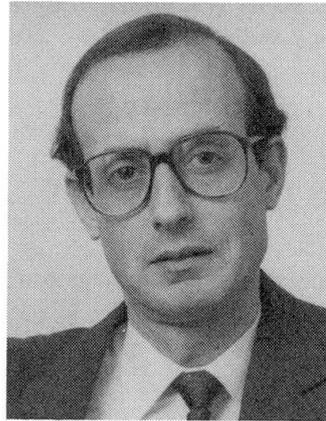# Recommendations on Reinforcement in Flexural and Compression Members

Recommendations concernant l'armature des structures en béton armé

Empfehlungen für die Bewehrung von Betonbauteilen

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SUMMARY

A set of recommendations related to the reinforcement in structural concrete flexural and compression members are presented. They address minimum and maximum levels of reinforcement, the percent of moment redistribution in continuous members, and the ultimate stress in the prestressing steel for bonded or unbonded tendons. The recommendations are tuned to lead to numerical results in accordance with the ACI Building Code; however, they are non-dimensional and can be applied to any code.

RÉSUMÉ

Sont présentées ici une série de recommendations concernant des structures en béton armé fléchies et comprimées. Elles concernent la qualité minimale et maximale d'armature, le pourcentage de la distribution des moments à considérer dans une structure continue, ainsi que la contrainte ultime à prendre en compte dans l'acier des câbles d'une précontrainte adhérente ou non. Les recommendations sont orientées dans le but d'obtenir des résultats numériques en accord avec le code de construction ACI; cependant, comme elles sont en fait adimensionnelles, elles peuvent s'appliquer à n'importe quel type de norme ou de code.

ZUSAMMENFASSUNG

Es werden einige Empfehlungen für die Bewehrung von Betonbauteilen unter Biege- und Normalkraftbeanspruchung dargestellt. Sie betreffen Minimal- und Maximalbewehrungsgrade, den Prozentsatz der Lastumlagerung von durchlaufenden Tragwerken und die Maximalspannung in den Spanngliedern bei Vorspannung mit oder ohne Verbund. Die Empfehlungen sollen numerische Ergebnisse in Übereinstimmung mit den ACI-Bauvorschriften geben, aber sie sind dimensionsfrei und können für jede Norm benutzt werden.

1. SCOPE

Unifying code recommendations to accommodate Structural Concrete (i.e. reinforced, prestressed, and partially prestressed concrete) in a simple and rational manner that does not violate the fundamental principles on which the provisions are based, should be an essential goal of future editions of any code of practice.

The recommendations proposed in this paper are related to the reinforcement of structural concrete members reinforced with conventional reinforcing bars, prestressing tendons, or any combination of them.   The numerical values derived from these recommendations are tuned to reflect, as a reference base, the current provisions of the American Concrete Institute's Building Code Requirements (ACI 318 - 1989). However, they are written in a non-dimensionalized form and could be easily adapted to any code of practice. Some related background information can be found in   [1-9].

2. FLEXURAL MEMBERS

2.1   Definition

The depth d_e from the extreme compression fiber to the centroid of the tensile force in the tensile reinforcement at nominal resistance of the section is given by the following expression (Fig. 1):

$$d_e = \frac{A_{ps}f_{ps}d_p + A_s f_y d_s}{A_{ps}f_{ps} + A_s f_y}\qquad (1)$$

where:

A_{ps}	=	Area of prestressing reinforcement in the tensile zone
f_{ps}	=	stress in the prestressing steel at nominal flexural resistance of the section (see Sections 2.5 and 5).
d_{ps}	=	distance from extreme compression fiber to centroid of prestressing steel
A_s	=	area of non-prestressed tension reinforcement
f_y	=	specified yield strength of non-prestressed tensile reinforcement
d_s	=	distance from extreme compression fiber to centroid of nonprestressed tensile reinforcement

The definition of d_e could also be easily extended to multi-layered systems, such as columns, having different layers of prestressing reinforcement and/or conventional reinforcing bars.

