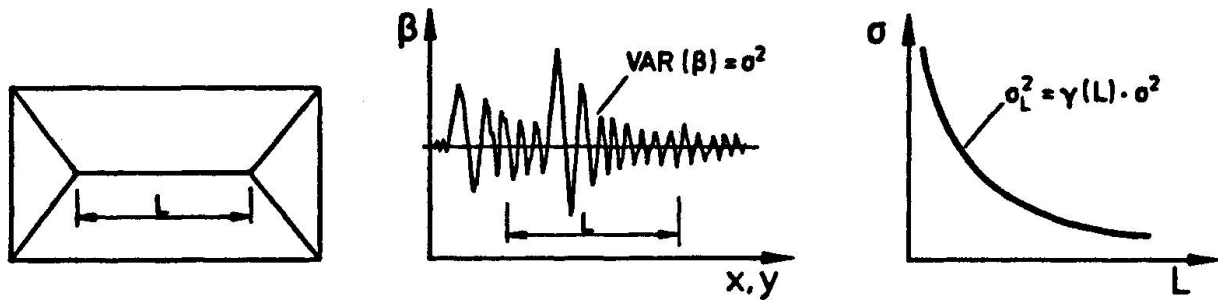


Fig. 2a: Plate as Example

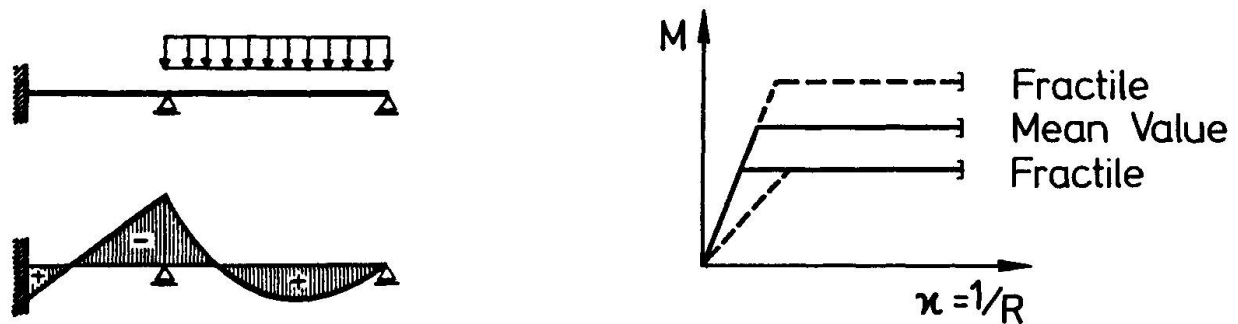


Fig. 2b: Continuous Beam as Example

Now nonlinear analysis allows to calculate the bearing capacity of a structure including local yield without overemphasizing it. This is done on the basis of only one physical correct set of constitutive laws A (Fig. 3) and allows a safety cheque comparing bearing loads that can be endured and acting outer loads.

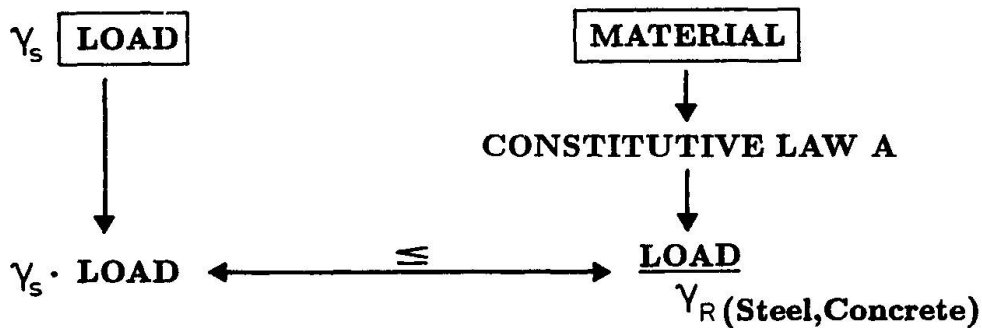


Fig. 3: Nonlinear Analysis and Design



In a 'Safety Format 2' then the bearing capacity is usually calculated by means of material mean values, which will be divided by a partial safety factor to account for materials scatter only at the final level of comparison. Similarly the acting load is increased by another partial load factor.

However instead of dividing the final bearing capacity at the stage of comparison and increasing the acting load both in a linear manner, one may also multiply both safety coefficients together and so end up with a global safety factor.

Proposals made in EC 2 in the CEB-Model Code for a safety concept in nonlinear analysis are not consistent with 'Format 2' and also not very rational.

It is proposed to compute in a nonlinear manner the inner forces of the structure by **m e a n** values of constitutive laws A and to do afterwards a cross-section design using now different constitutive laws B based on **f r a c t i l e** values. The consequence is that the inner forces and moments result from strain states that have nothing to do with the strain state of cross-section design and that the inner forces characterize just one arbitrary state of equilibrium as in 'Format 1'. This approach gives up the the advantage of nonlinear analysis to calculate system bearing capacity and does not justify the much higher amount of work necessary.

The opinion, it would be possible by iteration, to adjust the amount of reinforcement chosen first for the determination of inner forces and the one necessary for cross-section design is wrong of course. It is impossible to be in every cross-section (Fig. 2b) at the horizontal **m e a n** value branch and at the 5 % **f r a c t i l e** branch simultaneously.

When this was acknowledged it was then proposed to do the nonlinear analysis by only fractile laws to be on the 'safe side'. In general this is also not favorable as may be shown e. g. with Fig. 2b where too small support moments may lead to too small shear force at the inner supports. This proposal demands that one always has to decide what the 'safe side' is.

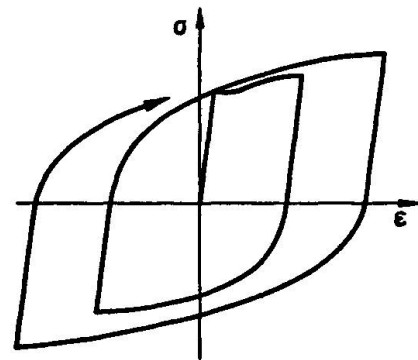
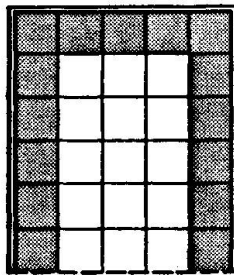
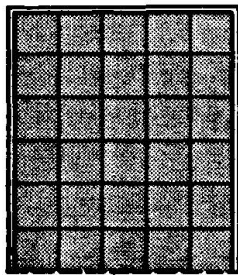


Fig. 4: Plate as Example

Fig. 5: Cyclic Constitutive Law

To save the concept of two partial safety coefficients, further proposals have been made to manipulate the constitutive laws in such a way as e.g. to take mean values for the first part of a bilinear stress - strain law to define stiffness and combine it with a reduced horizontal fractile branch.

It is obvious that such arbitrary manipulations cannot be a general answer to the problem. How should one manipulate a constitutive law in case of a nonlinear Finite Element computation for a plate (Fig. 4) e.g. to find a 'safe solution' ? If one reduces the whole stress strain law equally in all elements or alternatively only within the boundary elements, one finds quite different moments in the midst of the plate.

How does one have to manipulate the cyclic constitutive law (Fig. 5) to find a safe solution for what? These questions show clearly that in general a new approach, that means a new 'Safety Format' is necessary when calculating nonlinear and more realistic bearing capacities.

### 3. AN ADEQUATE SAFETY CONCEPT FOR NONLINEAR ANALYSIS

In the following a new adequate safety concept is derived by means of the nowadays possible stochastic Finite Element method – not to use it for practical design – for nonlinear analysis and design.

Beginning with the Equilibrium equations:

$$[\bar{\mathbf{K}}] + [\delta\mathbf{K}] \cdot \{ \bar{\mathbf{u}} \} + \{ \delta\mathbf{u} \} = \{ \bar{\mathbf{R}} \}$$

where

- $\mathbf{K}$  = stiffness matrix
- $\mathbf{u}$  = deformation
- $(\bar{\quad})$  = mean value
- $\mathbf{R}$  = bearing capacity

one finds after some manipulations (See [6], [7] e.g.):

$$[\mathbf{C}_u] = [\bar{\mathbf{K}}]^{-1} \cdot \left[ \frac{\partial \mathbf{K}}{\partial \alpha} \right] \cdot \{ \bar{\mathbf{u}} \} \cdot [\mathbf{C}_\alpha] \cdot \left[ \frac{\partial \mathbf{K}}{\partial \alpha} \right]^T \cdot \{ \bar{\mathbf{u}} \}^T \cdot [\bar{\mathbf{K}}]^{-T}$$

with

- $[\mathbf{C}_\alpha] = \delta\alpha \cdot \delta\alpha^T$
- $[\mathbf{C}_u] = \delta\mathbf{u} \cdot \delta\mathbf{u}^T$
- $[\quad]$  = Matrix
- $\{ \quad \}$  = Vektor

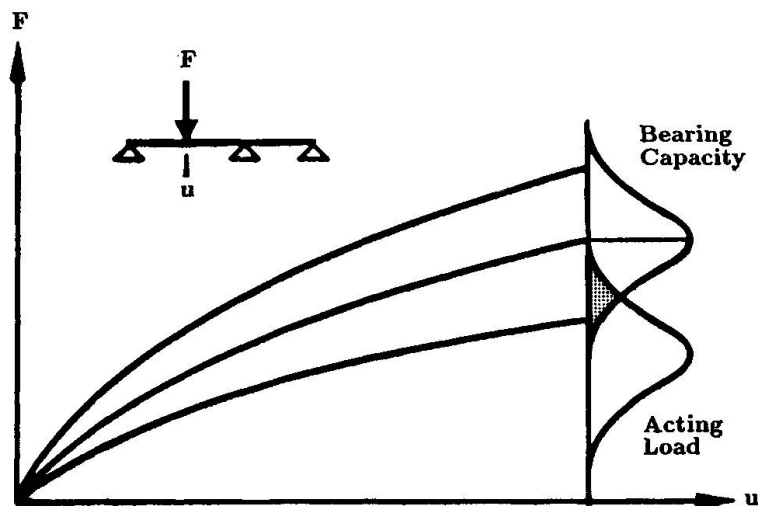


Fig. 6: Nonlinear Analysis including Probabilistic Approach

saying that from the given covariance matrix  $\mathbf{C}_\alpha$  of the scattering basic variables  $\alpha$  one can easily compute the covariance matrix  $\mathbf{C}_u$  of the deformations  $\mathbf{u}$  and by means of  $\delta\mathbf{u}$



finally the values  $\bar{\mathbf{R}}$  and the scatter of  $\mathbf{R}$ . So even in nonlinear analysis the scatter of the bearing capacity may be given in a  $\mathbf{R} - u$  plot (Fig. 6).

From these argumentations the following conclusions may be drawn :

What is necessary to achieve a desired probability of failure is just one **g l o b a l** safety factor to determine the distance of the acting load density function to the density function of the bearing capacity computed by mean material values. This may be a different factor in cases of steel or concrete failure of course. Even the different safety behavior of structural determinate and indeterminate structures is then inherently covered.

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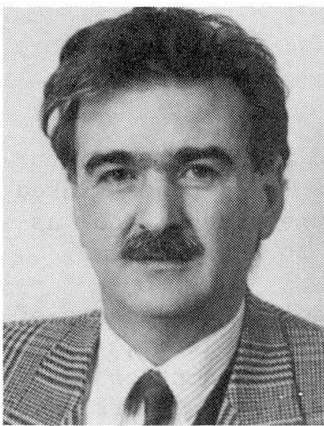
## Plane Elements Analysed Via a Simple Microplane Model

Analyse de structures planes selon un modèle simple par microplans

Analyse von Stahlbetonscheibenelementen  
mit einem einfachen Mikroebenen-Modell

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### SUMMARY

Panels subjected to monotonically increasing loads and deep beams with and without cutouts are analysed assuming a simple microplane model for concrete. The microplane model is incorporated into a finite element program based on an incremental-iterative procedure, which is well suited to the description of the highly non-linear behaviour of reinforced concrete elements. The reinforcement is smeared; bond-induced stiffening effects are included.

### RÉSUMÉ

Des parois en béton armé ainsi que des murs porteurs soumis à des charges continument croissantes sont examinés en recourant pour le béton à un modèle simplifié par microplans; ce dernier fait partie d'un programme par éléments finis basé sur une procédure itérative pas-à-pas. L'armature est disposée sur chaque élément, de même que l'on tient compte de l'adhérence acier-béton et de ses effets raidisseurs. Le modèle de microplans étudié s'adapte de façon satisfaisante aux résultats expérimentaux obtenus.

### ZUSAMMENFASSUNG

Stahlbetonscheiben bei stetig ansteigender Belastung und wandartige Träger mit Öffnungen werden mit Hilfe eines vereinfachten Mikroebenen-Modells für den Beton untersucht. Dieses Modell ist in ein Finite Elemente Programm eingebaut, das auf einem schrittweise wiederholenden Verfahren beruht. Die Bewehrung ist über jedes Element verschmiert und auch der Verbund zwischen Stahl und Beton ist mit seiner Steifigkeit vorhanden.



## 1. INTRODUCTION

The so-called "local models" for the description of the multiaxial behavior of concrete have become very appealing lately due to their intrinsic simplicity and to their strict connection with the micromechanics of aggregate materials.

Among the local models, the Microplane Model (Bazant and Oh [1], Gambarova and Floris [2]) has enjoyed special attention, since it seems well suited to the modellization of a variety of loading conditions, monotonic as well as cyclic (Bazant and Prat [3]).

Here an initial and relatively simple version for 2-D problems [2] is introduced into a pre-existing F.E. code and is applied to the analysis of several RC plane structures, such as Collins' panels [4], Cervenka and Gerstle's ribbed panels [5] and Kong's deep beams [6,7], in order to assess the ability of the model to describe the structural behavior (load-displacement response, cracking and path-dependency).

In a previous paper (Donida et al. [8]), Maier and Thürlimann's shear walls were successfully analysed with the proposed model.

## 2. MICROPLANE MODEL FOR CONCRETE SUBJECTED TO PLANE STRESSES

Concrete is considered as a system of randomly oriented planes (the microplanes), in which the elastic and inelastic deformations are concentrated (Figs.1a,b). In the simplified formulation adopted here, three fundamental assumptions are introduced, with a fourth assumption referring to 2-D problems:

- a) the local strains, acting on each microplane, are the resolved components of the applied strains (macroscopic strain tensor):  $\epsilon_n = n_i n_j \epsilon_{ij}$ , with  $i, j = 1, 2$ .
- b) the shear stiffness of the microplanes is neglected: this assumption has a physical explanation [1,2], but has been introduced mostly for its intrinsic simplicity, and may be dropped in a more general approach [3];
- c) only the normal stiffness of the microplanes is introduced, and the coupling between the normal microstress  $\sigma_n$  and the shear strain  $\gamma_{nt}$  is disregarded. Both the elastic and inelastic behavior of the concrete is described by assuming that  $\sigma_n$  is a function of  $\epsilon_n$ :  $\sigma_n = F(\epsilon_n) \epsilon_n$ ;
- d) in 2-D problems only the microplanes at right angles to the reference plane are considered (Fig.1b).

In order to work out the coefficients of concrete stiffness matrix it is necessary to formulate the constitutive law for the microplanes: then, by suitably superimposing the contributions from all microplanes, the stiffness characteristics of the concrete can be obtained [2], as well as the increments of the stresses in the general reference system. Under increasing loads, it suffices to specify the stress-strain relations for loading, unloading and reloading in tension and compression (Fig.1c). The model is -by its very nature- a kind of "rotating crack model" (crack planes coincide with the microplanes exhibiting the maximum normal strain in tension) and is path-dependent (the microplanes are activated independently of each other).

## 3. F.E. PROGRAM IMPLEMENTATION WITH THE MICROPLANE MODEL

A suitable F.E. code based on an incremental-iterative procedure, has been implemented with the microplane model. Quadrangular 4-node elements are used (Fig.3), a fifth inner auxiliary node being provided in order to subdivide each element into four constant-stress triangular elements. The fifth node does not contribute to the degrees of freedom of the structure, because it is removed before the stiffness matrix of the structure is assembled, by means of a condensation process [8]. The introduction of the microplane model, as well as the

evaluation and updating of concrete stiffness matrix, are worked out in a first subroutine dealing with the triangular elements; a system of 12 microplanes suffices for concrete description. The stiffness matrix of the quadrangular elements is evaluated and assembled in a second subroutine.

General properties of the F.E. code are: (a) the stiffness coefficients are formulated by the "direct method"; (b) the solution is based on the Gauss-Doolittle method; (c) the shape functions are of the polynomial type; (d) at the moment only monotonic load histories can be studied; (e) the reinforcement is smeared in two directions at right angles to each other.

Bond-induced tension stiffening effects are introduced by modifying the stress-strain law of the reinforcement: to this purpose, crack orientation, spacing and opening have to be evaluated at each load step and in each triangular element (Fig.2a). Once cracks are formed, their orientation remains fixed. In order to evaluate crack spacing, an "equivalent" steel ratio (Fig.2b) has to be defined [8]: the Young's modulus  $E'_s$  of the steel is a function of the average steel strain according to two different bond-stress situations (Fig.2c,d and Fig.4).

Finally, a very simple failure criterion has been introduced for the microplane system: as soon as 2/3 of the microplanes reach a prefixed limit strain in compression and/or in tension (where strain softening automatically diminishes the stiffness), the stiffness of the material is put to zero; subsequently, as soon as the solution process no longer converges, the whole structure fails.

#### 4. FITTING OF TEST DATA

Nine different cases are here examined (Figs.5,6 and 7) and no detailed comments are necessary, since the results are mostly self-explanatory; the principal material properties and the size of test specimens are reported in the figures or in the captions below. For further details see the references, which are easily

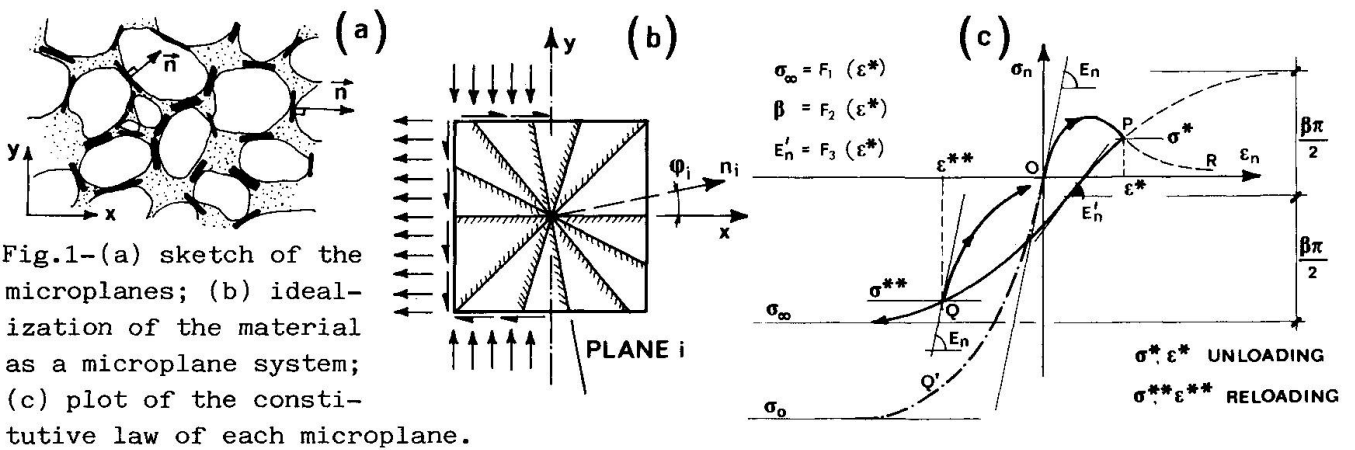


Fig.1-(a) sketch of the microplanes; (b) idealization of the material as a microplane system; (c) plot of the constitutive law of each microplane.

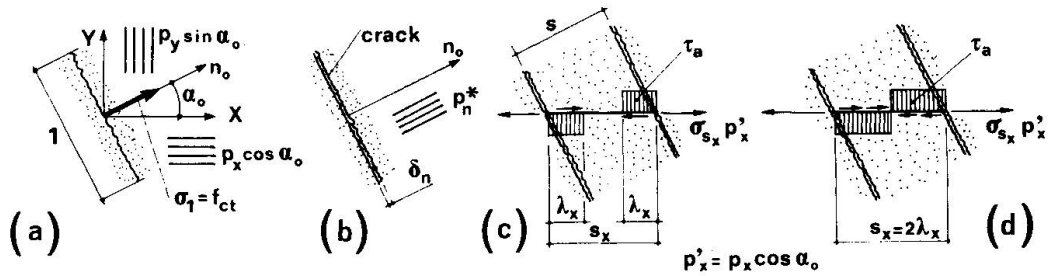


Fig.2 - Formation of a crack (a); equivalent steel ratio (b); bond stresses for mixed bond conditions (chemical adhesion and mechanical interaction) (c); bond stresses after the loss of chemical adhesion (mechanical interaction only) (d).

