Strut-and-Tie Modelling of Structural Concrete

Analogie du treillis dans les structures en béton

Stabwerksmodelle für Konstruktionsbeton

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SUMMARY

Strut-and-tie models can illustrate very well the internal flow of forces in structural concrete and thereby, provide valuable assistance to the designer who is striving for an appropriate and functional conceptual design. Moreover, such models are good enough to serve as a basis for the design and dimensioning of the modelled structure or structural detail for the cracked state. After some introductory statements this will be demonstrated by the example of a box girder, cantilevering from two supporting walls. Different arrangements of diaphragm walls and the effect of prestressing will be discussed.

RÉSUMÉ

L'analogie du treillis permet de visualiser de façon très claire le cheminement des forces dans les structures en béton; ceci constitue une aide précieuse pour l'ingénieur lors du dimensionnement approprié d'éléments porteurs ainsi que des détails de construction. De plus, de tels modèles représentent le fondement du calcul des constructions en béton armé et précontraint à l'état fissuré. Après quelques remarques introductives, ces faits sont démontrés à l'aide d'un exemple décrivant une poutre-caisson, dont deux parois constituent les appuis. Diverses positions d'un diaphragme intérieur ainsi que l'effet de la précontrainte dans le caisson sont successivement discutées.

ZUSAMMENFASSUNG

Mit Stabwerksmodellen kann der innere Kraftfluss sehr anschaulich dargestellt werden. Dadurch sind sie auch eine wertvolle Hilfe für den Entwurf zweckmässiger Tragwerke und Details. Sie sind ausserdem eine geeignete Grundlage für die Bemessung von Stahl- und Spannbetonkonstruktionen im gerissenen Zustand II. Dies wird im vorliegenden Beitrag am Beispiel eines auskragenden Hohlkastenträgers gezeigt. Dabei werden verschiedene Varianten von Querschotts und die Wirkung der Vorspannung diskutiert.

1. INTRODUCTION

Breen [4] proclaimed as a key topic of this conference "useful and transparent models, which can enhance the designer's realization of structural action" and he emphasized several times strut-and-tie models (STM) as such a tool. Schlaich, in his Introductory Report to "Modelling", integrated the strut-and-tie method into the framework of the design process of structural concrete [7]. Beginning with Ritter's truss model for beams such models were used for the visualization of forces in some specific cracked reinforced concrete elements and for proportioning their reinforcement. Thürlimann and his Zürich School developed a more general design concept using stress fields on the basis of theory of plasticity. More recently Schlaich and his co-workers proposed to generalize the strut-and-tie method for the application to all kinds of structural concrete elements or structural details and to compliment the method by a unified concept for the dimensioning of cracked structural concrete, including the node regions of struts and ties [1, 2, 3].

Such a concept is urgently needed considering the Codes of Practise, which give design rules only for elements with linear strain distribution (B-regions) but neglect all others, more complicated ones, where damage most frequently occurs. The lack of a consistent methodical approach for the design and dimensioning of such discontinuity regions (D-regions) was felt particularly bad when they were taught to the students. Considering the importance of the D-regions for the safety and endurance of structures their design cannot be left to the draftsman's skill or the engineer's good guess. Any rational approximate method is better than this state of dimensioning.

2. MODELLING METHODS

Basically three methods are available, which also may be combined:

- Orientation of the model, in particular the struts, at the linear theory of elasticity. Stress trajectories from FEM or stress diagrams in typical sections can be used for locating major struts and ties. A rough orientation at the elastic behaviour is necessary anyhow for compatibility and serviceability reasons.

- Analogy of the stresses with that of a fluid. This analogy - though mechanically not perfect - helps to find the "flow of forces" through the structure by the "load path method".

- Adaptation of known typical models to the specific case. This is facilitated by the fact that certain types of models repeat very frequently in different structures.

The general methods of finding and judging strut-and-tie models are published in some detail in [2, 3] and therefore will not be repeated here. Instead, an example will be presented.

3. SUPPORT OF A CANTILEVERING BOX GIRDER ON TWO WALLS

3.1 General Layout

How to carry the forces in the connection of the members shown in Fig. 1a? Normally diaphragm walls are introduced in the box girder, either two directly above the two supporting walls (Fig. 1b), or - because the inner one is difficult to construct - just one at the end of the box girder (Fig. 1c). The best solution, a diagonal wall, is not obvious in the beginning.

3.2 Frame Corner with Diaphragm Wall at the Box Girder's End only

The diagonal struts C_3 (Fig. 2a) which balance the chord forces T_1 (from the box girder's tensile flange) and T_2 (from the tensile wall) with the compressive forces C_1 and C_2 from the respective compression chords of the frame type structure can only be transferred within the two webs. Therefore, all the chord forces which in the adjacent B-regions are well distributed over the whole widths of the flanges have to be deviated and bundled into the small width of the webs.

First of all, this requires considerable transverse reinforcements in the four chord members according to the strut models given in Figs. 2b-e. The models are all of one type, which appears very frequently in D-regions of very different structures. The internal lever arm z of the transverse forces in Figs. 2c and 2e, oriented at theory of elasticity is approximately 0,6 b. In Figs. 2b and d the corresponding lever arm depends also on the length of overlap of longitudinal reinforcement. Standard lap lengths in Codes do not apply for this situation where the lapped bars are arranged at some distance from each other. A more detailed model (Fig. 2f) of this typical problem shows different transverse tie forces in different places.

Looking again at the struts C_1 and C_2 in plan and sections of Fig. 2 we realize that the corresponding stress fields must be squeezed through the bottleneck of the singular node 2 whose dimensions are restraint by the
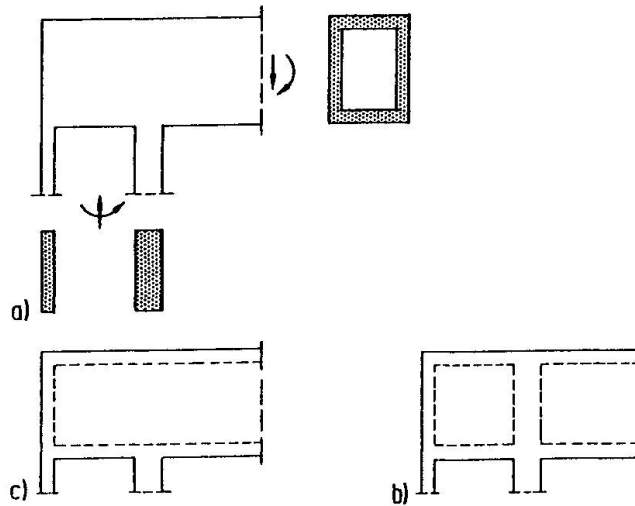
Fig. 1: a) View and cross-section of the box girder and support walls. b) Longitudinal section with interior diaphragm wall. c) Longitudinal section without interior diaphragm wall
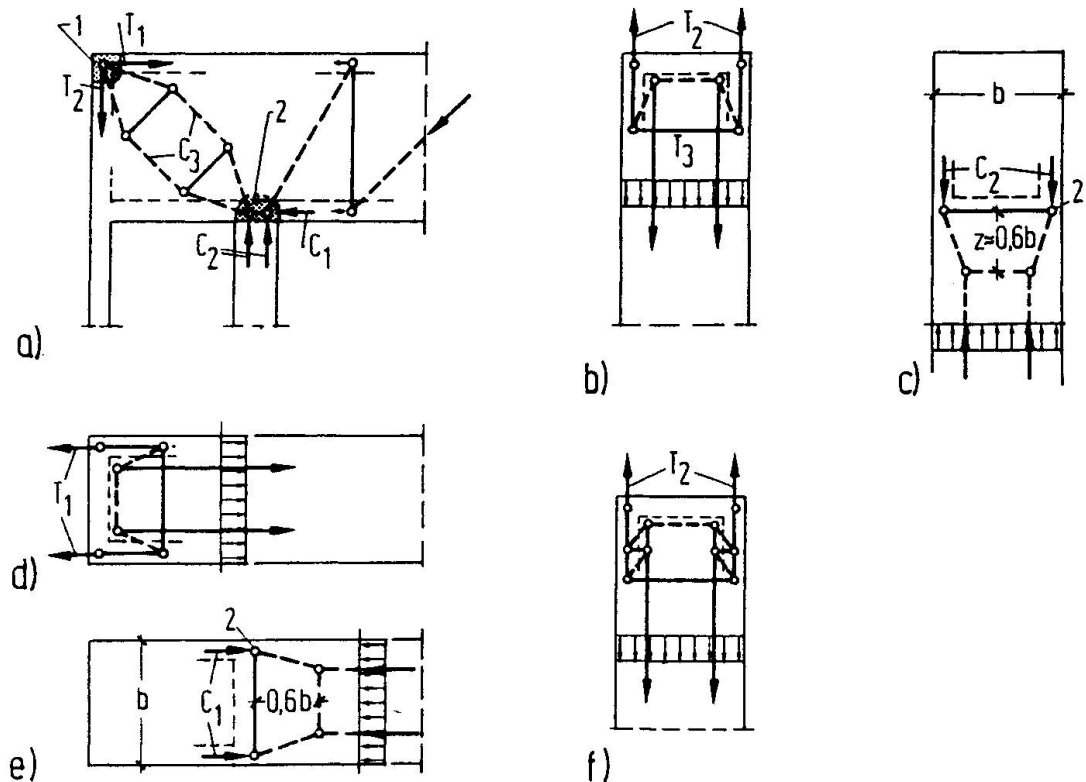