Note that while it is generally assumed that the reinforcing steel yields at ultimate behavior of the member, the stress, f_{ps}, in the prestressing steel is unknown and must be estimated separately (see Sections 2.5 and 5).

2.2   Maximum Reinforcement

The amount of prestressed and non-prestressed reinforcement, used for computation of moment strength of a member, shall be such that:

$$c/d_e \ \le \ 0.42.\qquad (2)$$

where c is the depth to the neutral axis at nominal resistance in bending, and d_e is as defined in Eq. 1.

The above provision requires the determination of c, which could be obtained from writing the two equations of equilibrium of the critical section at nominal bending resistance.
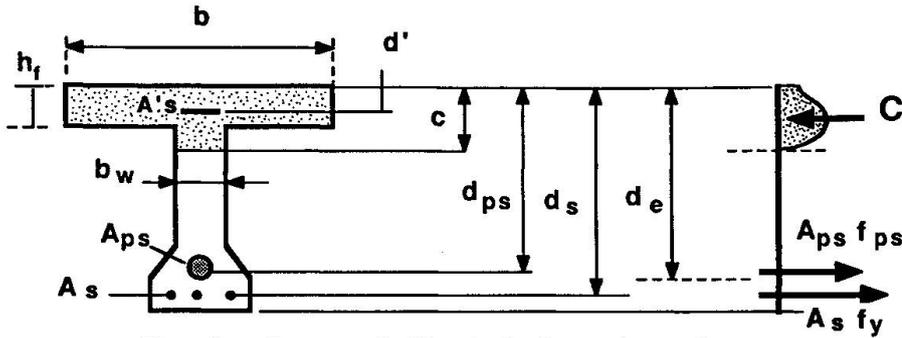
Fig. 1 Forces at ultimate in flexural members

2.3 Minimum Reinforcement

At any section of a flexural member, except where positive reinforcement is required by analysis, the amount of reinforcement shall be adequate to develop a design factored load, $\emptyset P_n$, at least 1.2 times the cracking load, P_{cr}, computed on the basis of the modulus of rupture f_r of the concrete material. Thus:

$$\emptyset \; P_n \; \geq \; 1.2 \; P_{cr} \tag{3}$$

For concrete members reinforced with conventional reinforcing bars only, this provision can be satisfied by providing a minimum reinforcement ratio given by:

$$\rho_{min} \; \geq \; 0.03 \; f'_c/f_y \tag{4}$$

where:

$$\rho \; = \; A_s/bd_s \tag{5}$$

in which $f'c$ is the compressive strength of concrete obtained from cylinder tests and other terms are as defined earlier. Note that b is taken equal b_w (Fig. 1) for T sections and joists where the web is in tension.

2.4 Moment Redistribution

Where bonded reinforcement is provided at supports in accordance with Section 18.9 of the ACI Code, negative moments calculated by elastic theory for any assumed loading arrangement may be increased or decreased by not more than

$$20(1 \; - \; 2.36 \; c/d_e) \qquad \text{in percent} \tag{6}$$

provided the value of c/d_e obtained from the design of the section at ultimate is such that:

$$c/d_e \; \leq \; 0.28 \tag{7}$$

2.5 Stress in Prestressing Steel at Ultimate

In lieu of a more accurate determination of f_{ps} based on strain compatibility, the following approximate values of f_{ps} shall be used if f_{pe} is not less than 0.5 f_{pu}

(a) Members with bonded tendons:

$$f_{ps} = f_{pu}(1 - k \frac{c}{d_p}) \tag{8}$$

where k is given by:

$$k = 2(1.04 - \frac{f_{py}}{f_{pu}}) \qquad (9)$$

If any compression reinforcement is taken into account when calculating f_{ps}, the value of c should be larger than or equal to 3d' to insure yielding of the compressive reinforcement. d' is defined as the depth from the extreme compression fiber to the centroid of the compressive reinforcement. If c is lesser than 3d', the contribution of the compressive reinforcement may be neglected. The basis for Eqs. 8 and 9 can be found in [1,6,7].

