Constitutive Laws

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ABSTRACT
1. The contribution of constitutive laws to the theory of structures
2. Constitutive laws and design stages – design and review
3. Are design and research models opposites?
4. Safety considerations resulting from constitutive laws
5. Open questions

1. THE CONTRIBUTION OF CONSTITUTIVE LAWS TO THE THEORY OF STRUCTURES

The role of constitutive laws in continuum theory, and for non-homogenous materials is demonstrated. Constitutive laws may start at micro level regarding e.g. the strength resulting from atomic interaction, may go to a meso level or macro level. How exact is exact? Problem orientation.

"A model has to be as simple as possible, but not simpler."

2. CONSTITUTIVE LAWS AND DESIGN STAGES - DESIGN AND REVIEW

Strength Design:
Compressive and tensile forces or their combination in moments for conceptual design (theme 2.3, A. Scordelis).
Simplified stress blocks, strut-and-tie models for preliminary or final design.
A "compressive strength of reinforced concrete" may be reasonable.
More sophisticated constitutive laws for review (theme 2.2, J. Schlaich).

Non-Strength Relevant Problems:
The order with regard to complexity - or what is declared as such - may be reversed when usual shorttime strength is not relevant as e.g. in cases of explosions, dynamic vibrations, temperature etc.

3. ARE DESIGN AND RESEARCH MODELS OPPONENTS?

A nowadays sophisticated dynamic or strain rate model may become a design model still trying "to be as simple as possible" e.g. in case of dynamic loads. If neglected, the behaviour of young concrete including shrinkage, creep and their temperature dependence may lead to obsolete serviceability checks in the usual design format.

4. SAFETY CONSIDERATIONS RESULTING FROM CONSTITUTIVE LAWS

Regarding non-linear material behaviour leads to a more economic design. Using simplified plasticity methods member capacities instead of cross sections may be the basis of design. An appropriate safety concept is necessary, as EC 2 is not consistent in this sense.

Unreliable tensile strength, multiaxial stress conditions, over reinforced concrete due to former load-histories need special safety considerations.

5. OPEN QUESTIONS

Among others we need stress-strain-laws regarding strain rate effects and constitutive laws regarding the influence of concrete age especially for young concrete. Extreme loading situations will be discussed.

The Need for Consistent and Translucent Models

Nécessité de modèles cohérents et intelligibles

Warum wir einheitliche und verständliche Modelle brauchen

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SUMMARY

Only through an intelligent model can a complex reality became translucent and understandable to a designer and there lies the key to quality in structural engineering. It must be the aim of this colloquium to agree on a modelling philosophy which covers consistently the whole range of structural concrete. The individual designer must acquire the art of finding the right model to suit each case, neither too special nor too general.

RÉSUMÉ

Une réalité complexe ne peut être comprise qu'à travers un modèle cohérent et intelligible; pour l'ingénieur ceci est la clef d'un dimensionnement efficace particulièrement dans le domaine des structures. Le but du Colloque est de mettre en évidence le sens de cette démarche, afin qu'elle concerne directement tout l'ensemble des structures en béton. Pour l'ingénieur projeteur, il s'agit d'apprendre dans chaque cas à appliquer le modèle, qui à juste titre, ne se doit de décrire la réalité ni d'une façon trop détaillée ni trop simplifiée.

ZUSAMMENFASSUNG

Weil man nur bearbeiten kann, was man versteht, ist die Wahl eines intelligenten Modells, das ein komplexes Tragwerk durchsichtig und verständlich macht, der Schlüssel zur Qualität im Konstruktiven Ingenieurbau. Im Rahmen dieses Kolloquiums sollen vor allem die Modelle diskutiert werden, die die ganze Breite des Konstruktionsbetons einheitlich beschreiben. Der entwerfende Ingenieur muss lernen, im Einzelfall das richtige Modell zu entwickeln, das die Wirklichkeit weder unnötig fein, noch zu grob vereinfacht beschreibt.

1. THE NEED FOR CONSISTENT AND TRANSLUCENT MODELS

The appeal of structural engineering is, that it combines rationality with creativity. The structural engineer, as does the crafts-man, forms material such that it serves its purpose,
i.e. - fulfills its function (utility)
      - during an expected lifetime (durability)
      - at a reasonable prize (economy)
      - with a pleasing form (beauty)
and today we are inclined to add
      - respectful of natural resources (harmony).

Obviously forming a material to this end can be successful only if one really knows and understands it. Since the real nature of all our materials, especially that of structural concrete, is very intricate and puzzling and therefore even more so whole structures made from such material and exposed to a natural environment, the structural engineer needs models as a medium between his capabilities and reality. Without such models of abstraction and simplification he would be completely subject to trial and error, a method which is especially worthless in structural engineering where each object is a prototype which has to be invented anew and whose behaviour has to be predicted.

The translation of a reality, which up to then existed only in his mind, into the right models which serve him to predict the utility, durability, economy and beauty of his structure to be build, is one of the main challenges to the structural engineer, a semi-rational intuitive step within the whole planning process, close in quality only to the creative conceptual design. Poor modelling results in poor quality of the structures, vice versa.

Looking at the planning process as a whole it becomes immediately clear, that one model would not help and only translate reality into a black box. Rather several additive models are required.

From that it follows that
- the different models which are to be applied in sequence must be consistent with each other
and that the individual models shall
- neither be too special
- nor too general.

A model and as a consequence to it a method of design or analysis, which is too special will not permit transferability and generalisation. If it serves only in points it will not augment or even multiply the experience of its user and therefore not deserve the attribute to be a practical or design model. Too spezialized models cannot be consistent amongst each other. Consistency and specialisation are contradictions in themselves.

It cannot be a design model. In certain cases however and thanks to the efficiency of modern computers such black box models (and analysis or programs associated with them) through parameter variation and sensitivity studies may on a research level be very useful and such deserve the attribute of a research model.

The art of finding the right model and applying the right method of analysis consists in defining and asking for "just enough" and not in "as much as possible". Any redundant refinement is destructive as is any substantial ommission. There are certain data, which the designer will quite happily do without. False accuracy distracts the mind. The time wasted for it will be found wanting for the design task.

2. RESEARCH MODELS, DESIGN MODELS, EXPERIMENTS

Following the above and also earlier writing of Duddeck /1/ on this subject and in view of the ongoing sophistication of modelling techniques with structural concrete it appears useful from an engineer's point of view to differentiate between

- research models and
- design models.

The research model tries to be as close as possible to reality and tries to find an explanation for a phenomena. Therefore it will use real loads, the latest constitutive laws, sophisticated Finite Elements etc. If a test is available, analytical and experimental results must agree.

The design model will reduce reality to its most significant parameters, will idealize loads, the statical system, the safety concept etc. The only criteria is that the structure designed on this basis is clearly understood and, when built, behaves satisfactorily.

This shows, that the research model tries to play and can play a similar role as large scale testing does and that therefore some day experiments may be replaced by sophisticated modelling techniques /2/. The research model may even supply more information as can the experiment - assumed of course, that some day it is really possible to precisely substantiate by way of an analytical model the carefully documented experiment on a structural concrete member.

The experiment inseparably integrates all scattering variables so that strictly speaking it is "only" good for falsifying a theory, impossible of being neatly interpreted without additional theoretical modelling. The theoretical modelling of structural concrete has the character of a theory i.e. that of transferability and universal validity because here every single parameter is individually variable. Therefore we must insist, that the experiment shall never be used as a sole source of information but must be based on design model required anyhow for the design of the test specimen and if necessary further explained by a research model. If this is not observed misinterpretations or at least fruitless disputes are the consequence. In case of structural concrete the most frequent misinterpretation results from the fact, that concrete's tensile strength is there, especially in a well cured and protected test specimen, but that it should be made use of only under very specific conditions which can be incorporated in a design model but not in an experiment or its corresponding research model (see sub-theme 2.5 of this Colloquium).

Experiments as research models describe nature, something which is already there in nature or in a test specimen. They cannot teach us what we should do, but only what we should not do. For the creative design and for innovative thinking we need the aid of a design model.

3. SOME GENERAL REMARKS ON MODELS FOR STRUCTURAL CONCRETE

In order to discuss the different types of models and to understand how they are interrelated, a look at the different steps of the planning process, given in a simplified representation in table I may be helpful.
In the context of this Colloquium on Structural Concrete we may restrict ourselves to the constitutive models describing the material behaviour needed for "Dimensioning" and "Review".

Before doing so it should however be mentioned, that "Detailing" can be omitted here and done based on experience only if it is defined in such a way, that it does not contain any hidden part of dimensioning as it frequently does. Finding the required amount and layout of reinforcement and shaping the concrete down to the smallest detail of a joint or node is called dimensioning. Only respecting rules of spacing or cover etc. whilst doing the working drawings is called detailing and even then it is geometrically interconnected with dimensioning.

Dimensioning (or design) means to fix the dimensions of a member and the amount of reinforcement required to carry a given set of loads over a certain period of time.

Review or validitation or check means to find out the behaviour (deformations, cracking etc.) and amount of load a member is able to carry if all its dimensions and steel is known.

Dimensioning is linked to the conceptual design, a blend of rationality and intuition, where much of the subjective experience of the designer and the objective boundary conditions of a given case merge. The assessment of the dimensions requires simple and transparent models and methods. Therefore dimensioning is the playground of design models. In simple cases, the experienced designer will be satisfied with dimensioning only and proceed from there to the working drawings directly. In more delicate cases dimensioning may assume a more preliminary character and be followed by a refined review or check.
Review similar to experiments - if at all - will always have to follow dimensioning. It should serve no other purpose than to confirm what the designer already knows.
The models or methods applied for review must be at least as or more informative or disclosing as those used for dimensioning if a review should make sense at all. Therefore review is the playground of sophisticated modelling techniques and even research models may be applied there. If in the worst case the designer is not able to carry out such a review himself, by whatever reason, as he would usually also not do a confirming experiment himself, but finds himself confronted with its results, he will have no problem with them, if he really cared for a prediction of these results whilst doing the dimensioning. Thus the designer expects from research models nothing but a confirmation of what he already knows.

To repeat it: Especially in view of the progress in computeroriented analysis we must encourage the designer to do the dimensioning, the pre-diction with intelligent simplified models and methods and to compare them with the results of the "exact" analysis. Where significant disagreement is found, he must look for the cause. The searching and finding is very instructive and serves to train the designer's understanding of structural load behaviour.
With this in mind we can welcome the advancement of computer-oriented modelling techniques because they restore the significance of the simplified design methods by serving as a safety net. Thanks to that the structural engineer may once again revive his inventive talents.

4. THE SPECIFIC MODELS FOR STRUCTURAL CONCRETE

If the above could be agreed upon, then the choice of the specific models should not be a problem at all: As accurate as necessary (not as possible) for the intended use which means that accuracy or density of information may increase if we step down along table 1.

Table 1.
The different steps of the planning process for a concrete structure and the type of models associated with them:

Functional Requirements including social and economical environment	Defined, given
Conceptual design: definition of overall shape and of individual members, choice of materials main details, construction concept.	Experience, intuition simple, mechanical models of subjective character, geometrical models (architectual mock-up).
Loads environmental (climatic impact)	Measurements, idealisation probalistic models
Material laws	constitutive models
Dimensioning Member stiffness	Force-deformation-models
Analysis (sectional or inner forces)	mechanical models (simplified)
fixing of dimensions	safety concept, experience
Detailing	experience, rules
Review Analysis (check of load capacity) if unsafe	mechanical model safety concept (refined)
Beauty check if ugly	architectural model
Working drawings	geometrical model
Specifications and tender documents	model
Submission (economy check) if too expensive Ready for construction	model

With respect to the <u>constitutive models</u> the degree of accuracy should clearly correspond with the particular application i.e. design or research respectively dimensioning or review. It must further be clearly differentiated between a consideration of the overall behaviour of a structure under service condition, when in principle mean values apply, or the search for a local failure, when basically extreme values must be combined. This further shows, that the choice of the appropriate constitutive model is closely related to the safety concept. Since Josef Eibl accepted the invitation of the Scientific Committee to prepare a special report for this Colloquium on this important subject /3/, it will be sufficient to touch the constitutive models in this paper only whilst discussing models for dimensioning and review.

<u>For dimensioning</u> the classical stepwise approach
- choice of a statical system
- definition of member stiffnesses (with respect to the determination of the inner forces only necessary for redundant systems)
- analysis of sectional or inner forces.
- dimensioning remains valid, even, if for simplification inconsistencies are to be accepted.

The most common or classical inconsistency derives from applying as a material model Hooke's law (linear - elastic) for the analysis but modelling the real behaviour of cracked structural concrete whilst dimensioning. However it is not only justified by simplification but because it also gives satisfactory results since a reinforcement layout which is oriented at the linear elastic stress distribution will be right for serviceability and simultaneously concerning safety the lower bound method of the theory of plasticity is satisfied.

<u>For the review,</u> as against that the chances of applying consistent models are much better. Whilst analyzing the inner stresses or the overall load capacity realistic non-linear or idealized elasto-plastic or purely plastic constitutive material models may be used.

This shall be carried out in some more detail for the most common types of structures
- structures made from linear or one-dimensional members such as beams, frames and arches
- two-dimensional plane structures with in-plane loading such as deep beams
- two-dimensional plane structures with transversal loading i.e. slabs
- three-dimensional structures with broken or curved surfaces i.e. folded plates and shells.

It should be stressed, that this conventional way of defining structures is not really helpful and did cause a lot of confusion and useless discussion, at least as far as the separation-line between the first two is concerned. It is in fact not only the global geometry but the local geometry as well and of course the type and distribution of the loads resp. reactions, which governs the load bearing behaviour and thus the approach to it via a mechanical model.

The approach through the subdivision of a structure in its B- and D-regions however is rational, straightforward and simple.

It clearly leads the way towards the appropriate mechanical model(s) and analysis - though of course it is not at all a compulsory or the only possible approach, but obviously the most practical and convenient. So this definition will further be used additionally. [1]

4.1 Models for structures consisting substantially of B-regions e.g. beams, frames and arches

This concerns the majority of the daily building activity (even more if we include the slabs, see sect. 4.3). Though these structures consist widely of B-regions (fig. 1 shows a typical example) they only in very rare cases can do without any D-regions. In fact, because there exist well established methods for dimensioning and review of the B-regions, the majority of problems, poor performance and even failures appear in D-regions. From that point of view the former "shear battle" and ongoing strive for further refinements of the B-region design appears disproportionate.
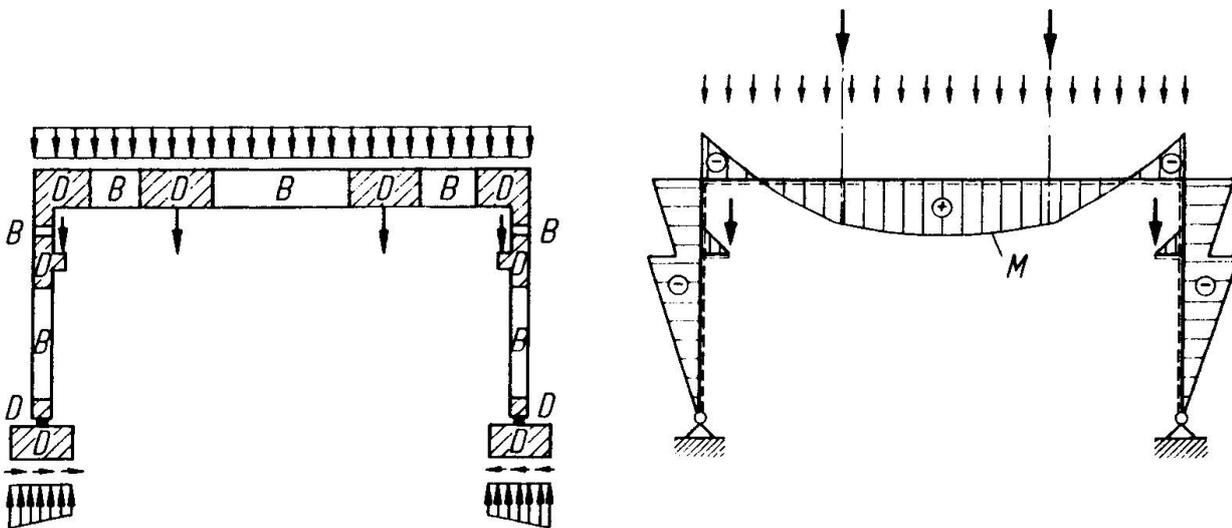

Fig. 1:   A frame structure containing a substantial part of B-regions, its statical system and its bending moments.

1.This method was first introduced in /4/ and further published in /5, 6/ and later referred to by other authors. It shall therefore not be repeated here.

In B-regions the Bernoulli-hypothesis of plain strain distribution is valid (B stands for beam or Bernoulli). Their internal state of stress is easily derived from the sectional forces (bending and torsional moments, shear and axial forces) through clearly defined models as discussed below.

Regions in which the strain distribution already for a linear-elastic stress-strain law is significantly non-linear due to static (e.g. concentrated loads) or geometric (e.g. corners, bends, openings) discontinuities are called D-regions (where D stands for discontinuity, disturbance or detail).

