Zeitschrift: IABSE reports = Rapports AIPC = IVBH Berichte
Band: 62 (1991)

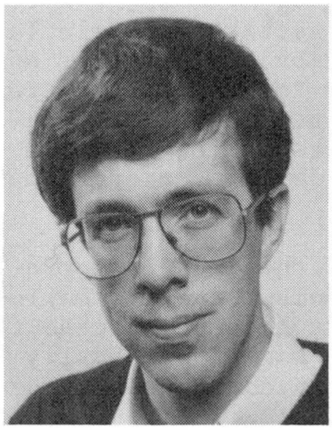
Artikel: Simple design method for partially prestressed concrete structures
Autor: Frénay, Jerome W. / Braam, Cornelis R.
DOI: https://doi.org/10.5169/seals-47641

Simple Design Method for Partially Prestressed Concrete Structures

Méthode simple de calcul pour structures en béton partiellement précontraint

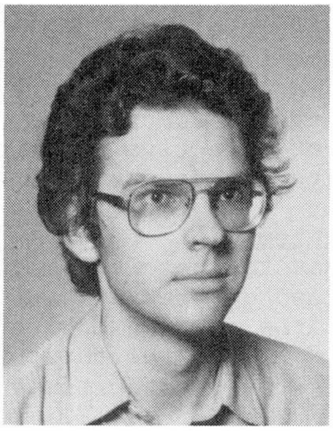
Einfache Rechenmethode für den Entwurf von teilweise vorgespannten Betonkonstruktionen

Jerome W. FRÉNAY
Dr. Eng.
Inst. of Agricultural Eng.
Wageningen, The Netherlands



Jerome W. Frénay, born 1956, is head of the structures department of IMAG (Institute of Agricultural Engineering) in Wageningen. He received M.Sc. and Ph.D. Civil Engineering degrees from the Delft University of Technology where he was with the concrete structures group from 1982 – 1988.

Cornelis R. BRAAM
Dr. Eng.
Molenbroek Civil Eng. Inc.
Rotterdam, The Netherlands



Cornelis R. Braam, born 1961, is senior-consultant at Molenbroek Civil Engineers Inc. in Rotterdam. He graduated as Civil Engineer M.Sc. at the Delft University of Technology. After joining the concrete structures group from 1985 till 1990, he received his Ph.D. degree in 1990.

SUMMARY
This paper presents an overview of a simple calculation method for concrete structures provided with a combination of reinforcing steel bars and post-tensioned prestressing tendons. The approach chosen relates closely to present theoretical modelling techniques aimed at a satisfactory approximation of the actual behaviour of structural concrete. The proposed method is illustrated by means of two statically indeterminate concrete structures: a rectangular girder for a warehouse and a box-girder used for a motorway.

RÉSUMÉ
Cette publication donne un aperçu d'une méthode de calcul simple pour des structures en béton pourvues d'armatures ordinaire et de câbles de post-contrainte. Cette approche a pour but d'obtenir une approximation satisfaisante pour le comportement du béton. La méthode est illustrée pour deux structures hyperstatiques en béton soit une poutre rectangulaire de magasin et un caisson de pont d'autoroute.

ZUSAMMENFASSUNG
In diesem Aufsatz wird eine einfache Rechenmethode für teilweise vorgespannten Beton vorgestellt. Die gewählte Vorgehensweise schliesst bei modernen Rechentechniken an, die zum Ziel haben, das Verhalten von Betonkonstruktionen so wirklichkeitsnah wie möglich zu beschreiben. Die vorgeschlagene Methode wird an zwei Beispielen illustriert: An einem rechteckigen Träger für ein Lagerhaus und an einer Hohlkastenbrücke für eine Autobahn.

1. INTRODUCTION

The design and behaviour of partially prestressed concrete structures have been discussed for many years [1,4,6,12,13]. However, the actual number of structural applications in The Netherlands is limited. Today, a similar situation exists in many other European countries with the exception of Switzerland [1]. Technical and economic reasons may hinder to practise research efforts, such as:

- Are cracks in concrete allowed if crossed by prestressing steel?;
- How should rather complicated calculation methods be coped with?;
- Which solution should be chosen and how does it affect the building-costs?

This paper pays attention to a rather simple calculation method applicable to crack formation in one-dimensional elements. First, the basic assumptions of the approach are briefly dealt with. Next, two structural applications are presented in chapters 3 and 4. A few conclusions are summarized in chapter 5.