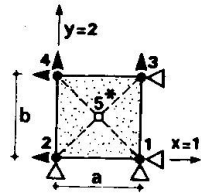


Fig. 3 - Typical 4-node element used in F.E. analysis; (\*) means node condensation.

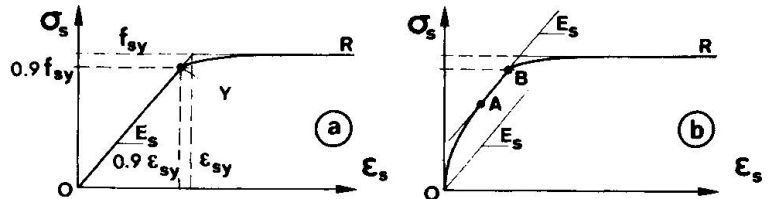


Fig. 4 - Constitutive law of the reinforcement without tension stiffening (no bond) (a) and with tension stiffening (embedded bars) (b).

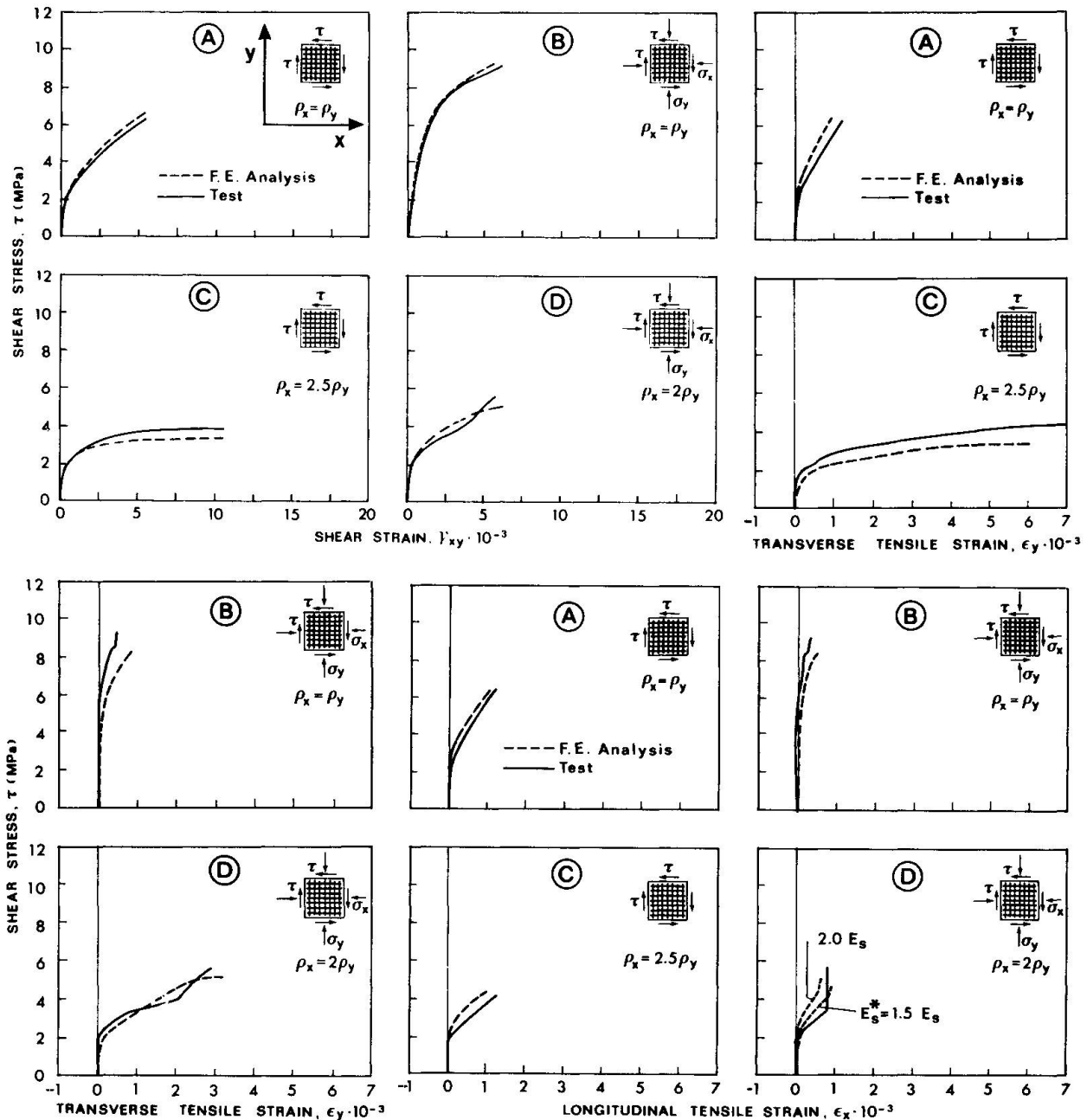


Fig. 5 - Fits of Collins' test results [4]: shear stress  $\tau = 0 - \tau_u$ ;  $\rho_x = 0.01785$ .  
 Panel A :  $f'_c = 20.5$  MPa,  $f_{sy} = 442$  MPa,  $\sigma_x = \sigma_y = 0$ ; Panel B :  $f'_c = 19.3$  MPa,  $f_{sy} = 466$  MPa,  $\sigma_x = \sigma_y = -0.7\tau$ ; Panel C :  $f'_c = 19.0$  MPa,  $f_{sy} = 458$  MPa for x-bars and 299 MPa for y-bars,  $\sigma_x = \sigma_y = 0$ ; Panel D :  $f'_c = 21.7$  MPa,  $f_{sy} = 441$  MPa for x-bars and 324 MPa for y-bars,  $\sigma_x = \sigma_y = 0$  for  $\tau \leq 3.9$  MPa and  $\sigma_x = \sigma_y = -(\tau - 3.9)$  MPa for  $\tau > 3.9$  MPa. Panel size : 890 x 890 x 70 mm. F.E. discretization: 9 square elements.

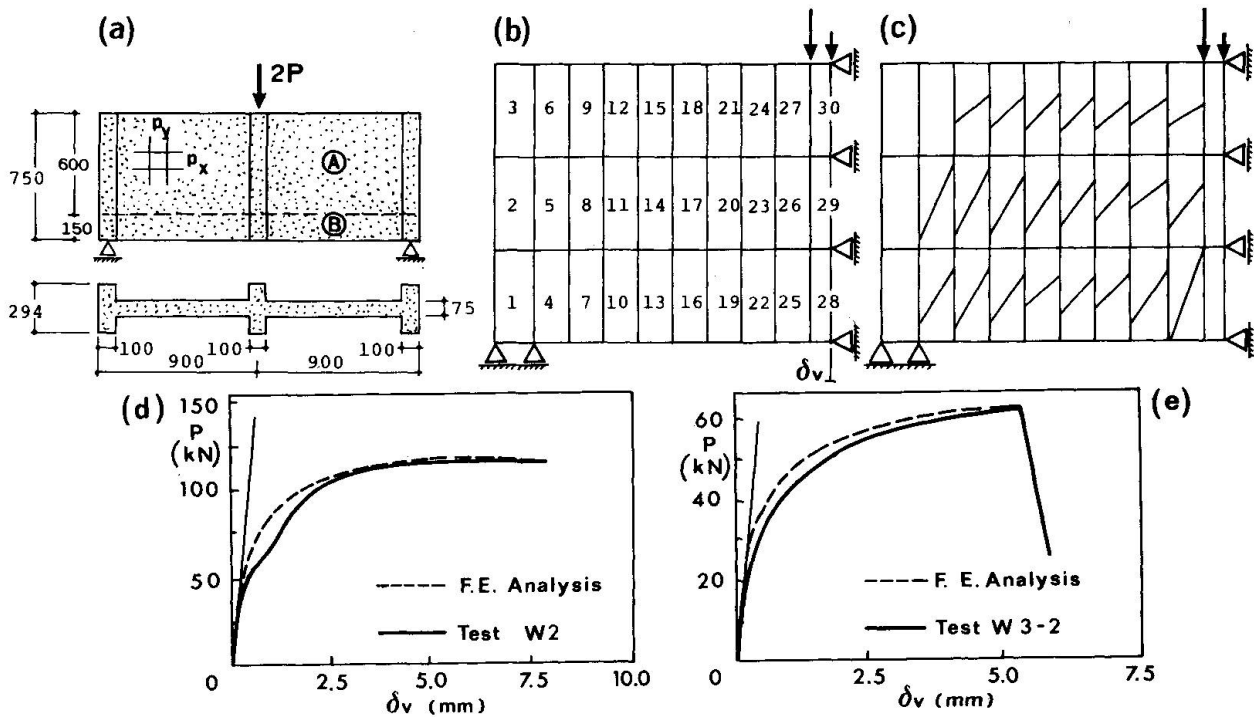


Fig. 6 - Fits of Cervenka and Gerstle's test results [5]: Test W2 (a,c,d) web thickness  $t = 75$  mm, rib thickness  $t' = 294$  mm, Zone A with  $\rho_x = \rho_y = 0.00916$ , Zone B with  $\rho_x = 0.01832$ ,  $\rho_y = 0.00916$ ; Test W3-2 (a,e)  $t = 50$  mm,  $t' = 269$  mm, Zones A and B with  $\rho_x = 0.0123$ ,  $\rho_y = 0$ ; (b) FE mesh, loads and boundary conditions; (c) directions of the principal compressive strain, at collapse; (d,e) load-displacement curves.  $f'_c = 27.4$  MPa,  $f_{sy} = 362$  MPa.

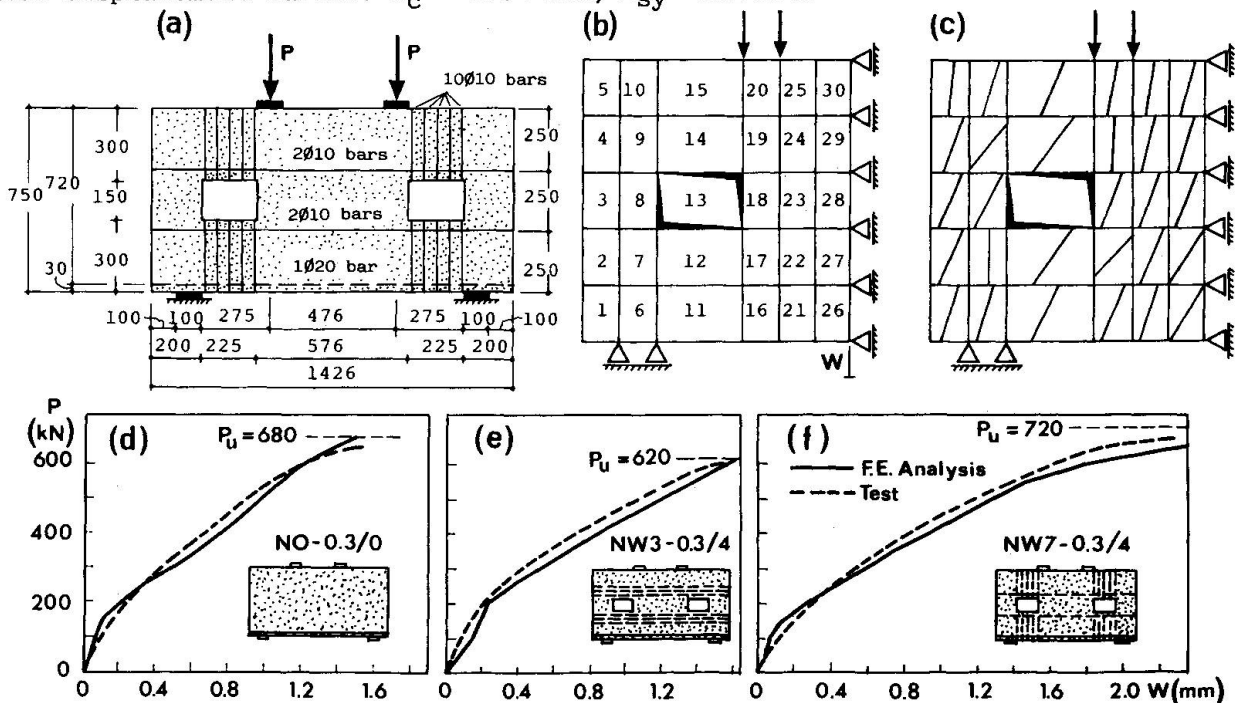


Fig. 7 - Fits of Kong et al. test results [6,7]: Beam NW7-0.3/4 (a) geometry, (b) FE discretization and (c) directions of the principal compressive strain at collapse; (d,e,f) load-displacement curves of Beams NO-0.3/0 ( $f'_c = 44.8$  MPa), NW3-0.3/4 ( $f'_c = 46.2$  MPa) and NW7-0.3/4 ( $f'_c = 42.9$  MPa);  $f'_{ct} = 3.75$  MPa,  $f_{sy} = 430-450$  MPa. Beam thickness = 100 mm.



found in the literature. As a rule, the rebars were smeared in one or two directions over the elements they go through; consequently, the "steel density" is often quite far from the real situation, but this fact does not seem to have a major impact on the results of the analysis. In Panel D tested by Collins ([4] Fig.5) significant unloading occurs after compressive stresses are applied; as a result, the "effective Young's modulus" of the embedded steel during unloading plays a relevant role ( $E_s^* = 1.5-2.0 E_s$ ).

On the whole the fits are more than satisfactory, but a more refined analysis with a better topological description of the steel arrangement is in progress.

## 5. CONCLUDING REMARKS

1. The Microplane Model can be introduced easily into available F.E. codes.
2. The Microplane Model can describe in a relatively simple way a few complex aspects of concrete mechanics, such as multiaxial behavior, path-dependency and cracking.
3. The necessity of storing a few data on the history of each microplane is offset by the limited number of parameters required by the formulation of the microplane constitutive relations.
4. The Microplane Model may be easily improved in order to describe concrete behavior under variable loads (for instance, cyclic and fatigue loads).

## ACKNOWLEDGEMENTS

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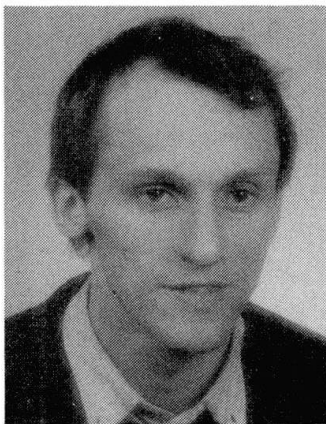
## Evaluation of the Safety of a Cracked Concrete Cooling Tower

Estimation de la sécurité d'une tour de refroidissement dont le béton est fissuré

Untersuchung der Sicherheit eines gerissenen Stahlbetonkühlturmes

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### SUMMARY

The limit of serviceability and the residual safety against structural collapse of a cracked concrete cooling tower, which was built 25 years ago, is evaluated numerically by the finite element method. The process of damage is simulated by means of different thermal load histories. Several assumptions concerning the degree of corrosion are made.

### RÉSUMÉ

La méthode des éléments finis a permis d'estimer numériquement la limite de service et la réserve de sécurité vis-à-vis d'une ruine de la structure, dans le cas d'une tour de refroidissement construite il y a 25 ans, dont le béton est fissuré. Le processus de détérioration est simulé à partir de considérations concernant différentes applications de charges de type thermique, ainsi que selon différentes hypothèses sur l'état de corrosion des armatures.

### ZUSAMMENFASSUNG

Mit Hilfe der Methode der Finiten Elemente wird die Gebrauchssicherheit sowie die Traglast eines 25 Jahre alten, durch Risse beschädigten Kühlturms bestimmt. Der Schädigungsprozess – Ausbildung vertikaler Risse infolge von Temperaturspannungen – wird mit verschiedenen Annahmen für die Abfolge der Temperaturbeanspruchung sowie für die Entwicklung der Korrosion des Betonstahles nachvollzogen.



## 1. PRELIMINARY REMARKS

Concrete structures which were designed and built in the 1950's and 1960's frequently show signs of damage such as cracks, caused, e.g., by thermally induced stresses, which usually were not considered adequately by the design provisions of that time [1]. This paper demonstrates the role of nonlinear finite element analyses in the context of the evaluation of the residual safety coefficient of a cracked natural draught concrete cooling tower, which was built 25 years ago. With regards to "safety", loss of serviceability is considered as well as the limit state, characterized by the ultimate load.

The cooling tower Ptolemaïs-III is part of a 125 MW power station located in Ptolemaïs, Greece. The present state of the cooling tower shell is characterized by an approximately uniform distribution of long meridional cracks of crack width up to  $\sim 1$  cm, which are distributed more or less evenly along the circumference (Fig.1a).

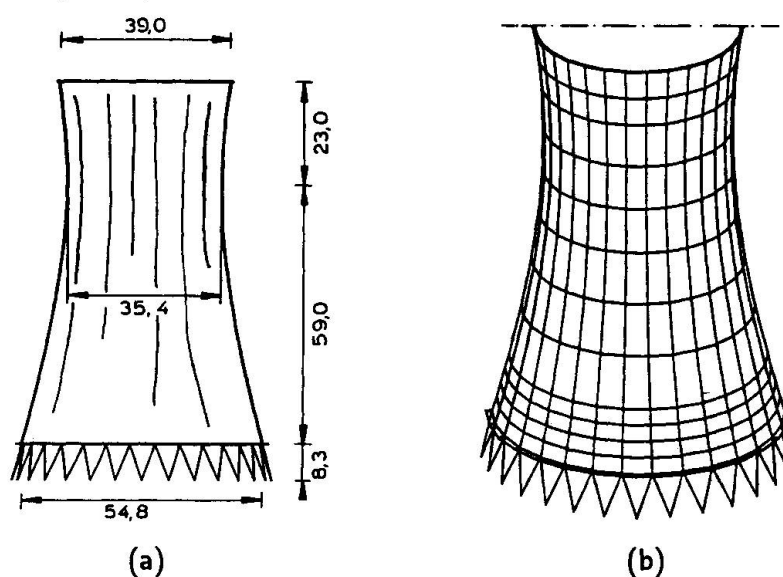


Figure 1: Meridional cracks, geometry, and finite element discretization of the Ptolemaïs cooling tower; (a) meridional cracks and geometry; (b) finite element discretization

The shell is supported by 30 pairs of concrete columns. Except for a small zone of  $\sim 9.3$  m height at the lower part of the shell, the structure is reinforced by only one layer of reinforcement located in the middle of the shell. The thickness of the shell is 10 cm with the exception of the aforementioned lower part of the shell, where the thickness is gradually increasing from 10 cm to 50 cm towards the base.

Following a detailed in-situ inspection it was concluded (having rejected improbable causes of the damage) that the temperature gradient between the inside and the outside surface of the cooling tower shell according to winter conditions is the most likely cause for the development of the observed meridional cracks.

## 2. OUTLINE OF THE NUMERICAL PROCEDURE

### 2.1 Finite Element Model

The numerical investigation is based on an incremental – iterative procedure with the step size being adapted to the degree of nonlinearity of the structural behaviour. Fig.1b shows the chosen finite element mesh. Because of symmetry of the wind loading, only one half of the shell needs to be considered for the analysis. The shell and the stiffening rings located at the crown and at

the bottom of the shell, respectively, are discretized by 225 quadrilateral shell elements [6]. Each element is subdivided into 13 layers such that approximately a plane state of stress may be assumed in each layer. As has been verified by a mesh sensitivity study based on a consistent refinement of the mesh [2], the selected discretization is sufficiently fine. The supporting columns are treated as beam elements (Fig. 1b) with axial as well as bending stiffness.

## 2.2 Constitutive Model

The numerical representation of cracked concrete is based on the "smeared-crack" approach. Cracks will open normal to the direction of the maximum principal stress when this stress exceeds the tensile strength  $f_{tu}$ . After crack initiation, tensile stresses are gradually released according to a linear post-peak stress-strain-relationship defined by a constant softening modulus  $E_S$  [2], [4]. The residual interface shear transfer across cracks, resulting from the roughness of the crack face and the dowel action of the reinforcing bars, is considered by means of a crack-strain dependent shear modulus  $G_c$  [2]. Tension stiffening is disregarded.

A linearly elastic - ideally plastic constitutive law is assumed for the reinforcement steel. The meridional and the circumferential reinforcement bars are smeared to mechanically equivalent, thin layers of steel which only have axial stiffness in the respective direction (orthotropic material). Corrosion is considered by multiplying the diameter of the reinforcement bars located near both faces in the lower part of the shell by 0.9.