Fig. 2: Strut-and-tie models for the structure without internal diaphragm wall. a) Web; b) tensile wall; c) compression wall; d) top flange; e) bottom flange. f) Refined model for the lap problem characterized in Figs. b and d

thicknesses of the compression wall, web and bottom slab of the box girder. This node will dictate the concrete dimensions, the large width b of the boxgirder slab cannot be used as compression zone. What an unreasonable structure! Who would have recognized this, applying the usual design rules?

A similar problem may arise in node 1, where tensile bars for the total chord forces T_1 resp. T_2 must be arranged within the thickness of the web or at least very close to it in order to avoid large "slab moments" in

the deck and wall. Also this node will become a singular node if reinforcing bars were bent sharply around the corner as shown in Fig. 2a. Consequently, the diagonal strut force C_3 in the web will spread out between nodes 1 and 2, thereby creating transverse tensile forces as indicated in Fig. 2a. Therefore it is much better to bend the chord reinforcement using a mandrel which is adapted to the dimensions of the frame corner (Fig. 3).
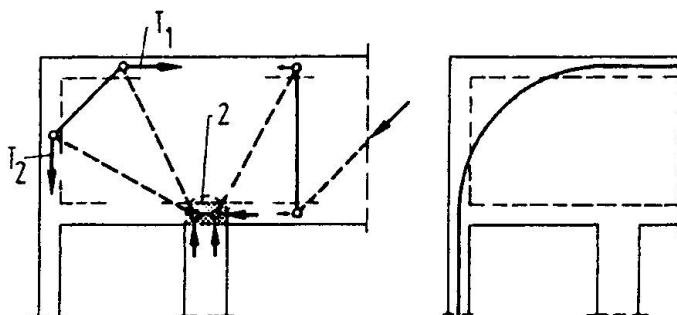
Fig. 3: Model and corresponding chord reinforcement with a well adapted mandrel diameter

By the way, omitting the lower diaphragm plate between the two walls would do no additional harm to the (poor) structural solution. Either none or two orthogonal disphragms are needed, as will be shown hereafter.

3.3 Frame Corner with Two Diaphragm Walls

The necessary transverse reinforcement in the boxgirder plates and the supporting walls is the same as before; but the singular nodes are avoided since the chord forces T_1, T_2, C_1, C_2 now enter the web reasonably well distributed over the whole length of the diaphragms (Fig. 4a). In other words: Each chord plate is no longer supported on two points only but rather along two lines (Fig. 4b). As a consequence the load bearing capacity of the frame corner is essentially increased by the additional diaphragm wall.
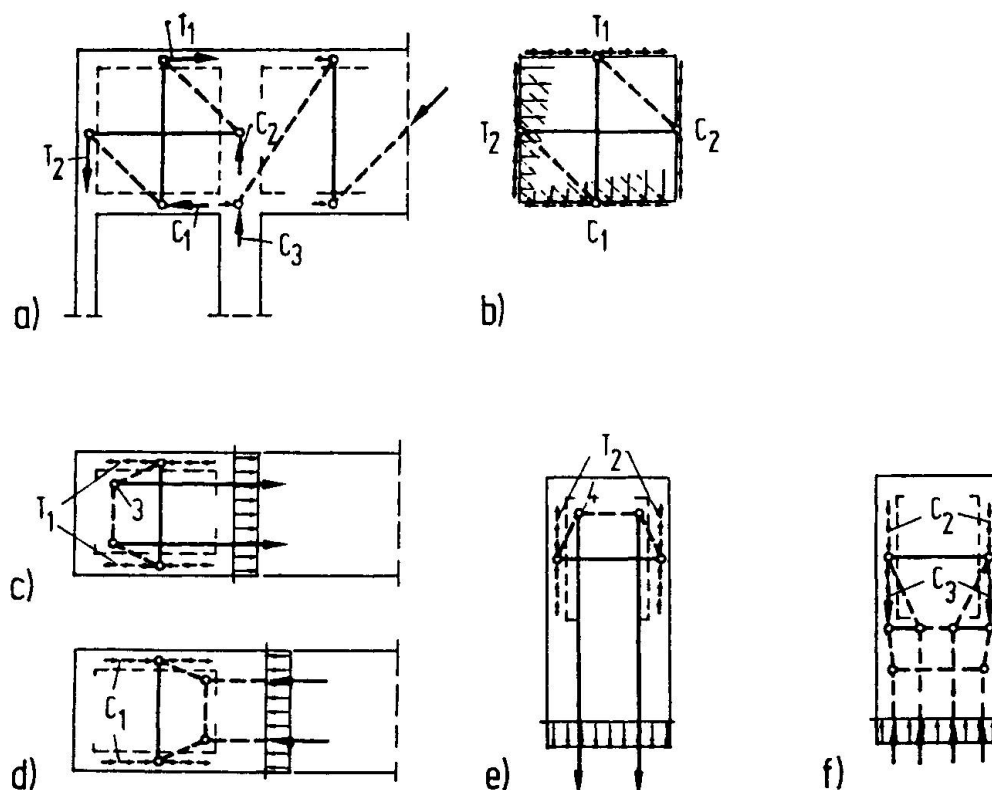
Fig. 4: Strut-and-tie models for the structure with internal diaphragm wall. a) Basic model for the web; b) refined model for the web with smeared forces. c) Top flange. d) Bottom flange. e) Tensile wall. f) Compression wall

3.4 Frame Corner with Diagonal Diaphragms

The best structural solution for the discussed problem is the diagonal diaphragm which follows the load path T_2 in Fig. 5a. This model avoids not only the singular nodes but also the transverse reinforcements in the flanges and walls. Only the spreading out of the support forces C_3, resulting from the shear forces of the webs, require some transverse reinforcement T_3 near the top of the compression wall (Fig. 5b).
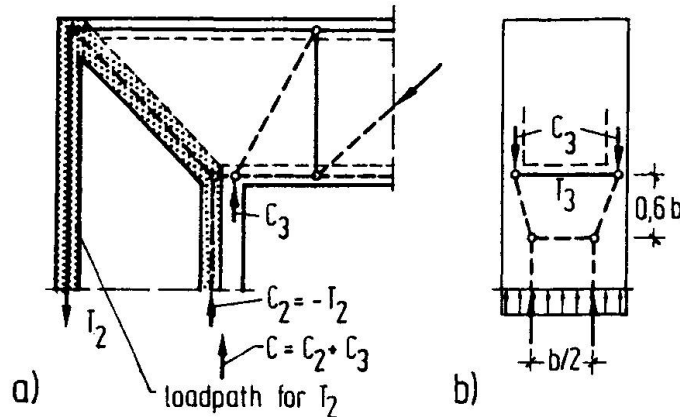

Fig. 5: a) Diagonal diaphragm following the load path for T_2 of the strut-and-tie model. b) Model for the compression wall.