2. BASIC ASSUMPTIONS

2.1 Distribution of forces

The amount of reinforcement needed in the various cross-sections is calculated on the basis of the theory of elasticity. No redistribution of forces is adopted in the ultimate limit state. The 'artificial' forces induced by prestress are either concentrated (anchorages) or distributed (pressure by curved cables). The level of effective prestressing includes losses due to friction and time-dependent material deformation which is assumed to develop unrestrained.

2.2 Cracking behaviour

The types of structural concrete may be characterized by the degree of prestressing K, defined as the ratio of the decompression moment and the maximum bending moment at the serviceability limit state [2]. Cracks are expected for $K < 1.0$ unless $\sigma_c < f_{ct}$ at the outer fibres. The cracking moment can easily be calculated if the effects of prestressing are taken into account, see figures 1a-b.

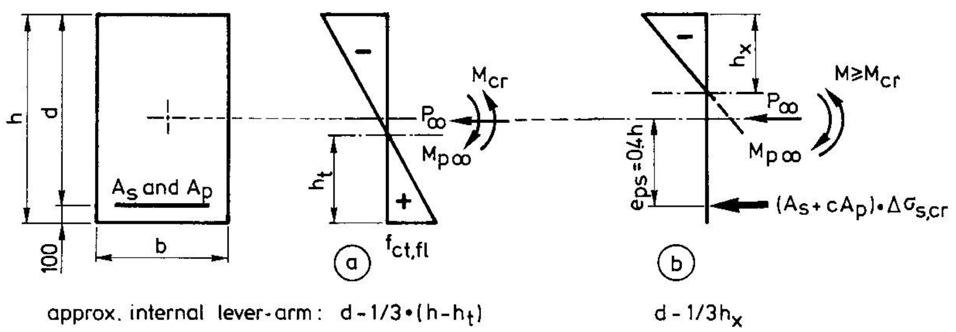


Fig. 1 Stress diagrams and internal forces (a) at and (b) after cracking.

The crack spacings and widths are found by means of theoretical models [3,4,10,13] provided that the concrete cover is at least 1.5-2 times the largest bar diameter used. Either the pure or the flexural tensile strength is adopted as a cracking criterion for concrete. The crack width is controlled by the reinforcement. Generally, the bond stresses developed between concrete and prestressing steel are relatively low which is represented by $c < 1$:

$$\Delta\sigma_{p,cr} = c.\Delta\sigma_{s,cr} \qquad (c < 1) \qquad [MPa] \qquad (1)$$

2.3 Bending moment and shear force

The bending moment at the ultimate limit state follows from $M^* = \gamma . M_{max}$ where γ denotes the structural safety factor including material as well as loading uncertainties. A minimum amount of reinforcement is used in each cross-section (see figure 2) to ensure a distributed crack pattern and a 'tough' structural behaviour. The contribution to shear transfer due to the tendon curvature demands a sufficient axial stiffness of the tension chord [7,8]. Thus:

$$A_s . f_{sy} + A_p . f_{pk} \geq V_d \quad \text{and} \quad A_s . f_{sy} \geq V_d/2 \qquad [N] \qquad (2a\text{-}b)$$

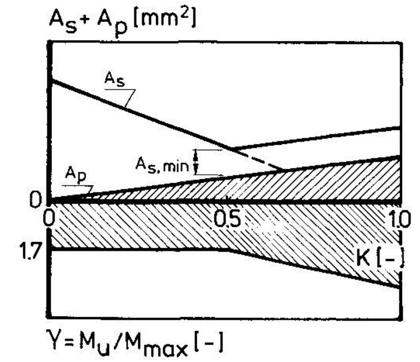


Fig. 2 Reinforcement ratio and γ as a function of the degree of prestressing.

3. STATICALLY INDETERMINATE GIRDER IN PARTIALLY PRESTRESSED CONCRETE

3.1 Introduction

The calculation method presented in chapter 2 is now illustrated by means of a continuous girder with three spans of 12.6 m each. Prefab-slabs are used supported by the beam which is part of a warehouse. The beam and the columns are monolithically connected: their bending stiffness may be neglected for the design. The dimensions of the rectangular beam are restricted to 450x1000mm²: its spacing amounts to 4.5 m. The characteristic ($\gamma = 1.0$) distributed loading amounts to: $q_p = 27$ kN/m (dead load of girder and slabs) and $q_q = 64$ kN/m (live load on slab: approx. 12 kN/m²). A safety factor $\gamma = 1.7$ is applied. The concrete cover is 50mm for the prestressing ducts. The 95% upper-bound characteristic crack width is restricted to $w_k = 0.30$mm (reinforcement) or 0.20mm (prestressing steel) [5]. Material properties: 150mm cube compressive strength $f_{cck} = 35$; $f_{sy} = 500$ and $f_{pk} = 1860$ MPa.