On the other side, even in times of ever increasing computer efficiency, it would be unreasonable to begin immediately to model these structures with strut-and-tie-models (STM) or even with finite elements. Rather the common practice should be maintained to model the real structure by its statical system, i.e. one-dimensional elements following the center lines of the real sections, and to analyse its support reactions and sectional effects, the bending moments (M), normal forces (N) shear forces (V) and torsional moments (T). It should be emphasized, that this analysis in cases of structures with predominantting B-regions such as in fig. 1 yields satisfactory results for the deformations and forces if it is carried through the D-regions even, i.e. if even the D-regions are for that purpose treated as B-regions - but only for this overall analyis, not for the dimensioning of the D-regions themselves! In cases of doubts, i.e. if the D-regions appear to dominate against the B-regions, the method described in sect. 4.2 should be followed.

As already mentioned, for calculating the deformations and, in case of a stati-cal indeterminate structure, the sectional effects, one will certainly start applying sectional values (bending stiffness EI, torsional stiffness GI_t, axial stiffness EA etc.) on linear-elastic basis.

If the sectional forces are known the dimensioning of the B-regions, especially of their reinforcement may follow standard procedures. As long as a section is uncracked (e.g. in columns or due to prestress), the inner forces are calculated with the help of section properties like cross sectional areas and moments of inertia. If the tensile stresses exceed the tensile strength of the concrete the truss model[1] applies (fig. 2). Since for B-regions with light transverse rein-forcement, the truss model yields unrealistic low inclinations for the struts, efforts have been made to explain the mechanical meaning of the V_c-term, applied for correction by several codes, because the inclined compression chord explana-tion can apply only to D-regions. It has been shown by several authors and a paper by Reineck submitted for this Colloquium will go further into details, that by considering the concrete tensile strength it is possible to model the load bearing behaviour of the webs of a B-region consistently /7/.

The overall analysis and the B-regions dimensioning provide also the boundary forces for the D-regions of the same structure. As long as the D-regions are uncracked, they can be readily dimensioned and analyzed by standard procedures including finite elements analysis (FEA) applying Hooke's law. If they are cracked the STM design has to be applied for dimensioning /4,5,6,8/. For finding the geometry of the strut-and-tie-models especially for unusual cases, an ela-stic analysis on FE basis is helpful (Table 2). The loadpath method supports the finding of the model geometry and trains the designer's understanding of the flow of inner forces /5,6,10/. However, the number of D-region types for beams and frames is rather limited and the experienced designer will soon be able to rely on his STM-collection. Efforts are being made to provide practice with a reliable collection of such cases (further comments on D-regions see sect. 4.2).

1.Here the expression truss model is used to define the special application of the general STM to B-regions. A truss has compres-sion and tension chords parallel to the surface lines, inclined struts or compressive stress fields and transversal ties repre-senting the stirrup reinforcement and/or tensile stress fields.

simple span with cantilever

θ

θ₁ / D-region

z

D-region

a)

force in the bottom chord

+

truss

multiple truss

M/z

M/z

multiple truss

−

truss

force in the top chord

+

forces in the stirrups

+

single truss

multiple truss (steps)

b)

beam

$M(x-a)$

$V(x-a)$

M_x

V_x

x-a

x

b

ε

σc

z

Ts

model

$C_c(x-a)$

$C_c(x)$

$V(x-a)$

C_w

$T_w(x-\frac{a}{2})$

z·cosθ

$C_w(x)$

$T_s(x-a)$

x-a

θ

$T_s(x)$

$a = z \cdot \cot\theta$

$$C_c (x) = \frac{M(x)}{z} - \frac{V(x)}{2} \cot\theta$$

$$C_w (x) = \frac{V(x)}{\sin\theta} \longrightarrow \sigma_w(x) = \frac{V(x)}{b \cdot z \cdot \sin\theta \cos\theta} \text{ (smeared diagonal stress)}$$

$$T_w(x-\frac{a}{2}) = V(x) \longrightarrow t_w(x-\frac{a}{2}) = \frac{V(x)}{z \cdot \cot\theta} \text{ (per unit length of beam)}$$

$$T_s (x) = \frac{M(x)}{z} + \frac{V(x)}{2} \cot\theta$$

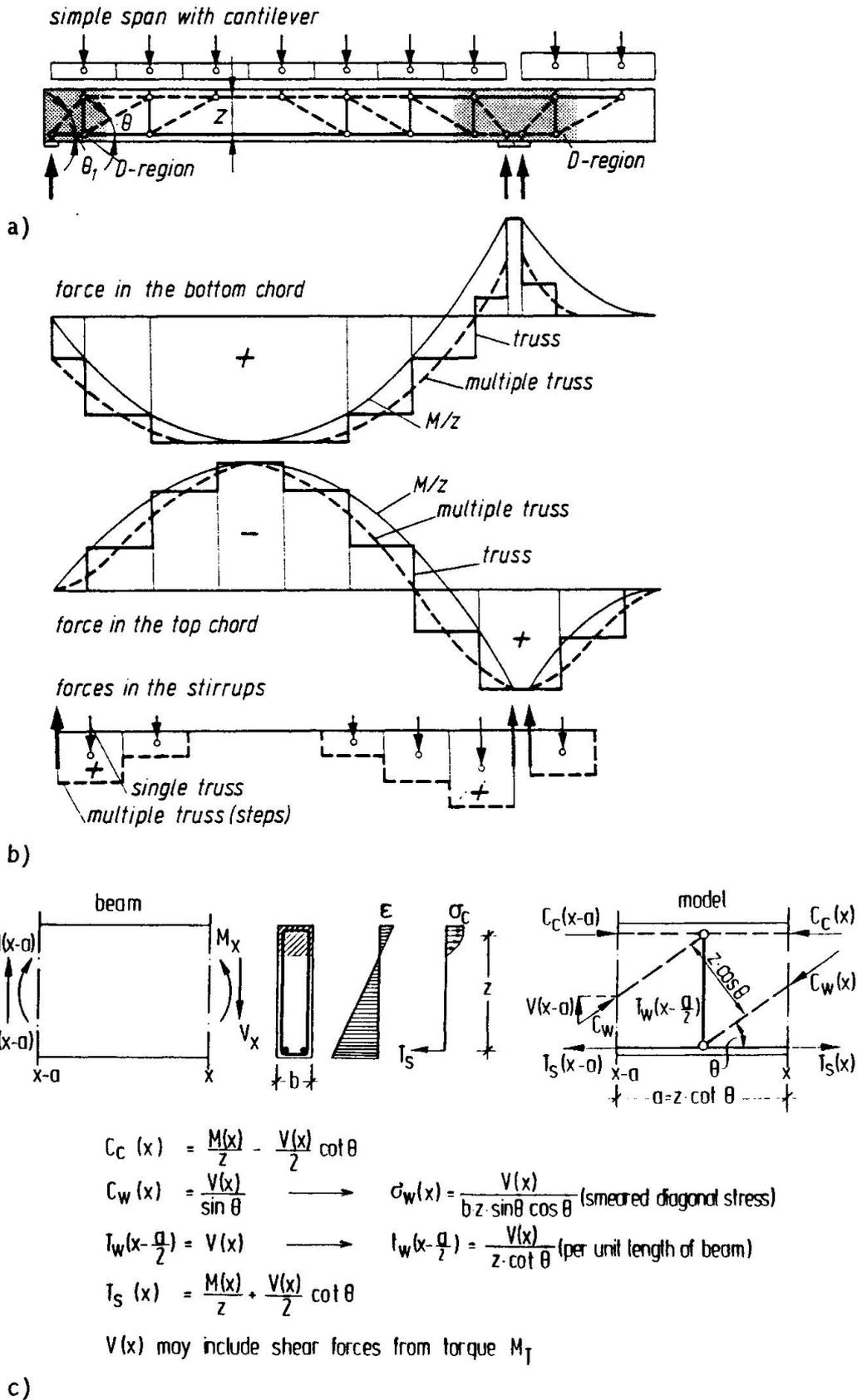$V(x)$ may include shear forces from torque M_T

c)

Fig. 2: Truss model of a beam with cantilever: (a) model; (b) distribution of inner forces; (c) magnitude of inner forces derived from equilibrium of a beam element.

Table 2. Analysis leading to stresses or strut–and–tie–forces

Analysis / Structure		Structure consisting of:		
		B– and D–regions e.g., linear structures, slabs and shells		D–regions only e.g., deep beams
		B–regions	D–regions	D–regions
Overall structural analysis (Table 3) gives:		Sectional effects M, N, V, M_T	Boundary forces:	
			Sectional effects	Support reactions
Analysis of inner forces or stresses in indivdual regions	State I (uncracked)	Via sectional values A, J_B, J_T	Linear elastic analysis* (with redistributed stress peaks)	
	State II (cracked)	Strut–and–tie–models and/or nonlinear stress analysis *		
		Usually truss		

* May be combined with overall analysis

For later improvement and review and with the real dimensions and reinforcement in hand, it may be necessary to repeat the analysis using non-linear moment-curvature relations. This will become a must for structures with strongly geometrically non-linear behaviour, with theory of second order effects or in case of buckling problems. Fortunately there are handy computer programs available today for that purpose.

It must indeed be warmly welcomed, that most instability problems can today be solved by a theory of second order analysis on basis of imperfections, whose assumption poses no problem to the experienced designer.

Finally as an overall review for statically redundant structures, there are further "closed" methods. After the above-mentioned revision of the sectional forces the designer has the choice to repeat a dimensioning with strut-and-tie-models, or to apply one of the "closed" methods as contained in table 3, mainly a plastic analysis with plastic hinges for finding the overall load capacity or a sophisticated non-linear FEA, which contains not only the non-linear material behaviour but also a realistic failure hypothesis.

Table 3. Overall structural behavior and method of overall structural analysis of statistically indeterminate structures

Limit state	Overall structural behavior	Corresponding method of analysis of sectional effects and support reactions	
		Most adequate	Acceptable
Service-ability	Essentially uncracked	Linear elastic	——
	Considerably cracked, with steel stresses below yield	Nonlinear	Linear elastic (or plastic if design is oriented at elastic behavior)
Ultimate capacity	Widely cracked, forming plastic hinges	Plastic with limited rotation capacity ·or elastic with redistribution	Linear elastic or nonlinear or perfectly plastic with structural restrictions

4.2 Models for structures consisting of D-regions only e.g. deep beams

In this case the analysis of sectional effects by a statical system makes no sense anymore and the inner forces or stresses can be determined directly from the applied loads following the principles outlined for D-regions above, already.

In /5,6/, where the modelling and dimensioning of D-regions with STM is described in all details, it is proposed to orientate the geometry of the STM at the elastic stress fields, which means to utilize the same model for the serviceability and the ultimate stress check (fig. 3). Of course this does not exclude adjusting the model geometry whilst approaching failure towards an increase of the internal lever arms (fig. 4).
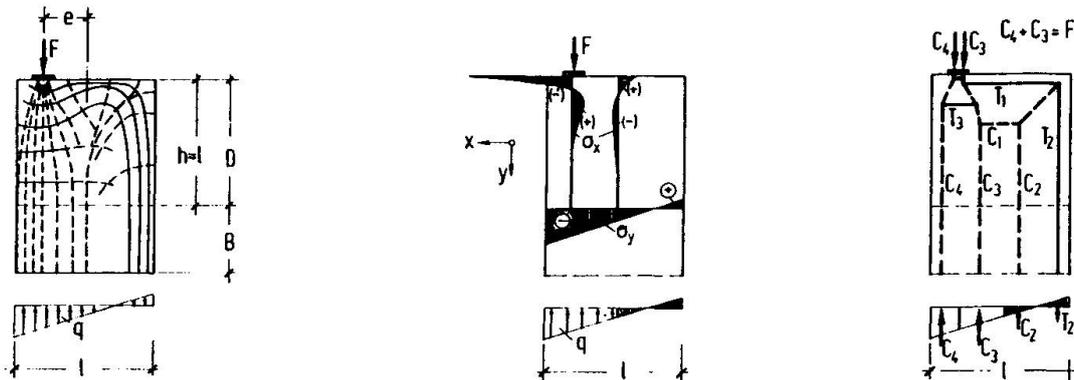

Fig. 3:   A typical D-region: (a) elastic stress trajectories; (b) elastic stresses; (c) strut-and-tie-models.
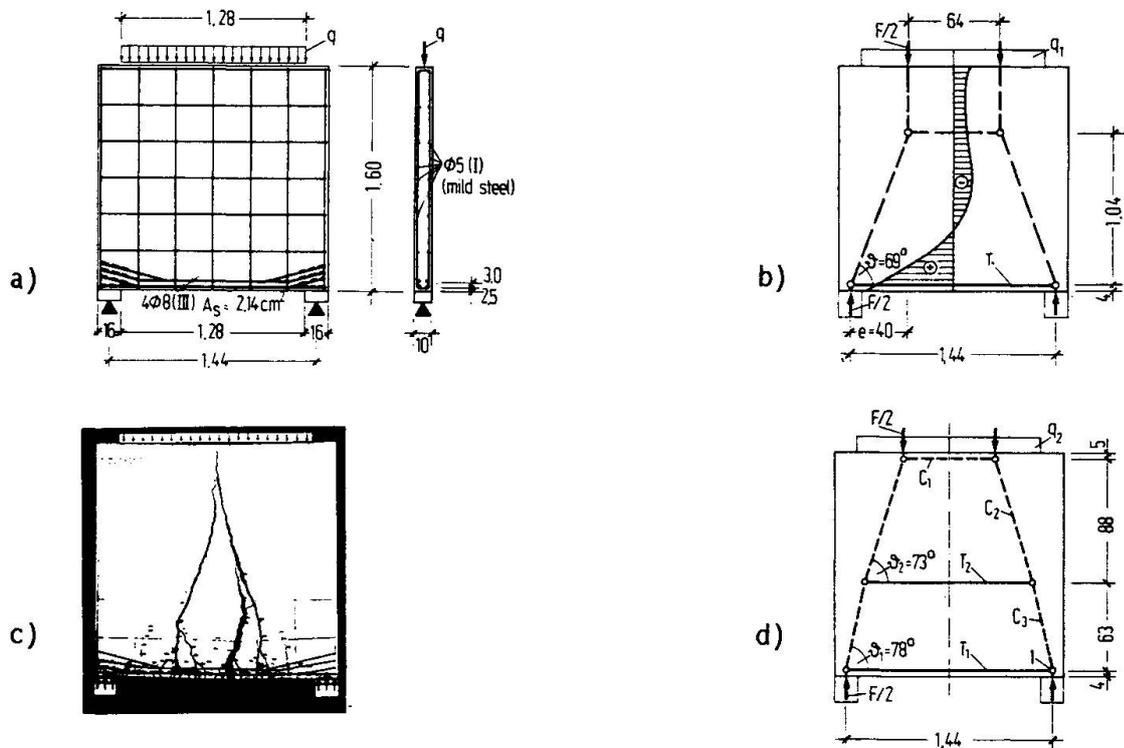

Fig. 4:   Deep beam: (a) Tested specimen WT2 /16/; (b) model oriented at the theory of elasticity; (c) crack pattern from test; (d) model adjusted to the failure mechanism.

The designer will decide in the individual case, whether he finds his STM on his own, where the "load-path method" will be a valuable tool (fig. 5), or if he wants in a more complicated case to start with a linear elastic FEM analysis (fig. 6).
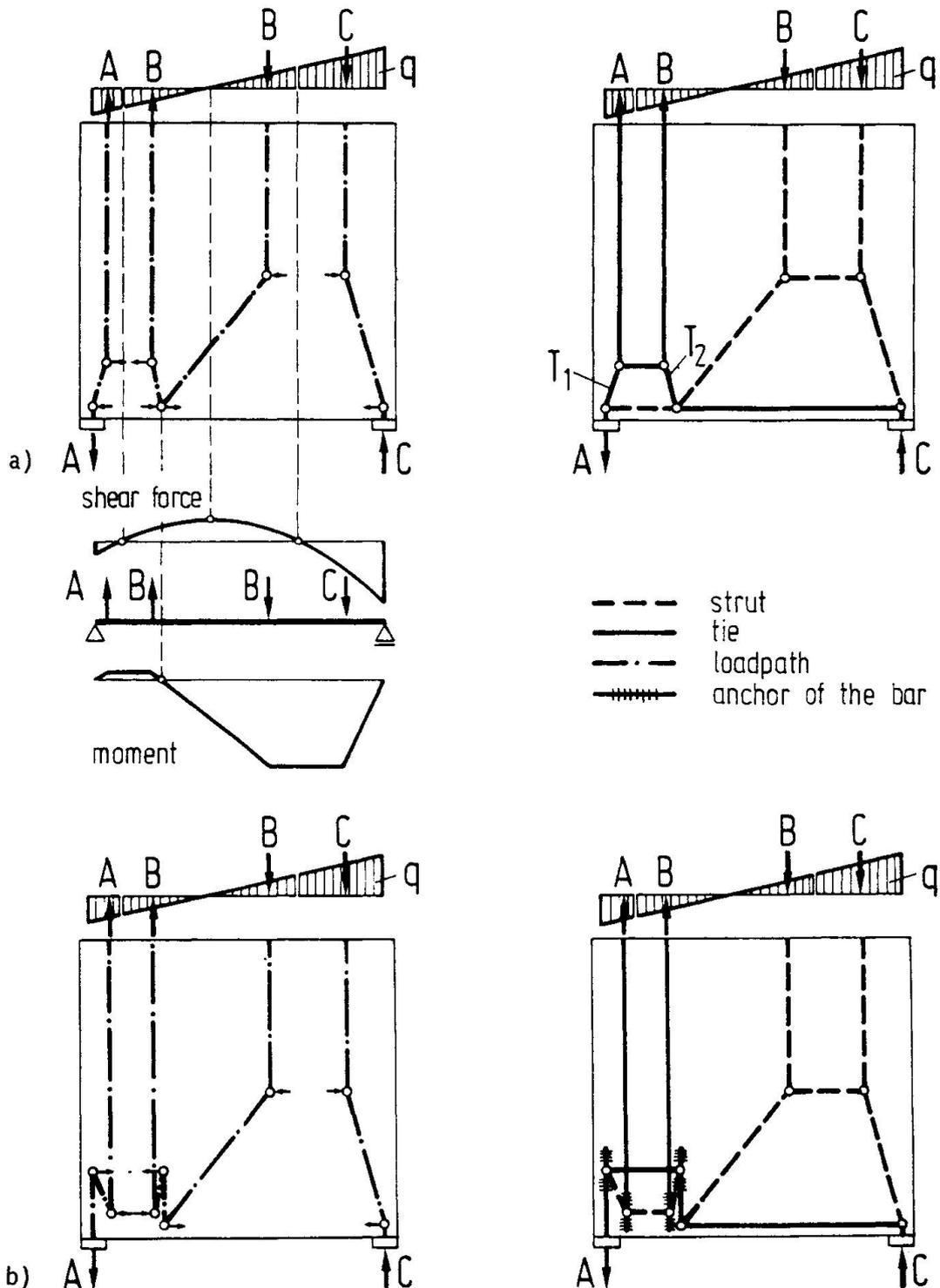