We can conclude from this example that strut-and-tie models are not only suitable for dimensioning but are also very helpful for the conceptional design of good structural solutions.

3.5 The same Structure with Prestress

Let's prestress now the top plate and tension wall of the structure without inner diaphragm wall (see Fig. 3.2) with prestressing forces $P_1 = T_1$, $P_2 = T_2$ just large enough to balance the concrete tensile forces T_1, T_2 due to dead load (Fig. 6a). At first sight one could think that thereby also the problem of transverse forces in the frame corner is cancelled, at least in the prestressed members. But it isn't at all! The model with prestress applied as external forces [4, 5, 6] discloses that the load paths of the compression forces C_1, C_2 have to squeeze as before (see section 3.2) through node 2 into webs in order to arrive at their "supports" provided by the anchor forces P_1, P_2 of the tendons. In order to avoid further detours of the load paths (see Fig. 2b and d), the tendons in the corner should be arranged within the web, either similar to Fig.3 or Fig.6b, thus balancing the compression strut in the web directly.

If the load is increased after prestressing, e.g. due to live loads or a safety factor for ultimate conditions, the tendons react like non-prestressed reinforcement with additional tendon forces ΔT_1, ΔT_2. These are anchored by bond according to the model shown in Fig. 6c.

In the structure with inner diaphragm wall (acc. to Fig.4) the tendons may be distributed over the whole width b of the structure and anchored near the edge, if transverse forces are carried according to the models given in Fig.4c and e. However, the position of the model nodes 3, 4, which in Fig.4c and e represent the centroid of bond forces, have to be reconsidered for the prestressing tendons (see Fig. 6c). The prestress force P is always applied at the tendon anchor. Only that part of the tendon force ΔT which exceeds the initial prestress force P is anchored by bond, and these bond stresses may develop at a considerable distance from the anchor.

Separating the prestressing loads from the additional tendon forces after prestressing as suggested by Breen, Bruggeling and Jennewein [4, 5, 6] is reasonable also for the application of strut-and-tie models to prestressed D-regions and leads to a clear understanding of structural behaviour.

4. OUTLOOK

Finding an adequate strut-and-tie model is not always as simple as in the examples shown above. It implies to have learned and practised the method for some time, like any other engineering skill or method of analysis. To develop an individual model still takes considerable time. But Rückert's contribution shows that in the future the computer can assist the design engineer also in this work [8]. And with an increasing number of published examples it will be easier to find a model which only has to be adapted to one's specific problem.
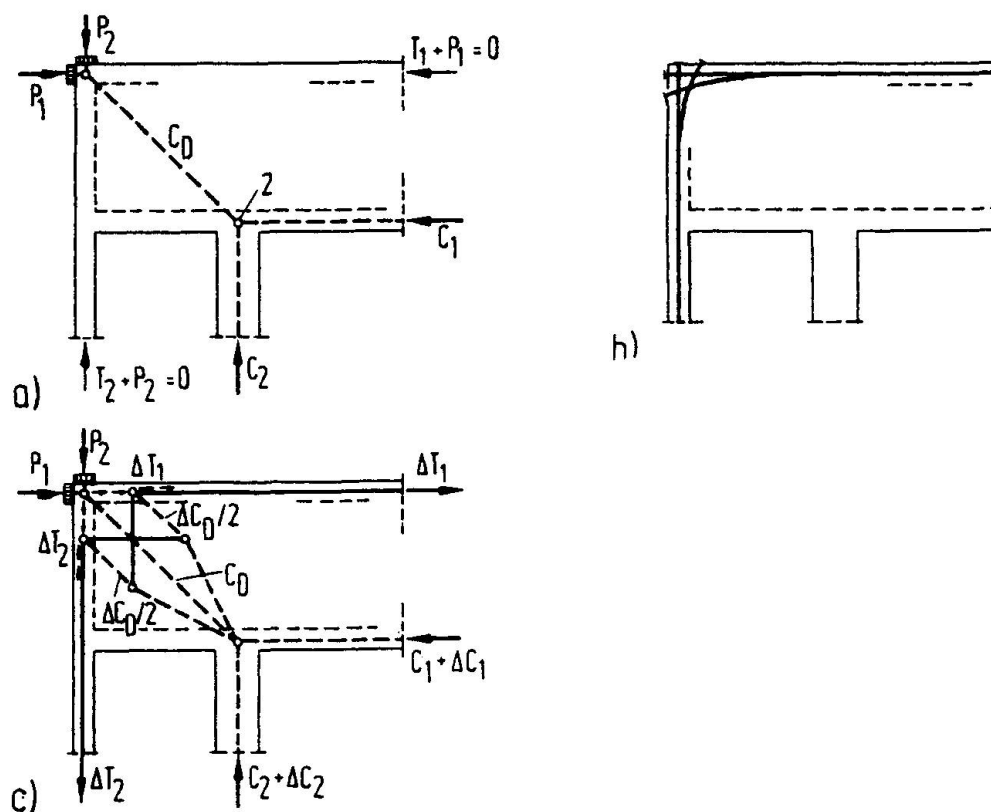
Fig. 6: Prestressed structure without internal diaphragm wall a) Top slab and tension wall prestressed under dead load to give zero concrete stresses. Model showing the load paths. b) Practical reinforcement layout c) Strut-and-tie model for increased loads, prestress as before.

At the moment standard models and procedures for dimensioning certain types of D-regions are being prepared by the authors. These include frame corners, beam-column connections, corbels and beams with openings.

In the future emphasis should be shifted from modelling techniques to a consistent design of node regions. Necessary anchorage lengths of reinforcement and permissible concrete stresses depend considerably on the type and geometry of nodes. Though the node problem is not specific for the strut-and-tie method but rather a problem of structural concrete, the authors' experience shows that strut-and-tie models help to understand and explain also the nodes' intrinsic behaviour.

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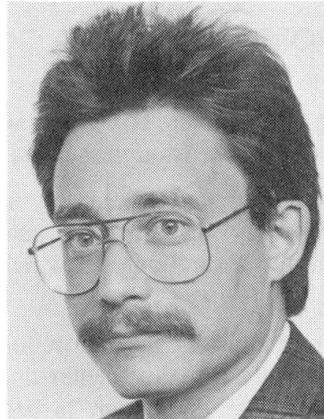
Design of the Support Regions of Concrete Box Girders

Dimensionnement des zones d'appui des poutres-caisson en béton

Bemessung der Auflagerbereiche von Beton-Hohlkästen

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SUMMARY

For dimensioning and detailing the support regions of concrete box girders, strut-and-tie-models are a valuable tool. In this paper it will be shown how a strut-and-tie-model for the transverse diaphragm can be developed and how in the model the interaction between the longitudinal and cross directions must be considered.

RÉSUMÉ

L'analogie du treillis faisant intervenir tirants et bielles constitue un outil très appréciable lors du dimensionnement des zones d'appui des poutres-caisson. Dans cet article, un exemple présente l'analyse d'un diaphragme à l'aide de cette analogie, ainsi que la nécessité d'une modélisation cohérente dans le cadre d'un calcul effectué dans les directions longitudinale et transversale.

ZUSAMMENFASSUNG

Für die Bemessung und konstruktive Durchbildung der Auflagerbereiche von Beton-Hohlkasten-trägern sind Stabwerksmodelle ein wertvolles Hilfsmittel. In diesem Aufsatz wird anhand eines Beispiels die Entwicklung eines Stabwerksmodells für das Querschott und die Notwendigkeit konsistenter Modelle für die Bemessung in der Längs- und Querrichtung gezeigt.

1. INTRODUCTION

Support regions of concrete box girders are important and highly stressed parts of the structure. For dimensioning and detailing these D-regions strut-and-tie-models are a valuable tool. They allow to follow up the forces consistently from the B-regions (webs, flanges) to the supports as strictly required by Schlaich et. al. in /1,3,4/. Examples of basic strut-and-tie-models for these regions are shown in /3,4/.

This paper shows how a strut-and-tie-model for the complicated diaphragm of a concrete box girder can be developed. The interaction between the load bearing behavior in the longitudinal and the cross direction is thereby considered.