3.2 Reinforced concrete girder

Fourteen 25mm diameter deformed steel bars are needed in Q, i.e. $\rho_d = 1.70\%$. In The Netherlands generally a ratio of 0.8-1.2% is economic for reinforced concrete beams with a rectangular cross-section. The computations according to [5] reveal crack widths $w_k \leq 0.30$mm which agreed with an analysis based on theoretical models [10]. Vertically placed 12mm diameter closed stirrups are applied at a minimum spacing of 110mm located at cross-section Q.

3.3 Girder in fully prestressed concrete

The schematic location of the prestressing ducts is presented in figure 3. The cable was stressed at both end blocks of the beam. A fully prestressed girder could not be achieved. A good approximation was found for:

$$e_1 = e_2 = e_3 = 400\text{mm}; \; R_0 = 5000\text{mm}; \; R_1 = 31100\text{mm}; \; R_2 = 19800\text{mm}$$

and P_0 = 2700.10^3 N. Three post-tensioned elements are needed of six 12.7mm (1/2 inch) diameter strands each, thus A_p = 3x600 = 1800 mm². Due to the 'secondary moment' the line of thrust is situated 85mm above the centre line of the tendon profile at cross-section Q.

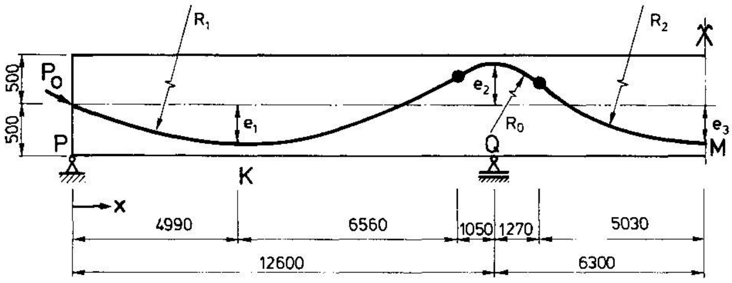


Fig. 3 Prestressing tendon profile.

3.4 Girder in partially prestressed concrete

It is proposed that no cracking may occur (or: cracks remain closed) for $q_p + 1/3q_q$ which resulted in two prestressing elements with A_p = 2x600 = 1200 mm². Additional reinforcement A_s should ensure the safety requirements. Next, the crack widths were checked: at Q, a surplus of seven 20mm diameter bars was needed. This is a reduction of 68% in comparison with the reinforced concrete girder, see also figure 4.

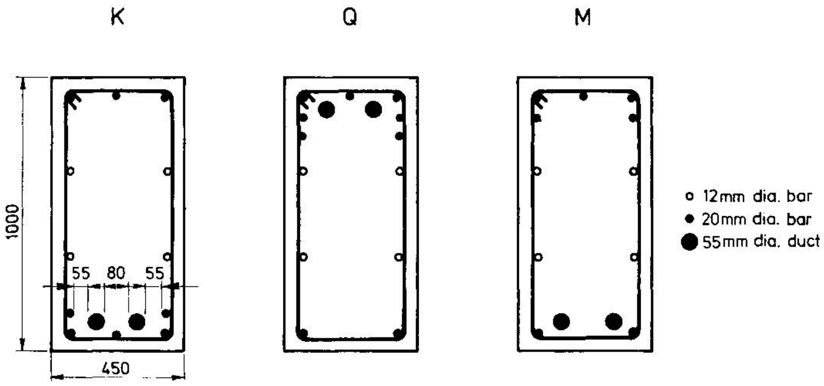


Fig. 4 Reinforcement in sections K, Q and M respectively for 0.0 < K < 1.0.