Fig. 5:   Application of the load path method for finding the appropriate strut-and-tie-model. Two models for the same case: (a) requiring oblique reinforcement; (b) for othogonal reinforcement.
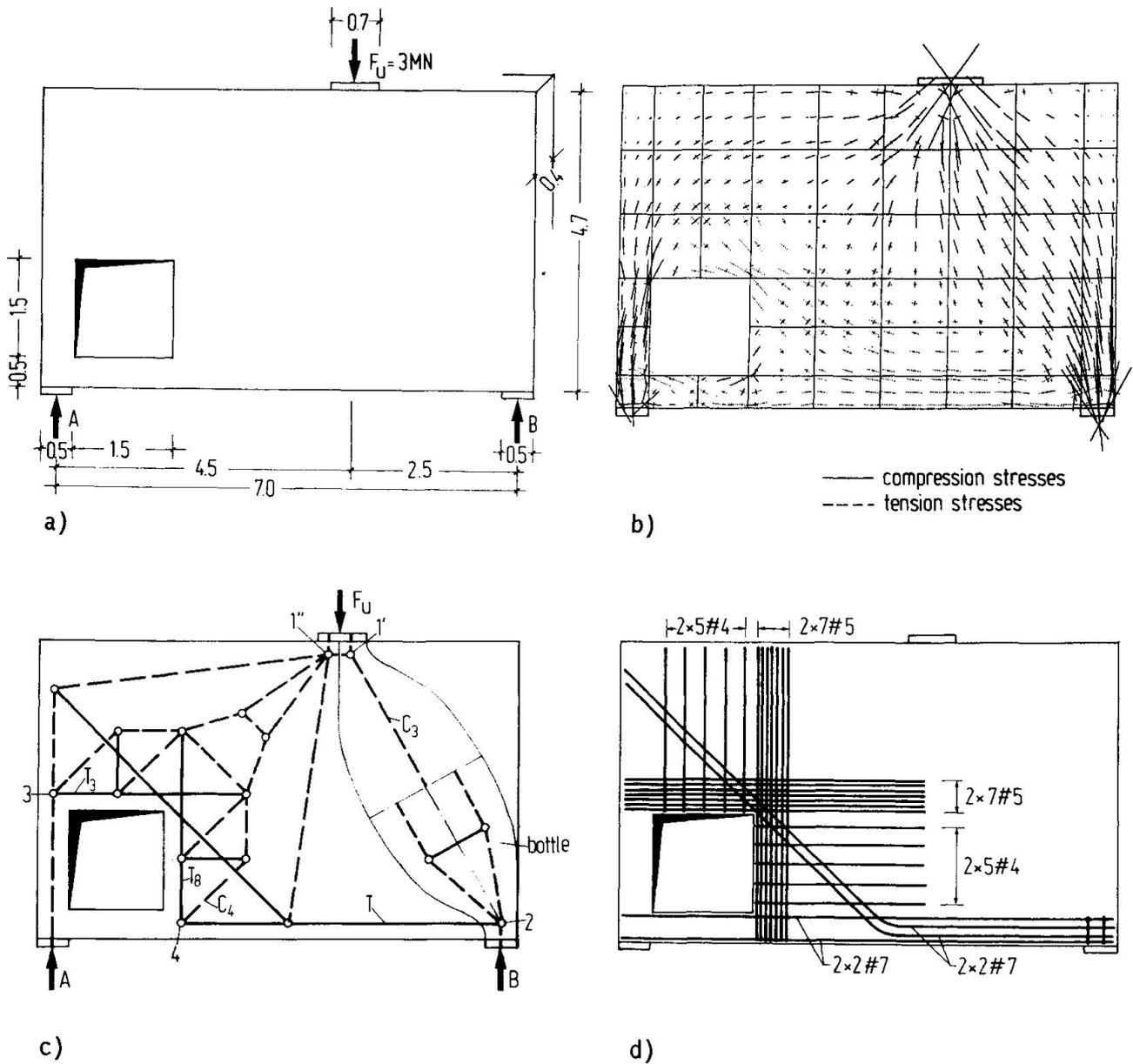
Fig. 6:   Deep beam with a large hole. (a) dimensions /m/ and load;
          (b) elastic stresses; (c) complete strut-and-tie-model;
          (d) reinforcement.

Recently the fact that the STM-dimensioning is a combined graphical and analyti-
cal method has led to very useful CAD programs, which permit to develop, optimi-
ze and dimension STM on the screen /9,10/. This also opens the door to not only
dimensioning D-regions but also to analyse them by attributing non-linear con-
stitutive laws to the struts and ties thus being able to evaluate the deformati-
ons and the redundant inner forces for statically indeterminate supported deep
beams /11/. Comparisons of such analyses with test results on the one, and non-
linear FEA on the other did yield promising results. Rückert and Sundermann will
specify that further during the Colloquium.

There has been some dispute on the so-called ambiguity of the STM, mainly from code-makers running after cookbook recipes. It's not the STM, it's reinforced concrete itself, which has fortunately the capability to adjust its inner flow of forces to the designer's reinforcement layout. A complex and intelligent material belongs into the hands of an experienced designer. He will find the right STM for his specific case and will keep serviceability and ductility requirements in mind, when optimizing it towards ultimate load capacity.

Fortunately there is a lot of progress with the non-linear FEA of cracked rein- forced concrete /12/. A. Scordelis will come back to that during his invited lecture on analysis. Thus the designer has the tool to review his STM results, from which he of course has to collect the reinforcement layout before doing a FEM check. Comparing both results will have a high pedagogical value and avoids misinterpretations of black-box computer outprints.

Such a procedure should be followed as a golden rule: Dimensioning on basis of relatively simple models, thereafter rewiew on a suitable level of sophistication.

Non-linear FEA appears to be of special value, if the overall deformational behaviour of a deep beam or the reactions of a statically indeterminate suppor- ted deep beam structure is asked for. It will also be able to describe and clearly trace failures of concrete in compression or tension as well as of the reinforcement. For that of course it must be possible to model the real crack pattern, especially discrete cracks often responsible for failure. But doubts arise with respect to its capability of describing the behaviour of nodes. For that purpose it would be necessary to computerize the concrete at a microlevel i.e. to follow with the finite elements down to the gravel and reinforcement ribs.

From that it follows, that even a FEM analysis should be followed by a STM check especially with respect to the safety of the nodes (fig. 7).

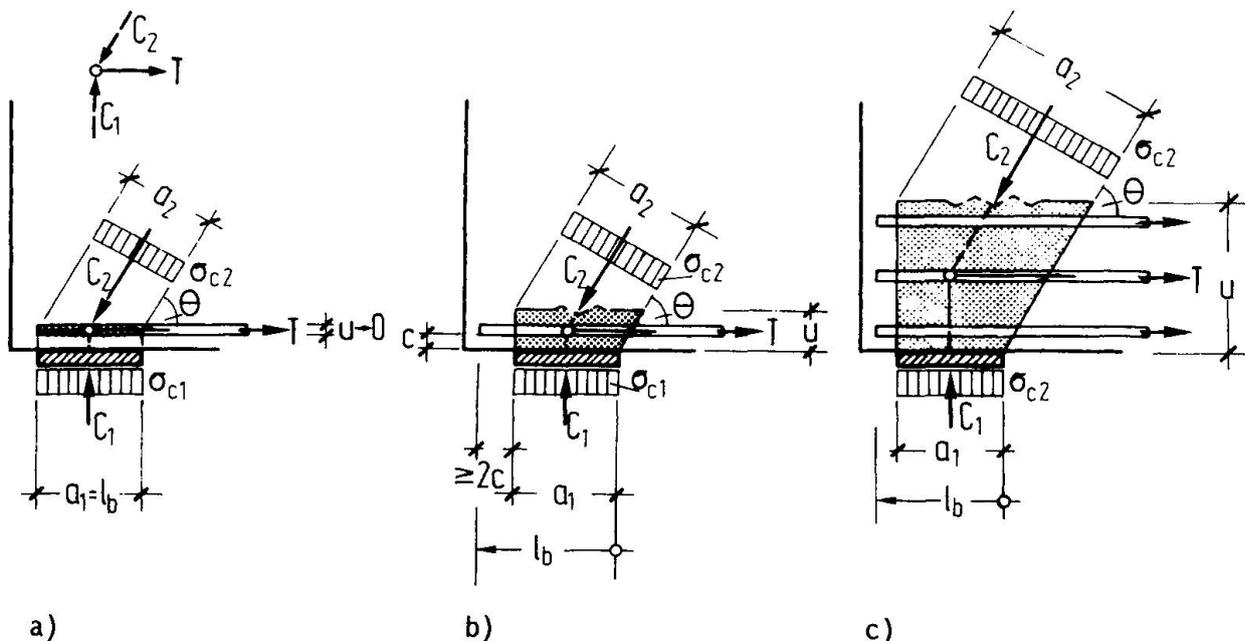a)   b)   c)

Fig. 7:   Typical node for the anchorage of reinforcement; (a) one layer; (b) one layer with additional length behind the node; (c) three layers with additional length behind the node.

4.3 Models for slabs and folded plates

Since these structures may as well be sub-divided into B- and D-regions, the same models and methods as discussed above may be applied as well. In fact they predominantly consist of B-regions (plane strain distribution). Starting from the sectional effects of the structural analysis, imaginary strips of the structure can be modelled like linear members.

However, it would be desireable to develop a real STM approach which considers that the principal moments and forces of slabs do not follow straight lines parallel to the edges.

Further there is no satisfactory model as yet describing punching of slabs. For the large variety of slab shapes with all kinds of openings it is very helpful, that today FEM programs on linear-elastic basis are available to any designer. Since slabs rarely do reveal substantial cracking, it may not be very desirable to repeat such an analysis with non-linear FEM. Rather will an overall ultimate capacity check by means of the yield line theory provide useful additional information.

4.4 Treatment of prestress

In a paper on modelling of structural concrete, a word on the treatment of prestress may be expected. However, it appears sufficient to mention that consistency between reinforced and all "types" of prestressed concrete can easily be reached if for the analysis of the sectional forces prestress is simply treated, what it really is: a self-equilibrated outer load, though artificially applied. Whilst dimensioning, its forces are treated as are other forces. After grouting the prestressing steel will then assume the role of reinforcement (with an initial prestressing force and with special properties).

In case of prestress without bond or of external prestress after prestressing the tendons take the role of free ties whose changes of forces due to loads may be estimated or analysed on basis of a statical indeterminate system /5, 15/. Jennewein, in a contribution to this Colloquium will give further evidence of that.

With this the same models and methods as already discussed apply also to the case of prestress (fig. 8). This treatment helps to avoid useless discussions as those, whether the statical indeterminate moments due to prestress are restraint forces which disappear due to cracking or not. Of course they are not, they are moments as those due to any other outer loads which cannot dissappear but of course be redistributed. This view of prestress is a valid basis for the consistent treatment of structural concrete and for a simplification of codes.

a)   b)
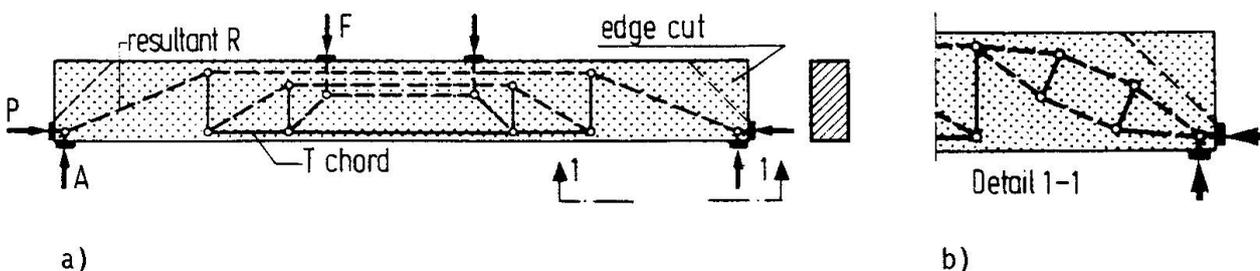
Fig. 8:   (a) Strut-and-tie-model of partially prestressed beam with
           rectangular cross section; (b) detailed strut-and-tie-model of
           the beam area, where the resultant is within the beam section.

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Modelling Philosophy for Structural Concrete

Logique de modélisation du béton structurel

Modelierungsphilosophie für Konstruktionsbeton

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SUMMARY

Firstly, the history of modelling in structural engineering is briefly covered. Subsequently, the basic features of a model are described and used as a guidance in assessing four packages of models: for plain concrete, for bond-related models, for force-transfer models through interfaces and for models depicting failure of compressive fields.

RÉSUMÉ

L'histoire de la modélisation dans le domaine du génie des structures est rappelée. Les propriétés exigées d'un modèle sont décrites et utilisées pour l'évaluation de quatre groupes de modèles influencés par l'adhérence, le transfert d'efforts à travers des interfaces, et des phénomènes de la rupture de zones comprimées.

ZUSAMMENFASSUNG

Die geschichtliche Entwicklung des Modellierens im Konstruktiven Ingenieurbau wird zunächst behandelt. Dann werden die grundlegenden Eigenschaften eines Modells beschrieben und als Richtschnur für die Beurteilung von vier Modellierungsvorschlägen genommen: für unbewehrten Beton, für Verbund, für die Kraftübertragung über Kontaktflächen und für das Versagen von Betondruckfeldern.

1. PREAMBLE

a) Design may be carried out just through experience, i.e. via a trial and error process. This used to be the way of structural engineering in the past; however, the very many of the structures of the past which had fallen down, cannot anymore tell us how risky and uneconomical such a procedure used to be.

The next step in design history, seems to be a hybrid procedure. Much was done by experience, but several structural parts were c h e c k e d by simple computations. As rudimentary as these checks might have been, for the first time they have made use of "modelling": Instead of building something and just see if it stands, little arithmetics was used on paper, as a substitute of reality; and this is in essence a m a g i c process!

Nowadays, the blend is the same but the second stage is getting stronger: A conceptual design always precedes, and an analytical procedure comes after (only an "apprentice sorcerer" would cancel the first stage, out of fanatism for just arithmetics). However, actual modelling keeps its somehow magic character as an i n t e r f a c e between the designer and reality.

b) What, then, is a model: A mathematical tool <u>predicting</u> the structural behaviour of a critical region or of a structural assemblage (*).

And how it functions: As an interface between the designer and reality, making use of an acceptable degree of abstraction and simplification.

Last but not least, how it may be built? Fig. 1 reminds the anatomy of modelling. "Formalistic" models are based on empirical data
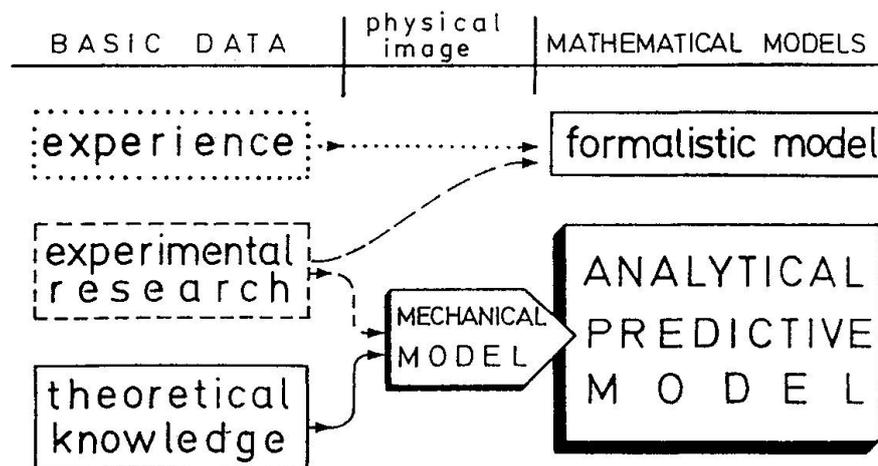

Fig. 1: The anatomy of model building

(*) Within this paper, distinction is made between a particular "analytical model" and a "design philosophy" which includes a system of compatible models; thus, the term "model" here is not used as in [1].

only, whereas "analytical-predictive" models are making use of physical knowledge on the function of the system considered. Despite the progress made, structural concrete may still be studied by means of formalistic models, especially in new fields; but the related lack of global understanding and the risk of possible gross-errors cannot be overemphasised.