2. DESIGN OF THE END-SUPPORT REGION OF A CONCRETE BOX GIRDER USING STRUT-AND-TIE-MODELS

Fig. 1 shows the end-support region of a concrete box girder subjected to shear and torsion. The bearings are spaced apart by means of "corbels" to avoid tension in the bearings under the given loading condition.

It is common use to treat "shear" and "torsion" separately and to superimpose the results later on. This, however, is unsatisfactory, as generally in structural concrete design the design-models cannot be superimposed. It is necessary to develop a single model which takes into account all the forces at the same time, as required by Breen /2/.

According to the basic assumption of a constant shear-flow for circulatory torsion, the in-plane forces due to shear and torsion are distributed uniformly along the center-lines of the individual webs V'_w and flanges V'_f. The following strut-and-tie-model for the diaphragm is based on the assumption that these forces are transfered evenly to the diaphragm. A corresponding model for the webs is shown in chapter 2.8.
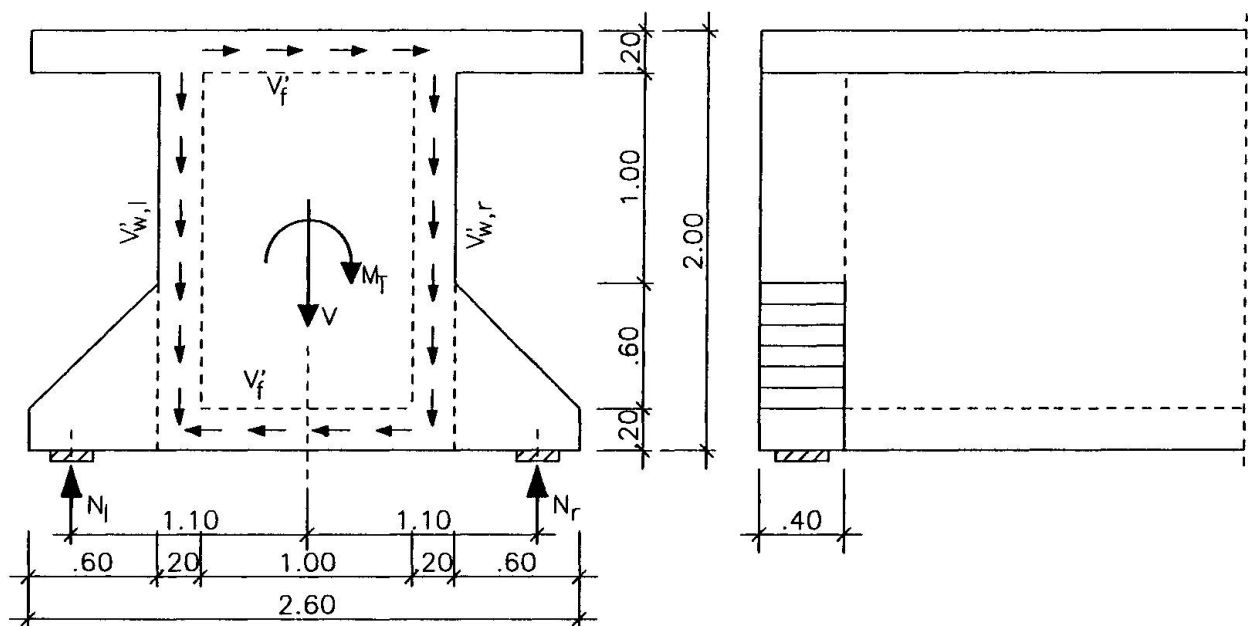
Forces at support:

M_T = 960 kNm

V = 1200 kN

Support reactions:

N_{left} = 1200/2 - 960/2.2 = 164 kN

N_{right} = 1200/2 + 960/2.2 = 1036 kN

Distributed forces in the webs and flanges due to shear and torsion:

V'_f = 960/(2·1.8·1.2) = 222 kN/m

$V'_{w,left}$ = 1200/(2·1.8) - 222 = 111 kN/m

$V'_{w,right}$ = 1200/(2·1.8) + 222 = 555 kN/m

Fig. 1: End-support region of the box girder
Support reactions and forces in the webs and flanges due to shear and torsion

2.1 Modelling technique

To develop the strut-and-tie-model of the whole D-region it is sometimes helpful to compose this model of already wellknown and established "sub-models". Thus parts of the structure can be treated separately. However, at the intersections of the different, parts the sub-models have to correspond with each other and must compose to the overall consistent model. Equilibrium must be satisfied for each of the sub-models as well as for the resulting overall model. In the following a strut-and-tie-model for the diaphragm will be developed in such a way.

Those parts of the structure in which the internal forces are reasonably well known are :
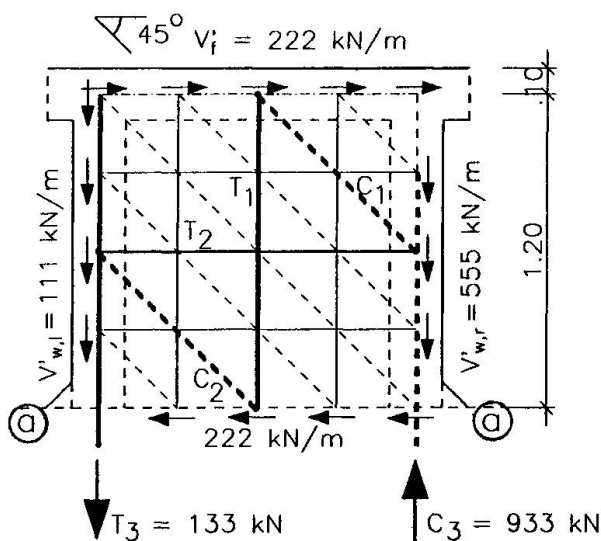- the upper part of the diaphragm
- the corbels.

In the lower part of the diaphragm the state of stress is primarily unknown due to the geometric discontinuity.

2.2 The upper part of the diaphragm

The upper part of the diaphragm is shown in fig. 2 . Equilibrium requires that the horizontal shear force along line a-a has to balance the total shear force in the upper flange. The shear force along line a-a is also assumed evenly distributed.

The state of stress in this part of the diaphragm and the appropriate model is that of an ordinary rectangular diaphragm loaded by shear forces from the webs and top flange and directly supported under the webs /4/. The forces from the flanges V'_f have to be diverted by inclined struts C'_1 and vertical ties T'_1. For simplicity the inclination of the strut is assumed to be 45°. To balance the horizontal components of the strut forces additional horizontal ties T'_2 are necessary. The vertical components of the strut forces and the forces of the webs $V'_{w,l}$, $V'_{w,r}$ sum up to give T_3 and C_3.

Note, that the struts and ties crossing the diaphragm represent stress-fields, therefore the reinforcement covering the tie forces has to be distributed accordingly.

Tie forces per unit length in the diaphragm
(for a 45° inclination of the compression field):
$T'_1 = T'_2 = 222$ kN/m

Forces in the webs:
$C_3 = 555 \cdot 1.2 + 222 \cdot 1.2 = 933$ kN
$T_3 = -111 \cdot 1.2 + 222 \cdot 1.2 = 133$ kN

Fig. 2: Upper part of the diaphragm with forces and model

2.3 The corbels

Fig. 3 shows the corbels loaded by the bearing forces and by the vertical forces from the webs necessary to obtain equilibrium in the vertical direction. To ensure overall equilibrium the models require inclined struts C_6 resp. C_7, horizontal ties T_4 resp. T_5 and horizontal compression forces C_4 resp. C_5.