(b)   Members with Unbonded Tendons

$$f_{ps} = f_{pe} + \Omega_u \, E_{ps} \, \varepsilon_{cu} \, (d_{ps}/c - 1) \, L_1/L_2 \leq 0.94 \, f_{py} \qquad (10)$$

where:

E_{ps} = elastic modulus of prestressing steel
ε_{cu} = assumed failure strain of concrete as per code used (i.e. 0.003 for ACI Code)
L = span length
L_1 = length of loaded span or spans affected by the same tendon
L_2 = length of tendon between anchorages
Ω_u = 3 / (L/d_{ps})   for uniform or third point loading
Ω_u = 1.5 / (L/d_{ps})   for one point midspan loading

In order to solve for the value of fps in Eqs (9,10), the equation of force equilibrium at ultimate is needed. Thus two equations with two unknowns (f_{ps} and c) need to be solved simultaneously to achieve a numerical solution. The background and basis for Eq. 10 can be found in [8,9].

3.   COMPRESSION MEMBERS

3.1 Maximum Reinforcement in Compression Members

The areas of prestressed and nonprestressed longitudinal reinforcement for non-composite compression members shall satisfy the following two conditions simultaneously (Fig. 2):

$$\frac{A_s}{A_g} + \frac{A_{ps}}{A_g} \times \frac{f_{pu}}{f_y} \leq 0.08 \qquad (11)$$

and:

$$\frac{A_{ps}f_{pe}}{A_g \, f'_c} \leq 0.3 \qquad (12)$$

Equation 11 limits the percentage of total reinforcement in the section, while Equation 12 limits the allowable uniform compressive stress in the concrete due to prestressing, if any.
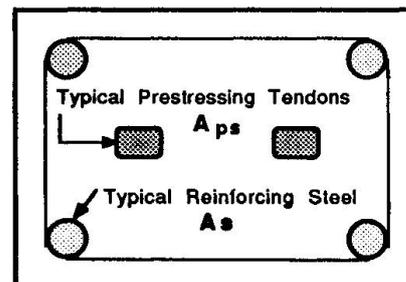

Typical Prestressing Tendons
A_{ps}

Typical Reinforcing Steel
A_s

Fig. 2 Typical section of compression members

3.2 Minimum Reinforcement for Compression Members

The areas of prestressed and nonprestressed longitudinal reinforcement for non-composite compression members shall satisfy the following condition (Fig. 2):

$$\frac{A_s f_y}{A_g f'_c} + \frac{A_{ps} f_{pu}}{A_g f'_c} \geq 0.12 \qquad\qquad (13)$$

where A_g is the gross area of the compression member, f_{pu} is the ultimate strength of the prestressing tendons and other terms are as defined above.

4. PRESTRESS LOSSES - STRUCTURAL CONCRETE?

This is a subject where the general term "structural concrete" may have to be broken down into three groups, namely, reinforced, prestressed and partially prestressed concrete. Prestress losses affects only the last two groups. The accurate determination of prestress losses in prestressed and partially prestressed concrete should be based on a time step analysis. However, lump sum estimates can be used for partially prestressed as well as for fully prestressed concrete. The following remarks may be in order for partially prestressed concrete:

1.   The average stress in the concrete in a partially prestressed member is generally smaller than that in a fully prestressed member. Thus the loss of prestress due to creep is also expected to be smaller.

2.   If the prestressing steel is tensioned to the same initial tensile stress as in the case of fully prestressed concrete, the intrinsic relaxation loss would be the same. However, since prestress loss due to creep is smaller in a partially prestressed member, and since loss due to creep influences that due to relaxation, the relaxation loss in partially prestressed concrete members is expected to be slightly higher than in fully prestressed concrete members.

3.   Everything else being equal, the loss of prestress due to shrinkage of the concrete should be the same for prestressed and partially prestressed concrete members.

4.   Other instantaneous prestress losses such as friction, anchorage set, and elastic shortening can be computed in the same manner as in prestressed members.