Thus, every effort is justified towards **rational modelling**. And this Symposium is one of the best opportunities to enhance developments to this end.

2. REQUIRED FEATURES OF A MODEL

a) In Table 1 an attempt is made to inventorise the required features of a structural model in general. It is not within the scope of this lecture to elaborate on them; however, the same Table 1 offers a short justification of each requirement, as well as a description of the ways towards their achievement.

No	FEATURE	WHAT FOR?		HOW?
1	Rationality	■ Adaptability to broader fields or to future developments ■ To enchance communication and consciousness		a) Non-equivocalness b) Based on Mechanics c) Sound constitutive law
2	Accuracy	■ Predictiveness		a) Sufficient number of basic variables along the life-time b) Sensitivity analysis c) Checking through experiments d) Calibration through practice (model "maturity")
3	Reliability vs. uncertainties	■ Uniformity of safety level		a) Probabilistic analysis b) Experimental parametric study
4	Simplicity	■ Applicability	per se	a) Selection of main variables b) Acceptance of a level of inaccuracy
5	Compatibility with other models		within the system	(Through rationality)

Table 1: Required features of a model and how to get them.

It is apparent that some of these desired characteristics of a model are **contradictory** with each other. The main contradiction is related with the understandable claim for simplicity, which seemingly may be opposed to accuracy, compatibility and eventually to rationality. Thus a certain o p t i m i s a t i o n   is needed: The concept of **efficiency** of a model emerges here, with the following qualitative definition:

$$E_f = \frac{P_r}{C} \tag{1}$$

where E_f = the efficiency of the model
   P_r = its predictive capacity, with however $P_r \nmid P_{ro}$
       (i.e. a minimum of necessary predictiveness required)
   C = the complexity (or the application costs) of the model.

b) Within the preceding short analysis, no distinction was made between "research" models and "design" models (see [2] §2): Depending on

- the importance of the structure,
- the complexity of expected actions, and
- the stage of design,

several accuracy and sophistication levels of models may be used in design. Schlaich [2] rightly points out that "review is the play ground of sophisticated modelling techniques, and even research models may be applied". To say the same thing in terms of Equ. 1, for a given efficiency level, higher complexity is tolerated if higher predictiveness in needed.
It is hoped that these introductory comments may be of some value in assessing the suitability of models of structural concrete to be used now or in the future.

c) We should not end this section without a clear statement regarding design "by testing". In fact, there is sometimes a tendency to skip-over modelling and go back to the rather ancient situation (§1.a) when design was based on "build and see" (in our case "test and see"). That is why I maintain that such a tendency is rather r e t r o g r a d e, despite its seemingly "pragmatic" appearance: Out of the nine prerequisites to achieve Rationality, Accuracy and Reliability (Table 1), only a couple of possibilities are offered by just direct testing....

But even if the intercession of a model is recognised, modelling by testing runs considerable risks, as i.a.:

- Several actions or influences expected during the intended lifetime, might be overlooked.
- The in-time variation of basic variables, may not be accounted for in laboratory testing (e.g. concrete tensile or compressive strength degradations, or cyclic nature of loading or hygrothermal conditions).

That is why, a "prior calculation model" should a l w a y s be sought (if unknown) by means of physical knowledge and appropriate parametric experimental investigations.

Last but not least, the reliability handling of the deterministic test-results should be appropriately carried out; and, of course, in-life uncertainties are not represented by the in-lab scattering!

3. COMPATIBLE PACKAGES OF STRUCTURAL CONCRETE MODELS

In what follows, examples of some relatively rational and compatible models are discussed. Independently of their apparent complexity, these models are a m e n a b l e to further simplifications, precisely because they are rational: From a rational and complicated model, we may easily get a simplified one; whereas from a set of rules of thumb we could never produce a rational model with a broader field of applications.

3.1. Modelling of concrete

It was too simple to be true what was hoped in the past, i.e. to produce R.C. models in which the behaviour of concrete itself was oversimplified. We now understand that fracture mechanics' considerations for concrete under tension and even under compression (see i.a. [3]), confined concrete constitutive laws (see i.a. [4]), as well as local compression of concrete end-faces, are sine qua non for physical understanding and for subsequent rational modelling. Time-dependent effects should also be realistically modelled (see i.a. [5]).

3.2. Bond related models

A performance oriented Code (see i.a. [6]) should address the following issues within the serviceability limit-state design:

- Crack width control (be it for aesthetics or for durability reasons under severe environments, or for tightness)
- Deflections' control (for functional reasons).

Similarly, ultimate limit-state considerations include:
- Anchorage checkings, and
- Rotational capacity control in case ductility is governing.

Besides, in every analysis, the value of stiffness (or, better, a knowledge of hysteretic behaviour) is needed.

Inspite of the fact that all these phenomena are strongly b o n d-dependent, a **fragmentaristic** modelling is normally followed: In each of these **five** areas, loosely related or totally u n r e l a-t e d models are used. It is said that this is dictated by "practical" necessity, which may be true. But this violates the 5th principle of model-making (s. Table 1), i.e. compatibility, and it may lead to inconsistencies or indeed to gross-errors.

An optimisation between compatibility and simplicity could be sought by adopting a basic model governing all these areas ("local bond vs. local slip" constitutive law, as I will maintain), and subsequently coming down to practical simplifications. Even

simple formalistic rules may be derived, which however will keep track of the input data of the same initial model.

As a matter of fact, it has been proved that, despite its large variability, a "local bond stress versus local slip" constitutive law, via appropriate algorithms, is able to rationally produce complete information on the following issues (see i.a. [7], Fig. 2):

— Tension stiffening effects
— Cracks' widths prediction
— Force/elongation diagrammes of a tie under both monotonic and cyclic actions.
— Pullout (anchorage) force-slip diagrammes.

Of course, flexural behaviour is also influenced by compressive behaviour, but the modelling of compression is relatively simpler, both under unconfined and confined conditions.
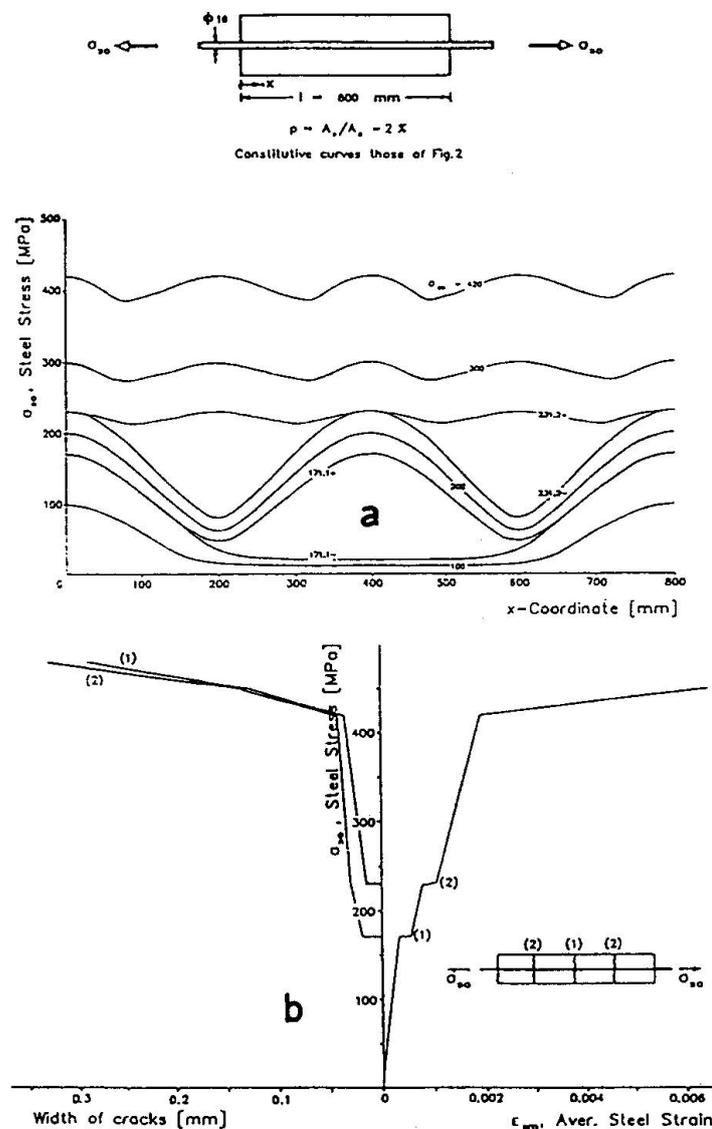
Fig. 2: Steel stress (a), and crack-widths (b) development during gradual loading of R.C. tie, up to post-yield levels

3.3. Force transfer through R.C. interfaces

Along predetermined interfaces (e.g. precast joints or repaired surfaces, etc) but above all along posteriorly cracked reinforced concrete areas, force transfer is secured by somehow complex mechanisms of:

- pull-out/push-in of steel bars,
- dowel actions,
- re-compression of precracked concrete, and
- concrete to concrete friction.

Modelling of the overall force (N, V) transfer across and along such discontinuous interfaces is of a paramount importance, since as a discrete crack approach (despite its seemingly complexity) offers considerable fundamental insight; and it is also amenable to further simplifications such as smear crack and the like. Among other problems elucidated by such a model development, the bearing capacity of biaxially loaded and cracked R.C. plate, may be better understood.

Based on appropriate input constitutive laws, such global modelling was described in [8], (Fig. 3).

Promising developments are expected along these lines, both for better insight and for more justified practical simplifications.

3.4. Failure of R.C. cracked compressive stress-fields

With the increasing tendency of using truss or struts and ties models in practical design, and with the tremendous development of non-linear finite elements method, the assessment of the bearing capacity of obliquely cracked R.C. region has become a crucial point in modelling.

Directly or indirectly, it has been repeatedly made clear that the bearing capacity of such compressive areas, both in the case of a web of beam or in a plate-element, is conditioned by essentially biaxial effects; one of the possible meso-levels interpretations, inspired by the model discussed in §3.3, is illustrated in Fig. 4. Actually, one of the most practical ways to account for these effects is to consider the transversal tensile strain, and reduce the longitudinal compressive strength accordingly [9].

However, it has to be admitted that for such an important issue, the actual state of knowledge and the level of rationality achieved is not the best we could hope. That is why, several solutions are offered and a continuously better insight is gained (see i.a. [10]).

It seems that all goes as if a macro-level constitutive law of concrete under compression were applicable, with modifications as suggested in [11], (see Fig. 5), which may lead to considerable reductions of both strength and ductility, However, the computational determination of an average transversal stress or strain (for different cases of crack angles, different patterns of reinforcement and different loading histories), remains a challenge.
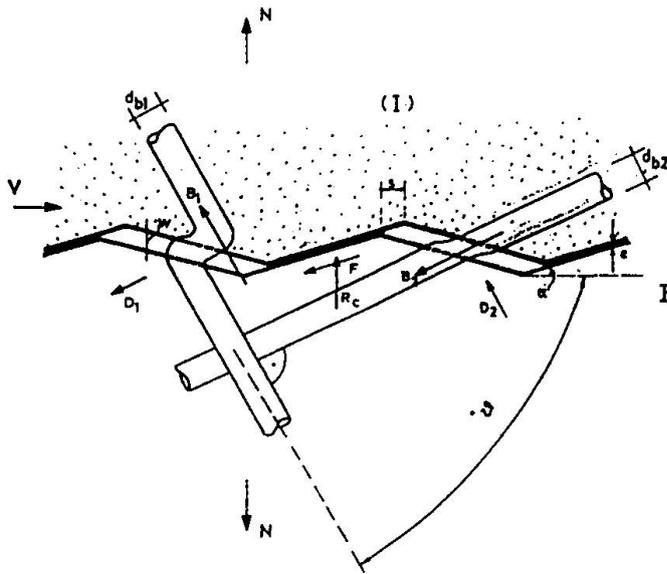
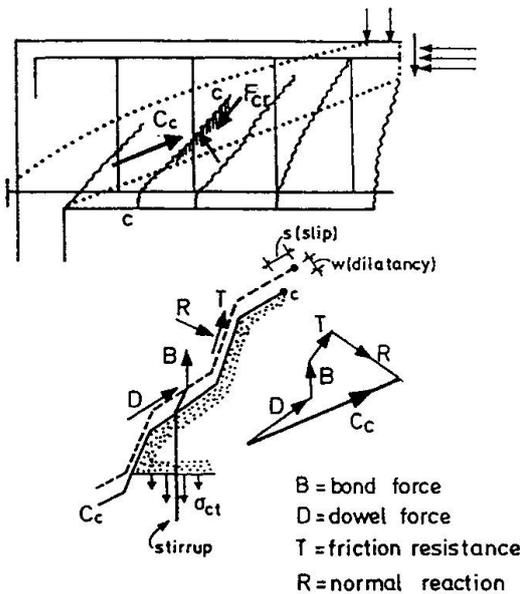**Fig. 3:** Topology (a) and force-displacement output (b) of an interface model

B = bond force
D = dowel force
T = friction resistance
R = normal reaction

Fig. 4: The pseudo-uniaxial compressive capacity C_c of the strut, is governed by the ultimate shear-transfer capacity F_{cr} along the initial crack c-c (and, consequently, by the angular distortion of concrete)
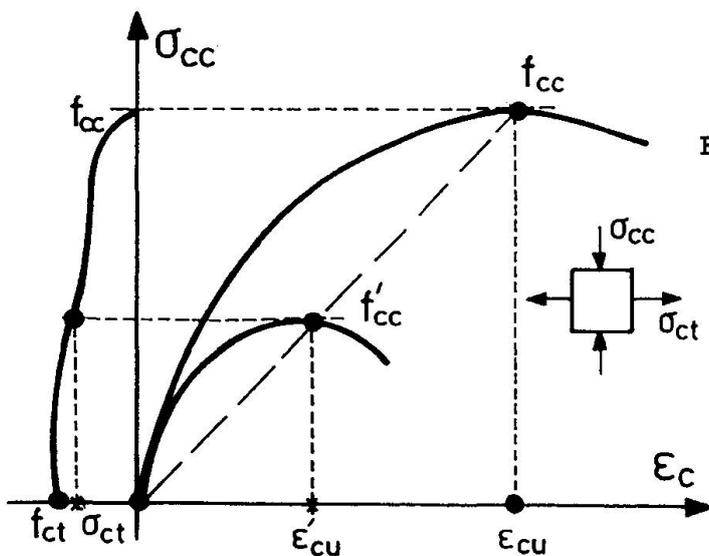
Fig. 5: Transversal tensile stresses, σ_{ct}, modify the constitutive law of plain concrete under longitudinal compression

In the meantime, some design applications are based on rather rough approximations (e.g. $f_c = 0,6.f_c$, etc). True, they are covered by calibrations against global experimental results of shear strength of R.C. beams. But the modelling needs definitely a further insight, especially in D-regions where compatibility cannot always be disregarded.

4. INSTEAD OF EPILOG

Modelling of structural concrete is now becoming a Science. But it has to fulfil so many, partly contradictory, requirements (s. Table 1) that it is not far from being an Art.

And that is precisely what makes modelling so attractive and so doubtful.

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Role of Experiments in a Consistent Dimensioning Concept

Rôle des expériences en vue d'un concept de dimensionnement

Experimentelle Prüfung von Stahlbetonbauteilen

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SUMMARY

The present paper deals with the possibilities and limitations of experiments in the field of structural concrete elements. With the background of the rapid development of computer, and simulation techniques, an attempt is made to define the importance of experimental tests both for past and future research in structural concrete.

RÉSUMÉ

L'article traite des possibilités et des limites de l'expérimentation dans le domaine des structures en béton armé. Dans le cadre du rapide développement des moyens informatiques, il s'afit de définir quelle a été l'importance des essais et expériences passées et quelle sera leur importance pour la recherche future.

ZUSAMMENFASSUNG

Der vorliegende Beitrag beschäftigt sich mit Möglichkeiten und Grenzen der experimentellen Prüfung von Stahlbetonbauteilen. Auf dem Hintergrund der sich stürmisch entwickelnden Rechen- und Simulationstechnik wird versucht, den Stellenwert der experimentellen Untersuchungen bisher und für zukünftige Forschungen auf diesem Gebiet zu umreissen.

1. INTRODUCTION

The present subject is tempting according to the given formulation to switch in a black and white mentality. On one hand there are practical engineers for which nothing than an experiment is better to confirm a constructive idea. On the other hand there are the computer specialists, becoming more and more numerous, which raise the impression to be able to simulate almost each experimental test. Encouraged by more and more powerful computers they propagate to replace each experiment by calculation.

I followed this development within the last 25 years with the eyes of an experimentator, therefore I am certainly a bit prepossessed. Nevertheless I believe that this activity allows to derive essential arguments for the role of experiments in the field of structural concrete.

I decided to restrict my argumentation to technical aspects. Therefore only some words at the beginning concerning financial problems. They are characterized by the situation in Stuttgart and therefore not completely transferable to other institutions.

Due to the separation of research and testing in two different organizations the costs for testing are expenses which have to be paid according to the fees of the testing institute. They concern mainly technical staff who is in general working without any personal interest in the research project. The theoretical and numerical work on contrary is performed by young engineers who were specially motivied by the task and are working more intensively therefore. Additionally only the immediate personal costs for this academic staff arize while their general costs are financed by the public budget. This unequal treatment of costs leads to an unbalanced comparison and should be taken into account in a realistic analysation of costs for economical research.