With respect to the calculated crack widths, a factor c = 0.40 was implemented in eq. (1). The empirical formula for the crack spacing presented in [2,5] was adapted to cope with prestressing effects:

$$\Delta l_m = 50 + \frac{k_2 . k_1 . d_s}{4\rho_{eff}} \qquad [\text{mm}] \qquad (3)$$

ρ_{eff} is calculated in accordance with various national codes. Eq. (3) is suitable for stabilized cracking. It followed that γ = 1.9 at K (midspan) and 1.8 at Q (support). The degree of prestressing K is at least 0.57. Vertical 12mm diameter stirrups at 300mm spacing are used throughout the structure.

3.5 Level of prestress and economy

The costs of materials (reinforcing and prestressing steel) and labour were estimated according to guide-lines presented by the Dutch building-industry (prices excl. VAT). An overview is shown in figure 5 for one girder.

-5-

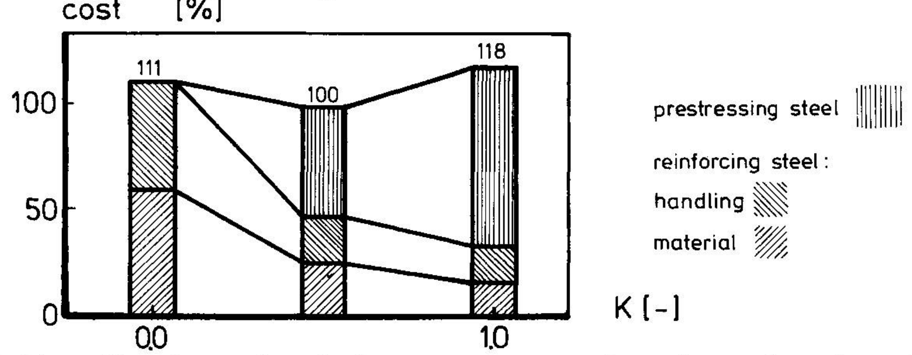


Fig. 5 Calculated distribution of reinforcement cost for three levels of prestress (100% = dfl. 5200,- = US$ 2500,- dated Oct. 1988).

4. DESIGN OF A BOX-GIRDER BRIDGE IN PARTIALLY PRESTRESSED CONCRETE

4.1 Introduction

The non-linear analysis concerns a continuous 50m span box-girder bridge with 2x2 traffic lanes subjected to dynamic traffic load, see figure 6. The design live load consists of two distributed line loads of 9 kN/m each and one 600 kN heavy-truck traffic load distributed over three axes. Load transfer of the box-girder is only considered in the longitudinal direction. Material properties: $f_{ccylk} = 36$; $f_{sy} = 400$ and $f_{pk} = 1770$ MPa.

Each of the three webs of the box-girder contains six prestressing elements of eight 15.3mm (5/8 inch) diameter strands so that $A_p = 3x6x1120 = 20160$ mm². See also figure 7. At the support and at the midspan $A_s^p = 44400$ and 33000 mm² which implies structural safety factors of 1.7 and 2.3 respectively.

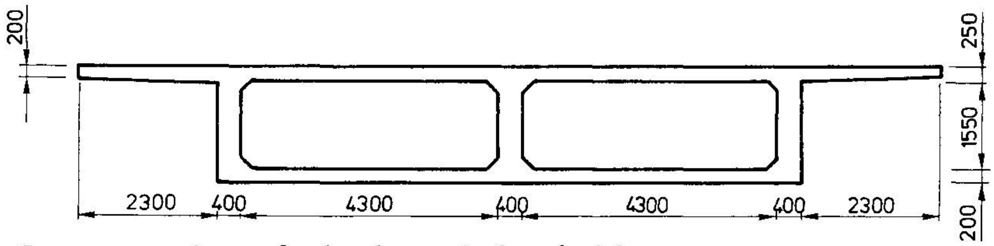


Fig. 6 Cross-section of the box-girder bridge.