Table 1: Material parameters

CONCRETE		
$E_c = 2600 \text{ kN/cm}^2$	Tensile Strength	$f_{tu} = 0.18 \text{ kN/cm}^2$
$\nu_c = 0.20$	Compressive Strength	$f_{cu} = 2.25 \text{ kN/cm}^2$
$E_S = 2000 \text{ kN/cm}^2$	Coeff. of Therm. Exp.	$\alpha = 10^{-5} / ^\circ\text{C}$
STEEL		
$E_{ST} = 206000 \text{ kN/cm}^2$	Yield Stress	$\sigma_y^{ST} = 40.0 \text{ kN/cm}^2$

## 2.3 Considered Load Cases

The investigation of the influence of the thermal preloading on the limit of serviceability and on the ultimate load of the structure comprises several thermal load histories. Two load cases refer to winter conditions. They are characterized by the incremental increase of the temperature difference  $\Delta T_W$  - the subscript "W" stands for "winter" - between the inside and the outside surface up to  $45^\circ\text{C}$  (LOAD CASE II) and, for one of these two load cases (LOAD CASE III), by subsequent thermal unloading. Thereafter, the wind load  $w$  according to [3] is applied incrementally. These load histories may be written symbolically as

$$g + \Delta T_W/h + \lambda w \quad (\text{LOAD CASE II}), \quad (1)$$

$$g + \Delta T_W/h - \Delta T_W/h + \lambda w \quad (\text{LOAD CASE III}), \quad (2)$$

where  $g$  represents the dead load,  $h$  is the thickness of the shell and  $\lambda$  is a dimensionless parameter ( $\lambda \geq 0$ ). For the purpose of including a possible weakening of the structure due to (micro)cracks on the inner surface, one additional load case (LOAD CASE IV) was considered. It allows simulation



of a winter-summer cycle ( $\Delta T_W = 45^\circ C, \Delta T_S = -20^\circ C$ ) with subsequent thermal unloading prior to the incremental application of the wind load:

$$g + \Delta T_W/h - \Delta T_W/h + \Delta T_S/h - \Delta T_S/h + \lambda w. \tag{3}$$

In order to assess the stiffness reduction due to the observed cracks, an ultimate load analysis of the originally uncracked shell was performed (LOAD CASE I):

$$g + \lambda w. \tag{4}$$

### 3. INFLUENCE OF THE THERMAL LOAD HISTORY

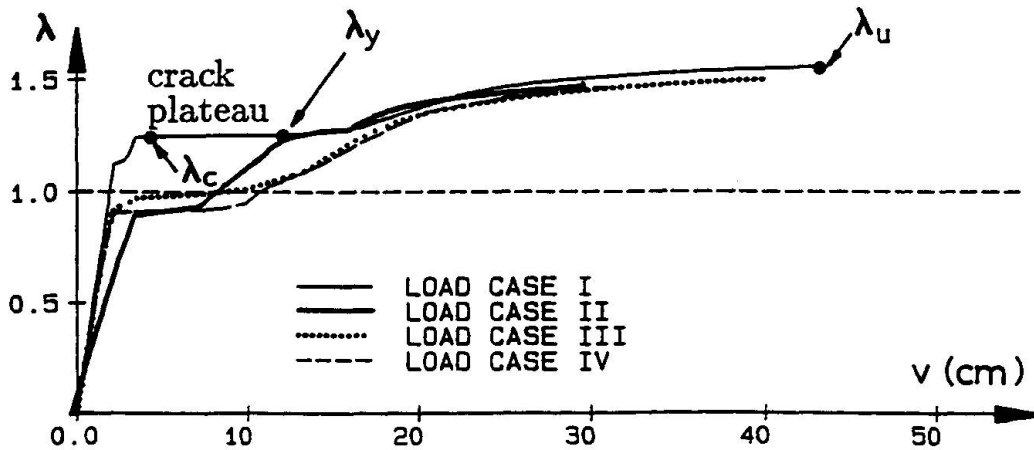


Figure 2: Load-displacement curves for load cases I, II, III and IV

In Fig.2, the horizontal displacement at the windward meridian, at the crown of the shell, parallel to the axis of symmetry (see Fig. 1b) is plotted as a function of the dimensionless factor  $\lambda$  for the load cases I, II, III and IV, respectively. Obviously, the damage induced by thermal preloading (LOAD CASES II, III and IV) has a significant influence on the level of the "crack plateau" ( $\lambda_c$ ). With respect to the ultimate load level ( $\lambda_u$ ), however, the influence of these temperature cracks is insignificant. The beginning of yielding of the reinforcement ( $\lambda_y$ ) may be considered as a *sufficiently conservative limit of the serviceability* of the structure. Compared to the uncracked structure, the values for  $\lambda_y$  obtained from analyses of the damaged shell ( $\lambda_y = 1.24$  for LOAD CASES II and III and  $\lambda_y = 1.20$  for LOAD CASE IV) are only slightly lower than  $\lambda_y = 1.29$ , corresponding to LOAD CASE I. Table 2 summarizes the values for  $\lambda_c, \lambda_y$  and  $\lambda_u$  resulting from the numerical investigation.

Table 2:  $\lambda_c, \lambda_y$  and  $\lambda_u$  for load cases I, II, III and IV

Load Case	$\lambda_c$	$\lambda_y$	$\lambda_u$
I	1.26	1.29	1.56
II	0.92	1.24	1.495
III	0.98	1.24	1.515
IV	0.915	1.20	1.47

It is noteworthy that the mode of considering the history of the temperature loading does not have a great influence on the structural resistance of the shell when being subjected to wind loading.

#### 4. INFLUENCE OF CORROSION

In the investigation described so far, corrosion of the reinforcement was considered by multiplying the diameter of those reinforcement bars by 0.9, which are located near both surfaces at the lower part of the shell. This assumption was regarded as not sufficiently conservative. Therefore, three different

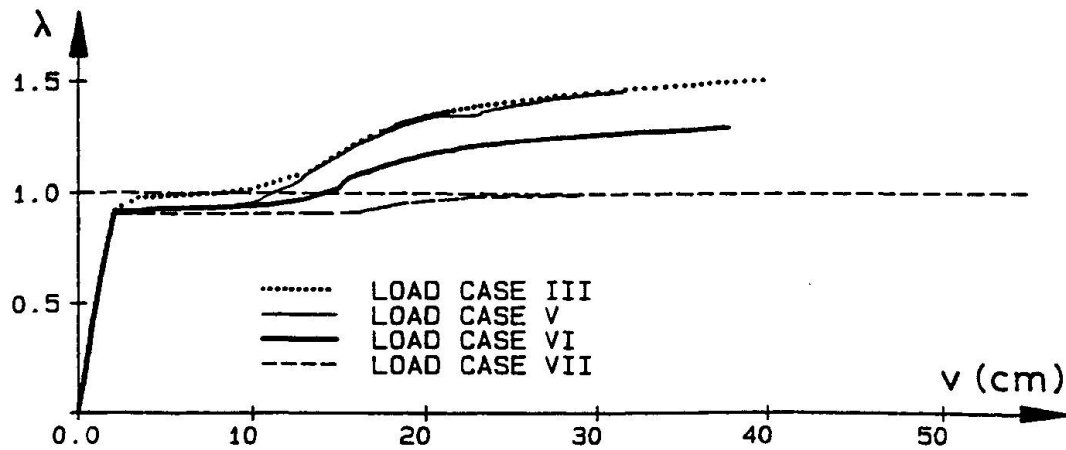


Figure 3: Load-displacement curves for load cases III, V, VI and VII

assumptions were made concerning the amount of corrosion that may occur in the remaining lifetime of the cooling tower. (The temperature loading process was chosen according to LOAD CASE III). In LOAD CASE V, only corrosion of the reinforcement bars located near both faces of the lower part of the shell is considered. According to [5], a reduction of the diameter of the reinforcement bars by 0.1 mm per year is a realistic assumption. Hence, after 25 years, the total reduction of the diameter will be 2.5 mm. In LOAD CASE VI, also the corrosion of the reinforcement bars located in the middle of the shell is taken into account. The diameter of these bars is multiplied by the factor 0.9. LOAD CASE VII is based on the worst assumption. Corrosion is considered by a 2.5 mm reduction of the diameter of *all* reinforcement bars.

Fig.3 contains load-displacement diagrams obtained from LOAD CASES III to VII. The respective values for  $\lambda_c$ ,  $\lambda_y$  and  $\lambda_u$  are summarized in Table 3.

Table 3:  $\lambda_c$ ,  $\lambda_y$  and  $\lambda_u$  for load cases III, V, VI and VII

Load Case	$\lambda_c$	$\lambda_y$	$\lambda_u$
III	0.98	1.24	1.515
V	0.925	1.21	1.465
VI	0.93	1.07	1.30
VII	0.905	0.915	0.995

For LOAD CASE VI,  $\lambda_y$  and  $\lambda_u$  are obtained as 1.07 and 1.30, respectively, as compared to 1.24 and 1.515 for LOAD CASE III. For LOAD CASE VI the ratio  $\lambda_u / \lambda_y$ , is equal to 1.22. In view of the character of this ratio as a "residual safety factor", the reduction of the diameters of the reinforcement through multiplication of the diameter by 0.9, as considered in LOAD CASE VI, can be regarded as a tolerable *upper limit* for the corrosion. For a significantly worse state of corrosion, as represented by LOAD CASE VII, almost no safety margin between the cracking plateau and the onset of yielding exists. This load case represents a critical state of corrosion, where neither safety



against loss of serviceability nor against structural failure is guaranteed! Such a critical state of corrosion of the reinforcement located in the middle of the shell, however, is unlikely to occur.

## 5. INFLUENCE OF THE TENSILE STRENGTH

The sensitivity of the structural response with respect to the assumed value for the direct tensile strength  $f_{tu}$  is demonstrated by reinvestigating LOAD CASE VI, on the basis of the experimentally obtained value  $f_{tu} = 0.26 \text{ kN/cm}^2$  (LOAD CASE VIII) instead of the original value  $f_{tu} = 0.18 \text{ kN/cm}^2$ . Table 4 contains the values for  $\lambda_c$ ,  $\lambda_y$  and  $\lambda_u$  for the two load uses. For an increase of the tensile strength by 44.4 %,  $\lambda_c$  increases by 28 %,  $\lambda_y$  by 13 % and  $\lambda_u$  by 2.7 % !

Table 4:  $\lambda_c$ ,  $\lambda_y$  and  $\lambda_u$  for Load Cases VI and VIII

Load Case	$\lambda_c$	$\lambda_y$	$\lambda_u$
VI	0.93	1.07	1.30
VIII	1.205	1.21	1.335

## 6. CONCLUSIONS

It is concluded that the investigated cooling tower shell will be sufficiently safe against structural failure even if no provisions for a repair of the concrete shell are taken, provided the reduction of the diameter of the reinforcement bars will be less than 10 % within the remaining life-time of the structure. The degree of corrosion of the reinforcement and, consequently, the efficiency of the precautions taken to protect the reinforcement from further corrosion are the relevant criteria for the safety of the cooling tower against loss of serviceability and structural collapse.

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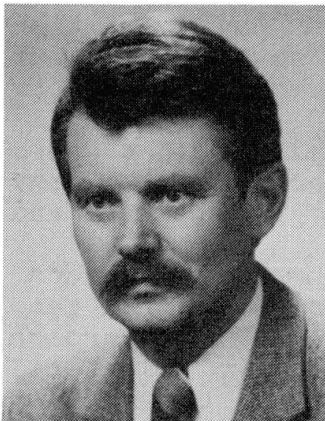
## Remarks on Failure of a Reinforced Concrete Cooling Tower

Considérations sur la ruine d'une tour de refroidissement en béton armé

Bemerkungen über den Einsturz eines Stahlbetonkühlturms

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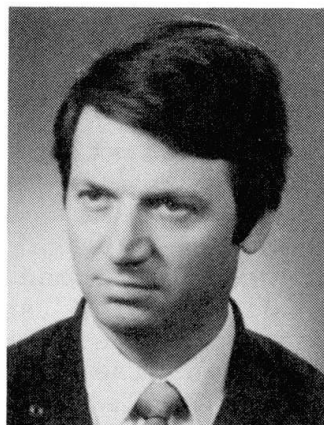
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### SUMMARY

In February 1987, a reinforced concrete cooling tower, 100 m high, collapsed at the power station Turow in the Lower Silesian region. As the process of the collapse, its effects and the results of later research on its cause revealed a series of interesting findings and conclusions, we decided to describe them in our paper.

### RÉSUMÉ

C'est en février 1987 que le tour de refroidissement en béton armé de la centrale thermique de Turow (Basse Silésie, Pologne), haute de 100 m, s'effondra brusquement. Le processus de l'écroulement, ses effets et les résultats de la recherche qui s'ensuivit afin d'en déterminer les raisons, ont apporté une série de remarques intéressantes ainsi que certaines conclusions. Ces considérations font l'objet du présent article.

### ZUSAMMENFASSUNG

Im Februar 1987 ist ein 100 m hoher Stahlbetonkühlturm des Kraftwerks Turow eingebrochen. Der Verlauf des Unfalles, seine Auswirkungen und die Ergebnisse späterer Forschungen über die Ursachen waren so interessant, dass wir uns entschlossen haben, sie in dieser Arbeit zu beschreiben.



## 1. Introduction.

Increasing in the last years computing abilities and intensive research in modeling and structure analysis, brought the formulation of the new idea of concrete structure definition a term called structural concrete. The term called total structural designing is tightly with this term connected. What is understood by these terms, we may find in the works of Bruggeling [1], Breen [2] and others.

First it is necessary to introduce a definition of the structural concrete. Bruggeling [1] gives the following definition of this term; "Structural concrete" refers to any structure built from concrete, and non-prestressed and/or prestressed reinforcement which can resist, in controlled way, all the actions exercised on these structures by loads, imposed deformations and other influences. Moreover, these structures must be constructed in the safe and economical way. In that definition, the article in a controlled way says about our possibility to control, for instance: some deformations, cracks, or durability during the designing process.

Breen [2] says that a structural concrete that is the term describing a wide range of concretes used for building any structure, involving plane concretes, normally reinforced and prestressed concretes.

The basic idea of what is termed the "Structural Concrete Approach" is to eliminate distracting and artificial barriers which tend to compartmentalize the designer's thinking. In the new approach it's necessary to emphasize more global attention to total load paths and resisting elements. That means that at the base of that definitions there lies a wish at overcoming the existing division of concrete structures on normally reinforced and prestressed. In other words it is a wish to enable an analysis of freely chosen structure independently of the way of its reinforcing and a type of acting loads.

Introduction of these definitions and satisfactory description of computing methods opens the way to the total structural design. Under this term it's necessary to take into account, building of special kind of structure model, (it may particularly refer to the concrete structure), connected with environment through all existing influences. Model like that may be totally analyzed, and the picture we receive of its internal work, gives the possibility of its effective and active forming and constructing. This process maybe repeated up to the moment of receiving optimal characteristic of designed structure. Especially important is the fact, that using this method, a designer is reducing (minimizing) a material usage and is controlling the work of the structure all the time. Using this method, it is possible to calculate and design any type of structure independently of the type of loads. So the principle of total design lies down now in looking on the structure as a whole, not as at the system of separated elements.

Scordelis [3] describes a process of total designing that may be fulfilled and presented as a collection of the following stages:

- first stage it is a conceptual stage,
- second is a predesignig,
- third: structure analysis,
- fourth: structure synthesis,
- fifth: a drafting stage.

Especially important is the third stage that means observations of

structure behaviour under the wide range of influences. Designing process on this stage may be realized in two ways:

First relays on calculating the internal forces and further on using the received results in detailing of structure. This way is a kind of imitation of traditional methods, fulfilled at the general look at the designing structure. Second way relays on examining the internal forces flow in structures involved in the wide range of influences connected with active and effective forming of this structure during the process.

There opens a wide range of possibilities and it is enough to show here only one of them that is possibility of modeling the process of structure collapse.

As an example the case of cooling tower collapse was analyzed.

## 2. THE COLLAPSE AND ITS EFFECTS.

On the 7th of February 1987 at the power station TUROW in Lower Silesia Region in Poland, there collapsed one of nine existing there cooling towers. On the height around 43 m over the ground level, that means between 38 and 39 m of tower shell, almost horizontally, the shell was cut and fell down into the tower. The construction of tower and its remains after the collapse are shown in fig. 1.

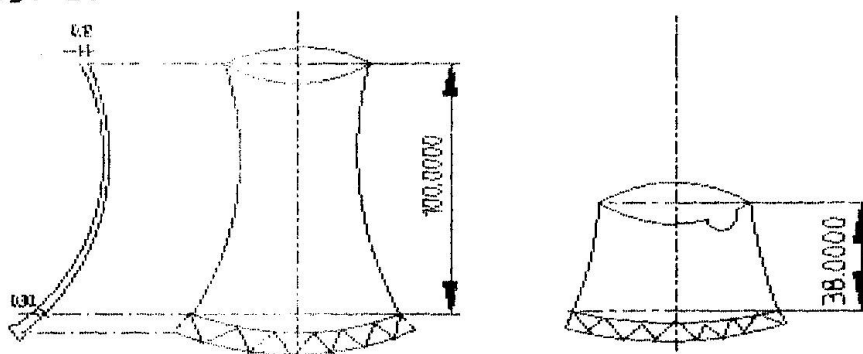


Fig.1  
Cooling tower before and after collapse.

The discussed cooling tower was built in 1963. The Main part of this structure make the reinforced concrete double curved shell, dimensioned as follows: total high 100 m ( cover shell 95 m), cover shell dimension 74.6 m, shell thickness in the lower ring 0.30 m and it is decreasing to 0.12 on the level of 30 m, further thickness is constant. The shell is reinforced by nets of bars on the distance  $0.20 \times 0.20$  m.

For over 25 years of exploitation, the discussed structure was very often controlled and examined and never any damages were found. That is why the researches on the collapse were very difficult.

There were many tests made of which most important were:

- tests of strengths characteristic for used materials,
- tests of physical and chemical properties of used materials,
- geodesic tests of the real geometry of the remaining part of the shell and comparison of the results with the ideal shell geometry,
- analysis of changes in the shell state during exploitation caused by the failures during the shell building,



- analysis of the influence of super loads on the shell work,
- analysis of correctness of designing solutions applicable to the tested shell.

Generally it was established that:

- concrete was generally technically good, that means its strength and quality coefficients were containing in the range demanded by codes of practice,
- position and quality of reinforcement was good,
- most sensitive points of the shell were the places of work breaks made during the shell concreting. It was established that reinforcement bond was decreased in these places. Also the corrosion of concrete developed there and it was increased by diffusion of the water through concrete. The weakest point developed in the region of 38th stripe of concreting. Moreover the stripe of the weak concrete developed on the great length. The increasing lixiviation of cement from concrete caused probably some rising of horizontal slit in the shell. This led further to a change of the static scheme, local overload of the structure and finally to a collapse.

The following hypothesis about the mechanism and causes of collapse were formulated:

- the post building imperfections of the wall-shell,
- the technological and technical disadvantages referring to the concreting process,
- a very bad quality of technological breaks during the concreting,
- the destructive influence of the environment eq.
  - strong wind blasts,
  - local earthquake caused by mine exploitation existing around,
  - thermal influences etc.

After the detailed analysis of the causes listed above, the main reason of the collapse was found. It was the destruction of concrete on the 30 m long stripe of shell circumference, caused by long term lixiviation of cement from concrete. There appeared a long slit in the wall, which changed the static model of the shell structure.

### 3. MODELING OF THE SHELL WORK.

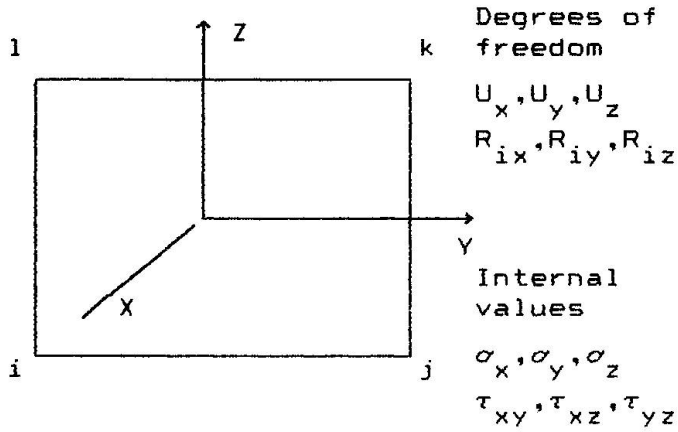
To check the hypotheses about the cooling tower collapse, mentioned in the previous section, the analytical model of structure was built. There was A series of Computer Programs for Static Finite Element Analysis of Structures called STRAINS used [4]. System STRAINS works on the base of the following assumptions:

- there exists the continuity of the matter,
- stresses in the structures develop in the moment applying some forces,
- the matter is linear,
- the state of stresses in examined point is marked by a state of deformations in this point,
- displacements and deformations are small,
- it is possible to use Saint-Venant's principle.