The concentrated node at the bearing plate is shown at the right corbel. The dimensions of this node are determined by the width of the bearing plate and by the reinforcement layout (node K6 acc. /4/). This critical concentrated node should already be checked at this early design state with respect to bearing pressure and anchorage of the reinforcement according to /4/.
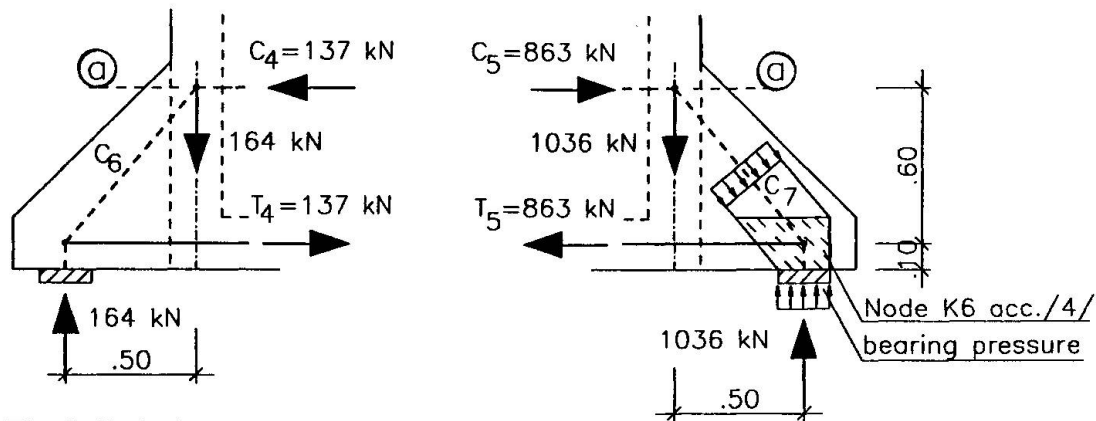
Fig. 3: Corbels with forces and models

2.4 The lower part of the diaphragm

The lower part of the diaphragm (fig. 4a) is loaded by the shear forces from the webs and bottom flange, by the forces from the upper part of the diaphragm and by the forces from the corbels.

A horizontal strut at the top C_8 and a horizontal tie at the bottom T_8 balance parts of the horizontal forces and further introduce evenly distributed forces along the horizontal edges of the lower part of the diaphragm. The concentrated vertical tension forces in the axis of the webs T_9, T_{10} are anchored in the diaphragm by reinforcement, and thus introduce distributed forces along the vertical edges of the diaphragm (fig. 4b).

What remains from all the forces (acc. to figs. 4a and 4b) is a "shear-wall" loaded along its edges (fig. 4c). The model for this shear-wall is of the same type as for the upper part of the diaphragm. For the inclination of the struts again 45° was assumed. Note, that the tie forces per unit length in the lower part of the diaphragm T'_{11}, T'_{12} are much higher than in the upper part $(T'_{11} = 1.73\ T'_1)$!
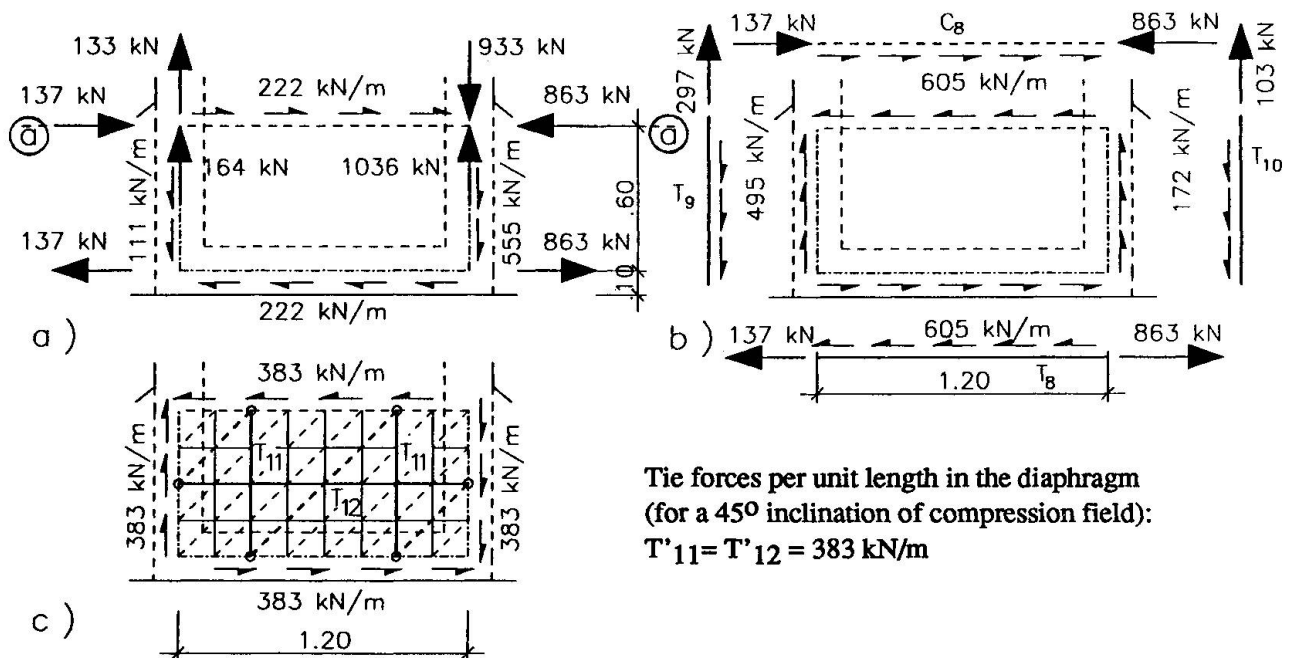
Tie forces per unit length in the diaphragm
(for a 45° inclination of compression field):
$T'_{11} = T'_{12} = 383$ kN/m

Fig. 4: Lower part of the diaphragm
  a) with forces acting on it
  b) additional shear forces onto the diaphragm
  c) remaining shear-wall with appropriate model

2.5 Complete model

Fig. 5 presents the overall model of the end-section resulting from the previous sub-models. The inclined struts in the corbels and the horizontal strut in the diaphragm along line a-a also require a certain width as indicated for the strut C_7 in the corbel (fig. 3).

2.6 Principal reinforcement layout

Fig. 6 shows the principal reinforcement layout for the end-section. Note again the additional horizontal and vertical reinforcement necessary in the lower part.
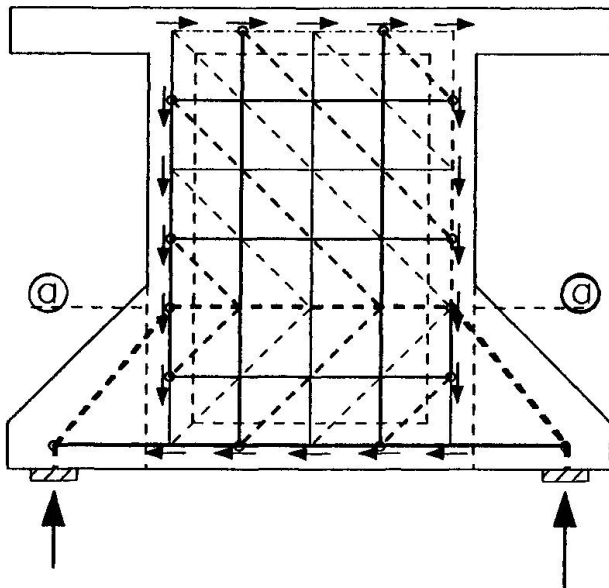

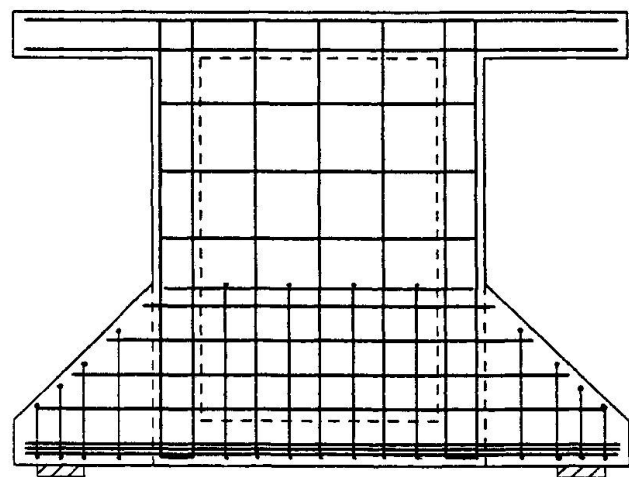**Fig. 5:** Complete model

Fig. 6: Principal reinforcement layout

2.7 Check of concrete compression stresses

The highly stressed and rather concentrated struts C_6 resp. C_7 in the corbels have to be checked at the nodes at the bearing plates. This can be done according to regulations given by Schlaich et. al. in /4/ or by Sundermann in /6/. Within the remaining parts of the diaphragm only stress-fields occur in which the concrete stresses are not critical.