2. MATERIAL TESTING

Now I want to continue with the technical part of the problem. The material properties are essential for the design of structural concrete. Besides the physical capacities more and more the thermal and environmental influenced properties become important. In this field experimental investigations are necessary to determine fundamental properties as

- coefficient of expansion
- conductance
- penetration of gases and liquids.

Already at this point it is necessary to emphasize that usable test results are only possible together with well founded theoretical investigations. For example test concerning the absorptive capacity of concrete are almost useless without accompanying calculations

concerning the chemical and physical correlations to this problem (Fig. 1).
Except quality control tests for reinforcing steel there are no tests necessary to determine the strength properties. That is different for the ductility. Modern manufacturing processes may change properties as geometry of ribs, ductility and sensitiveness against chemical attack. So the practicability of these bars is influenced by reduced bond quality, moment redistribution and increased concrete cover (Fig. 2). Instead of the well established tensil test we need specific tests for characterizing these properties.
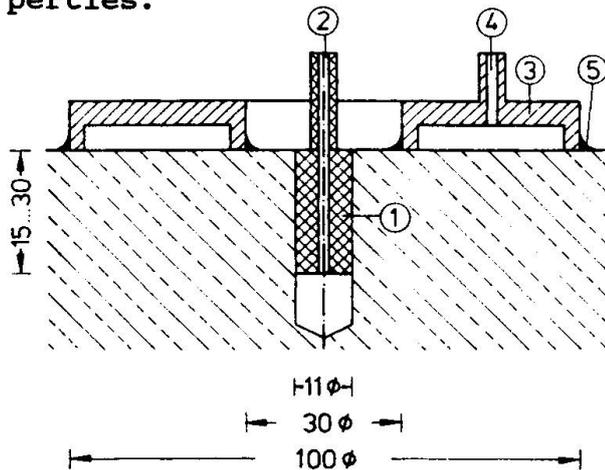
Fig. 1 MEASUREMENT OF PERMEABILIY
        ON CONCRETE SURFACES

1 Injection-padding
2 Gas-inlet
3 Gas-collector
4 Gas-outlet, to flowmeter
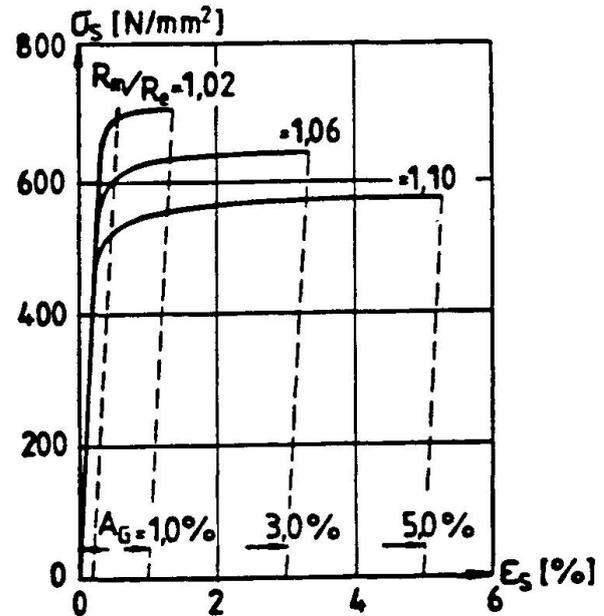5 Sealing

Fig. 2 SCATTER OF STRESS-STRAIN-
        RELATIONSHIP FOR WELDED
        WIRE-MESH

This necessity for specific tests is much more pronounced for concrete. Without regarding the problem of cube- or cylinder-strength there are only some words important concerning the tensile strength. Presuming their existance it is necessary to determine amount and scattering on the background of the different parameters. On the basis of statistical and probabilistic concepts this values have to be adopted by dimensioning concepts.

The correlations become even more complicated concerning the bond qualities of re-bars. In the first approach bond stress-slip-relationship, determined on pull-out-test were used for a realistic modelling of bond. Meanwhile the test specimen and the accompanying results became much more sophisticated according to the local situation near cracks (Fig. 3). This approach already indicates the problem of modelling for test specimen. How close to the real element should the model be designed, covering then only a narrow part of application? On contrary, if the model comprises a wide practical applicability are then the test results enough evident? These material properties are the tools for the designer to calculate the dimensions to carry the internal forces. There is no feed back to the determination of these material properties.
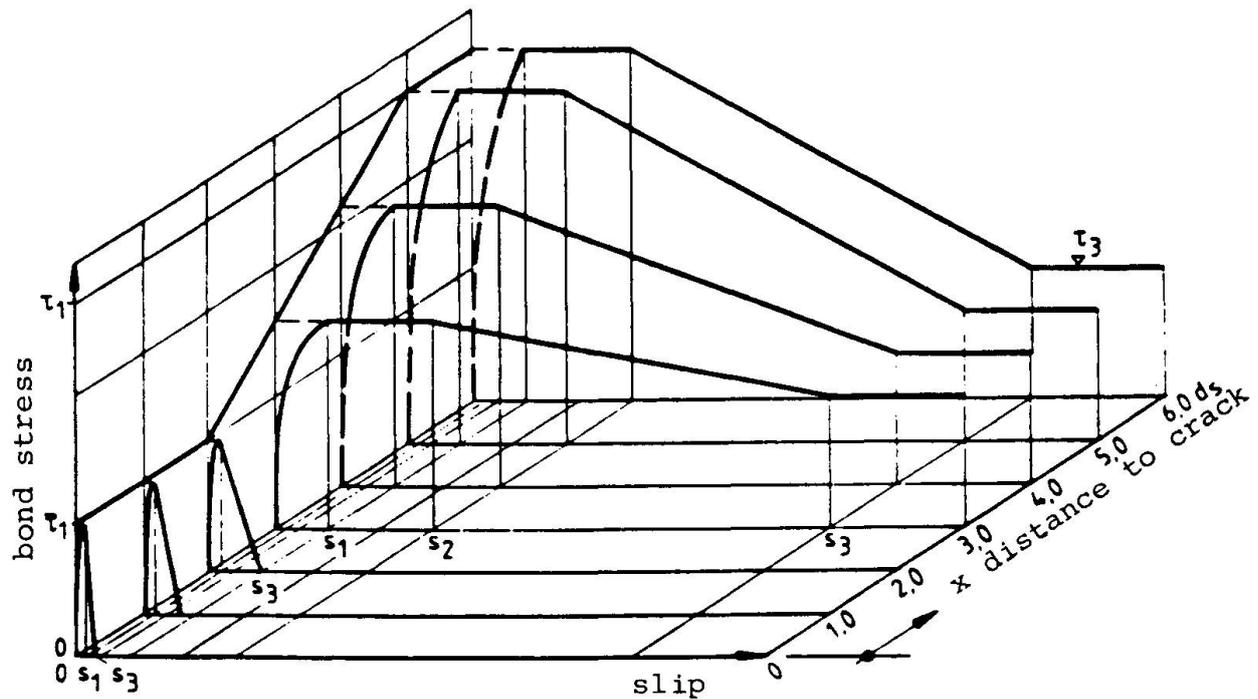
Fig. 3   BOND STRESS-SLIP RELATIONS

3. TEST ON STRUCTURAL ELEMENTS

3.1 General remarks

The main part of this article concerns tests on structural elements. This means all experimental investigations, where simple and complicated structural elements are modelled to test specimen in order to find out and explain the flow of the internal forces. Examples for such tests are:

> capacity of lapped splices;
> punching resistance of flat slab-column-connection;
> moment redistribution for beams and plates;
> indirect supporting and loading;
> combined loading of moment, shear force, torsion...;
> capacity of columns and long term loading;
> tensile stiffening of concrete;
> determination of restraint forces;
> combined action of loads and restraint forces;
> test on complete structures.

In the beginning of the application of reinforced concrete already the confirmation of theoretical research activities by experimental tests was quite usual. Especially Stuttgart was well known by the close cooperation between Mörsch and Graf.

The influence of the above mentioned experimental investigations on
the existing codes and dimensioning concepts are evident and it is
not necessary to explain this in detail. But at least from today's
point of view it is worth to ask if these results could also have
been achieved with other methods and if future problems should be
solved with more appropriate tools.

3.2. Some Remarks to conventional testing techniques

A critical review concerning the conventional testing techniques is
pointed out on 3 elements:

- designing of test models
- choise of appropriate measurements
- limits of validity for the rest results.

A suitable designed test model should be idealized from the real
element so that secondary structural effects are excluded
consciously to emphasize the specific subject. Unintentional re-
straints and mainly the concrete tensile strength are the most
important disturbing factors. The tensile strength can not always
artificially be excluded although it is neglected for almost all
dimensioning concepts.

The second problem concerns the measurements. During planning of
tests is has to be judged responsibly which kind of measurement are
appropriate without disturbing the internal flow of forces in an
unacceptable way. Remembering the fact that each measurement
includes a disturbance either these measurements have to be reduced
to a minimum or the amount of disturbance should be checked
realistically.
Furthermore the time for taking the measurements should be
realized. Improvement of measurement-equipment (strain-ganges,
LVDT, load-cells...) and modern computer controlled multi-switches
have solved many problems. Quite modern methodes as Laser, Holo-
graphie and Photogrammetrie techniques are also working rapidly and
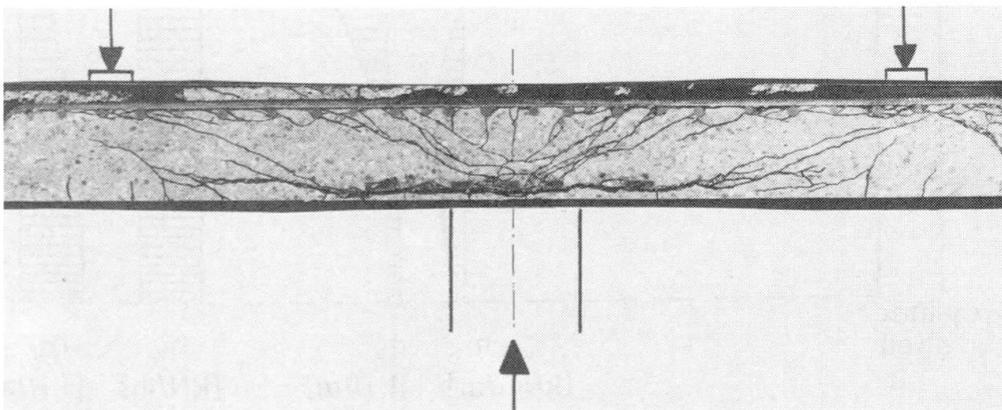guarantee a quick taking and storing of experimental data.

Fig. 4   CRACK PATTERN WITHIN A FLAT SLAB AFTER PUNCHING

Only crack measurements are a major problem in spite of the fact, that crack pattern represents a fundamental key to understand the internal flow of forces in structural concrete elements. There are existing some optic-electronic methods to take the crack pattern of visible concrete surfaces. But almost no method exists to determine pattern and crack width inside the test models. There is in general only one way to saw the test specimen after the test (Fig. 4). This is not only quite expensive but gives also severe problems in analysing the load-depending crack pattern and crack width.

The third remark concerns the evaluation of the test results. A single tensile test on a reinforcing bar out of a specific lot gives single results. Together with the well known scatter of this results the responsible engineer is able to classify this results. In case of doubt he has to test additional specimen. This judgement is not so easy for test results on structural elements. The purpose of such test mainly is to point out a specific aspect by means of different measurements without knowing the scatter of such measurements. Very often the interdependance of the different aspects are not well known during planing of the tests. This means, that not always the appropriate measurements were performed compared to the final stage of knowledge. Also secondary effects as loading or deformation rate, testing time for visco-elastic effects, restraint effects... remain unknown. Very often people are forced to publish numerous test data to demonstrate expensive experimental work. In this field the computer specialist is in a much better situation. Inopportune results of calculations may easily disappear in the waste paper box to be replaced by a new course of calculation.
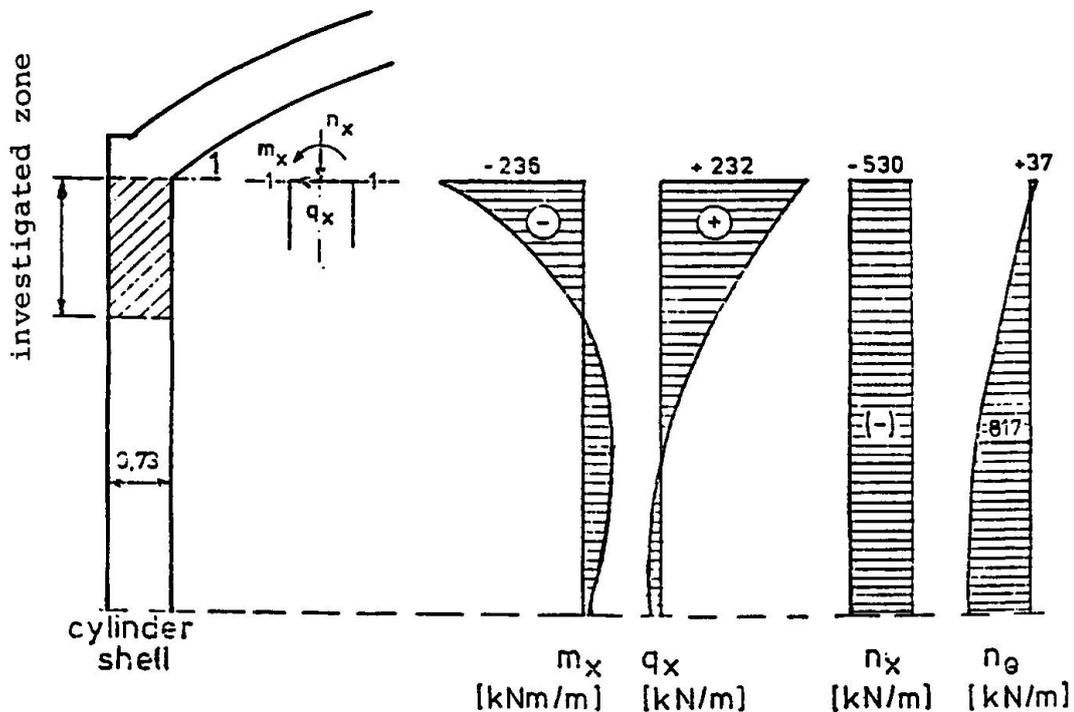

Fig. 5 DISTRIBUTION OF FORCES IN THE CYLINDER SHELL

3.3. Example

It is obvious that each example presented at this place will be criticized heavily and of course there will be more arguments if the tests of this example are carried out by the author in Stuttgart. In spite of this facts I want to present this example since only fruitful criticizme will help to overcome old way of thinking and helps to find better solutions.

Starting point of this research project was the problem of shear transfer at the intersection between cylindrical shells and cones or domes, which is often used for Off-Shore platforms (Fig. 5). Linear elastic calculations produced high shear forces at this corner; the required shear reinforcement was so high, that there was no practical solution to install this reinforcement. The aim of the theoretical and experimental work was to demonstrate
- that the bearing capacity of such high stresses regions are higher than given by present code and resp. or
- that the peak values of the shear stresses according to the elastic calculation were reduced due to the redistribution of the internal forces after cracking of the shells.

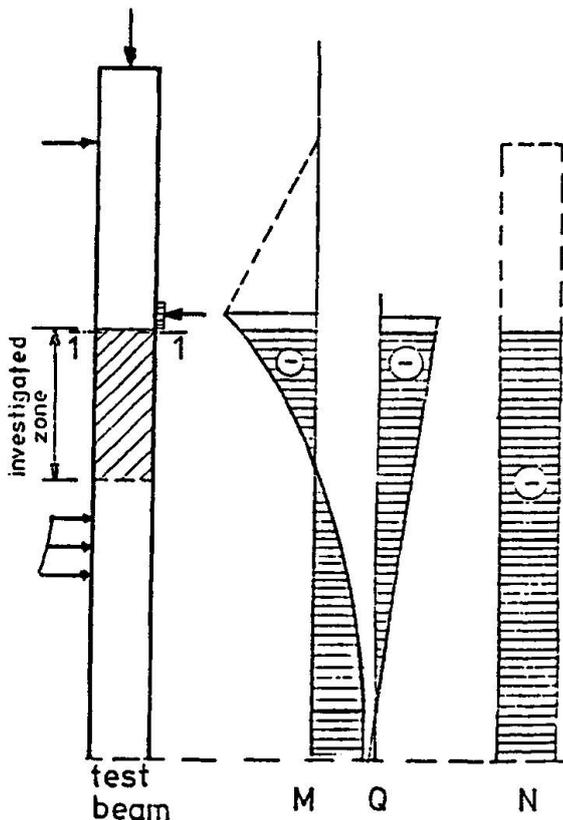The experimental work for this project was done in three steps.