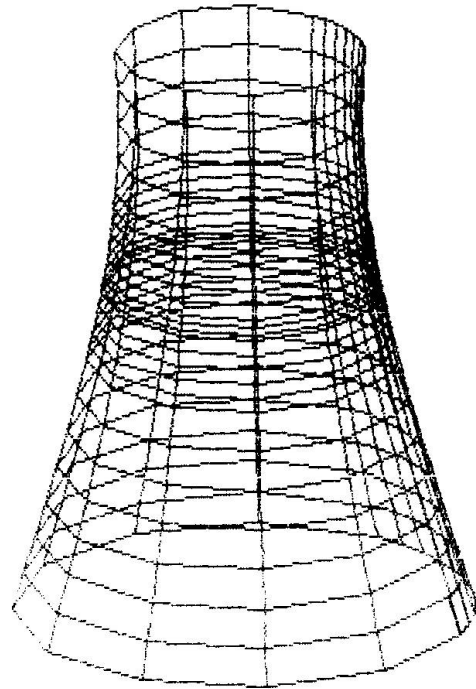
To build the model of the discussed structure there were used plane surface elements as shown in. Fig. 2a . The received node mesh is shown in fig. 2b.

On the shell surface, the horizontal slit was modelled. The length of the slit was changeable. The model was loaded by wind forces and several times examined while changing the slit length.

The results of the calculations are shown in figure 3a,b,c.

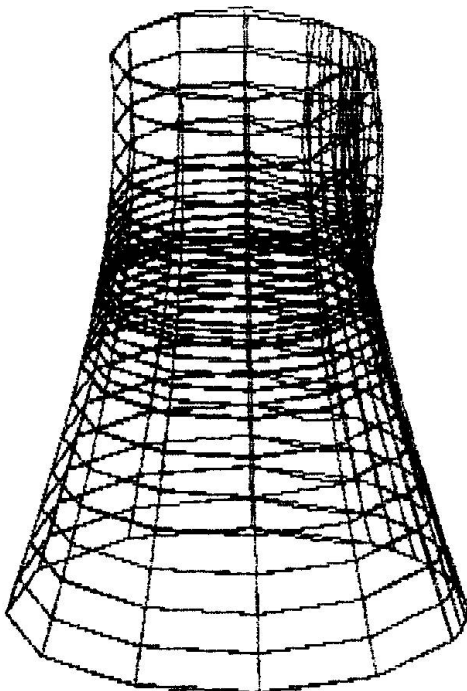


a. Used shell element

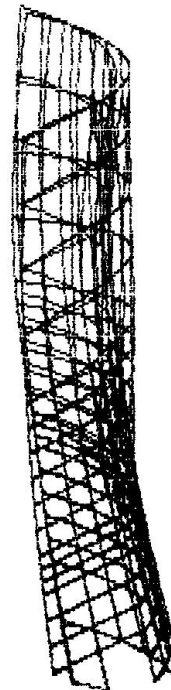


b. FEM model of the shell

Fig. 2 a,b.  
 Finite Element model of the  
 discussed cooling tower.



a. Displacements of the shell under the  
 wind load. Length of the slit 40 m



b. Zoom of the deformed  
 place

fig. 3 a,b  
 Results of the computer calculations.



On the base of the upper listed results it was possible to establish the limited length of the slit which gave enough values of the displacements to cause destruction of the shell.

During the calculation process the limitations of used computer program were seen. Mainly they may be listed as follows:

- it was possible to use (in the applied system STRAINS) only 500 elements. It was only enough to cover the tower coat, by the mesh dimensioned 6x6m. Only around the slit, the mesh had smaller dimensions 0.20x0.2 m. The mesh like that was too rare and received results were only a far approximation of reality,
- there were used only isotropic elements (concrete) to build model of the reinforced shell. In the used system there were no special reinforced concrete elements,
- the used system wasn't connected with dimensioning systems, so it was necessary to transfer data manually.

Instead of so many limitations and disadvantages the gained results make the evidence that such method of structure analysis has significant future.

#### 4. CONCLUSIONS

The presented way of the total structural design of structures, leads to a wider evaluation of their work. That is a way of connecting in the one system, on the same level, methods of structure analysis and synthesis, with the possibility of the data exchange. During the process of the structural designing it is possible to take into account many new aspects and observe a structure response. Also it's possible to design wanted features of structure, e.g. material, shape or ways of reinforcing. In the presented example the structural analysis of work of the cooling tower shell work with slit permitted to check several hypotheses about collapse reasons.

This way of structure analysis is possible only while using computers. Growing calculation power of computers gives a lot of hope about progress in that field. This trend is bound to continue.

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## **Cracking Analysis of a Prestressed Concrete Containment Structure**

Analyse des fissures dans un réservoir en béton précontraint

Analyse der Rissbildung bei einem Spannbetonbehälter

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### **SUMMARY**

A practical problem-related to the assessment of structural cracking is considered. The structure is analysed using two finite element models, i.e. two separate computer codes. Although the same numerical model for simulation of nonlinear concrete behaviour is employed, certain discrepancies in the numerical results are observed. This is mainly due to different modelling of boundary conditions and prestressing. Regardless of the problems mentioned, FEM is proving to be a most powerful engineering tool.

### **RÉSUMÉ**

L'article traite de l'évaluation des fissures dans une construction. La construction a été analysée par deux modèles d'éléments finis. Bien qu'on applique le même modèle numérique pour la situation du comportement non-linéaire du béton, une divergence des résultats numériques a été observée. C'est dû surtout aux modèles différents des conditions limites et des forces de précontrainte. Cependant, la méthode est efficace pour l'analyse structurale.

### **ZUSAMMENFASSUNG**

Der Beitrag behandelt die rechnerische Erfassung der Rissbildung eines Spannbetonbehälters. Bei der Analyse der Konstruktion wurden zwei entwickelte Modelle der Methode der finiten Elemente und das zugehörige Rechenprogramm verwendet. Obwohl das nichtlineare Verhalten des Betons in beiden Modellen gleich simuliert wurde, konnten Differenzen in den numerischen Ergebnissen festgestellt werden. Die Ursache liegt in den verschiedenen modellierten Randbedingungen und in der Vorspannung. Trotz dieser Probleme hat sich die Methode der finiten Elemente als ein sehr erfolgreiches Ingenieurmittel erwiesen.



## 1. INTRODUCTION

The 1st Conference on Computer-Aided Analysis and Design of Concrete Structures [1] held in Split in 1984 successfully summarized a large number of numerical models and techniques, particularly these based on the finite element method, for the analysis of concrete structures. Since then, numerical modelling and solution techniques have been significantly improved. On the other hand, the advent of powerful computers, as well as microcomputers with parallel processing, endowed with the exciting prospect of developing an interactive design system gives the possibility that such techniques can be efficiently employed to the solution of complex practical problems. However, it is true to say that the numerical capability is nowadays in advance of the knowledge of concrete constitutive behaviour. The knowledge of concrete behaviour under triaxial stress states is incomplete; or at least no consistent constitutive relation has yet been established. Nevertheless, in spite of this fact, satisfactory results can be achieved in engineering practice. The 2nd ICC conference [2], together with another recent conference has proved this trend. However, having experience in both the fields, practical structural concrete design and finite element analysis, we are aware of all the problems, or quoting Ref. [3], "the current inconsistencies" well summarized in Section 3 of the mentioned colloquium's introductory report. In this paper showing results of F.E. analysis of a practical problem we intend to point out certain issues primarily related to F.E.M. as an efficient design-oriented analysis tool.

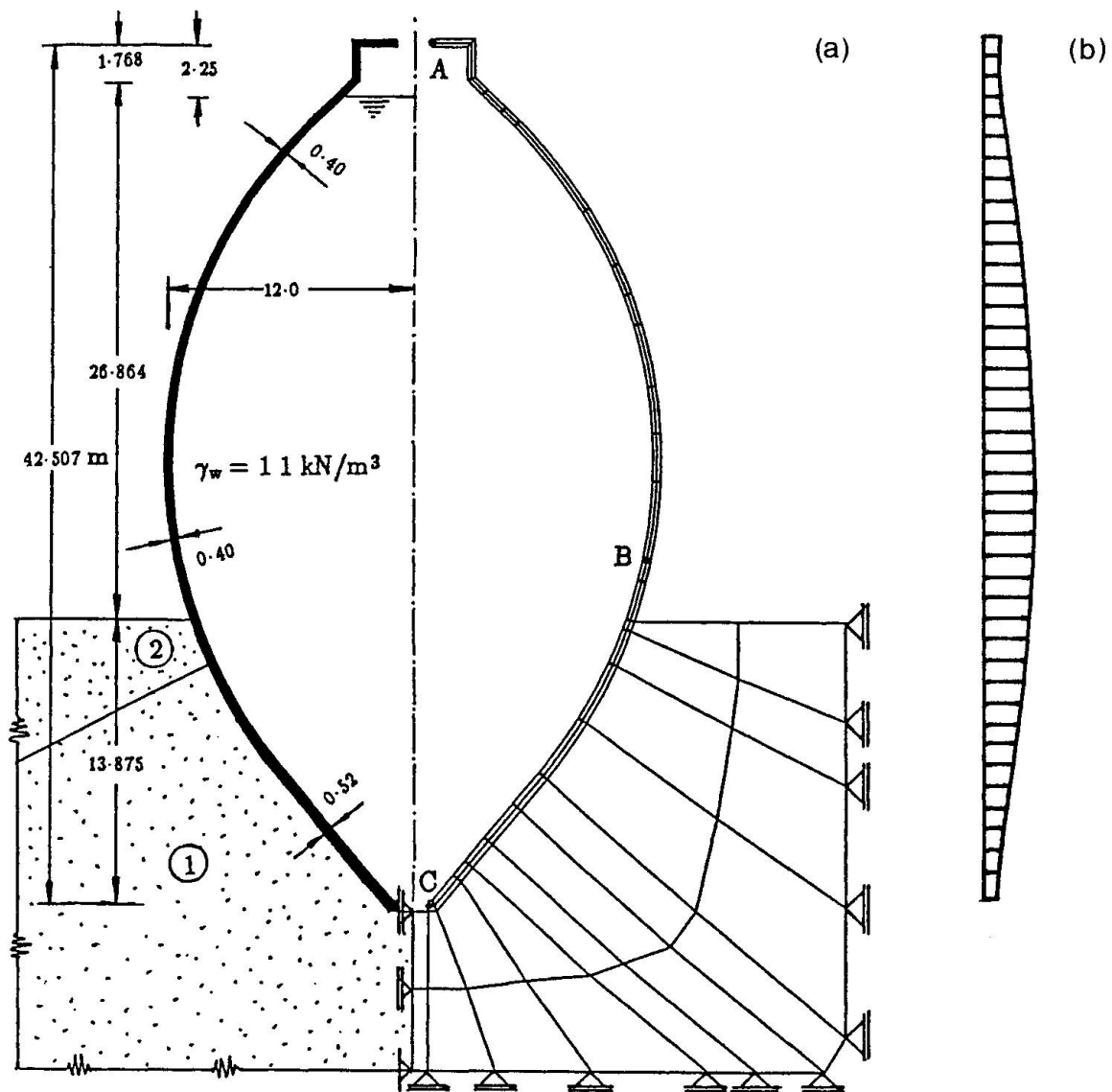
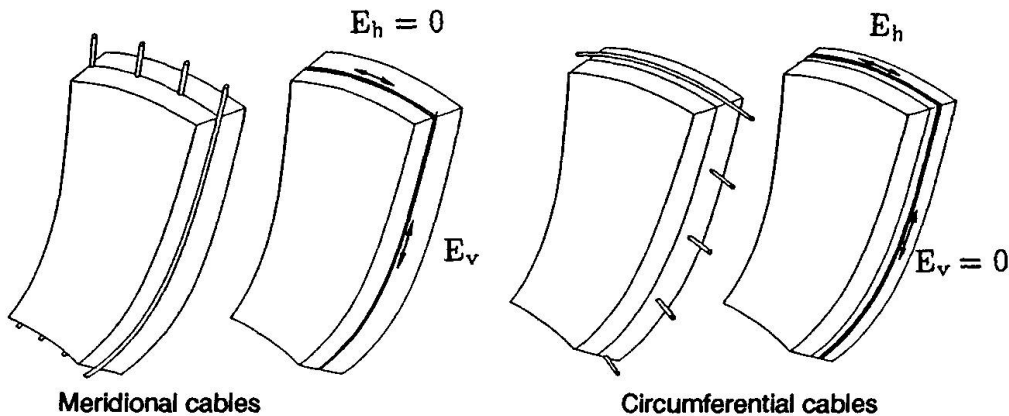
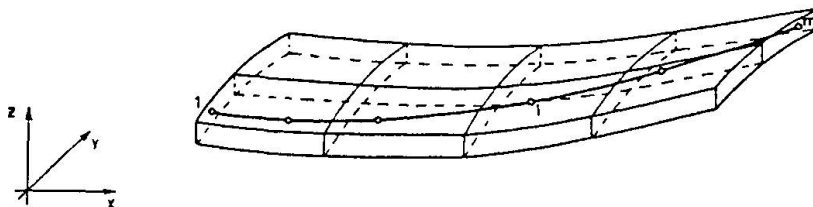


Fig.1 Prestressed concrete septic containment showing problem details and finite element idealisations: (a) axisymmetric mesh, and (b) shell element mesh.



**Fig.2** Prestressing cable position and finite element idealisation (Axial elements).

The example here presented is related to the assessment of structural cracking of a prestressed septic containment under normal service loading. As a matter of fact, cracking of concrete structures very often occurs even far below service load conditions, and represents probably the main, and also undesirable, feature of concrete behaviour. Permitting insight into the cracking phenomenon (initiation, spreading and closing of cracks) the finite element method makes possible a rational analysis of concrete structures.



**Fig.3** Geometry of the prestressing tendon (Shell elements).

The structure is analysed using our two finite element programmes (the first one based on the 2D/axisymmetric formulation [4] and the second based on the shell formulation [5]) for the non-linear analysis of reinforced and prestressed concrete structures. Numerical model employed in the programmes accounts for the most dominant nonlinear behaviour of concrete, e.g. multiaxial elastoplastic compressive behaviour including crushing, initiation and spreading of cracks etc. The prestressing cables are represented with discrete elements allowing uniaxial elastoplastic modelling of steel behaviour. The model is earlier developed [6-8] and further improved and extended. Results of several engineering studies [9-13] illustrate the applicability and practical merit of both the model and the codes.

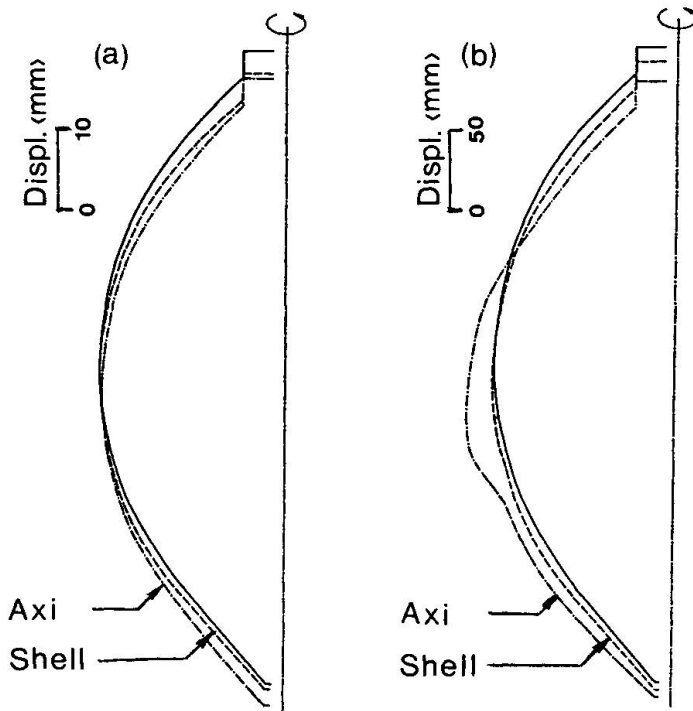
## 2. ILLUSTRATIVE EXAMPLE

### 2.1 Problem description

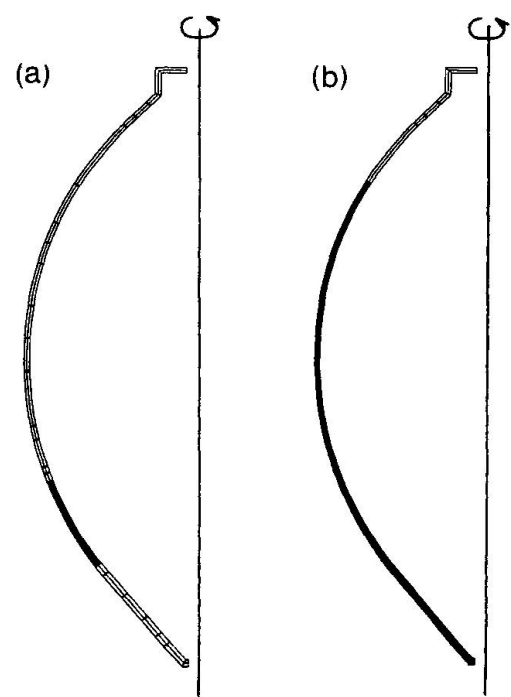
The containment, a prestressed concrete structure shaped in an axial symmetric "amphora" form, was earlier designed for the sewage treatment plant of the city of Ljubljana. The design followed a "standard" engineering manner; a linear elastic finite element code was employed to determine internal forces, than reinforcement arrangement was defined. Essential details of the structure designed are given in Fig.1. Approximately one third of the structure, the lower conical part, is "implanted" into the soil. No additional structural foundation elements are designed. The structure is reinforced with vertical (meridional) prestressing cables located in the middle of the structural wall cross section, and horizontal (circumferential) prestressing cables are close to the outer surface of the wall. There are 96 meridional cables all together. However, the length of 24 cables is practically from the top to the bottom of the structure. The rest of them are located in the mid-height of the structure to reinforce the widest part of the structure. The total number of circumferential cables is 142. The effective prestress forces (after initial prestressing losses) of 600 kN/cable and 500 kN/cable are expected to act in meridional and circumferential cables respectively. One of the major requirements for the structure is that structural cracking is not permitted due to very aggressive contents of the waste water. Since the structure



was designed assuming prestressed concrete as a homogeneous isotropic linear elastic material we reanalysed the containment using two non-linear F.E. codes (Axisymmetric and shell formulation). Finite element mesh discretizing the structure and a part of surrounding soil for the axisymmetric analysis is shown in Fig.1(a). The prestressing cables are represented by a pair of membranes of equivalent thickness (Fig.2) and the influence of the prestress forces due to curved shape of the structure by external equivalent pressure (as suggested in Ref.(14). For the shell analysis a vertical section (6 degrees) of the structure is discretized with 44 shell elements shown in Fig.1(b). Prestressing cables are modelled as indicated in Fig.3 using a tendon formulation [ 12 ]. Soil influence is simulated by equivalent springs at the appropriate nodes.



**Fig.4** Deformed shapes due to: (a) Self-weight and prestressing, and (b) servis load.



**Fig.5** Tensile cracking zones using: (a) Shell formulation, (b) Axisymmetric formulation.

## **2.2 Numerical results**

The structure is analysed under the conditions of prestress and internal hydrostatic pressure equivalent to the containment filled up to the maximum level (See Fig.1). The deformed structure profiles, i.e. the prestressed configuration and the configuration at the full service load compared to the initial configuration of the structure are illustrated in Fig.3. No cracks are predicted under the prestressing conditions. However, vertical cracks throughout the thickness of the structure are predicted at the service load conditions. The zones of vertical cracks are indicated in Fig 5. The max. crack width (supposing a crack distribution at the position of meridional cables only) is estimated in the range of 1.13 mm (axisymmetric analysis) and 0.5 mm (shell analysis). Displacements obtained using the two codes are compared in Table 1. Significant stresses in cables and in concrete are listed in Tables 2 and 3 respectively. Considering two set of the results it is clearly seen that: (a) vertical cracks will occur, and (b) the shell formulation predicts a rather stiffer structural response. The differences in the results are primarily due to different modelling of: (a) prestressing and (b) influence of the surrounding soil. More realistic prestress modelling is applied in the shell analysis (Equivalent load in the axisymmetric). On the other hand, soil interaction in the axisymmetric is modelled in a more appropriate way analysis.



Displacements [mm]		SHELL	AXISYMMETRIC
Load case 1: g + p	top	$u_r = 0.004$ $u_z = -3.08$	$u_r = \approx 0$ $u_z = -3.59$
	bottom	$u_r = -0.016$ $u_z = -0.837$	$u_r = -0.005$ $u_z = -3.03$
Load case 2: g + p + w	top	$u_r = 0.016$ $u_z = -9.88$	$u_r = \approx 0$ $u_z = -22.84$
	bottom	$u_r = -0.912$ $u_z = -5.17$	$u_r = 0.271$ $u_z = -8.86$

Table 1: Radial and vertical displacements.

Min&Max stresses in prestressing cables $\cdot 10^6$ [kN/m <sup>2</sup> ]		SHELL	AXISYMMETRIC
Load case 2: g + p + w	circum.	0.90 - 1.04	0.89 - 1.23
	meridional	1.07 - 1.08	1.07 - 1.50

Table 2: Extreme stresses in the cables.