2.8 The strut-and-tie-model for the webs

An appropriate model for the webs for the above assumed evenly distributed vertical forces at the connection to the diaphragm V'_w is shown in fig. 7. This model has been developed on the basis of stress-fields and is explained in detail by Reineck et. al. in /5/. Here only the results are presented.

According to this model distributed longitudinal reinforcement T'_1 is necessary over the full height of the web. Furthermore increased vertical reinforcement T'_w is required in the D-region. Locally the concrete compression stresses in the inclined stress-fields are twice as high as in the B-region.

The flanges are loaded by the in-plane shear forces V'_f due to torsion as well as by the longitudinal forces V_w cotϑ resp. V'_w cotϑ from the webs. Developing the complete model for the flanges would exceed the limits of this paper. Separate models for either the shear force or the longitudinal forces are shown in /4/.
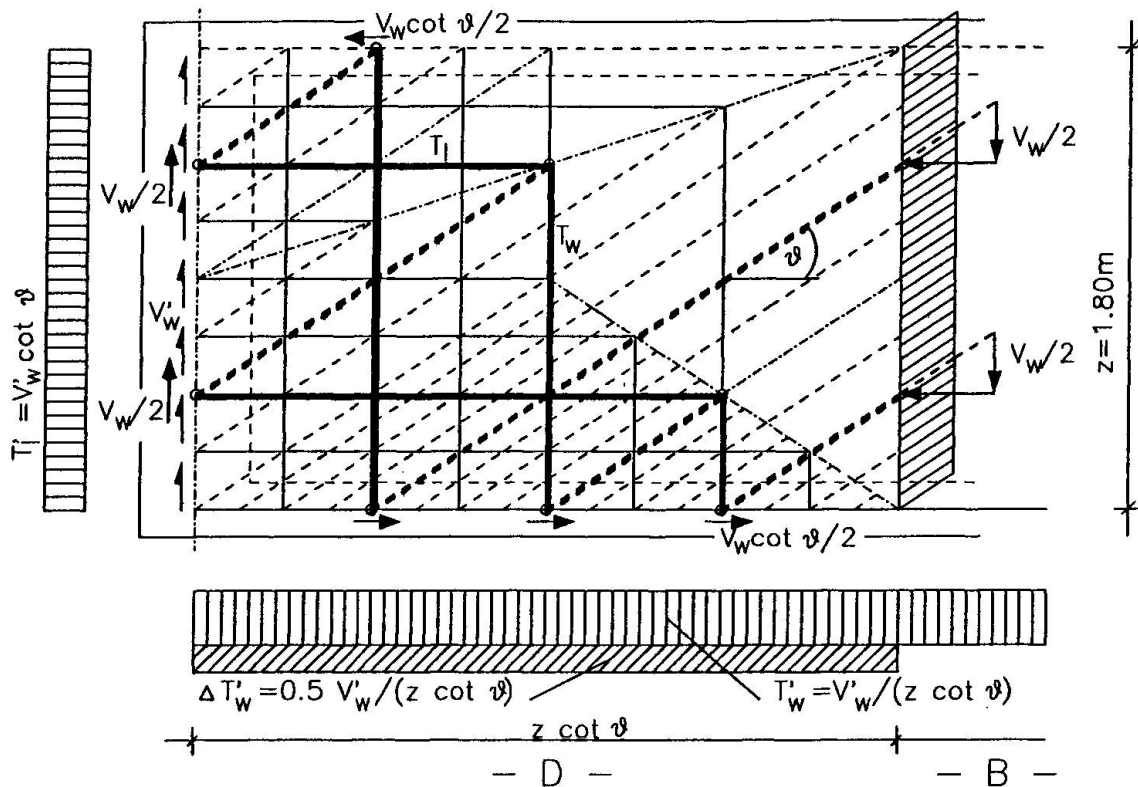
Fig. 7: Strut-and-tie-model for the webs acc. to /5/

3. CONCLUSIONS

This example demonstrates how a consistent model for the complicated diaphragm of a concrete box girder can be developed. For proper modelling and detailing of the D-region it is necessary to consider distributed forces by using stress-fields, in order to distribute the reinforcement accordingly. The complete model for the diaphragm can be composed of established and wellknown "sub-models". The models for the webs and flanges must be consistent with the model for the diaphragm. With this modelling-technique a safe design and a proper detailing of this complicated D-region can be guaranteed.

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Practical Experience with Modelling of Structural Concrete Members

Expérience pratique avec des modèles de barres pour des constructions en béton

Praktische Erfahrung bei der Anwendung von Stabmodellen im Betonbau

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SUMMARY

This report deals with examples of three actual projects where strut models helped to clarify the global flow of forces, to simplify the calculation and to develop structural details.

RÉSUMÉ

Trois projets vus à la lumière de différents modèles de barres permettent d'aider à comprendre le cheminement des efforts, de simplifier les calculs et de développer la conception des détails constructifs.

ZUSAMMENFASSUNG

Dieser Bericht zeigt am Beispiel von drei ausgeführten Projekten, wie bei der praktischen Arbeit das Modellieren von Stabwerken zum Verständnis des Gesamttragverhaltens, zur Vereinfachung der Bemessung und bei der Detailausbildung helfen kann.

1. INTRODUCTION

In the CEB-Bulletin d'Information 150: Detailing of concrete structures [1] a method was proposed how to perform methodically the practical design and detailing of concrete structures. Its main idea is expressed there: a deep understanding of the flow of forces is needed to design good structures, that means materially sound as well as beautiful.

This idea is still valid and over the last 10 years has caused further activities [2] ./. [5] to transfer this way of thinking into practical work.

Did it succeed? Is it accepted and does it help? Three examples will demonstrate that strut models can indeed help to understand the global flow of forces, to replace sometimes extensive calculation and to develop structural details.

2. MODELLING OF THE GLOBAL FLOW OF FORCES

Project: University of Kassel, Technik III [6]

The new building "Technik III" of the University of Kassel is divided in two: The southern part consists of halls for engineering tests and the northern part serves as an institute- and laboratory building (Fig. 1). It is about 170 m long and 6 storeys high. Its northern side exists of columns, which are inclined between level 0 and + 2. At the kinks high forces occur due to change of direction, and the effects of these forces have to be followed carefully (Fig. 2).

According to Fig. 3 the vertical force V, which results from the column load at level + 2 creates an equally distributed groundpressure σ_b below level - 1.

To transfer the load from the top to the bottom the following members are available (for one half of the building): The slab I at level + 2 collects the horizontal forces due to change of direction at the kink of the columns and transfers them concentrated into the vertical wall II. There they are taken to slab III, creating equilibrium with the horizontal forces at the kink of the columns at level $\pm$ 0. The vertical forces at that point are transferred continuously into the wall IV and form there into the bottomplate VI. In the walls V the horizontal forces are transferred into vertical ones, creating a moment at the edges of the bottomplate VI. This moment together with the vertical forces from the columns causes an equally distributed groundpressure σ_b, provided that the bending stiffness of the plate VI is big enough.

All the members, which are necessary to carry the loads, are shown in Fig. 4. They each can now be designed and detailed separately according to their geometry and loadings. Joined together the single members form a 3-dimensional strut model, which shows the global flow of forces of the whole structure in a transparent and demonstrative way (Fig. 5).

3. STRUT MODELS AS A REPLACEMENT FOR EXTENSIVE CALCULATIONS

Project: Extension of the Casino building of the Bayerische Rückversicherung [7]

The Casino building of the Bayerische Rückversicherung in Munich was extended in 1989 by three additional storeys. It is a cylindrical suspension house with slabs, which are suspended by 6 steel bars equally distributed along their edges. The suspension elements hang straight down from the edge of the ceiling of the last storey, above which they are inclined to the top of the concrete core (Fig. 6). During the erection stage this part existed only of radial girders between a concrete ring, which was formed polygonally at its interior edge.