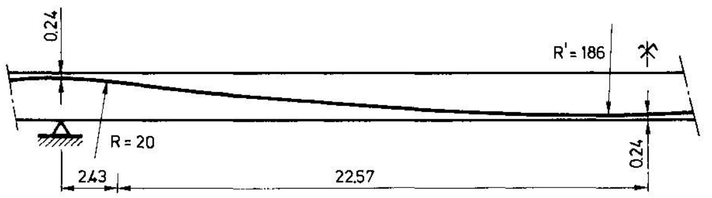


Fig. 7 Tendon profile of the prestressing cables. dimensions in m

4.2 Development of cracks and steel stress variations

The average crack widths were calculated in two ways. A first approximation is based on an assumed cooperation of reinforcing and prestressing steel leading to a uniform cracking pattern. A second approach takes account of a concentrated location of the prestressing elements at the flange-web connection of the box-girder, causing a distributed cracking pattern. The average and the characteristic (95% upper-bound value) crack widths are respectively [3]:

$$w_m = \Delta l_m . \epsilon_{sm} = \Delta l_m . (\epsilon_s - \beta\Delta\epsilon_s) \quad \text{and} \quad w_k = 1.7 w_m \quad [\text{mm}] \qquad (4)$$

where β incorporates reduced tension-stiffening ($\Delta\epsilon_s$) by a dynamic or a sustained loading. A sensitivity analysis revealed that $\beta = 0.5$ fits closely to the actual structural behaviour. The permissible crack widths (section 3.1) are not exceeded. A variation of the complete live load is related to $\Delta\sigma_p = 70$ and $\Delta\sigma_s = 130$ MPa at midspan, see also eq. (1). The permissible values ($\Delta\sigma_p = 104$ and $\Delta\sigma_s = 180$ MPa) are still empirically based.

4.3 Finite element analysis

The non-linear finite element program 'DIANA' was used in order to study the detailed structural behaviour of the girder, see in [2,9,11]. The computed longitudinal moment distribution was compared with a simple linear elastic approximation: the differences were less than 5%. The program provides a prediction of the cracking pattern at the very instant of structural failure.

5. CONCLUSIONS AND RECOMMENDATIONS

The analysis focused on partially prestressed concrete. Satisfactory results were achieved in comparison with reinforced concrete, such as: reduced crack widths and deflections, more simple detailing of the reinforcement. Moreover, the structure reacts rather insensitive to imposed deformations due to differential settlements and restraint of temperature movements or shrinkage. Applications may often be advantageous for high ratios of live to dead load or in case of a limited structural height. Partially prestressed concrete may also exhibit good economic prospects.
As stated in [4,13], extended research is needed to attain simple, consistent and reliable models which predict the behaviour of structural concrete. It may also enhance the introduction of uniform, realistic and clear design codes.

REFERENCES

1. BACHMANN H., Design of partially prestressed concrete structures based on Swiss experiences. PCI Journal, vol. 29, no. 4, 1984, pp. 84-105.
2. BRAAM C.R. & FRENAY J.W., Simple design method for partially prestressed concrete structures. IABSE Periodica, No. 3, 1989, pp. 77-94.
3. BRAAM C.R., Control of crack width in deep reinforced concrete beams. Dissertation, Delft University of Technology, Dec. 1990, 96 pp.
4. BRUGGELING A.S.G., An engineering model of structural concrete. Invited paper to IABSE Coll. 'Structural concrete', Stuttgart, April 1991, 10 pp.
5. CEB-FIP Model Code 1990. CEB Bull. d'Inf. nos. 195/196, March 1990, 14 ch.
6. COHN M.Z., Partial prestressing. NATO ASI Series, 1986, pp. 405-426.
7. DARWIN D., Shear component of prestress by equivalent loads. PCI Journal, March/April 1977, pp. 64-77.
8. FRENAY J.W., REINHARDT H.W. & WALRAVEN J.C., Time-dependent shear transfer in cracked concrete (2 parts). To be published in Journal of the structural div., Proceedings of the ASCE, 1991, 32 pp.
9. MIER J.A.M. van, Examples of non-linear analysis of reinforced concrete structures with DIANA. Heron, vol. 32, no. 3, 1987, 147 pp.
10. NOAKOWSKI P., Verbundorientierte, kontinuierliche Theorie zur Ermittlung der Rissbreite. Beton- und Stahlbetonbau, vol. 80, nos. 7/8, 1985, pp. 185-190/215-221.
11. ROTS J.G., Computational modeling of concrete fracture. Delft University of Technology, Dissertation, 1988, 132 pp.
12. SCHIESSL P., Einfluss von Rissen auf die Dauerhaftigkeit von Stahlbeton- und Spannbetonbauteilen. D. Aussch. für Stahlb., Heft 370, 1986, pp. 9-52.
13. SCHLAICH J., The need for consistent and translucent models. Invited paper to IABSE Coll. 'Structural concrete', Stuttgart, April 1991, 10 pp.