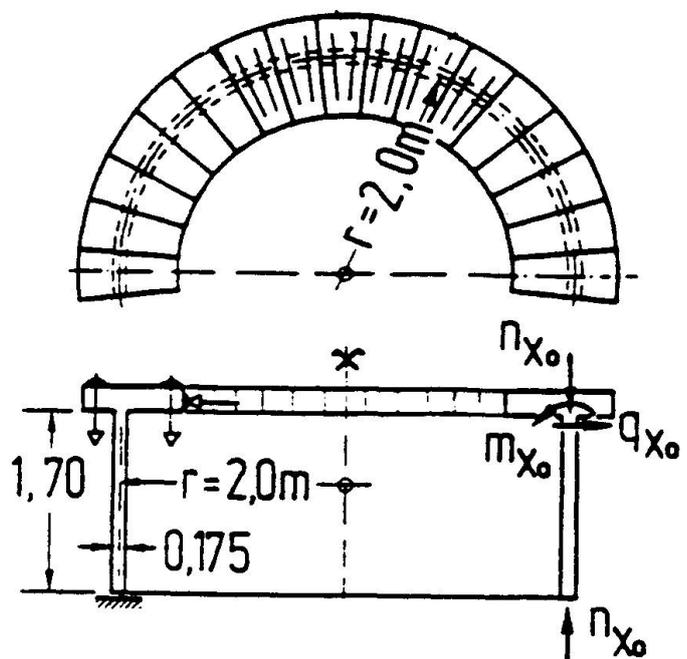Fig. 6   FORCES FOR THE BEAM-
CANTILEVER SPECIMEN

Fig. 7   CYLINDRICAL SHELL SPECIMEN
FOR UNIT FORCES

Step 1) Loading tests on wall-strips under the combined action of moment, shear- and axial force (Fig. 6).

Step 2) Loading tests on cylindrical shells under unit forces acting at the edge of the shell (Fig. 7).

Step 3) Loading test on a complete structure of a cylindrical and a conical shell (Fig. 8).

It is not the right moment to explain the numerous measurements and their results but I would like to explain some ideas which were predominant for the choosen solution of the test models.

Within step 1 the spherical problem should be linearized by neglecting the hoop action. By a realistic variation of the moment-shear-ratio the capacity of this corner should be determined. For the choosen test specimen the part of the conical shell was folded in the surface of the cylinder. So the test specimen was a single span plate strip with a cantilever arm.
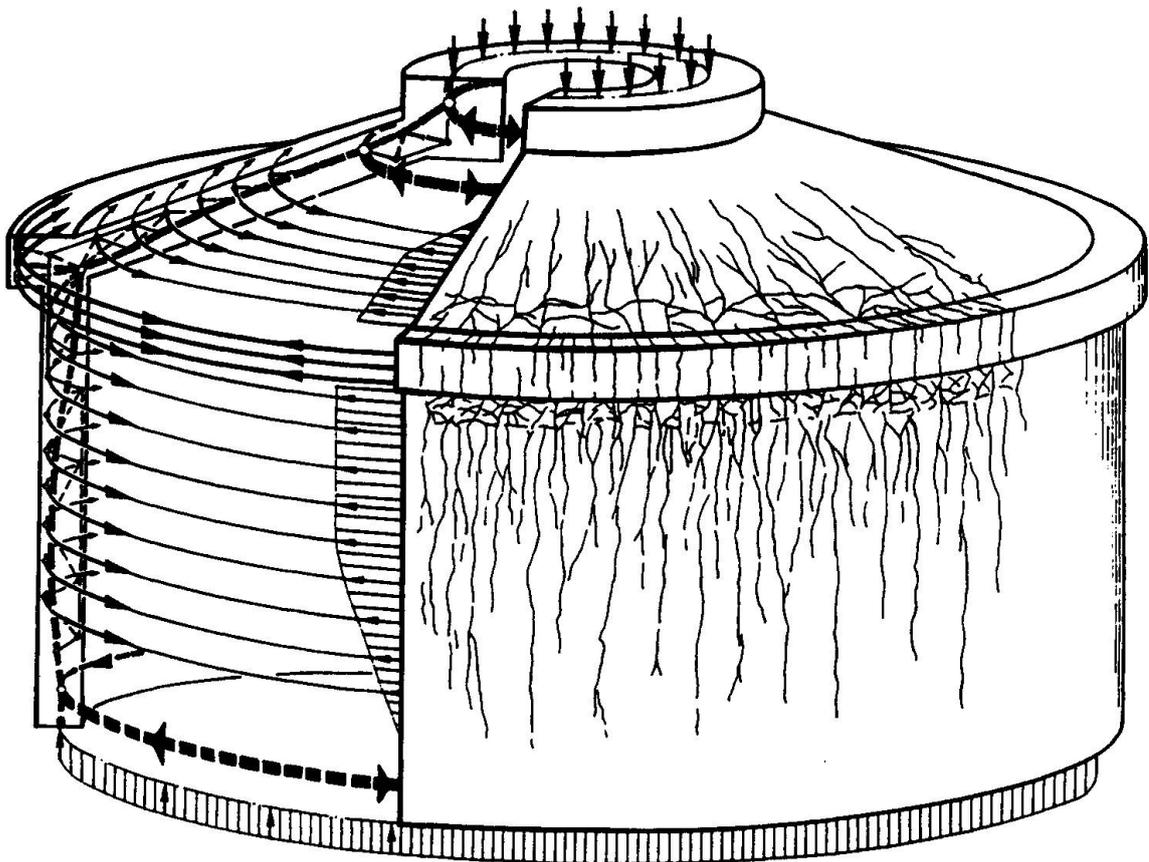

Fig. 8 CYLINDRICAL AND CONICAL SHELL-STRUCTURE

In step 2 besides meridian forces also hoops forces were produced.
For this reason cylindrical shells were loaded with edge-forces.
The three forces

- bending moment in meridian direction
- shear forces in radial direction
- normal meridian forces

could be applied either separately or in realistic combinations.

The final test step 3 demonstrated the load-deformation behaviour
of a complete structure where par example the edge forces and their
distribution into the different elements could develop according to
the actual stiffness after to cracking.

From a distance of several years since we performed these tests it
is quite difficult to judge if the former decision to carry out
this kind of tests was correct or not. There are no indications
concerning the economical alternatives to the choosen solution to
have objectiv arguments for this judgement.

4. SUMMARY AND OUTLOOK

During the pioneer time of reinforced concrete structures and also
during the huge application after the second world war tests on
structural concrete elements were necessary but also most expensiv
to show the flow of internal forces in order to draft realistic
codes and dimensioning concepts. Quite a lot of these results were
taken over without critical review. Many of this test can now be
simulated by computer programs. Some of this test results and their
interpretation seem to be problematic today.
For further problems there remains the question how to achieve new
knowledge and which will be the range of experimental investi-
gations. Obviously there are some main fields of activities.

- Stressing of structural elements not only caused by forces
  (temperature, environment, chemical...).

- Replacing of common materials by new materials.

- Better and more economical use of available materials including
  improved methods for calcuations and designing.

- Industrialisation of the building process.

It is unbelievable for me that all research activity which is
necessary for these fields can be simulated by computers. There
must be performed additional experimental work to do fundamental
research and to confirm other results.
The numerical checking of test results has indicated the weakness
of some kind of experimental work. For the future it is therefore
necessary to combine both research methods to achieve optimal
results.

There are three main fields for this cooperation:

1. Simulation of the behaviour of model specimen and real
   structural elements under realistic loading conditions.

2. Computer aided testing to comprehend complicated loading and
   deformation conditions.

3. Experimental testing of structural elements as a spot check of
   numerical calculations.

The testing institutes have to devote themselves to this dual job,
either by establishing a own group which is specialized for
simulation problems or they have to look for partners for such a
cooperation.

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Partial Prestressing with and without Bonding in Bridge Decks

Précontrainte partielle par câbles adhérents ou non dans les tabliers des ponts

Teilweise Vorspannung mit und ohne Verbund bei Fahrbahnplatten

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SUMMARY

Prestressed structures can be treated consistently with all degrees of prestressing. Basic criteria are given on how to select the appropriate solution as compared to the present practice of thinking in separate classes. This consistent approach is demonstrated for the design of bridge decks so that an optimum solution with bonded and unbonded strands can be given. It is shown that the amount of prestressed and non-prestressed reinforcement used in the slabs can be selected for various boundary conditions with consideration on aspects of reliability and economy.

RÉSUMÉ

Les structures précontraintes peuvent être traitées d'une façon cohérente à tout degré de précompression. Les critères de base permettant de sélectionner la solution optimale sont donnés, en comparaison avec la pratique actuelle des classes distinctes. Cette approche cohérente est démontrée dans le cas du dimensionnement des dalles de roulement, afin de présenter une solution optimale par câble adhérents au non. On montre ainsi que la quantité d'armature passive et précontrainte dans les dalles peut être sélectionnée pour divers conditions aux limites tout en tenant compte de la sécurité et de l'économie d'ensemble.

ZUSAMMENFASSUNG

Spannbetonkonstruktionen können einheitlich mit verschiedenen Vorspanngraden untersucht werden. Grundsätzliche Kriterien werden erläutert, wie eine geeignete Lösung zu wählen ist, im Vergleich mit dem augenblicklichen Denken in getrennten Güteklassen. Dieses einheitliche Vorgehen wird erläutert für den Entwurf von Fahrbahnplatten, damit eine optimale Lösung für Litzenspannglieder mit und ohne Verbund erreicht wird. Der Anteil an vorgespannter und schlaffer Bewehrung in den Fahrbahnplatten kann für verschiedene Randbedingungen unter Beachtung der Zuverlässigkeit und der Wirtschaftlichkeit gewählt werden.

1. Introduction

The question which appears every time when designing bridge-superstructures is: Which construction of the roadway slab in prestressed concrete has to be chosen for different boundary conditions taking into account durability and economy?

The general opinion which assumes that there is an increase of quality from reinforced concrete to partial prestressing and up to limited or even full prestressing needs to be corrected, because this simple point of view is not correct. This thinking in different quality classes must be overcome by summing up the whole range to structural concrete [1].

2. Degree of prestressing

The sign of the so far still differently named structures is the degree of pre-stressing κ. This degree is defined as the fraction of the whole sum of actions, which - together with the chosen prestressing - is leading to decompression at the unfavourable cross-section-fibre, this means a concrete tension zero.

Regarding beam structures under bending with axial force, this definition corresponds to the ratio of the internal forces - decompression moment to load moment -, which are related to the relevant kern point. The degree of prestressing has the following clearly defined boundaries:

$\kappa = 0$ reinforced concrete
$\kappa = 1.00$ full prestressing
$\kappa \approx 0.70$ to 1.00 limited prestressing

The partial prestressing covers the range of $\kappa = 0$ to approximately 0.70, because only for the κ-fold part of the complete actions in the service state there are compressive stresses in the whole examined cross-section.

The practical application shows that especially the degree of prestressing from 0.40 to 0.70 often results in constructively and economically favourable solutions. However, values of $\kappa \approx 0.40$ can only be used efficiently to improve the properties of reinforced concrete in the service state [2].

$$\text{degree of prestressing } \kappa = \frac{\sigma_{c,p}}{\sigma_{c,g+q}} = \frac{M_0 + N_0 \, k_0}{M_{G+Q} + N_{G+Q} \, k_0}$$

```
0              (0,4)           ≈0,7              1,0
|----------------|---------------|----------------|-------> κ
reinforced      partial         |     limited    | full
concrete                        |                |
                                prestressing
```
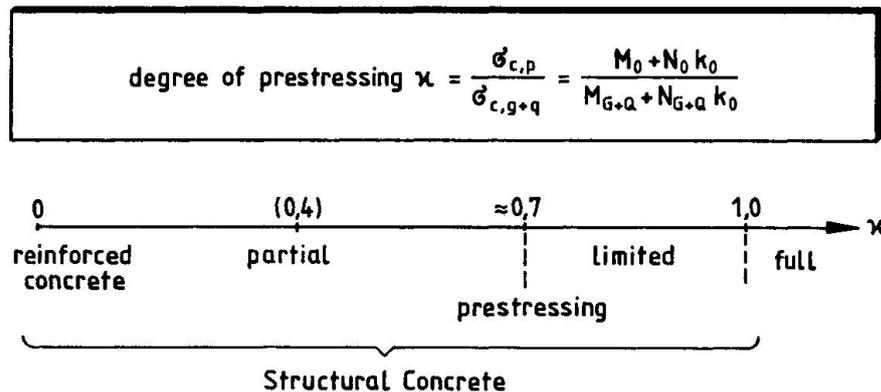Structural Concrete

Fig.1: Definition for the degree of prestressing and the range of structural concrete

For the whole range of the structural concrete, one could give the following recommendation: Only as much prestressing as necessary, in order to achieve a favourable behaviour in the service state by means of additional axial and transverse forces with regard to deflections and reduced crack formations. But not less reinforcement as reasonable, in order to assure the durability and the reliability of the prestressed reinforced concrete concerning crack width control.

3. Restrictions of the standards

The rules of the DIN 4227, which are actually used in Germany, contain different restrictions which limit the application of the partial prestressing to a great extent. In the following they are explained by the limiting values of the concrete stresses at the unfavourable edge of the cross section:

DIN 4227, Part 1 (full and limited prestressing) prescribes that the tensile stresses - resulting of dead load, imposed deformations and 1.0-fold live load - may not exceed the given values ($\approx2,5\div3.5$ N/mm^2) and that for the sum of the actions - including 0.5-fold live load - no tensile stresses appear.

In Part 2 (partial prestressing with bond) no stress checks are required, but it must be checked that for the actions including 0.5-fold live load the sheathing of the tendons is situated in the compressive area of the cross-section.

Part 6 (unbonded tendons) gives no limitations for the degree of prestressing. However, instead the general demand of the bridge authorities is relevant. This lays down that for the sum of actions and 0.3-fold live load - as the quasi permanent live load - at the unfavourable edge of the cross-section no tensile stresses might appear. This leads to the unintentional result that usually for all actions and 1.0-fold live load the tensile stresses do not exceed the values of DIN 4227, Part 1 and a planned crack formation does not occur (see the following examples).

Fig.2 shows the relation between the degree of prestressing κ and the usual ratio of live load moment to the permanent load moment, which in general is placed between 0.5 and 2.0. Moreover the required degree of prestressing, which is necessary to ensure that for the decompression moment the edge stress is zero, results with the quasi-permanent combination value ψ_Q [1] from the formula:

$$\text{decM} = M_G + \psi_Q\, M_Q = \kappa\, (\, M_G + M_Q\,)$$

This degree of prestressing results from the given simple hyperbolic formula. One can recognize three facts:
1. The transition from limited prestressing and partial prestressing does not appear at a constant value, but varies between 0.8 and 0.7 with increasing ratio of M_Q to M_G.
2. If - including 0.3-fold live load - no tensile stress is required, the degree of prestressing can only be reduced by the hatched part. This means κ between 0.7 and 0.55.
3. If one follows a proposal of Menn [3] for the normal design of roadway slabs in Suisse - prestressing only for permanent load, i.e. $\psi=0$ - the result would be a considerably greater constructive possibility. Then the degree of prestressing could be reduced to approx. 0.4.
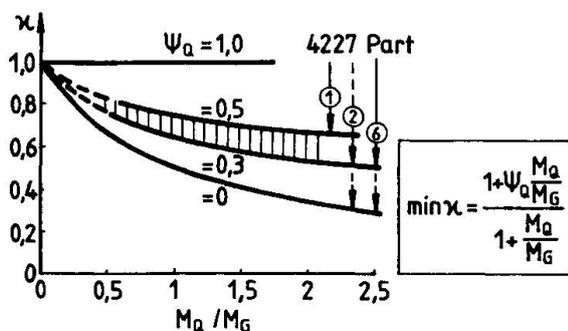

Fig.2:
Degree of prestressing for the decompression moment $M_G+\psi_Q M_Q$ with variation of ψ_Q
For bridges:
according to DIN 4227,P.1: min $\psi_Q \geq 0.5$
for Part 2+6 usually required: $\psi_Q > 0.3$

4. Application to roadwayslabs

For the construction and the design of roadway slabs one has to regard some peculiarities, such as the high percentage of live loads - M_Q/M_G - 1+2 -, the dynamic actions and the attack of deicing salt. Therefore different answers are possible to satisfy the three principal requirements: load capacity, durability and economy.

For the problematic characteristics of the different constructions, examinations were carried out [4]. Fig.3 shows the cross-sections - box girder and double T-beam - which were half of the size of a normal German motorway (BAB). Especially, the results of middle and large dimensioned cantilevers are very expressive as a decision-making help if and how much prestressing is necessary for the transverse direction of the bridge.

The length of the cantilever were changed from 3 to 4.5 m and the depth of the connection from 0.4 to 0.65 m. The other dimensions were adapted. They do not lead to unfavourable results.
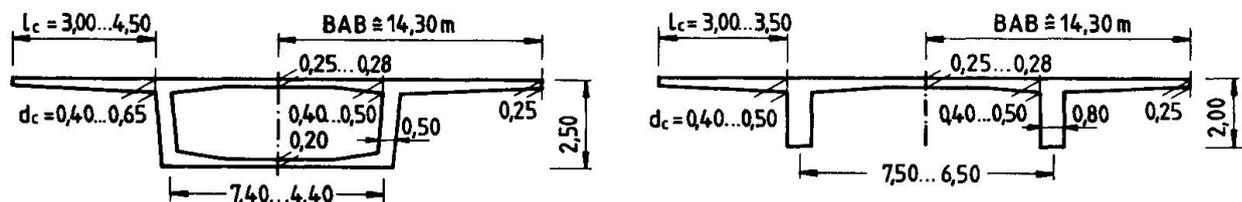

Fig.3: Form and dimension of the examined cross-sections

If the designer makes use of the partial prestressing, he has the best possibility to reach a construction with a reasonably increased amount of reinforcement and a sufficient amount of prestressing steel regarding load capacity and durability and, at the same time, a minimum of deformation.