The critical loading case for this member was an eccentric load, which created two horizontal forces of about 1100 kN each at the edge of the concrete ring. A strut model according to Fig. 8 was taken as a basis and yielded a reinforcement 13 ⌀ 20, which was to be arranged polygonally from one support to the next one.

Unfortunately this result was just accepted as a predesign, but for the official calculations some more efforts were expected!

Therefore a computer calculation for the ring with radial girders and the geometric according to Fig. 7 was carried out. The resulting diagrams for the bending moments, normal- and shear forces in fact were quite impressive (Fig. 9) and stimulated to do a proper design calculation for support-, span- and especially shear reinforcement.

But the ambition was even stronger to show that strut models are competitive. Therefore the direction and the value of the resultant forces out of N and Q were determined and their location calculated from M and N. The result is shown in Fig. 10 and is now satisfying the strut model designer too.

4. DETAILING

Project: Ice Skating Hall, Munich

In the Munich Olympia area a further hall was built for the Skating-World-Championship in 1991 (Fig. 11). The hall is standing on concrete columns above a parking place with a raster of 10.80 m x 5.40 m. The slab is made out of prestressed concrete, with cantilevers on both ends of 6.75 m length, which are formed as T-beams (Fig. 12). Its webs are 0.60 m wide and 0.40 m high with a slab on top, which is 0.20 m thick. The distance of the webs amounts to 5.40 m. The cantilever is inclined to the horizontal level by 21° with the kink in a distance to the last column row of 1.35 m. There the value of the bending moment amounts to M = - 1800 kNm per T-Beam. With an internal lever arm z = 0,45 m the horizontal forces become 4000 kN each. Due to the change of direction at the kink, vertical forces occur, which are named $U_{(T)}$ and $U_{(C)}$ and are 1440 kN big. They are to be connected by stirrup reinforcement (1) according to Fig. 13.

The effective width of the tension area in the slab was determined to 1.40 m and therefore a part of the bending reinforcement ($\approx 1/3$) is arranged there. The reinforcement is shown in Fig. 14 and the expert, who already possesses long-term experience with reinforcing structures, which are - at least at the first glance - similar, e.g.: framecorners and staircases, will normally judge the reinforcement as correct and complete straight away.

But the careful study of the flow of forces in cross direction shows that important reinforcement is missing: the forces due to change of direction $U_{(T2)}$ in the side parts of the slab, are not taken over by any reinforcement (Fig. 15).

With the stirrup reinforcement (3) - (Fig. 16) - in the slab, which amounts to 65 cm^2/m^2 in this example and with the bending reinforcement (4) in cross direction as bottom reinforcement 13 cm^2/m the design is complete and equilibrium is now installed between the forces due to change of direction $U_{(T)}$ and $U_{(C)}$.

5. CONCLUSION

I am convinced that the method of strut modelling is able to produce the necessary knowledge to design good and harmonious structures. Unfortunately the method is constantly being underestimated: even the inexperienced designer expects quick results with a solution, which he can represent. That does not fit together! Confidence in the solution develops only after studying the problem intensively. This method needs its time for application too!

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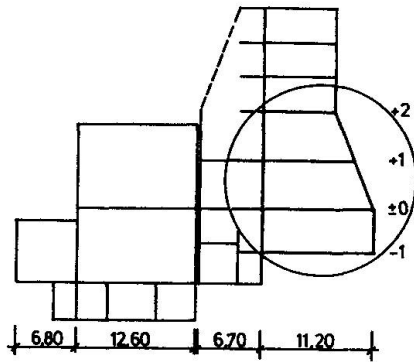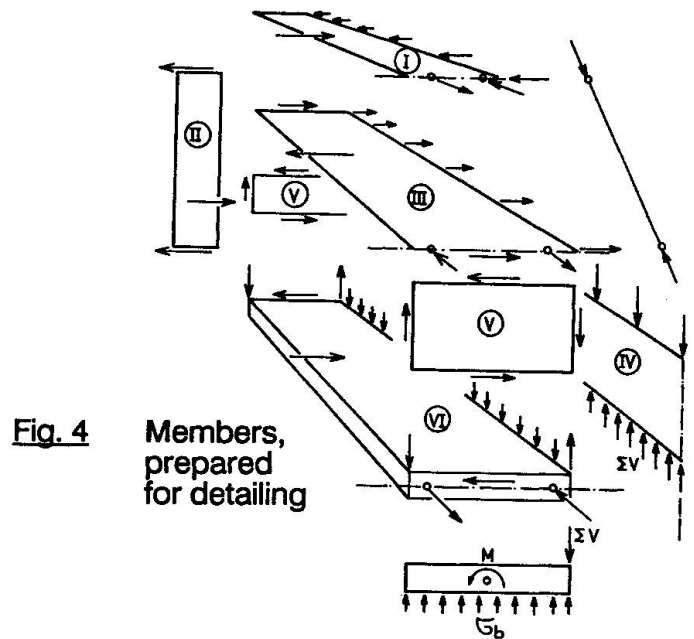
Fig. 1   Project: University of Kassel
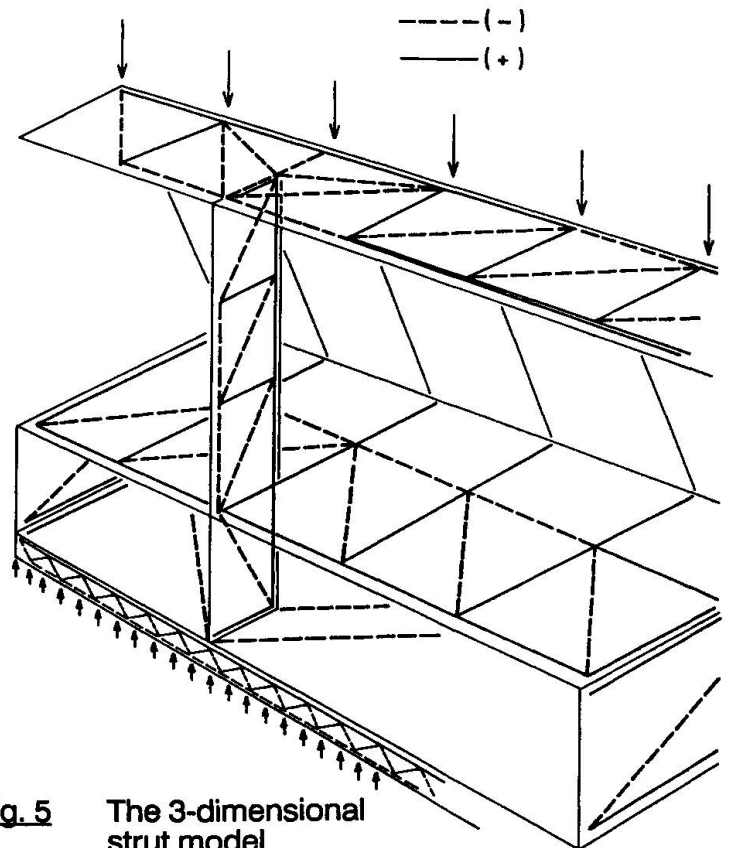Technik III
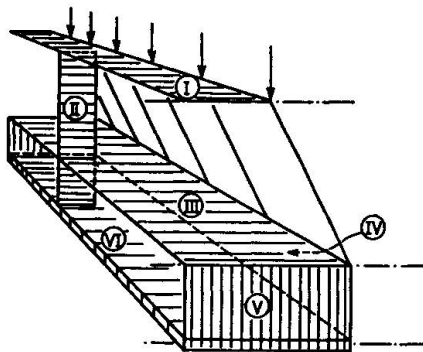
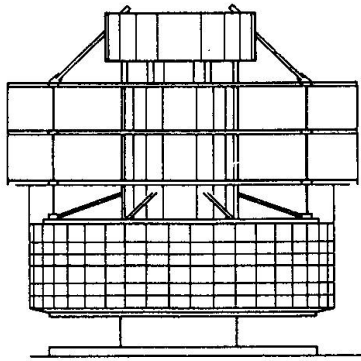**Fig. 2**   Part of the laboratory building
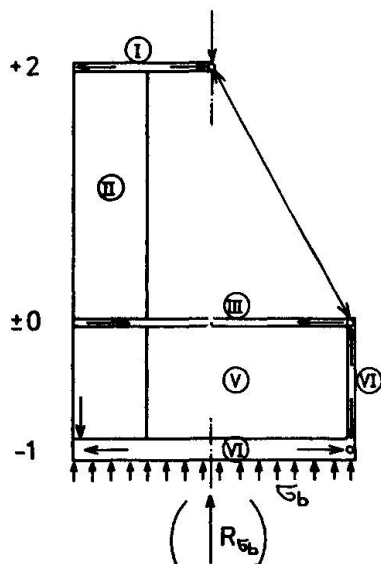
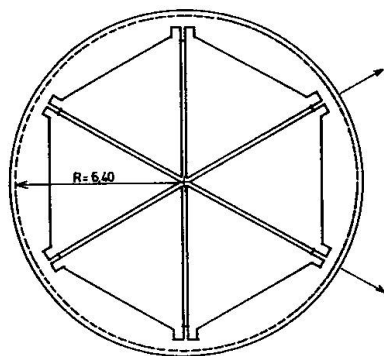**Fig. 3**   Section through the
laboratory building