Max. compressive stress in concrete [kN/m <sup>2</sup> ]		SHELL	AXISYMMETRIC
Load case 1: g + p	circum.	-8720	-735
	in plane	-3201	-1138
Load case 2: g + p + w	circum.	-3596	-486
	in plane	-2926	-3891

Table 3: Max. principle compressive stress.

### 3. DISCUSSION AND CONCLUSIONS

The brief description of the results obtained analysing one single structure using non-linear FE analysis and applying practically the same constitutive model for the simulation of R.C. behaviour, but based on two different formulation indicates the following:

- \* Non-linear FE analysis is (or if there is any doubt, it will be definitively very soon) one efficient reliable and practical design-oriented analysis tool which is perfectly consistent with "strut-and-tie model" (STM) concept.
- \* It seems that using FE structural analysis a step-by-step procedure (Firstly linear FE analysis, dimensioning, and than fully non-linear FE "control") has to be used.
- \* Although the same concrete constitutive model is employed in the two analyses presented the results are a rather different. This is not due to two different formulations applied, but primarily due to different boundary condition interpretations.
- \* In addition to the above, we can conclude that nowadays a large number of concrete constitutive models, from a very simple one (if we do not take into account the "historical" linear elastic) to very sophisticated are spreaded and applied in practice. In this case, applying different models included in a FEA, a spectrum of different results for the same example would be obtained. An immediate question is: What is



a real "truth"? Which model is the most reliable, appropriate, or correct? This is a natural question or dilemma of any researcher and designer. From the FEA aspects we may conclude, as we mentioned earlier, that the numerical capability is in advance of the knowledge of structural concrete behaviour.

Therefore we strongly support the initiative to consider the list of "challenges" (Section 5, Ref. [3]) in the colloquium. Furthermore, we expect from the colloquium to make a general, but an organized step forward in structural concrete design.

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## Serviceability Analysis of Reinforced Concrete Slabs

### Analyse à l'état de service de dalles en béton armé

### Berechnungsverfahren für die Gebrauchsfähigkeit von Stahlbetonplatten

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#### **SUMMARY**

Proposed analysis of reinforced concrete slabs in the serviceability limit states is based on the finite element method and the effective stiffness model of the plate cross-section. The nonlinear effects of concrete cracking, tension stiffening, and simplified creep behaviour are included. Examples of the analysis of real structures are described.

#### **RÉSUMÉ**

La méthode proposée pour l'analyse de dalles en béton armé à l'état de limite de service est basée sur la méthode des éléments finis, ainsi que sur un modèle de rigidité effective de la section transversale de la dalle. Les effets non-linéaires de la fissuration du béton, ainsi qu'un modèle simplifié pour la fissure sont également inclus. Des exemples d'analyse de structures réelles illustrent ces propos.

#### **ZUSAMMENFASSUNG**

Das vorgeschlagene Berechnungsverfahren für Stahlbetonplatten im Gebrauchszustand beruht auf der Finite Elemente Methode und dem Modell effektiver Querschnittssteifigkeiten. Es werden die nichtlinearen Effekte der Rissbildung, der tension-stiffening-Effekt und, auf einfache Weise, das Kriechen des Betons berücksichtigt. Es werden Beispiele für die Berechnung realistischer Bauwerke gegeben.



## INTRODUCTION

The authors have developed a method for the computer analysis of reinforced concrete plates under serviceability conditions. The purpose of this work was to provide an efficient tool for practical design which would enhance current engineering practice. It is known that deflection of reinforced concrete structures is greatly affected by concrete cracking and by the long-term effect of creep. However, such effects are non-linear and they can complicate analysis to the extent that it becomes impractical. Therefore, it was decided to simplify the material and numerical model as much as possible and to focus on efficiency at the expense of unnecessary accuracy.

In view of the above considerations the following assumptions are made:

1. Thin-plate theory based on the Kirchhoff hypothesis is used for plate mechanics. This theory is based on the action of bending moments only. Normal forces are not considered and shear forces must be derived from moments.
2. An effective stiffness model is used in which the material behavior is defined for the whole cross-section of the plate. The constitutive law is defined for moments and curvatures.

The work presented here is based on the first author's Ph.D. dissertation [1].

## CONSTITUTIVE MODEL

A smeared approach is used to model the material non-homogeneity, i.e., the material properties are constant within a considered element volume but can vary between the elements. The moment-curvature relation for the plate bending element is as follows:

$$\mathbf{m} = \mathbf{D} \cdot \mathbf{k} \quad (1)$$

where  $\mathbf{m} = \{m_x, m_y, m_{xy}\}$  is the vector of internal moments per unit width and  $\mathbf{k} = \{\kappa_x, \kappa_y, \kappa_{xy}\}$  is the vector of corresponding curvatures. The constitutive matrix  $\mathbf{D}$  for uncracked concrete has the form of the elastic matrix for isotropic plates. The cracking criterion is given by the cracking moment  $m_c = h^2 R_t / 6$ , where  $R_t$  is the tensile strength of concrete. When the principal moment reaches the cracking moment, the constitutive matrix takes the form of the orthotropic elastic plate matrix. The axes of orthotropy are aligned with the crack direction, Fig.1:

$$\mathbf{D} = \begin{bmatrix} d_{nn} & d_{nt} & 0 \\ d_{tn} & d_{tt} & 0 \\ 0 & 0 & d_{qq} \end{bmatrix} \quad (2)$$

The coefficients of the constitutive matrix are calculated according to Jofriet et al. [2]:

$$\begin{aligned} d_{nn} &= \frac{B_n}{1 - \nu^2} & d_{nt} &= d_{tn} = \nu \sqrt{d_{nn} d_{tt}} \\ d_{tt} &= \frac{B_t}{1 - \nu^2} & d_{qq} &= \frac{1 - \nu}{2} \sqrt{d_{nn} d_{tt}} \end{aligned} \quad (3)$$

where  $B_n, B_t$  are the flexural stiffnesses normal and parallel to the cracks, respectively, and  $\nu$  is the Poisson's ratio. The flexural stiffness of the cracked cross section is modeled according to Hajek [3] as:

$$B_r = \frac{h_o z}{\frac{\psi_s}{E_s A_{st}} + \frac{2\psi_b}{E_c A_{ci}}} \quad r = n, t \quad (4)$$

in which  $n, t$  are indices for the directions normal and parallel to cracks.  $E_s$  and  $E_c$  are the elastic moduli of the steel and concrete, respectively;  $A_{st}$  and  $A_{ci}$  are the areas of steel and

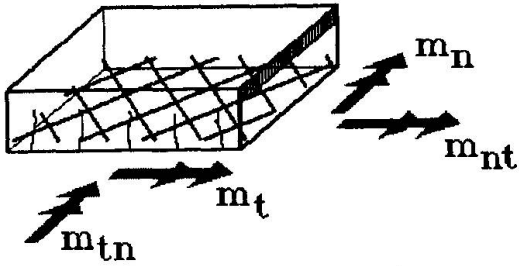


Fig.1 Moments on the cracked plate element.

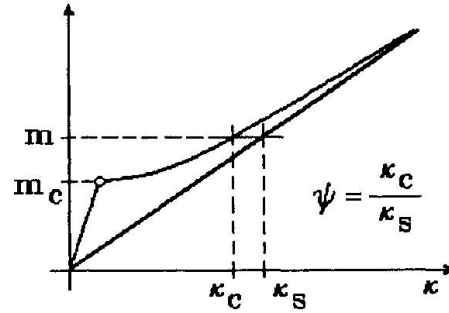


Fig.2 Moment-curvature diagram. Mean of tension stiffening coefficient  $\psi$ .

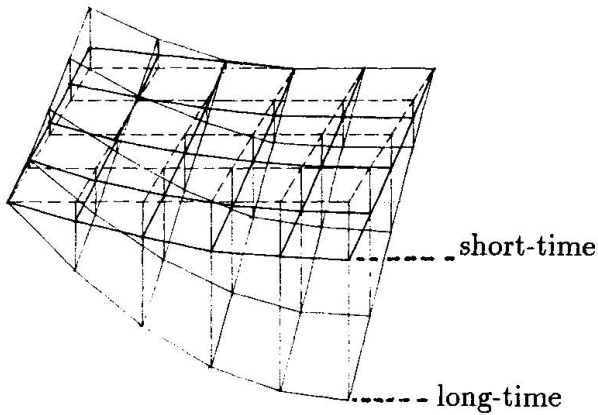


Fig.3 Long- and short-time deflection shapes in Hajek's experiments [3].

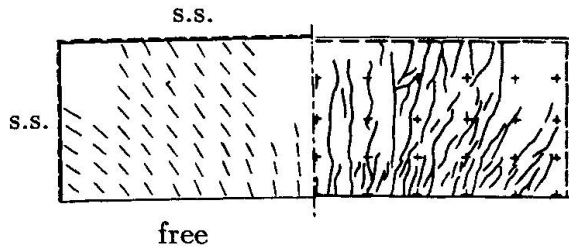


Fig.4 Comparison of experimental and analytical crack patterns. Legend: free - free edge, s.s. - simply supported edge

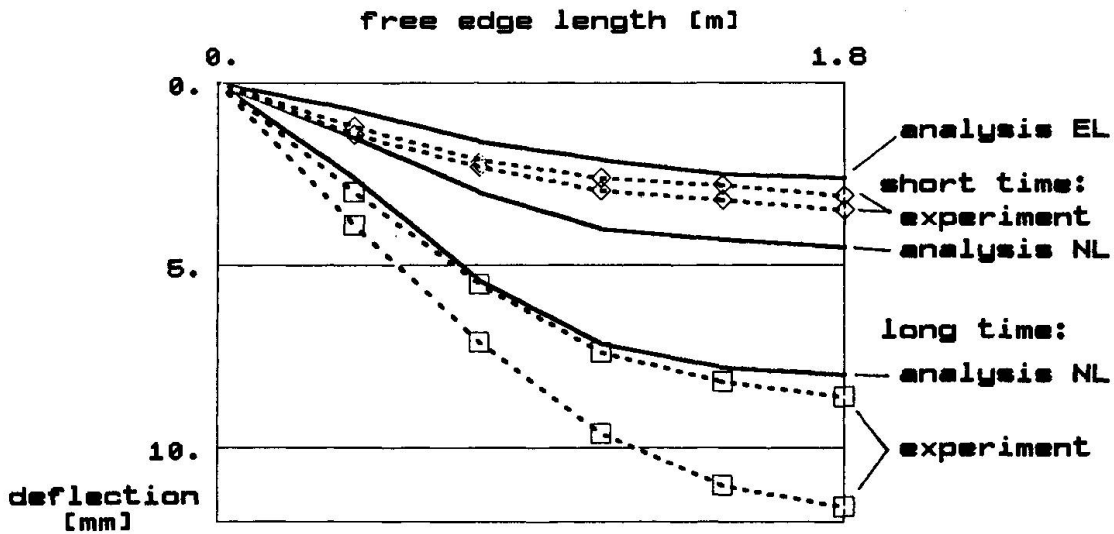


Fig.5 Comparison of analytical and experimental deflections of the free edge for Hajek's slab [3].



concrete, respectively;  $h_o$  is the effective depth and  $z$  is the lever arm of the internal forces. The tension-stiffening coefficient  $\psi_s$  represents the reduction of steel strains due to concrete tensile stiffness after cracking. A similar effect in the compression zone is described by  $\psi_c$ . These coefficients are defined as follows:

$$\psi_i = 1 - \rho(1 - \psi_{ir}), \quad i = s, c, \quad \rho = \frac{1}{4} \left( 5 \frac{m_c}{m} - 1 \right) \quad (5)$$

$$\psi_{sr} = \frac{6}{0,85} n \frac{A_s (2h_o - h) z}{bh^3}, \quad b = 1m \quad (6)$$

$$\psi_{cr} = \frac{6}{1,7} \frac{A_{ci} z}{bh^2}, \quad n = \frac{E_s}{E_c} \quad (7)$$

in which the indices  $s, b$  refer to steel and concrete, respectively;  $\rho$  is an interpolation parameter for tension stiffening and  $\psi_{sr}, \psi_{cr}$  are initial values of  $\psi_i$  after cracking. The moment-curvature diagram resulting from this model is schematically shown in Fig.2. In the case of an inclined crack, the effective reinforcement area,  $A_s$ , is calculated using appropriate transformation of the two reinforcing directions passing through the cracked cross-section. Three crack modes are recognized: 1. one crack, 2. two orthogonal cracks on the same surface, 3. two orthogonal cracks on opposite surfaces.

The time effect is included as a simplified method based on an extensive experimental investigation by Hajek of real plates [3]. He found an affinity relationship between the short-term and the long-term deflected shapes of plates (Fig.3). The deflection  $w(t, x, y)$  at the time  $t$  is governed by the simple formula

$$w(t, x, y) = (1 + \beta) \cdot w(in, x, y) \quad (8)$$

in which  $w(in, x, y)$  is the initial deflected surface of the plate and  $\beta$  is the creep coefficient. The creep function is taken from the Czechoslovak National Standard for concrete structures as follows:

$$\beta = (0,15 + 0,008e^{-0,015t_1}) (1 - e^{-0,07\sqrt{t-t_1}}) \phi \quad (9)$$

where  $t$  is the age of concrete at the considered time,  $t_1$  is the age of concrete at the time of load application, and  $\phi$  is the coefficient of creep.

## NUMERICAL SOLUTION

The numerical solution of the plate analysis is performed using the finite element method. The Clough-Felippa [4] quadrilateral finite element is used. The element is composed of four linear-strain subtriangles. It has four external nodes with three degrees of freedom in each node, one deflection and two slopes. This element has piece-wise linear moment distribution and thus gives quite good results even for coarse meshes. The non-linear solution is done by the secant stiffness method. The total loading is applied, then iteration is performed until the constitutive laws and equilibrium are satisfied. Finally, the creep effect is included by scaling the deflected shape using the creep law, Eq.9.

The program can handle irregular geometrical shapes, internal hinges, elastic supports, elastic foundations, concentrated and distributed loadings and other features.

The program is written in FORTAN77 and can be installed on various computer systems (mainframes, workstations, personal computers). It is equipped with efficient pre- and post-processors by means of which numerical results can be presented on graphical terminals, plotters, or printers.

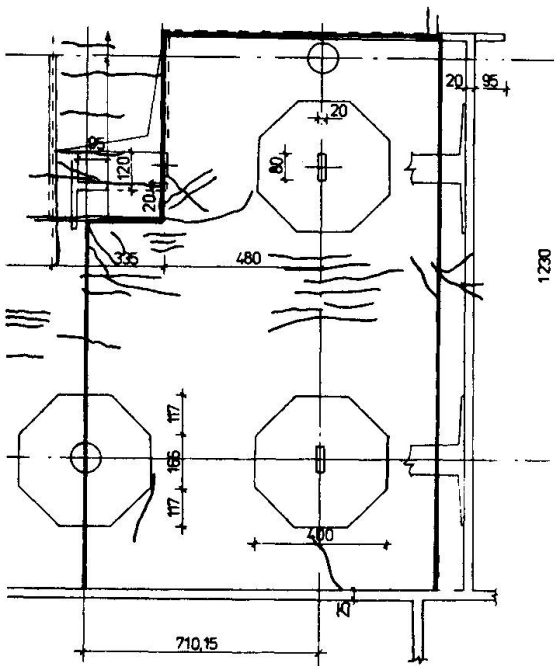


Fig.6 Garage slab. Plan view of the slab section with observed cracks.

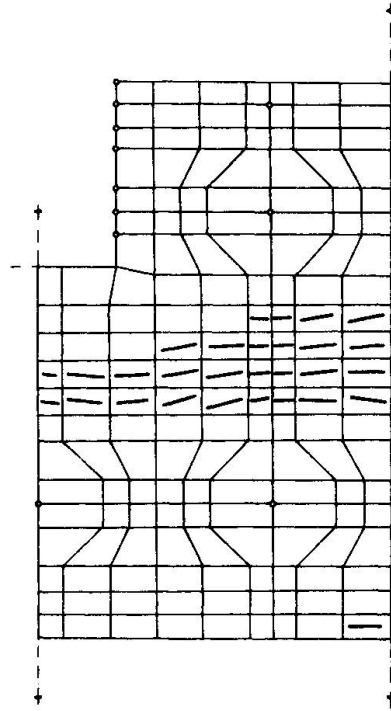


Fig.7 Garage slab. Finite element model with calculated crack pattern.

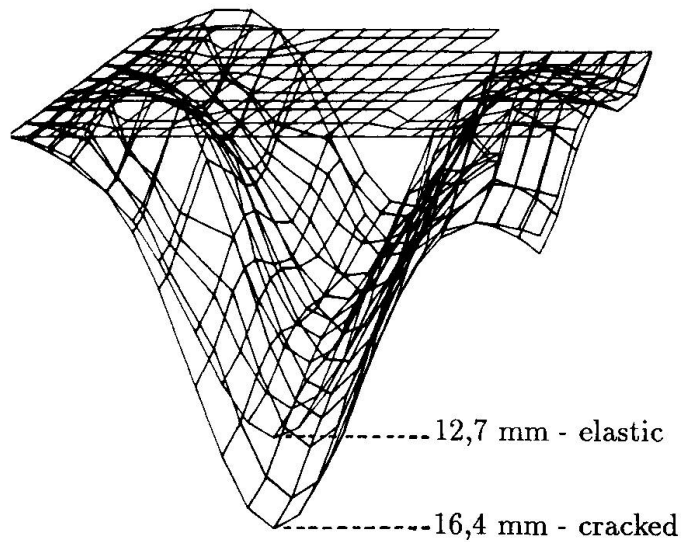


Fig.8 Calculated deflection surfaces.



## COMPARISON WITH HAJEK'S EXPERIMENTS

The experimental slabs in ref.[3] were supported on three sides. The slab dimensions were 3,6 x 1,2 x 0,12 m. They were reinforced by mesh with reinforcing ratios in both directions equal to 0,0112. The concrete quality was 30 MPa. The slabs were loaded 28 days after casting by a uniform load of 14,7 kN/m<sup>2</sup>. The load was applied by plastic bags under air pressure, and was kept constant during the entire loading history. The slabs were kept under controlled laboratory conditions where deflections under sustained load were monitored during a period exceeding one year. An example of measured deformed shapes is shown in Fig.3. Long term deformations are considered at 400 days.

The slab was analyzed by the above described program. The deflections of the free edge are compared in Fig.5 with experimental results for short-term and long-term effects. The results from two nominally identical experimental slabs A, B are shown to indicate the scatter of the experimental results. The experimental and analytical crack patterns are compared in Fig.4. Good agreement of the analysis with the experiment is found.

## EXAMPLE OF A SLAB IN PARKING GARAGE

The program was successfully used for checking deflections of several slab structures in design practice. One example is shown here. It concerns an existing floor structure of a parking garage which did not performed satisfactorily after the first few years of service life. It was required to check the deflections of the slab. The project also included the design of repair provisions. Here we are showing only partial results for illustration. Fig.6 shows a section of the floor structure comprised of an irregular slab, supported by columns. The slab, 20 cm thick, is cast-in-place and the octagonal precast shear heads are 35 cm thick. The existing cracks are also recorded. Fig.7 shows the finite element model and crack pattern as calculated by analysis. The calculated deflected shapes are shown in Fig.8. The elastic and cracked shapes are compared. The analysis was used to check the serviceability performance of the slab before and after the designed reconstruction.

## CONCLUSION

The proposed analysis of reinforced concrete plates based on the finite element method and the effective stiffness of the slab cross section gives a fair approximation of the real performance of slabs under serviceability conditions. It can model complex geometrical shapes and important effects of cracking, tension stiffening, and creep.

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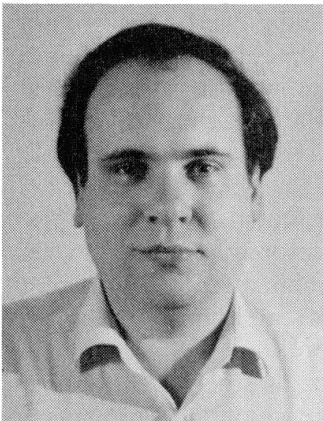
## Computer-Aided Automatic Construction of Strut-and-Tie Models

Génération automatique de modèles conformes à l'analogie du treillis

Automatisches Entwickeln von Stabwerkmodellen

### Argiris HARISIS

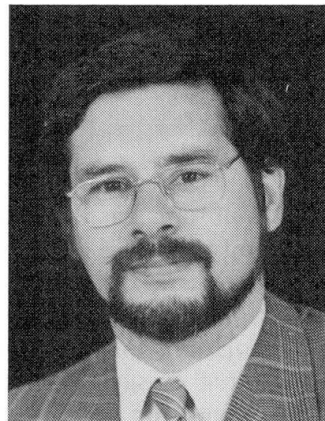
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### SUMMARY

A multi-step algorithm is presented for the automatic construction of strut-and-tie models of in-plane loaded structural concrete plates. The algorithm is implemented in a user-friendly post-processing mode into a plane-stress linear elastic finite element program, and results are compared to manually constructed strut-and-tie models.