The employment of tendons with bond requires special examinations regarding fatigue resistance and special corrosion problems - e.g. fretting corrosion and deicing salt effects.

If the internal transverse tensioning is carried out without bond, one will have the advantage of a durable corrosion protection and a larger allowable prestressing steel stress, but this construction leads to greater costs. I will leave the question beside, whether, at a later point of time, the tendons are actually changed or lengthened with widening of bridge superstructures.

Fig.4a explains the interaction between prestressing steel and reinforcing steel with different altitudes in the acceptance of the ultimate moment M_u for the concrete cantilever dimensions l_c=3.7 m and d_c=0.5 m. The sum of the A_p and the proportional A_s is shown versus the chosen degree of prestressing κ. The amount of the reinforcing steel is reduced with the ratio of the yield stresses of the reinforcing steel to the usual prestressing steel. This ratio - approx. 1:3.1 - nearly corresponds to the ratio of the costs.

You can recognize that for κ=0 and A_s=3.1·7=22 cm^2 the limit of a rational design in reinforced concrete is nearly reached. On the other hand you can see that from κ=0.5 the existing safety against rupture is greater than 1.75. If you take the additional contribution of the minimum reinforcement into consideration, the hatched saving of the prestressing steel will enlarge about this proportional amount of reinforcing steel. An economical and at the same time technical optimum is clearly situated at the partial prestressing with κ=0.5 to 0.6.
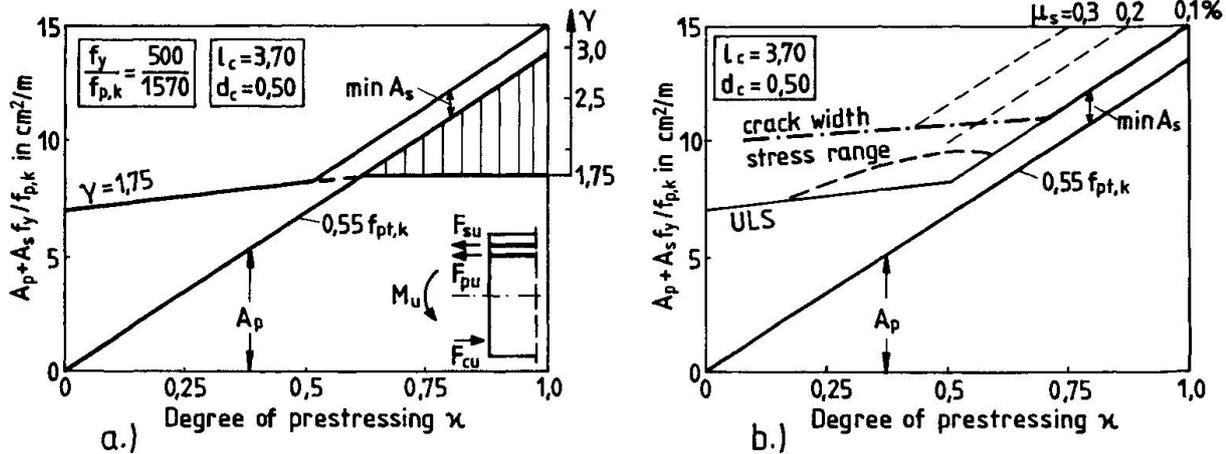
Fig.4: Proportional amount of reinforcement in a posttensioned cross-section
    a.) from the check of the load capacity (ULS)
    b.) from the check of the crack width and the stress range (SLS)

Among the transferred results from the ultimate limit state (ULS) the demands of the durability in the service state are supplementary analysed in Fig.4b. A substantial greater amount of reinforcing steel for the partial prestressing is the result of the crack width control according to DIN 4227 - paragraph 10.2, including ΔM - which is actually not yet conforme to DIN 1045. However, the crack width control is not responsive for limited prestressing (κ-0.75), so that you cannot make use of a reasonable percentage of reinforcement μ_s-0.2 to 0.3% for the load carrying capacity.

In the case of partial prestressing, the range of the stress amplitude of the prestressing steel might not exceed the reduced value of 110 N/mm^2 to guarantee the fatigue resistance. Moreover, Fig.4b shows that for roadway slabs this checking is not decisive and the required amount of reinforcing steel is smaller than the amount which results from the crack width control.

The demands and the knowledge which are explained in Fig.4a and 4b can be transmitted into design nomographs. These nomographs can be used as a help for the decision on the choice of the quantity and the sort of prestressing as well as for the design of cantilevers.

In the nomograph in Fig.5a you can directly see the amount of the prestressing steel which belongs to the chosen cantilever length, the cantilever depth and the requested degree of prestressing. The left dimensional line applies to $0.55f_{pt,k}$ for bonded prestressing, the right dimensional line is applicable to $0.7f_{pt,k}$ for unbonded prestressing. For the bridges which are carried out in Germany the following values result: Wannebach with κ-0.61: A_p=8.4 cm^2/m, Berbke with κ-0.66: A_p=7.4 cm^2/m.

From Fig.5b you can get the amount of reinforcing steel for the crack width control for prestressing with bond. For the example Wannebach-Bridge with l_c-4.25 m and d_c=0.62 m, you can pick out the value of 11 cm^2/m and with this the solid reinforcement of $\varnothing$12, e=10cm at the cantilever connection, whereas for κ>0.75 only 5 cm^2/m would be necessary.

From the nomograph in Fig.5c you can get the tensile stresses in the uncracked state. For reasonably chosen dimensions and κ=0.5 to 0.7 the maximal stresses are not greater than the tensile stresses which are allowed for the limited prestressing. For the Wannebach- and Berbke-Bridge they reach with 2 N/mm^2 only 60% of the allowed stresses from DIN 4227, Part 1. For this reason, a planned crack formation does not appear. Simultaneously, you can directly pick out the limit of a construction in reinforced concrete with κ-0 and the maximum permissible values of transverse bending.
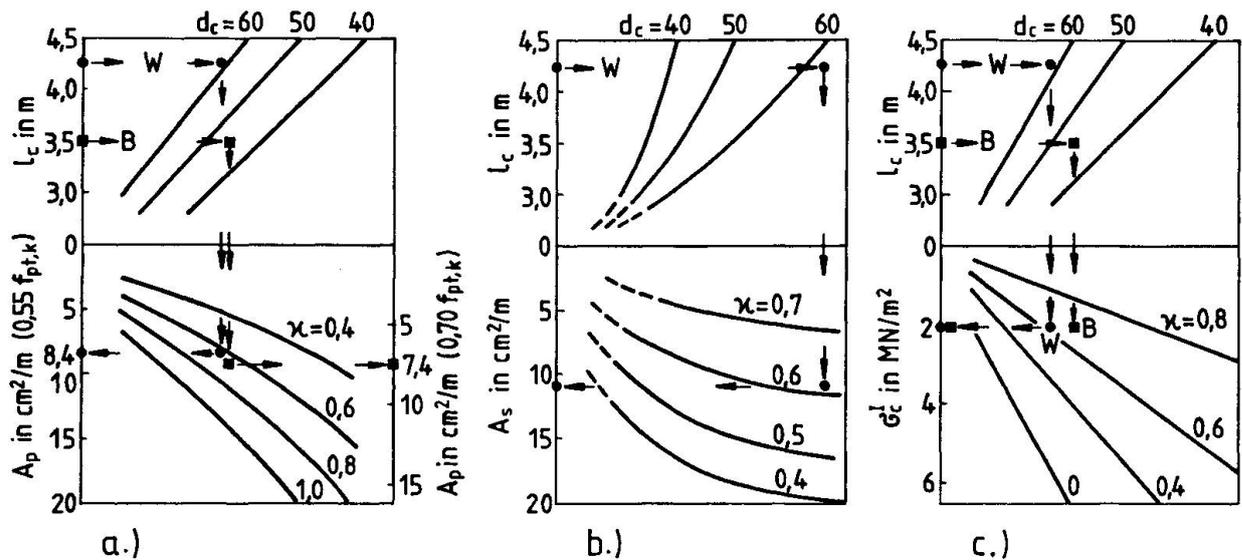
Fig.5: Nomographs for chosen cantilever dimensions (l_c and d_c) and the degree of
prestressing (W=Wannebach-Bridge, B=Berbke-Bridge as examples)
a.) Amount of prestressing steel at prestressing with bond ($0.55f_{pt,k}$) and
without bond ($0.70f_{pt,k}$)
b.) Amount of reinforcing steel of partial prestressing with bond
resulting from the crack width control (w_{min} for ø12 mm)
c.) Edge stresses of the concrete in the uncracked state

5. Summary

With these general diagrams, which are based on the examinations of M. Empelmann
[4], the designer has fundamental decision-making helps at his disposal to
answer the question, which was submitted at the beginning: Whether or how much
prestressing in combination with a sufficient reinforcement has to be chosen. In
order to reach an economical and technical optimum, the degree of prestressing
can be recommended to κ=0.5÷0.7 and the percentage of reinforcing steel to
μ_s=0.2%÷0.3%. Further details about the carrried out Wannebach- and Berbke-
Bridge with the consequences of the degree of prestressing can be taken from
[5].

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Simple Design Method for Partially Prestressed Concrete Structures

Méthode simple de calcul pour structures en béton partiellement précontraint

Einfache Rechenmethode für den Entwurf
von teilweise vorgespannten Betonkonstruktionen

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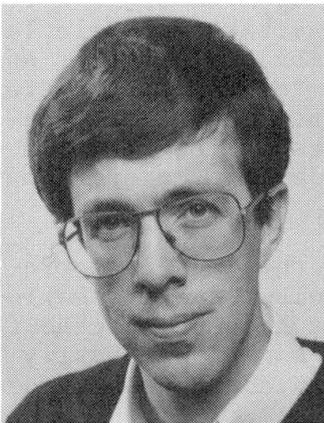
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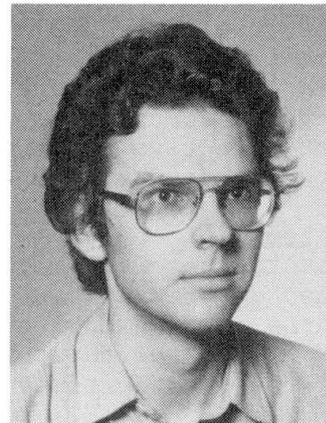
Cornelis R. Braam, born 1961, is senior-consultant at Molenbroek Civil Engineers Inc. in Rotterdam. He graduated as Civil Engineer M.Sc. at the Delft University of Technology. After joining the concrete structures group from 1985 till 1990, he received his Ph.D. degree in 1990.

SUMMARY

This paper presents an overview of a simple calculation method for concrete structures provided with a combination of reinforcing steel bars and post-tensioned prestressing tendons. The approach chosen relates closely to present theoretical modelling techniques aimed at a satisfactory approximation of the actual behaviour of structural concrete. The proposed method is illustrated by means of two statically indeterminate concrete structures: a rectangular girder for a warehouse and a box-girder used for a motorway.

RÉSUMÉ

Cette publication donne un aperçu d'une méthode de calcul simple pour des structures en béton pourvues d'armatures ordinaire et de câbles de post-contrainte. Cette approche a pour but d'obtenir une approximation satisfaisante pour le comportement du béton. La méthode est illustrée pour deux structures hyperstatiques en béton soit une poutre rectangulaire de magasin et un caisson de pont d'autoroute.

ZUSAMMENFASSUNG

In diesem Aufsatz wird eine einfache Rechenmethode für teilweise vorgespannten Beton vorgestellt. Die gewählte Vorgehensweise schliesst bei modernen Rechentechniken an, die zum Ziel haben, das Verhalten von Betonkonstruktionen so wirklichkeitsnah wie möglich zu beschreiben. Die vorgeschlagene Methode wird an zwei Beispielen illustriert: An einem rechteckigen Träger für ein Lagerhaus und an einer Hohlkastenbrücke für eine Autobahn.

1. INTRODUCTION

The design and behaviour of partially prestressed concrete structures have been discussed for many years [1,4,6,12,13]. However, the actual number of structural applications in The Netherlands is limited. Today, a similar situation exists in many other European countries with the exception of Switzerland [1]. Technical and economic reasons may hinder to practise research efforts, such as:
- Are cracks in concrete allowed if crossed by prestressing steel?;
- How should rather complicated calculation methods be coped with?;
- Which solution should be chosen and how does it affect the building-costs?

This paper pays attention to a rather simple calculation method applicable to crack formation in one-dimensional elements. First, the basic assumptions of the approach are briefly dealt with. Next, two structural applications are presented in chapters 3 and 4. A few conclusions are summarized in chapter 5.

2. BASIC ASSUMPTIONS
2.1 Distribution of forces

The amount of reinforcement needed in the various cross-sections is calculated on the basis of the theory of elasticity. No redistribution of forces is adopted in the ultimate limit state. The 'artificial' forces induced by prestress are either concentrated (anchorages) or distributed (pressure by curved cables). The level of effective prestressing includes losses due to friction and time-dependent material deformation which is assumed to develop unrestrained.

2.2 Cracking behaviour

The types of structural concrete may be characterized by the degree of prestressing K, defined as the ratio of the decompression moment and the maximum bending moment at the serviceability limit state [2]. Cracks are expected for K < 1.0 unless $\sigma_c < f_{ct}$ at the outer fibres. The cracking moment can easily be calculated if the effects of prestressing are taken into account, see figures 1a-b.
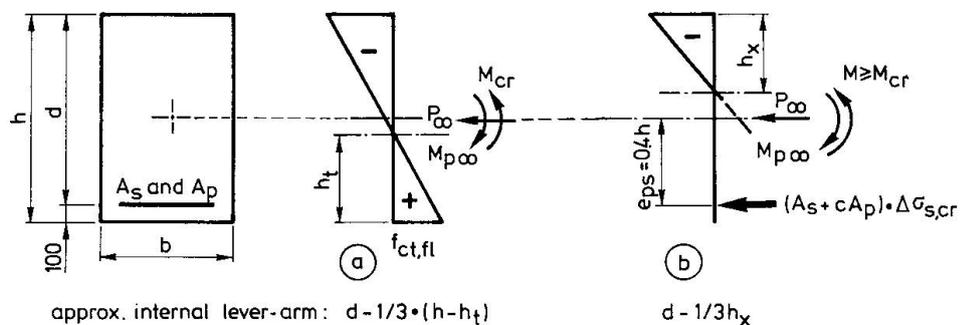

approx. internal lever-arm: d-1/3·(h-h$_t$)   d-1/3h$_x$

Fig. 1 Stress diagrams and internal forces (a) at and (b) after cracking.

The crack spacings and widths are found by means of theoretical models [3,4, 10,13] provided that the concrete cover is at least 1.5-2 times the largest bar diameter used. Either the pure or the flexural tensile strength is adopted as a cracking criterion for concrete. The crack width is controlled by the reinforcement. Generally, the bond stresses developed between concrete and prestressing steel are relatively low which is represented by c < 1:

$$\Delta\sigma_{p,cr} = c.\Delta\sigma_{s,cr} \qquad (c < 1) \qquad\qquad [MPa] \qquad (1)$$

2.3 Bending moment and shear force

The bending moment at the ultimate limit state follows from $M^* = \gamma \cdot M_{max}$ where γ denotes the structural safety factor including material as well as loading uncertainties. A minimum amount of reinforcement is used in each cross-section (see figure 2) to ensure a distributed crack pattern and a 'tough' structural behaviour. The contribution to shear transfer due to the tendon curvature demands a sufficient axial stiffness of the tension chord [7,8]. Thus:

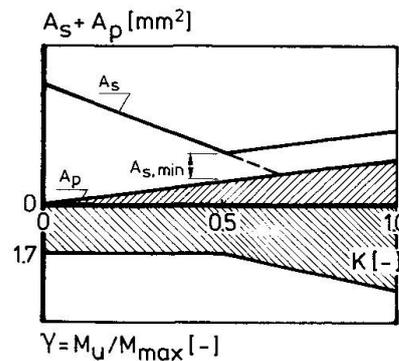$$A_s \cdot f_{sy} + A_p \cdot f_{pk} \geq V_d \quad \text{and} \quad A_s \cdot f_{sy} \geq V_d/2 \qquad [N] \qquad (2a\text{-}b)$$

Fig. 2 Reinforcement ratio and γ as a function of the degree of prestressing.

3. STATICALLY INDETERMINATE GIRDER IN PARTIALLY PRESTRESSED CONCRETE

3.1 Introduction

The calculation method presented in chapter 2 is now illustrated by means of a continuous girder with three spans of 12.6 m each. Prefab-slabs are used supported by the beam which is part of a warehouse. The beam and the columns are monolithically connected: their bending stiffness may be neglected for the design. The dimensions of the rectangular beam are restricted to $450 \times 1000 mm^2$: its spacing amounts to 4.5 m. The characteristic ($\gamma = 1.0$) distributed loading amounts to: $q_p = 27$ kN/m (dead load of girder and slabs) and $q_q = 64$ kN/m (live load on slab: approx. 12 kN/m²). A safety factor $\gamma = 1.7^q$ is applied. The concrete cover is 50mm for the prestressing ducts. The 95% upper-bound characteristic crack width is restricted to $w_k = 0.30$mm (reinforcement) or 0.20mm (prestressing steel) [5]. Material properties: 150mm cube compressive strength $f_{cck} = 35$; $f_{sy} = 500$ and $f_{pk} = 1860$ MPa.