Fig. 4   Members,
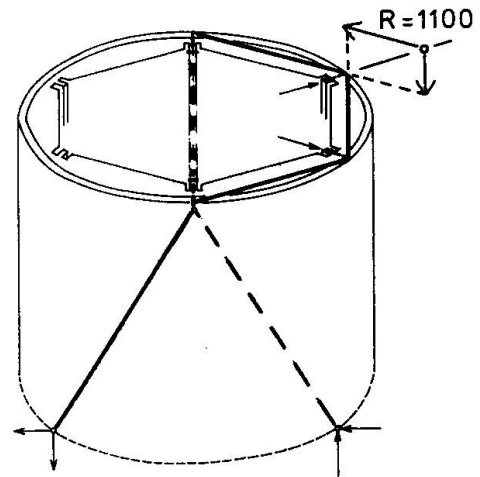prepared
for detailing

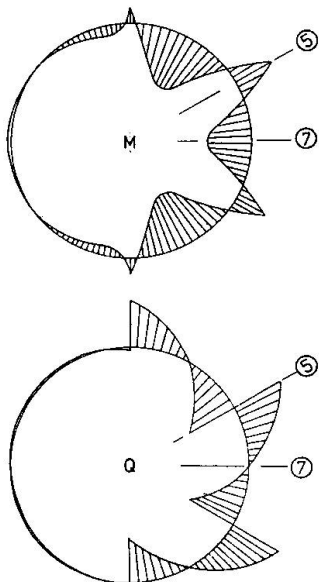**Fig. 5**   The 3-dimensional
strut model

Fig. 6   Project: suspension building
of the Bayerische Rückversicherung

Fig. 7   Top slab at erection state
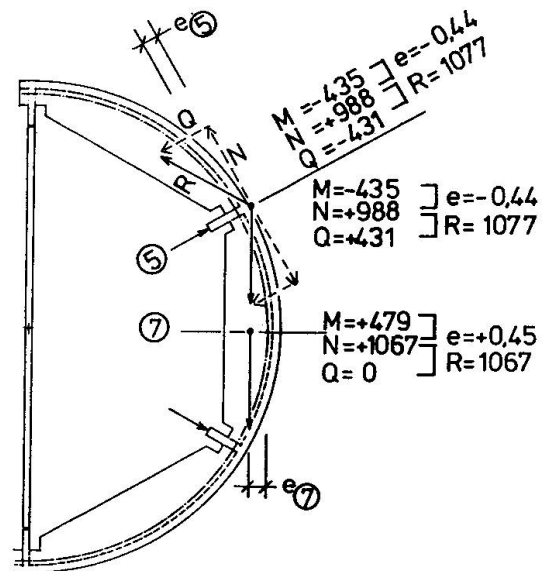with eccentric loading
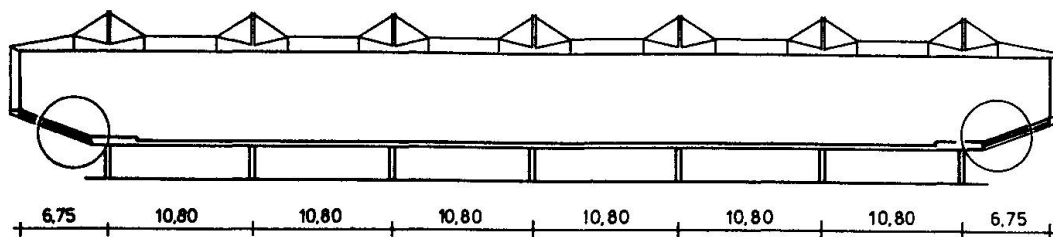

Fig. 8   Strut model for
loading case acc. to Fig. 7

M
N $\Big]$ e

N
Q $\Big]$ R

Fig. 9   Action effects acc. to Computer
calculations
- Bending moments M
- Shear forces Q

R =1100

M =-435   e=-0,44
N =+988   R= 1077
Q =-431

M=-435   e=-0,44
N=+988   R= 1077
Q=+431

M=+479   e=+0,45
N=+1067 R=1067
Q= 0

Fig. 10   The resultant forces and.
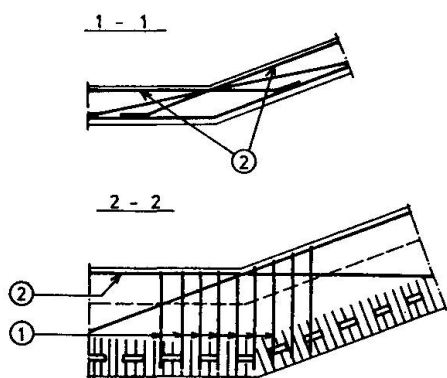eccentricities.
Compare with Fig. 8!

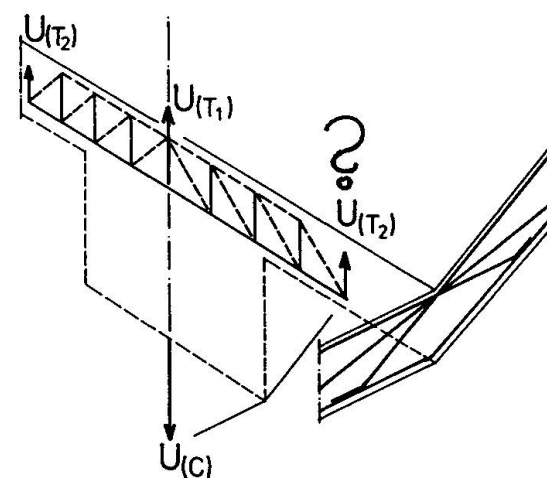**Fig. 11**   Project: Skating hall in Munich
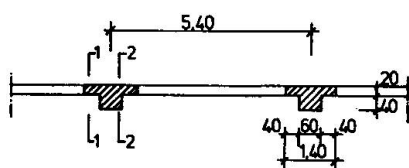
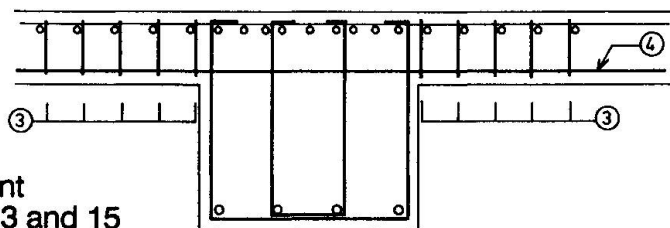**Fig. 12**   T-beams at the cantilevers
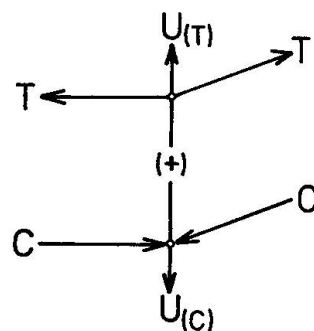
Fig. 13   Forces U due to change of direction

Fig. 14   Reinforcement acc. to Fig. 13

Fig. 15   A 3-dimensional effect!

Fig. 16   Reinforcement acc. to Fig. 13 and 15