### RÉSUMÉ

Cet article présente un algorithme destiné à générer automatiquement des treillis conformes à l'analogie, en vue de l'étude de structures planes en béton. Pour faciliter l'utilisateur, l'algorithme est muni d'un système de post-processing qui l'amène dans un programme d'éléments finis linéaires-élastiques exprimant les contraintes dans le plan. Les résultats obtenus sont comparés à des modèles de treillis dessinés sans l'intervention de l'ordinateur.

### ZUSAMMENFASSUNG

Ein Algorithmus zur automatischen Entwicklung des Stabwerkmodells einer Stahlbetonscheibe wird vorgestellt. Dieser Algorithmus ist in benutzerfreundlicher Weise in einem linear-elastisch Finite Elemente Programm zur Berechnung der ebenen Beanspruchung eingebaut. Die Ergebnisse werden mit manuell aufgestellten Stabwerkmodellen verglichen.



## 1. INTRODUCTION

Recent years have seen the emergence of Strut-and-Tie models as a powerful general approach for the rational and consistent design of structural concrete plates and of two-dimensional regions of static or geometric discontinuity (the so-called D-regions) [1], [2]. Strut-and-Tie models have been originally proposed and developed as a hand-calculation design procedure, in which the structural engineer uses his experience and intuition to draw load paths through the structure in the form of a (usually statically determinate) truss, which is then analysed for the design loads and proportioned according to the applicable Code and to other appropriate rules of practice. In the manual exercise of the construction of the Strut-and-Tie model the engineer may be aided by knowledge of the magnitude and of the directions of principal stresses, obtained by a linear Elastic plane-stress Finite Element analysis of the structural concrete plate or D-region under the design loads. The Struts-and-Ties of the model may then be drawn collinear to the resultants of the principal stresses. However, even when such Finite Element results are available, development of a Strut-and-Tie model for a nontrivial case requires from the designer not only a certain experience and expertise but also considerable time. This may work against the wide acceptance of this new and powerful design tool by the Structural Engineering community, and in favor of the old-fashioned detailing rules-of-thumb advocated by traditional Codes of practice. This is more so as a high design cost for structural concrete plates and discontinuity regions is not economically justifiable, as they are of relatively low cost and of the one-of-a-kind type.

Development of computational tools for the construction of Strut-and-Tie models will reduce the total design time and cost, and therefore may contribute to their more widespread application. To date, and despite the fact that Strut-and-Tie models have drawn considerable attention in recent years, progress in this direction has been limited. As a notable exception, researchers at ETH have developed computational algorithms for the automatic verification of the nodes of a Strut-and-Tie model [3], and for the selection of such a model so that the total weight of steel in the ties is minimized [4]. Nevertheless, selection of the topology of the Strut-and-Tie model still remains a manual task. In the present paper an attempt is made to develop and apply a computational algorithm that automatically generates the topology of Strut-and-Tie models. Development of such an algorithm is facilitated by the fact that the primary information normally used by the engineer to sketch a Strut-and-Tie model, i.e. the stresses obtained from a Finite Element Analysis, is in a systematic digital form that can be directly processed by the computer for further utilization.

## 2. ALGORITHM FOR THE SELECTION OF THE STRUT-AND-TIE TOPOLOGY

The first stage of the proposed algorithm consists of a linear Elastic Finite Element Analysis of the two-dimensional element or region, subjected to the force and displacement Boundary Conditions of the problem. The Finite Element mesh should be relatively fine and is defined by automatically generated nodes with user-selected constant spacing in the two orthogonal directions  $x$  and  $y$ . The Analysis yields nodal stresses, by averaging over the neighboring elements, and from them computes (and plots, if the user so desires) the magnitude and the direction of the principal stresses,  $\sigma_1$  and  $\sigma_2$ , at the nodes. Two databases are then formed, one referring to the positive principal stresses and the other to the negative ones. The generic record in each database includes the coordinates  $x$  and  $y$  of the node, the value of  $\sigma_1$  or  $\sigma_2$  there, and the angle  $\theta_1$  and  $\theta_2$  between its direction and axis  $x$ . Using these databases, the program performs the following tasks, separately for the positive and the negative stresses :

- 1) It identifies the group of nodal points where the value of the principal stress of interest (positive or negative) lies within a certain user-specified range of values with respect to the mean value of this stress over the entire two-dimensional region, e.g. the group of points where this stress exceeds the mean value by a user-specified multiple of the standard deviation of the stress in question (positive or negative) within the two-dimensional region, but not by more than another such user-specified multiple. In this way the points with relatively high nodal stresses are identified, irrespective of their location in the two-dimensional region.

2) The points in the group identified in 1) above are ordered according to the magnitude of the corresponding angle  $\theta_i$  ( $i=1$  or  $2$ ), as this angle varies between  $0^\circ$  to  $180^\circ$ . In other words, the sample histogram (cumulative distribution function) of  $\theta_i$  is constructed. Next we compute the difference between successive angles in this ordering, which is inversely proportional to the derivative of the histogram (i.e. to the probability density function) of  $\theta_i$ . A range of angles where this difference is small (i.e. where the probability density function assumes high values) consists of points where the principal stress in question (positive or negative) not only assumes relatively high values but also has nearly parallel directions. So, if they are close, such points may form a strut (for negative stresses) or a tie (for positive). A user-specified number  $N$  of groups is formed, covering the entire range of angles between  $0^\circ$  and  $180^\circ$ , each group bounded by two successive local maxima of the difference between the angles  $\theta_i$  (i.e. by two successive local minima of the probability density function of  $\theta_i$ ). To avoid cases of very small subgroups between two local maxima at almost equal values of  $\theta_i$ , the variation of these differences is smoothed, by taking their 10-point moving average. In this way closely spaced local maxima merge into a single one.

3) All pairs of points in each group of nearly parallel principal stresses formed in 2) above, are examined for geometric proximity, by comparing the distance of the two points in the group to a certain percentage of the maximum size of the two-dimensional region. In this way each group is partitioned into subgroups of neighboring points with nearly parallel principal stress directions.

4) Provided that it contains a minimum number of points, each subgroup is replaced by a provisional straight-line strut or tie. This straight line passes through the center of gravity of the group points weighted by the value of the principal stress of interest (i.e. positive or negative). Its direction is chosen to coincide with the average principal stress direction of the points in the group, again weighted as above. An option has been included for turning the direction of the ties (or of some of them) to the closest one among a set of user-preferred reinforcement directions (e.g. at  $0^\circ$ ,  $45^\circ$ ,  $90^\circ$ , or  $135^\circ$  to the x-axis).

5) The provisional struts and ties are replaced by final ones, which form a statically determinate truss consisting of triangles. This is accomplished by finding triads of neighboring points of intersection of three different struts or ties, merging these three points into a single one, and shifting accordingly the corresponding struts or ties. Ties which have originally been arranged to be parallel to one of the pre-defined directions mentioned above (i.e. at  $0^\circ$ ,  $45^\circ$ ,  $90^\circ$ , or  $135^\circ$ ), may be excepted from this shifting operation. Concentrated applied loads or reactions are also included in this part of the algorithm as struts or ties of fixed direction and position.

The algorithm has been implemented in a FORTRAN program that runs under DOS. Both the algorithm and the Finite Element program have been installed on IBM-compatible PC/XT's, and PC/AT's.

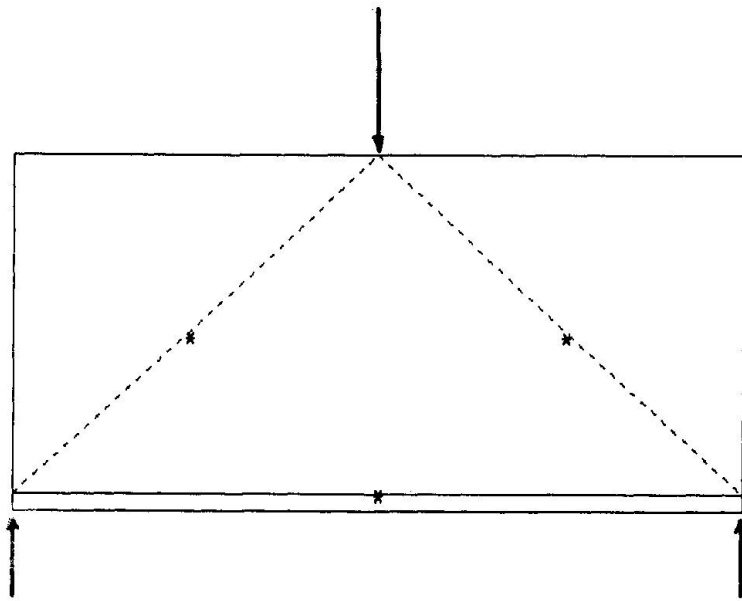
### 3. APPLICATION EXAMPLES

The computational procedure described above is applied to three simple examples: a) A simply supported deep beam with  $l/h=2.0$ , subjected to a concentrated force at midspan (Fig.1); b) a simply supported deep beam with  $l/h=1.56$ , with a square perforation extending over  $h/3$  near the left support, subjected to a concentrated force at a distance  $l/7$  to the right of midspan (Fig. 2, and [1], [4]); and c) the D-region of a stepped beam (Fig. 3) extending to both sides of the step by one beam depth, subjected to pure bending [1]. The Finite Elements used for the analysis were 8-noded square ones, with side about equal to one tenth of the maximum height of the two-dimensional region.

In the first problem the primal group of principal tensile stresses formed by step 1 of the algorithm was chosen to include all stresses exceeding the mean value of the principal tensile stresses by two standard deviations, whereas that of principal compressive stresses was selected to cover all compressive stresses from half a standard deviation below the mean (principal compressive) stress to infinity. For a user-specified maximum number of compressive and tensile stress subgroups equal to 3 each, the Strut-and-Tie model in Fig. 1 was finally obtained. This figure, and the ones to follow, shows the struts by dotted lines, and the ties by continuous ones. In addition, it shows as points the

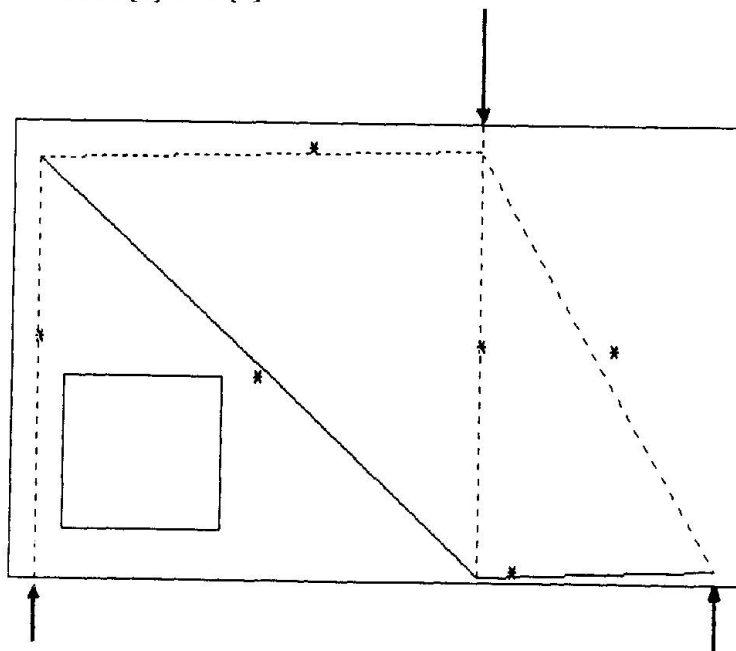


centroids of the stresses which correspond to the nearby strut or tie.



**Fig.1** Strut-and-Tie Model for a deep beam

In the second problem the primal groups of principal compressive stresses has been selected as in the first problem, whereas that of the principal tensile stresses has been chosen to include all stresses exceeding the corresponding mean value. For a user-specified maximum number of subgroups of compressive and tensile stresses equal to 4 and 3 respectively, the Strut-and-Tie model is as shown in Fig. 2a. This model is almost identical to the simplest Strut-and-Tie models manually drawn for this problem in [1] and [4].



**Fig.2** Strut-and-Tie Model for deep beam with perforation

In the third problem, the primal group of principal compressive stresses was chosen as in the previous two problems. A similar selection was made for the group of principal tensile stresses. The

partitioning user had specified each of these groups into a maximum number of 4 subgroups of the type of step 2) of the algorithm. This latter step produced only 2 such subgroups for each type of stresses, which were further partitioned into 3 subgroups of neighboring points in Step 3) of the algorithm. The final Strut-and-Tie model, shown in Fig. 3a, is stable only for the specific loads of the problem. The same holds for the more refined Strut-and-Tie model proposed for this case in [1]. So, this Strut-and-Tie model can be analysed only by equilibrium of the nodes, and not by a general purpose computer program based on the Direct Stiffness Displacement approach. To make the truss stable, the user should insert additional Struts-and-Ties to the model. An extension of the present algorithm to automatically augment the Strut-and-Tie model in such cases is currently under development.

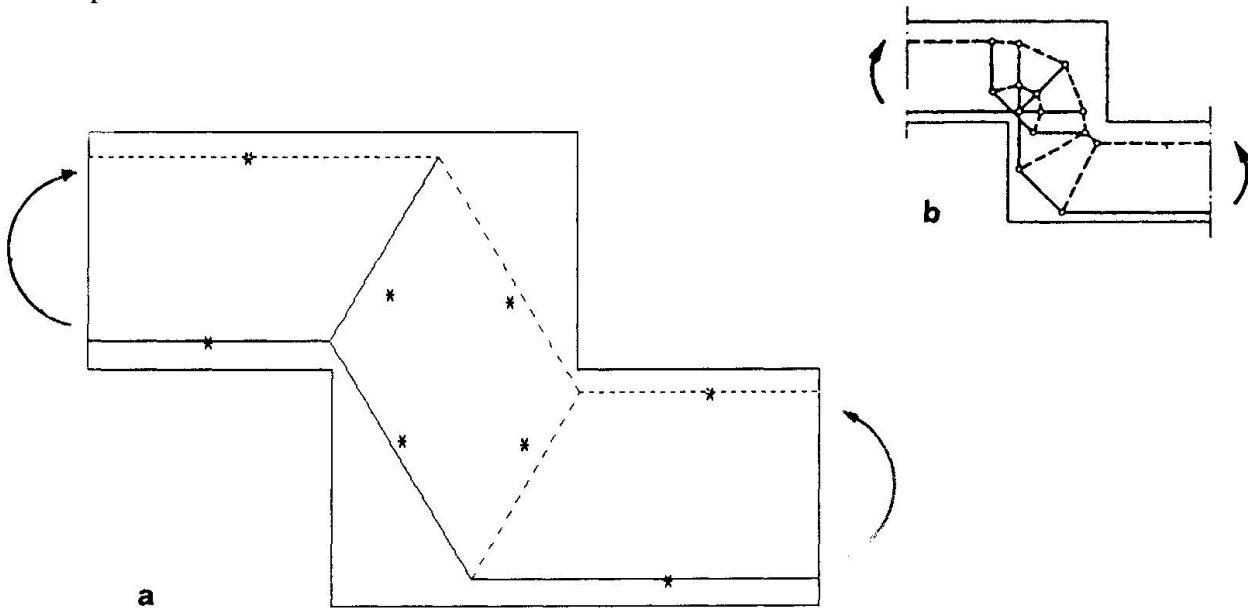


Fig.3 a) Strut-and-Tie Model for stepped beam; b) Strut-and-Tie Model after [1].

The automatic construction of the Strut-and-Tie model in each one of the 3 examples herein required less than 1 min. of computer time on a PC/AT. (To be compared to the few minutes required for the Finite Element Analysis of each problem on the same computer). This time is relatively short, allowing the engineer to try various alternative selections of user-specified parameters in order to improve or refine the Strut-and-Tie model.

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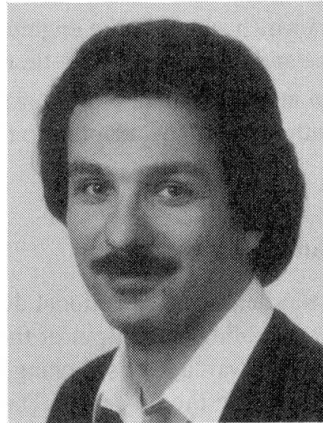
## Design and Analysis with Strut-and-Tie-Models – Computer-Aided Methods

### Méthodes assistées par ordinateur pour la conception basée sur l'analogie du treillis

#### Methode der Stabwerkmodelle: Umsetzung in einem Computerprogramm

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#### **SUMMARY**

The design method for strut-and-tie models was implemented in a computer program both to increase user-friendliness and to use the graphical and numerical capabilities of the computer. Therefore, a program system was developed which consists of several independent modules, which are linked through interprocess-communication and which run under a uniform user interface. The strut-and-tie model method was extended especially in the fields of: Modelling (strut-and-tie models can automatically be constructed out of trajectory fields) and Analysis (the use of a nonlinear analysis program together with nonlinear material laws and an algorithm for optimization, allows to calculate forces and displacements for the design as well as for the serviceability state).

#### **RÉSUMÉ**

La méthode de conception qui se base sur l'analogie du treillis a été traduite par un programme d'ordinateur afin d'augmenter le confort de l'utilisateur et de profiter des capacités graphiques et numériques de la machine. Le programme qui a été développé dans ce but comprend plusieurs modules indépendants liés à une méthode de communication interactive faisant l'interface avec l'utilisateur. L'analogie du treillis a été spécialement étendue dans les domaines de la modélisation (à partir du champs des trajectoires, on peut construire automatiquement des modèles de treillis) et de l'analyse (un programme d'analyse non-linéaire permet, conjointement aux lois non-linéaires de comportement des matériaux et d'un algorithme d'optimisation, de calculer forces et déplacement nécessaires au projet, aussi bien que pour l'évaluation de l'état de service).

#### **ZUSAMMENFASSUNG**

Die Methode der Stabwerkmodelle wurde in ein Computerprogramm umgesetzt, um die Benutzerfreundlichkeit zu erhöhen und die grafischen und numerischen Möglichkeiten des Rechners auszunutzen. Dafür wurde ein Programmsystem entwickelt, das aus mehreren unabhängigen Modulen besteht, die durch Interprozess-Kommunikation gekoppelt sind und unter einer einheitlichen CAD Benutzeroberfläche ablaufen. Die Methode der Stabwerkmodelle wurde insbesondere erweitert auf den Gebieten der Modellfindung (aus Trajektorienfeldern können automatisch Stabwerkmodelle abgeleitet werden) und der Berechnung (durch Verwendung eines nichtlinearen Rechenprogrammes zusammen mit nichtlinearen Materialgesetzen und einem Optimierungsalgorithmus können realistische Kräfte und Verformungen sowohl für die Bemessung als auch für den Gebrauchszustand berechnet werden).



## 1. Introduction

During the last years the strut-and-tie model (STM) design was developed as a method to unify the design of structural concrete for all kinds of concrete members and details /1,2,3/. One can consider this method as a combination of graphical and analytical techniques which had traditionally been applied 'by hand'. As the development and application of hard- and software has proceeded rapidly in the last years and a computer is on nearly every desk, it became clear that this method should also benefit from the graphical and numerical capabilities of the computer /7,9/ since a program based on a consistent design concept would be more logical for a CAD program than the effort to program only codes. Developments took place at different locations in various directions, i.e. based on theory of plasticity /5/.

The goal of bringing the strut-and-tie models 'onto the computer' included the following two tasks:

- Development of a program system which supports the engineer in an easy to use, graphical manner and allows to display, input, edit, analyze and design strut-and-tie models on a workstation.
- Enhancement of the strut-and-tie model method, especially in the fields of model finding and calculation of forces and displacements. The method had to be adapted to the computer, but this yielded also new possibilities.

## 2. Development of the Program System

The overall design process includes the fields of conceptual design, idealization of the structural model, analysis, design of the structural members, detailing and output of the drawings. Specialized programs exist for each of this fields with the user having the disadvantage of switching between different programs and the difficulty of transferring and translating the data between the programs. There was no integrated system which supports not only single steps but the whole process. To fulfil this requirement the developed integrated system needed the following modules within 'one' program:

- A CAD program to draw and edit the models and drawings and to display the results of the calculations.
- A FE program to analyze the structures.
- Programs to design the structural members with the STM method.
- A database to save all input and calculated data for easy access through the other modules.
- A interactive graphical user interface and control program which allows ease of use, portability and expandability.

An 'automatic' program was surely not the goal, as '.. it is imperative that an experienced and qualified engineer be involved in the interpretation of the results using their knowledge of structural behavior ..' /9/. Therefore the program was developed as a toolbox or method-base of application programs which is used similar to a database. The user communicates with the program through a uniform user interface (the CAD program) and can choose among several ways (simple to more demanding) and modules according to the specific needs of the problem. The results of each step can be checked as they are immediately displayed on the screen and if needed can be calculated repeatedly. The user must not be concerned about the compatibility of the program modules and the integrity and transfer of the data as the program system takes care of this.