5.   The presence of a substantial amount of non-prestressed reinforcement (conventional reinforcing bars) such as in partially prestressed concrete, influences stress redistribution along the section due to creep of concrete with time, and generally leads to smaller prestress losses.

6.   It is advisable to estimate creep loss on the basis of the ratio of average stress in the concrete to its compressive strength.

5. STRESS IN PRESTRESSING STEEL AT ULTIMATE - SIMPLIFIED APPROACH

In the above Section 2.5, the latest developments known to the author regarding prediction of the stress at ultimate in prestressed flexural members have been described in Eqs. 8 to 10. Such equations, combined with the equations of equilibrium at ultimate, allow for the computation of nominal bending resistance. This is as close in accuracy to a strain compatibility analysis as can be achieved to date. In an analysis or investigation situation, the combination of Eqs. 1, 8 or 10, with the two equations of force and moment equilibrium at ultimate, leads to solving four equations with four unknowns. In a design situation where, for instance, the non-prestressed steel is to be determined, an additional unknown is present. The solution becomes unnecessarily messy (involved) and its accuracy may not be needed in many design cases.

Thus, it is tempting to suggest very simplified and safe recommendations to estimate the stress at ultimate in prestressed and partially prestressed concrete. The following approach is proposed:

(a)   Members with bonded tendons:

$$f_{ps} = f_{py} \tag{14}$$

This equation is always on the safe side since the limit on maximum reinforcement (Eq. 2) does not allow for the design of overreinforced members; thus actual f_{ps} will always be larger than f_{py}.

(b)   Members with unbonded tendons:

$$f_{ps} = f_{pe} + 70 \quad\quad MPa \tag{15}$$

This is generally on the safe side as observed for most of the 143 beams analyzed in [10].

Thus Eqs. 14 and 15 may be used in a first step analysis and, only if additional accuracy is needed to satisfy the design, one may revert to Eqs. 8 and 10, or to a non-linear analysis procedure.

ACKNOWLEDGEMENTS

REFERENCES

1.   NAAMAN, A.E., "Partially Prestressed Concrete: Review and Recommendations," PCI Journal, Vol. 30, No. 6, Nov.-Dec. 1985, pp. 30-71.

2.   NAAMAN, A.E., proposal to ACI Committee 423, Prestressed Concrete, on "Proposed Revisions to ACI Building Code and Commentary (ACI 318-83)," first draft March 1987.

3.   NAAMAN, A.E., "Partially Prestressed Concrete: Design Methods and Proposed Code Recommendations," Proceedings, International Conference on Partially Prestressed Concrete Structures, T. Javor, Editor, Bratislava, Chechoslovakia, June 1988.

4.   SKOGMAN, B.C., TADROS, M.K., AND GRASMICK, R., "Ductility of Reinforced and Prestressed Concrete Flexural Members," PCI Journal, Vol. 33, No. 6, Nov.-Dec. 1988, pp. 94-107.

5.   DISCUSSION of Ref. 4 by NAAMAN et al., PCI Journal, Vol. 35, No. 2, Jan.-Feb. 1990, pp. 82-89.

6.   LOOV, R.E., "A General Equation for the Steel Stress for Bonded Prestressed Tendons," PCI Journal, Vol. 33, No. 6, Nov.-Dec. 1988, pp. 108-137.

7.   DISCUSSION of Ref. 6 by NAAMAN, A.E., PCI Journal, Vol. 34, No. 6, Nov.-Dec. 1989, pp. 144-147.

8.   NAAMAN, A.E., "A New Methodology for the Analysis of Beams Prestressed with Unbonded Tendons," in ACI SP-120, External Prestressing in Bridges, A.E. Naaman and J. Breen, Editors, American Concrete Institute, Detroit, 1990, pp. 339-354.

9.   NAAMAN, A.E., and Alkhairi, F.M., "Stress at Ultimate in Unbonded Prestressing Tendons - Part I: Evaluation of the State-of-the-Art; Part II: Proposed Methodology;" Submitted for Publication, ACI Structural Journal, September 1990.