It was unreasonable to combine everything in one program because of program size, complexity and expandability with other modules. Therefore programming interfaces were developed for graphics, database and interprocess communication (IPC). All stand-alone programs (CAD, FE, design, ...) were only extended with these interfaces. The CAD program with its graphical capabilities for input, editing and output was used as the main and control program. The others, which can run invisible in different processes or windows, were linked to it through IPC and can exchange commands and data very quickly. By this way the programs can be developed (and used) separately and new or improved modules can be implemented easily. This structure is invisible to the user as he has the impression of only one single menu-driven program. An overview of the program system is shown in Fig: 1 .

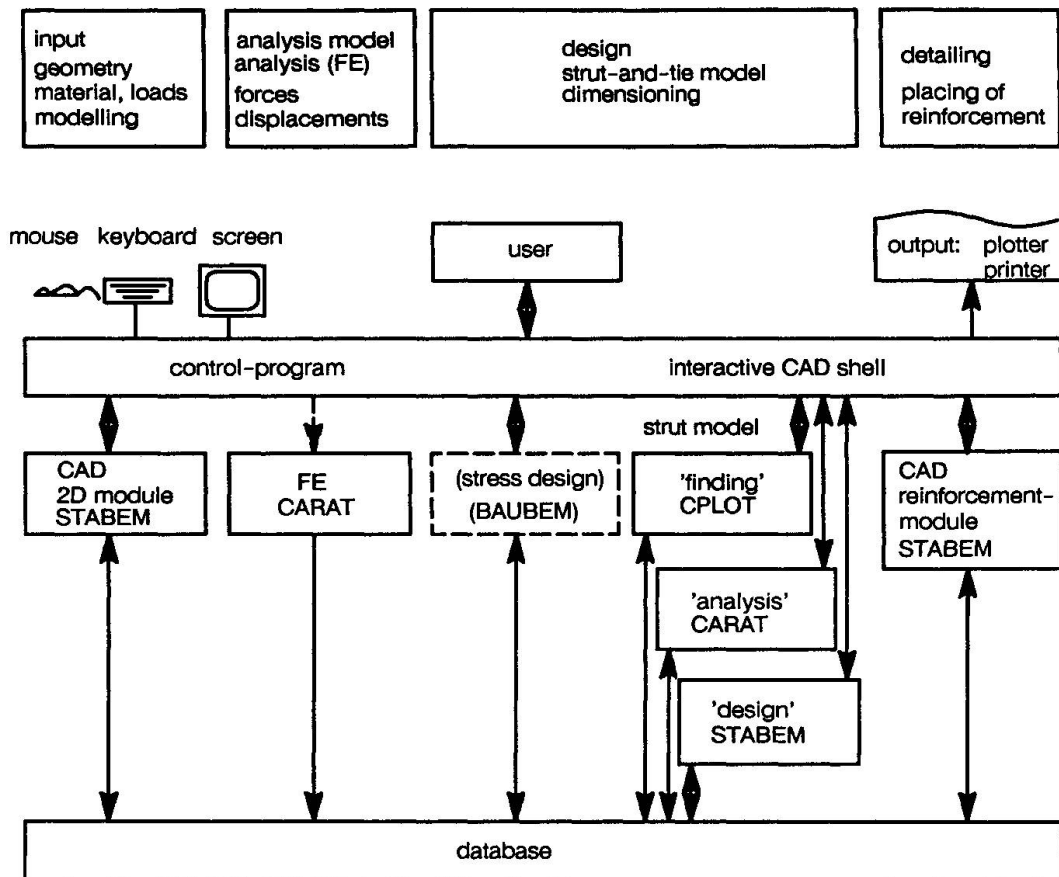


Fig. 1: Structogram of the program system

### 3. Strut-and-tie model design

#### 3.1. Elements

The struts as basic elements represent both the resultant forces and the corresponding stress fields. In the design program they are therefore defined as truss elements with varying shapes (Fig: 2).

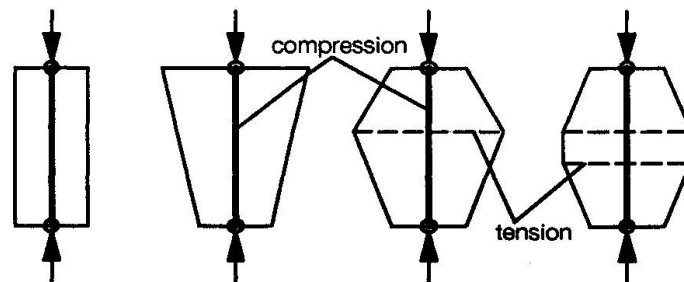


Fig. 2: Elements

The struts have dimensions according to the thickness of the structure and the width of the stress fields. This together with realistic material laws for concrete (compression and tension) and reinforced concrete (tension) allows the calculation of the nonlinear behavior (stress, strain, displacement, energy) of struts and ties.

#### 3.2. Modelling

The finding of a suitable model is one of the most important points for the design as the model has to represent the actual flow of forces in the structure and is the starting point for all following calculations.

The simple modelling methods:

- 'free hand' drawing of simple models
- adaptation of known typical models to a specific case



are supported by the computer through the graphical drawing and editing possibilities and a 'database' of known models.

A simple preliminary elastic FE analysis of the structure helps the modelling methods:

- orientation of the model at the linear theory of elasticity,
- finding of the 'load path'

by underlaying the principle stress fields and possible calculations of stress resultants anywhere in the structure.

The most interesting and newly developed part is the model finding with trajectory fields. Trajectories can be generated at a desired density (which can but need not increase accuracy) and automatically be transformed into a strut-and-tie model (Fig.3 ). This represents a model which is closely oriented at the theory of elasticity and is inherently in a state of equilibrium.

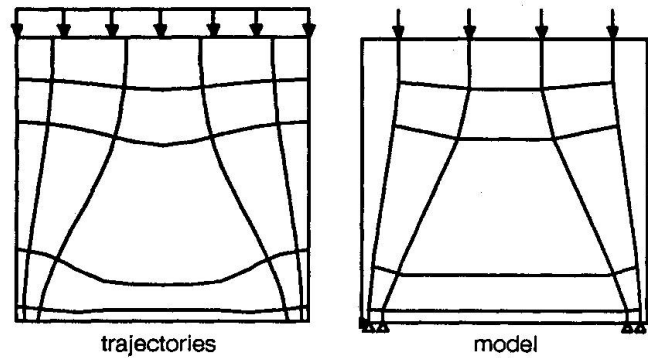


Fig. 3: Strut model out of trajectories

The net of trajectories is equivalent to a 'shear free' net of (FE-) elements which have only biaxial normal stresses. It is possible to calculate the forces in a 'strip' bordered by two trajectories (Fig. 4 ). Therewith the 'flow' of forces in the structure (e.g. from the load to the support and perpendicular) becomes visible. These forces and their direction in the continuum are then equivalent to the forces and directions of the struts resp. stress fields and therewith the analogy between continuum and strut model becomes obvious.

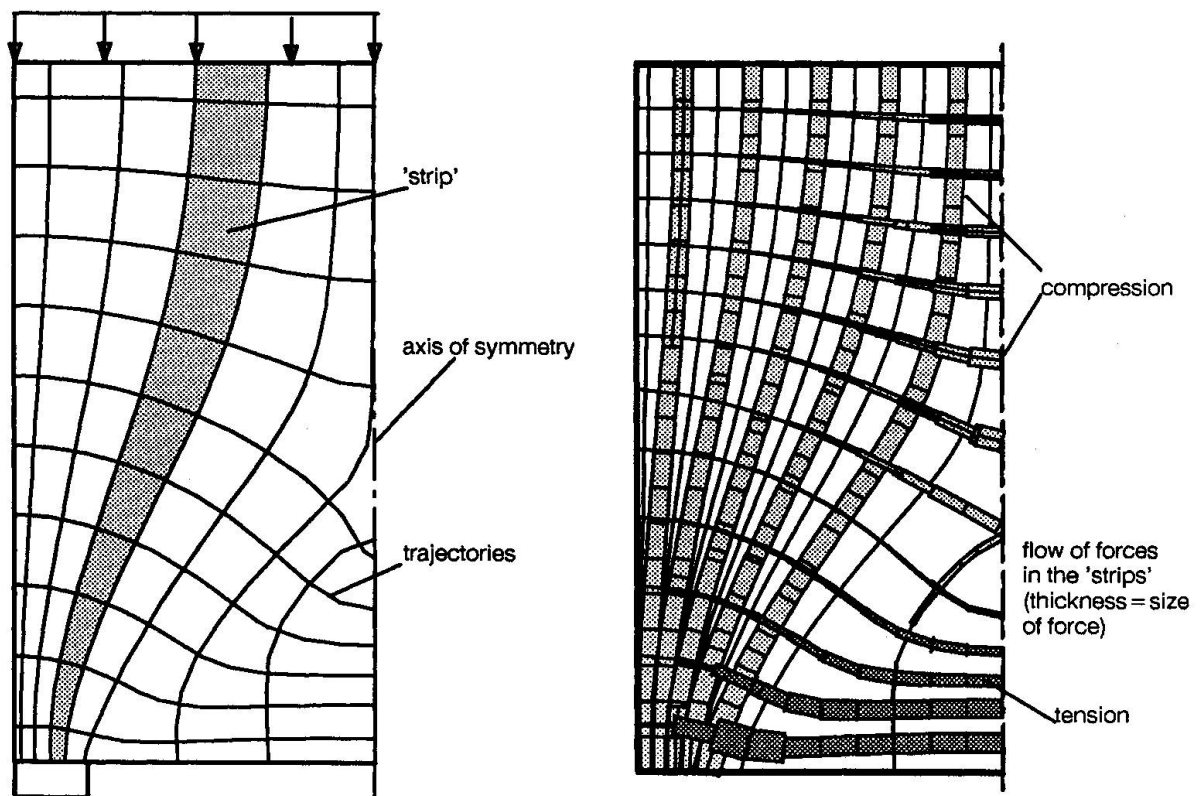


Fig. 4: 'Trajectories and flow of forces'

The 'density' of the model is up to the designer and depends on the design stage /9/. Often it is sufficient for design purposes to use a simple model (Fig. 3 ) and leave the less important trajectories out. The program pro-

vides numerous possibilities to edit the automatically created model and to adapt it to the specific needs, as for example given positions and directions of reinforcement.

### 3.3. Analysis

For statically determinate and indeterminate models it is difficult to verify that they represent the load bearing of the structure as the geometry can be (almost) arbitrarily chosen. For kinematic models there is (for a given load) only one stable geometry which then reflects the load bearing.

The difficulty of analyzing a kinematic model was solved by using a geometric nonlinear program which was extended with initial stiffnesses to calculate 'kinematic cable structures' and which can therefore also be used to find a stable geometry (Fig: 5). If the model is generated through trajectory fields, the advantage is that it is already in a stable position and also represents the actual flow of forces.

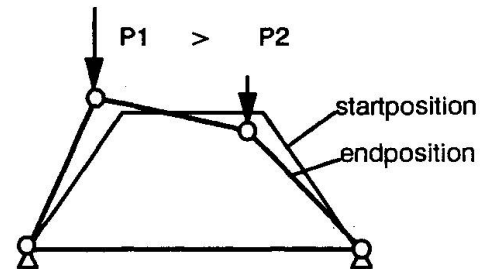


Fig. 5: Analysis

The dimensioning of concrete and reinforcement is a relatively simple task and could for every model also be done by the theory of plasticity [4,5]. A model which is oriented at the theory of elasticity is both well suited for the design of struts and ties, as the calculated amount of reinforcement and concrete stresses are on the safe side according to the lower theorem of plasticity, and the requirements for compatibility and serviceability are also approximately fulfilled.

If however, one wants to know the state of stress or the displacements for any loading stage from cracking up to ultimate load, the geometry of the model has to be adapted to the load bearing behavior resp. 'load path' of this state. The struts must also have the according properties (width, material, etc.). The implemented strut elements with dimensions corresponding to the stress fields and nonlinear material laws allow the calculation of forces and stresses as well as displacements (Fig: 6)

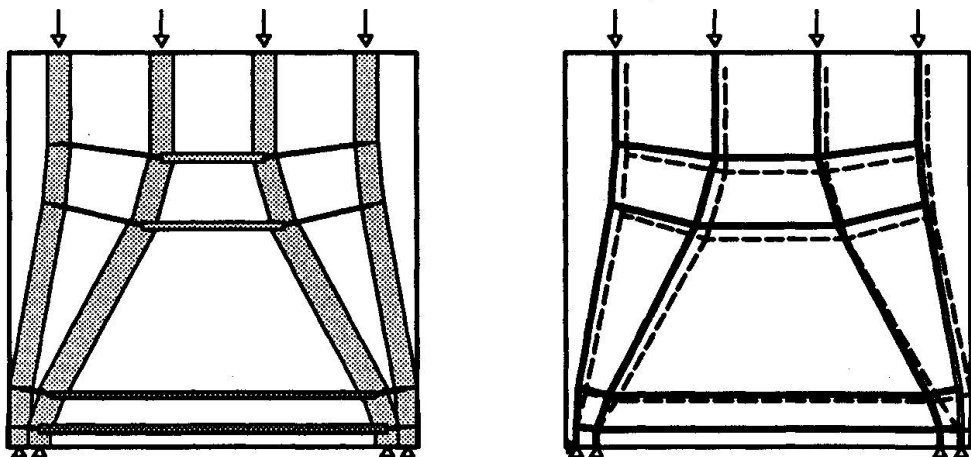


Fig. 6: Forces and displacements

This is done with the criterion of optimizing/minimizing the internal energy of the total system as it tends to undergo the smallest possible stresses and strains. This together with an iterative nonlinear analysis allows to adapt the model, i.e. find the right geometry, for increasing loads as shown in Fig: 7. Comparisons with tests show satisfying results.

This is a tool for the experienced engineer to achieve results for the design and its verification in a rather simple and quick way. An additional nonlinear FE analysis for the verification (but not for the design itself) could still be done, but to achieve possibly better results it has to be much more elaborate.

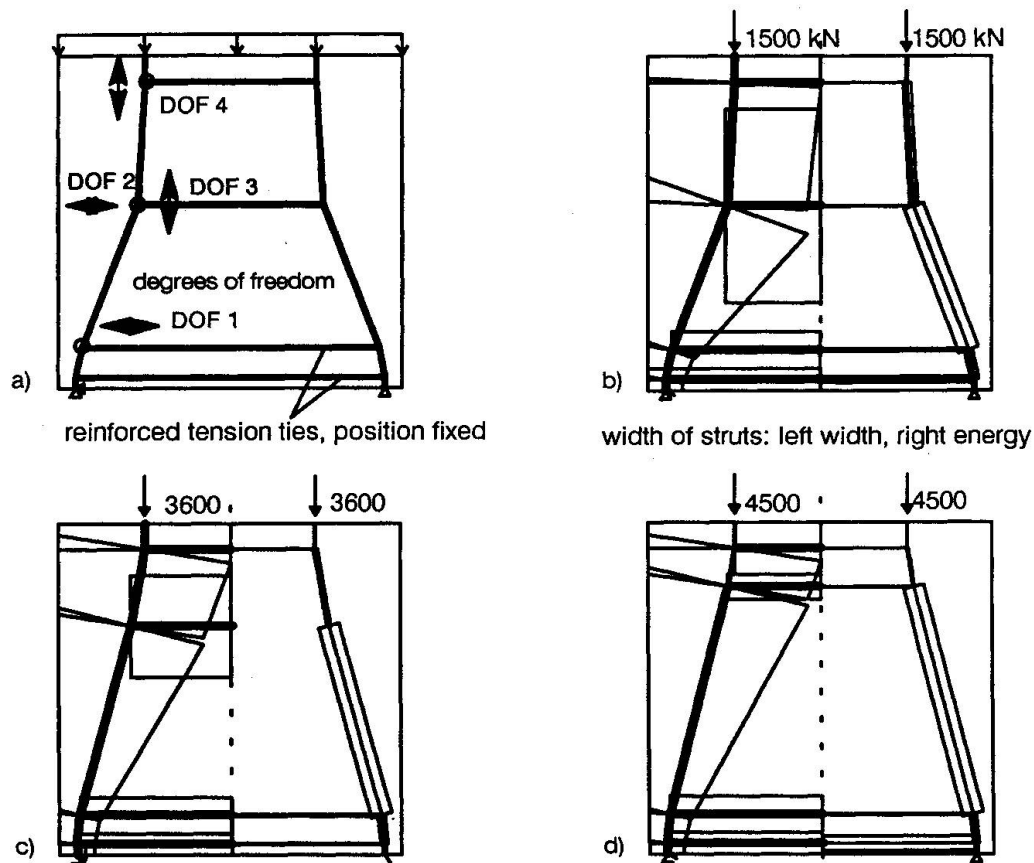


Fig. 7: Adaptation of model under increasing load

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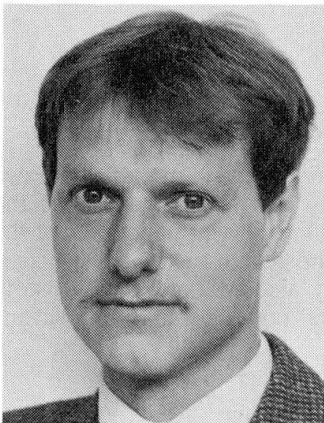
## Nonlinear Behaviour of Deep Beams

Comportement non-linéaire d'un élément porteur de type cloison

Nichtlineares Tragverhalten von Scheiben

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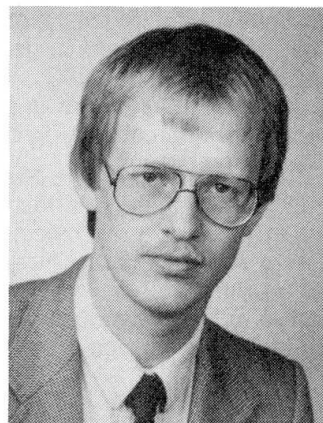
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### SUMMARY

This paper deals with an analytical method, which is an extension of the method of Strut-and-Tie-Models. It enables the design engineer to calculate the nonlinear behaviour of deep beams in a simple way. It yields approximate values for the ultimate load, the support reactions of statical indeterminate systems, the important stresses and strains and the load deflection behaviour, respectively. Additionally, a simultaneous control of locally high stressed regions, e.g. near supports or loadings, is possible. The results of this analysis can be used also as an independent verification of the results of a nonlinear finite analysis.

### RÉSUMÉ

On présente dans ce rapport une méthode dérivée de la modélisation par bielles (analogie du treillis) qui permet à l'ingénieur de calculer simplement le comportement non-linéaire d'éléments porteurs de type cloison. On obtient ainsi des valeurs approchées de la charge ultime, des réactions d'appui de systèmes hyperstatiques, des contraintes et des déformations déterminantes ainsi que des diagrammes force-déplacement. Le contrôle de régions particulièrement sollicitées, comme le voisinage des appuis ou le point d'application d'une force concentrée est de plus rendu possible. Les résultats obtenus par cette méthode peuvent être employés pour vérifier une analyse non-linéaire par éléments finis.

### ZUSAMMENFASSUNG

In dieser Abhandlung wird ein Verfahren vorgestellt, das eine Erweiterung der Methode der Stabwerkmodelle darstellt und dem entwerfenden Ingenieur ermöglicht, das nichtlineare Tragverhalten seines Tragwerks vereinfacht zu untersuchen. Dabei können z.B. die tatsächlich erreichbare Bruchlast, Auflagerkräfte statisch unbestimmter Systeme, massgebende Spannungen, Dehnungen und Last-Verschiebungskurven ermittelt werden. Zusätzlich ist eine Kontrolle lokal hoch beanspruchter Knotenbereiche, z.B. im Auflager- oder Krafteinleitungsbereich, möglich. Das Verfahren kann z.B. auch als ingenieurmässiges unabhängiges Kontrollinstrument für eine nicht-lineare FEA verwendet werden.



## 1. INTRODUCTION

For the analysis and dimensioning of deep beams of reinforced concrete, especially for statical indeterminate structures, the theory of elasticity still is the main basis. However, the real behaviour of these structures under increasing load is determined by the nonlinear behaviour of the materials. This has a major influence on the real ultimate load capacity and the load-deflection-response, respectively.

Up to now the calculation of such effects is only possible with the Nonlinear Finite Element Analysis (NLFEA) and therefore requires an enormous amount of computing. Moreover the results are very difficult to check. Additionally the sophisticated calculation of stresses and strains in every point of the structure is often of no interest (/1/).

This paper presents a practical tool, which enables the design engineer to analyse his concrete structure with respect to some interesting points, e.g. (s. also Fig.1):

- Magnitude of the real ultimate load after the mobilizing of all the bearing capacities within the structure;
- Determination of the structural elements which are probably responsible for the failure of the whole structure, e.g. node regions, parts of reinforcement etc.;
- Load-Deflection-Behaviour with increasing load;
- Distribution of the support reactions in statical indeterminate structures;
- Sensitivity of a structure to restraint;
- Response of the structure to variations in amount and arrangement of reinforcement (e.g. partially prestressed ties, redistribution of reinforcement between span and support-regions etc.).

The presented method is an extension of the Strut-and-Tie-Model (STM) analysis and therefore draws attention to those elements, which mainly influence the load bearing capacity and the behaviour of the structure. The assumptions for the numerical calculation, e.g. for the strength and nonlinear behaviour of the materials, are always present because of the small number of elements. So the response of the structure remains transparent and the results are easy to control. Therefore this tool may also become a very useful educational aid (/3/). Because of the permanent check of the highly stressed local regions this tool performs both at global as well as at local level, which is strictly demanded in /4/.

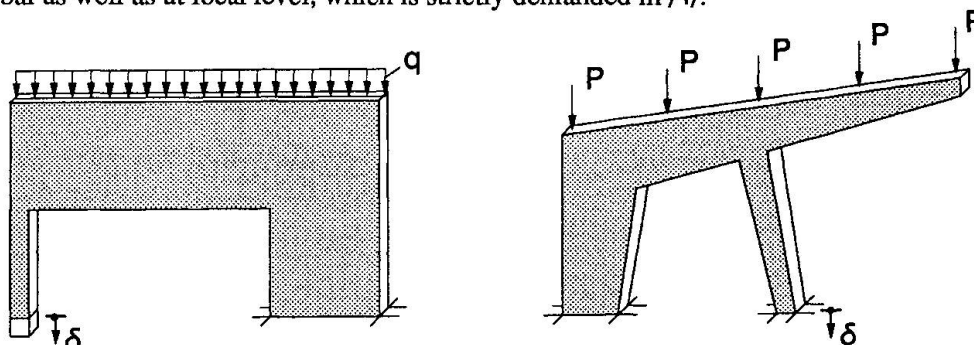


Fig.1 Two typical deep beams, for which a study of the load carrying behaviour due to load ( $q$  and  $P$ ) and settlement ( $\delta$ ) may lead to a better estimation of the real structural safety.

## 2. FORMULATION OF THE METHOD

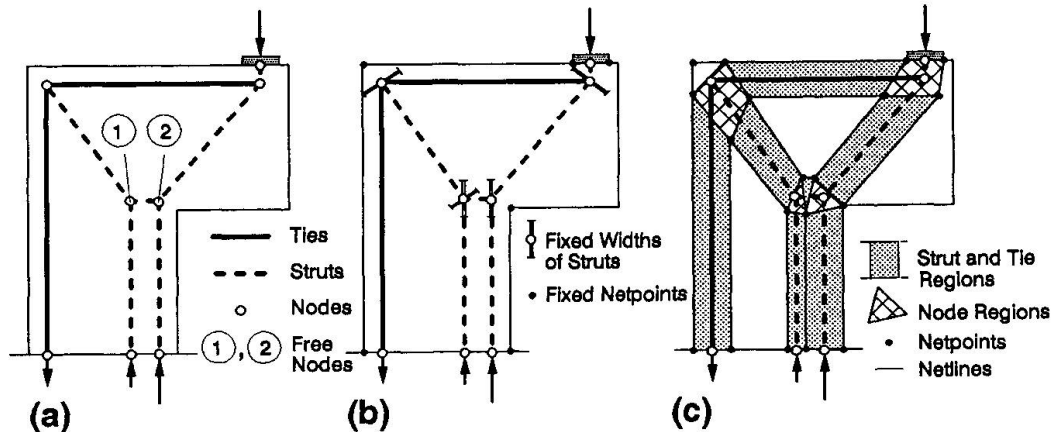
The Flow of Forces in the structure can be simply modeled either by hand ("Load Path Method", /2/) or by a more refined analysis of the structure with a Linear Finite Element Analysis (/5/). The result is a STM with struts, reinforced ties and nodes, s. Fig 2(a).

The Strut-and-Tie-Net separates the predominantly one-dimensional stress fields of the struts and ties from the two-dimensional stress fields of the nodes. With the fixed netpoints and the assumption of some effective widths of the struts, s. fig 2(b), the geometry of the Strut-and-Tie-Net can be found as a result of the current geometry of the STM, s. fig 2(c). Now the effective widths of the struts and ties and consequently their stresses, which govern the capacity and behaviour of these elements, can be computed.

The bearing capacity of the individual Ties is determined by the amount and strength of the reinforcement. The nonlinear behaviour can be calculated either according to the well-known regulations in codes (e.g. MC 90, EC 2 etc.) or according to appropriate publications. In this paper a simple formulation is used, which is an extension of the tension stiffening relations given in /7/. With its help the number of cracks, the crack-width and the average deformation of the ties can be calculated. Additionally an early state of cracking can be assumed. The different kinds of reinforcing-steel (e.g. in a partially prestressed ties) can be taken into account by their stress-strain-curves including the strain hardening, s. fig. 3(a).

The capacity of the Struts is governed by the stresses at the borderline between the node and strut regions.

For simplification the stresses are calculated in the so called "Transition-point", s. fig. 4. The description of the nonlinear behaviour follows a modified rule in the MC 90 and the DIN 1056, respectively. With the help of 3 factors, which determine the actual strength ( $\alpha_f$ ), the tangent modulus at the origin ( $\alpha_E$ ) and the strain reached at ultimate stress ( $\alpha_\epsilon$ ) a wide range of nonlinear response of the struts can be covered, s. fig. 3(b). The values of these factors depend e.g. on the geometrical form of the struts (fan- or bottle-shaped) and their structural design (longitudinal or transversal reinforcement).

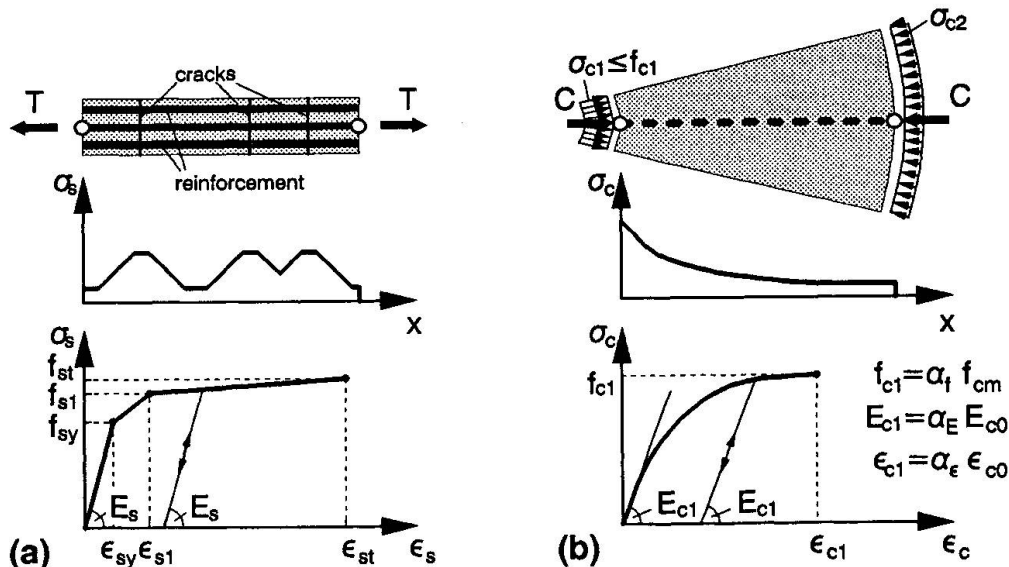


**Fig.2** Evaluation of the Strut-and-Tie-Net demonstrated for a cantilever wall:

- (a) The structure with its borderline, external forces and the STM
- (b) Determination of the fixed netpoints and some widths of struts
- (c) Complete Strut-and-Tie-Net with the effective idealized fields of struts, ties and nodes

The bearing capacity of the **Nodes** is also determined by the stresses in the "Transition Point", s. fig. 4. It depends on the kind of node (e.g. pure compression- or bond-node) and its detailing (e.g. kind of anchorage, amount of transverse reinforcement etc.) and is described by a strength factor ( $\alpha_n$ ), too.

The **time dependent behaviour** is determined by the creep coefficient, which can be adopted from code instructions. For this method the behaviour of struts and ties is considered differently: the struts have a creep strain, whereas the cracked ties have a time dependent decrease of the bond-stresses.



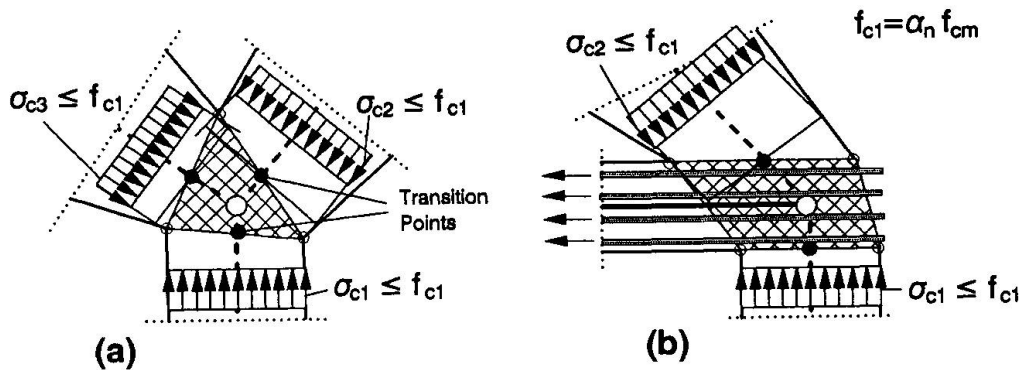
**Fig.3** Calculation of the nonlinear behaviour of Ties (a) and Struts (b)

The positions of the **Free Nodes** are determined by applying an energy-criterion. This yields the best simulation of the real load bearing behaviour regarding to the accuracy of the chosen STM. The applied energy-criterion of the extreme value of the overall potential is computed by the internal deformation energy and the potential of external forces and support settlements. The compatibility of this equilibrium state is thus automatically guaranteed.

For the **numerical calculation** the loading, settlements and creep coefficients can be increased step by step,



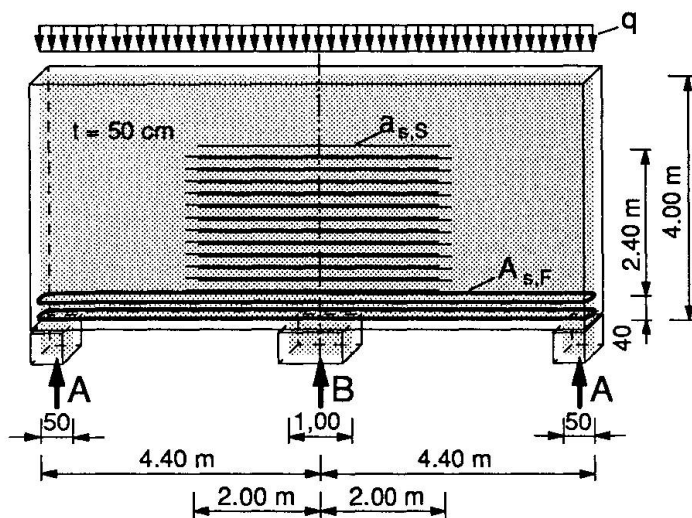
whereby the best position of the free nodes is computed in every loading step. If some struts or ties start to fail near the ultimate load, the increments are reduced to 1/5 of the original value. This ensures a sufficient redistribution of the inner forces. The results of the computation are directly presented in diagrams, which allow a fast and engineering analysis of the structure (e.g. load deflection curves, important strains and stresses, crack-widths, support reactions etc.).



**Fig. 4** Two examples for node regions and their governing stresses  
 (a) Pure compression node (CCC)  
 (b) Node with deviation of struts due to the anchorage of a tie (CTC)

### 3. ANALYSIS OF AN EXAMPLE

In fig. 5 a deep beam with 2 spans is shown with its geometry, loading and two cases of reinforcement. In case 1 the reinforcement is chosen according to a linear-elastic analysis. In case 2 the amount of the span-reinforcement is substantially increased, while simultaneously the reinforcement at support is reduced as against case 1. The load carrying behaviour for both cases now will be examined with the above presented method.



Concrete:  $f_{cm} = 36.0 \text{ MN/m}^2$   
 $f_{ctm} = 3.20 \text{ MN/m}^2$   
 $E_{co} = 37,000 \text{ MN/m}^2$   
 $\epsilon_{co} = 2.2 \%$

Reinforcement:  
 $f_{sy} = 500 \text{ MN/m}^2$   
 $f_{st} = 550 \text{ MN/m}^2$   
 $E_s = 210,000 \text{ MN/m}^2$

Case 1: Reinforcement due to linear-elastic analysis  
 $A_{s,F} = 8 \phi 20 = 25.12 \text{ cm}^2$   
 $a_{s,S} = \phi 12/20 \text{ cm} = 11.3 \text{ cm}^2/\text{m}$

Case 2: Increased span reinforcement  
 $A_{s,F} = 12 \phi 25 = 58.92 \text{ cm}^2$   
 $a_{s,S} = \phi 10/25 \text{ cm} = 6.28 \text{ cm}^2/\text{m}$

Loading:  $q = 1.0 \text{ MN/m}$  (service load)

**Fig.5** Geometry, reinforcement and loading of the example

The structure is modeled with a very simple STM, s. fig. 6(a), which was found by the "Load Path Method". The geometry of this STM determines the initial position of the free nodes for both cases. In order to leave open the relations of the support reactions (A and B) and also the internal lever arm in the span (distance between tie 1 and strut 13) the free nodes 7 to 10 can vary in any direction. To allow a free distribution of the tie forces 7 and 17, the nodes 5 and 6 can vary in their x-coordinates. Regarding the symmetry and the internal coupling of nodes there are altogether 3 degrees of freedom for the whole system. The ultimate stresses at the supports are set to  $f_{c1} = 1.20 f_{cm}$  ( $\alpha_n = 1.20$ ) because of the very proper node design with loops, while all other factors ( $\alpha_f$ ,  $\alpha_E$  and  $\alpha_\epsilon$ ) are simply set to 1.0. To avoid a contribution of the concrete tensile strength over the middle support, the Ties 7 and 17 are assumed to be precracked with one crack, whereby the tie 17 is assumed to be reinforced like tie 7. The load increment is  $\Delta q = 0.50 \text{ MN/m}$ .

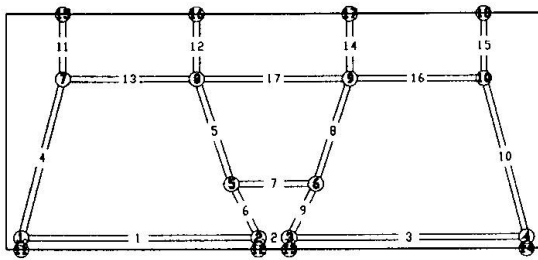
In the first load step ( $q = 0.50 \text{ MN/m}$ ) the structure behaves in a linear-elastic mode. The position of the free nodes, calculated by the energy-criterion, lead to the geometry of the STM shown in fig. 6(b). The internal

lever arm and the distribution of stresses over the middle support agree very well with a linear-elastic FEA.

With increasing load the geometry of the STM changes, which is caused by a redistribution of the inner forces. The internal lever arm increases in both cases, until the geometries for the STMs at the ultimate loads are obtained, see fig. 6(c). In case 1 the ultimate load is  $q_u = 3.60 \text{ MN/m}$  and the failure of the structure is initiated by a simultaneous failure of the reinforcement in the span and at support. In case 2 the ultimate load is  $q_u = 4.60 \text{ MN/m}$  and the failure of the structure is caused by a failure of the nodes at the supports, which have nearly all the same pressures at the ultimate load.

The relation of  $B/A$ , see fig 6(d), shows a distinct increase for the wall in case 1. This is due to a redistribution of forces from the span to the support region because of the relatively weak stiffness of the tie in the span. In case 2 the ratio  $B/A$  remains nearly constant, it even deminishes nearby the ultimate load.

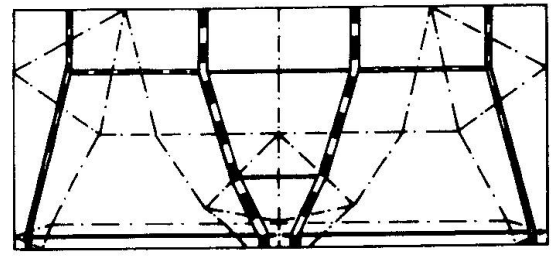
Numbers of Struts, Ties and Nodes:



(a)

Case 1

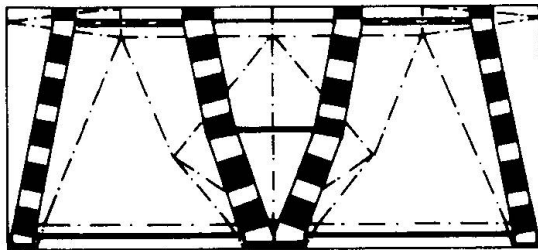
Linear-Elastic Calculation:



(b)

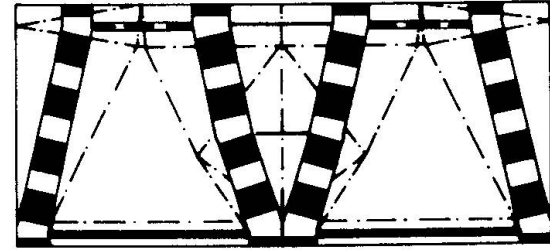
Case 2

Load  $q_u = 3.60 \text{ MN/m}$

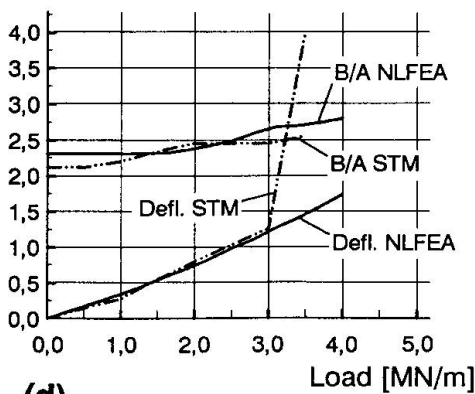


(c)

Load  $q_u = 4.60 \text{ MN/m}$



Deflection [mm]  
 $B/A$



(d)

Deflection [mm]  
 $B/A$

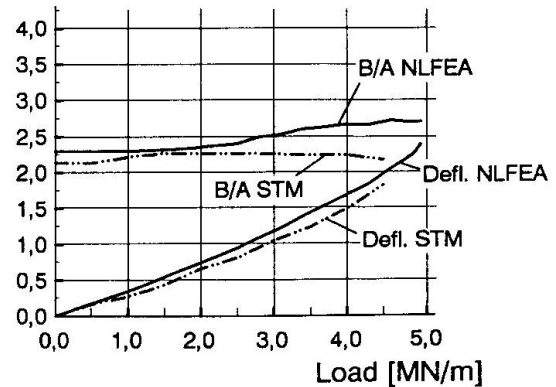


Fig. 6 Response of the structure



The **load-deflection-curves** in fig. 6(d) show a quite linear increase of the deflections because of the overall stiffness of the structure. Only in case 1 the deflection rapidly increases shortly before the ultimate load is reached. This is due to the yielding of the span- and support-reinforcement.

Additionally a NLFEA was carried out using the program SBETA described in /8/. The aim was to proof the reliability of the demonstrated STM method. Some results are shown in the diagrams (s. fig. 6(d)) in comparison to the calculations with the STMs. The ultimate loads of the NLFEA ( $q_{ul} = 4.10 \text{ MN/m}$  for case 1 and  $q_{ul} = 5.20 \text{ MN/m}$  for case 2) are slightly higher than the values of the STMs. In case 1 the increase of  $B/A$  occurs at higher loads in the NLFEA than in the STMs. This is due to the uncracked state, which is conserved up to higher loads in the NLFEA. In case 2 the relation  $B/A$  is higher in the NLFEA, which is also caused by uncracked areas especially at the top region over the middle support. The **load-deflection-curves** are in good agreement, apart from the case 1, where a difference occurs near the ultimate load, because of a greater stiffness of the wall in the NLFEA. A more refined STM would of course improve the simulation of this deep beam. More details about the non-linear calculations with STM and the NLFEA including further examples can be found in /6/.

#### 4. CONCLUSIONS

It could be demonstrated that the overall load carrying behaviour is simulated quite well with the presented tool. The discussed method especially provides the following advantages:

- The flow of forces remains transparent throughout the whole nonlinear analysis because of the small number of load bearing elements.
- The capacity of locally high stressed regions can be adapted separately according to their structural design.
- The whole numerical analysis runs on a PC with small equipment and takes only a very short time.
- This tool enables the engineer to analyse the behaviour of his structure under varying boundary conditions (e.g. with or without concrete tensile strength, with or without creep effects etc.).
- An adaptation of this model to future code regulations is easily possible.

When applying this method, the following points should be paid attention to:

- The chosen STM must be able to follow all the possible internal redistributions of the forces.
- The method cannot cover all structural effects, especially in the uncracked state, because of the simple modelling.

This practical tool promotes a very good understanding of the whole structure and the interaction of its components. This leads to a better design and to a more reliable estimation of the safety of the analysed structure.

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