3.2 Reinforced concrete girder

Fourteen 25mm diameter deformed steel bars are needed in Q, i.e. $\rho_d = 1.70\%$. In The Netherlands generally a ratio of 0.8-1.2% is economic for reinforced concrete beams with a rectangular cross-section. The computations according to [5] reveal crack widths $w_k \leq 0.30$mm which agreed with an analysis based on theoretical models [10]. Vertically placed 12mm diameter closed stirrups are applied at a minimum spacing of 110mm located at cross-section Q.

3.3 Girder in fully prestressed concrete

The schematic location of the prestressing ducts is presented in figure 3. The cable was stressed at both end blocks of the beam. A fully prestressed girder could not be achieved. A good approximation was found for:

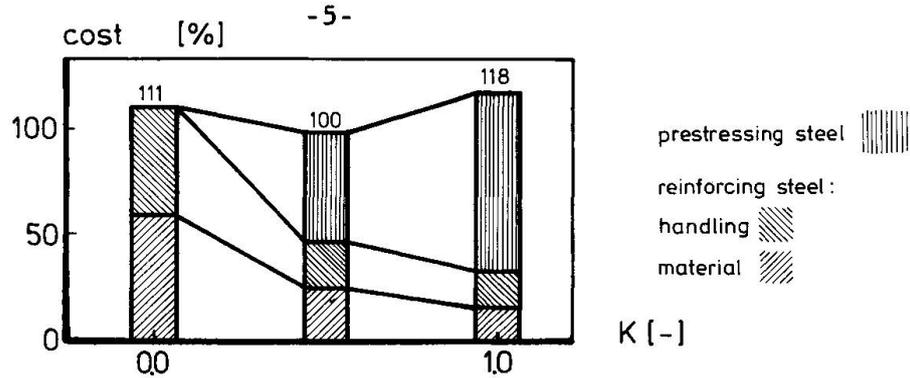$$e_1 = e_2 = e_3 = 400mm; \quad R_0 = 5000mm; \quad R_1 = 31100mm; \quad R_2 = 19800mm$$

and $P_0 - 2700.10^3$ N. Three post-tensioned elements are needed of six 12.7mm (1/2 inch) diameter strands each, thus $A_p - 3 \times 600 - 1800$ mm². Due to the 'secondary moment' the line of thrust is situated 85mm above the centre line of the tendon profile at cross-section Q.

Fig. 3 Prestressing tendon profile.

3.4 Girder in partially prestressed concrete

It is proposed that no cracking may occur (or: cracks remain closed) for $q + 1/3q_p$ which resulted in two prestressing elements with $A_p - 2 \times 600 - 1200$ mm². Additional reinforcement A_s should ensure the safety requirements. Next, the crack widths were checked: at Q, a surplus of seven 20mm diameter bars was needed. This is a reduction of 68% in comparison with the reinforced concrete girder, see also figure 4.

Fig. 4 Reinforcement in sections K, Q and M respectively for $0.0 < K < 1.0$.

With respect to the calculated crack widths, a factor $c - 0.40$ was implemented in eq. (1). The empirical formula for the crack spacing presented in [2,5] was adapted to cope with prestressing effects:

$$\Delta l_m - 50 + \frac{k_2 \cdot k_1 \cdot d_s}{4 \rho_{eff}} \qquad \text{[mm]} \qquad (3)$$

ρ_{eff} is calculated in accordance with various national codes. Eq. (3) is suitable for stabilized cracking. It followed that $\gamma - 1.9$ at K (midspan) and 1.8 at Q (support). The degree of prestressing K is at least 0.57. Vertical 12mm diameter stirrups at 300mm spacing are used throughout the structure.

3.5 Level of prestress and economy

The costs of materials (reinforcing and prestressing steel) and labour were estimated according to guide-lines presented by the Dutch building-industry (prices excl. VAT). An overview is shown in figure 5 for one girder.

Fig. 5 Calculated distribution of reinforcement cost for three levels of prestress (100% = dfl. 5200,- = US$ 2500,- dated Oct. 1988).

4. DESIGN OF A BOX-GIRDER BRIDGE IN PARTIALLY PRESTRESSED CONCRETE

4.1 Introduction

The non-linear analysis concerns a continuous 50m span box-girder bridge with 2x2 traffic lanes subjected to dynamic traffic load, see figure 6. The design live load consists of two distributed line loads of 9 kN/m each and one 600 kN heavy-truck traffic load distributed over three axes. Load transfer of the box-girder is only considered in the longitudinal direction. Material properties: $f_{ccylk} = 36$; $f_{sy} = 400$ and $f_{pk} = 1770$ MPa.

Each of the three webs of the box-girder contains six prestressing elements of eight 15.3mm (5/8 inch) diameter strands so that $A_p = 3x6x1120 = 20160$ mm^2. See also figure 7. At the support and at the midspan $A_s^p = 44400$ and 33000 mm^2 which implies structural safety factors of 1.7 and 2.3 respectively.
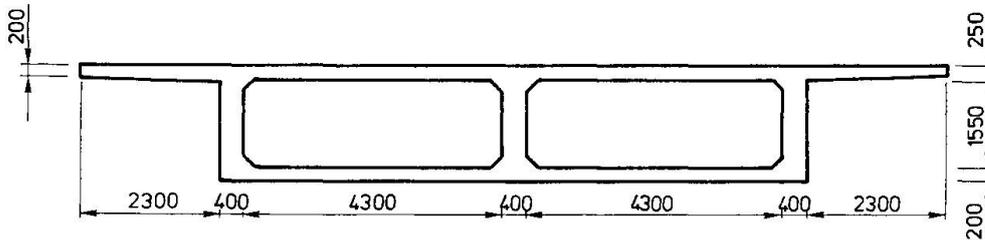
Fig. 6 Cross-section of the box-girder bridge.
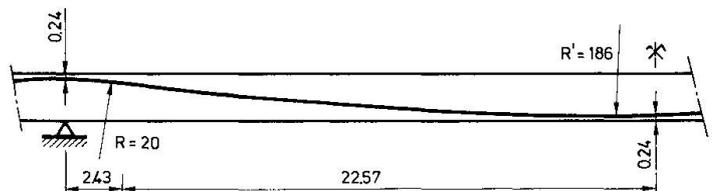
Fig. 7 Tendon profile of the prestressing cables.   dimensions in m

4.2 Development of cracks and steel stress variations

The average crack widths were calculated in two ways. A first approximation is based on an assumed cooperation of reinforcing and prestressing steel leading to a uniform cracking pattern. A second approach takes account of a concentrated location of the prestressing elements at the flange-web connection of the box-girder, causing a distributed cracking pattern. The average and the characteristic (95% upper-bound value) crack widths are respectively [3]:

$$w_m = \Delta l_m \cdot \epsilon_{sm} = \Delta l_m \cdot (\epsilon_s - \beta \Delta \epsilon_s) \quad \text{and} \quad w_k = 1.7 w_m \qquad [mm] \qquad (4)$$

where β incorporates reduced tension-stiffening $(\Delta\epsilon_s)$ by a dynamic or a sustained loading. A sensitivity analysis revealed that $\beta = 0.5$ fits closely to the actual structural behaviour. The permissible crack widths (section 3.1) are not exceeded. A variation of the complete live load is related to $\Delta\sigma_p = 70$ and $\Delta\sigma_s = 130$ MPa at midspan, see also eq. (1). The permissible values $(\Delta\sigma_p = {}^s104$ and $\Delta\sigma_s = 180$ MPa) are still empirically based.

4.3 Finite element analysis
The non-linear finite element program 'DIANA' was used in order to study the detailed structural behaviour of the girder, see in [2,9,11]. The computed longitudinal moment distribution was compared with a simple linear elastic approximation: the differences were less than 5%. The program provides a prediction of the cracking pattern at the very instant of structural failure.

5. CONCLUSIONS AND RECOMMENDATIONS
The analysis focused on partially prestressed concrete. Satisfactory results were achieved in comparison with reinforced concrete, such as: reduced crack widths and deflections, more simple detailing of the reinforcement. Moreover, the structure reacts rather insensitive to imposed deformations due to differential settlements and restraint of temperature movements or shrinkage. Applications may often be advantageous for high ratios of live to dead load or in case of a limited structural height. Partially prestressed concrete may also exhibit good economic prospects.
As stated in [4,13], extended research is needed to attain simple, consistent and reliable models which predict the behaviour of structural concrete. It may also enhance the introduction of uniform, realistic and clear design codes.

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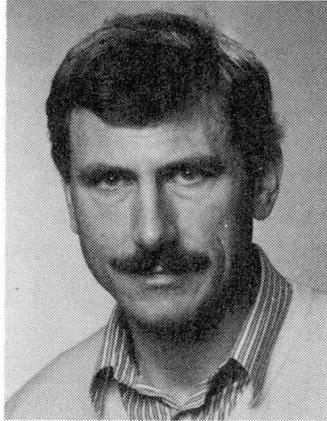
Some Remarks on the Analytical Treatment of Prestressing

Quelques remarques au sujet du traitement analytique de la précontrainte

Einige Bemerkungen zur analytischen Behandlung der Vorspannung

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SUMMARY

What is better: to handle prestressing as a self-strained condition or as a load? The answer to this question (shown by means of an example) is only obvious if prestressing is defined as the load which is produced by the hydraulic jack.

RÉSUMÉ

Vaut-il mieux considérer la précontrainte comme un état d'autocontrainte ou plutôt comme une charge? A l'aide d'un exemple, on montre qu'une réponse à ces questions peut être obtenue si l'on considére la précontrainte comme la charge produite par une presse hydraulique.

ZUSAMMENFASSUNG

Ist es besser, die Vorspannung als Eigenspannungszustand oder als Last zu behandeln? An einem Beispiel wird gezeigt, dass die Antworten auf alle Fragen nur einfach werden, wenn die Vorspannung als diejenige Last definiert wird, die mit der hydraulischen Presse erzeugt wird.

Some remarks on the possibilities to take "prestressing" into account in a beam with an unbonded cable (fig. 1) as an example.
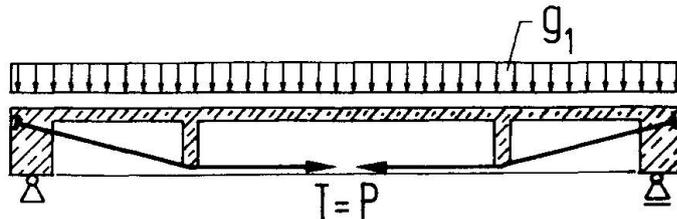
Fig. 1   Statically determinate structure during prestressing, loaded with P and g_1

During prestressing the structure is statically determinate, both internally and externally. The force $T = P$ in the cable is determined only by prestressing. During prestressing the dead load g_1 acts as well. You can calculate all the stresses due to prestressing $T = P$ and the load g_1. This is very simple, as you see.

However it is not simple at all if you look at it in the usual way, where prestressing is defined as the self strained condition, refering to the statically indeterminate structure after anchoring. To calculate the self strained condition, you must calculate the influence of the dead load g_1 separately in the statically indeterminate structure. You are only able to do so, if you give up the reality and if you imagine, that the load g_1 acts from the beginning (before prestressing) on the indeterminate structure. Then the force in the cable T_{g1} attributed to dead load g_1 is substracted from the real prestressing force P.

$$\text{"P"} = P - T_{g1}$$

This reduced force "P" degenerates conceptually into an imaginary parameter called "prestressing" without any practical quality. The prestressing force "P" is not a fixed value any longer. Don't tell the man at the hydraulic jack this value, if you want a correct prestressing! This parameter depends on the load g_1. If the cable will be bonded, this parameter will even change its value from one section to the other. Furthermore it depends on time. Shrinkage and creep due to the stresses of "prestressing" and even due to the dead load reduce the self strained condition, that is to say the value of the parameter "prestressing". It is common use to speak then about "losses" of prestress.

If the structure leaves the uncracked state, you get into more trouble. What's then the meaning of "prestressing" as a self strained condition? There is no meaningful explanation! The superposition or the subdivision in independant loadcases and a self

strained condition is no longer possible. Therefore the question, what does happen with the moment due to the prestressing, especially with the hyperstatic part of it in an externally indeterminate structure, and what does happen with the axial force due to prestressing, cannot be answered principally. It's pretty cold comfort to show, that the answer to that question is not very important with respect to the theory of plasticity. Equilibrium is still satisfied and compatibility is taken for granted on the "beautyfull" assumption that the materials are enough ductile.

As you can see now: the definition of prestressing as a self strained condition leads you to a complicate thinking, and you finish up a blind alley. It is able to lead you astray, if you want to have an answer on a question, which cannot be answered from this point of view. It is much simpler and furthermore generally valid to take facts as facts in their natural order. Let's start all over again.

During prestressing the structure in fig. 1 is statically determinate. The force $T = P$ in the cable is nothing but exactly the force of prestressing P in theory as well as in practice. The man at the hydraulic jack is only told this value. During prestressing the dead load g_1 acts as well. You can calculate all the stresses due to prestressing $T = P$ and the load g_1. You see, it is not at all necessary to know the self strained condition. By anchoring the cable, the statically determinate structure changes into an internally indeterminate one. Only the events which take place after anchoring as e.g. loading with additional dead loads or live loads and even shrinkage and creep act on that internally indeterminate structure and must be calculated accordingly for the actual stiffnesses of concrete, reinforcing and prestressing steel.

Any changes of stresses in the prestressing steel have to be added to its stresses due to the prestress P applied by the jack. For example, the loading with the additional dead load g_2 (fig. 2):
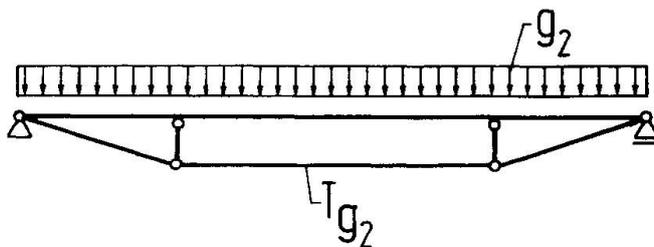

Fig. 2   Internally statically indeterminate structure loaded with g_2

The total force in the cable is now composed of the prestressing force P and of the tie force T_{g2} due to the load g_2 and amounts to

$$T_{total} = P + T_{g2}.$$

If the concrete is cracked, the actual stiffness of the beam can be taken into account. Evidently this stiffness also depends on the stresses in the beam caused by the earlier loading g_1 and P.

What about shrinkage and creep? They do not reduce the prestressing force P, which is defined as the value applied by means of the hydraulic jack once forever. There are no losses of prestress! In reality shrinkage and creep only cause a redistribution of forces between concrete and steel as it also happens in a reinforced column (fig. 3).
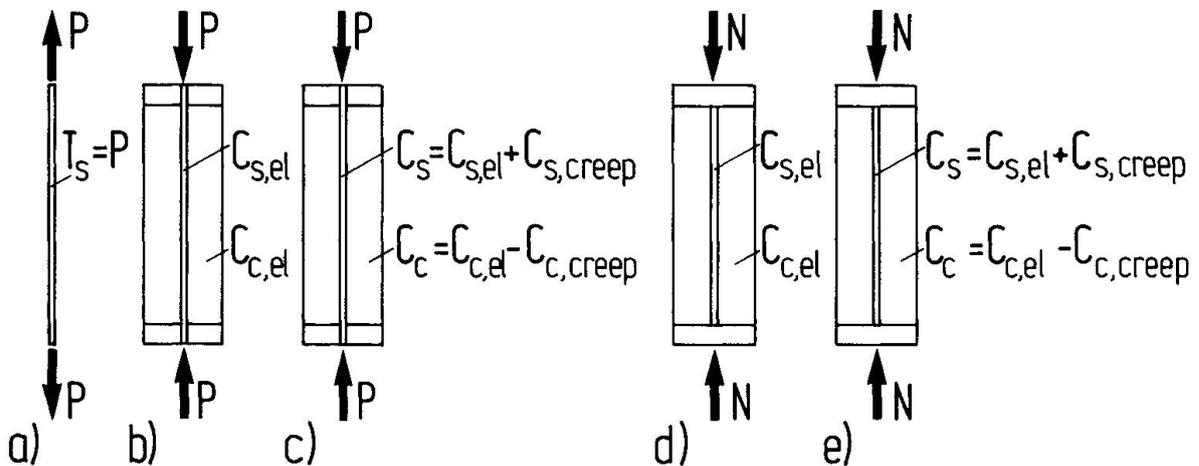
Fig. 3   a) Pretensioned steel member in the prestressing bed
   b) and d) same elastic behaviour of a reinforced
         concrete member loaded by the prestressing
         force P and a column loaded by the axial
         force N
   c) and e) same behaviour of the prestressed member and
         the column during creep

The steel member in fig. 3a shall be pretensioned in a prestressing bed with the prestressing force P. This prestressing force P acts as an external force on the reinforced concrete member (fig. 3b), reinforced with prestressing steel (fig. 3a) which therefore works like the steel in the column (fig. 3d). The compression force $C_{c,el}$ in the concrete and the compression force $C_{s,el}$ in the steel initially depend on the elastic stiffnesses of concrete and steel (fig. 3b and 3d).

$$C_{s,el} = P * (A_s*E_s) / (A_s*E_s + A_c*E_c) = P * n*A_s/A_i$$

$$C_{c,el} = P * (A_c*E_c) / (A_s*E_s + A_c*E_c) = P * A_c/A_i$$

Respectively in the column

$$C_{s,el} = N * (A_s*E_s) / (A_s*E_s + A_c*E_c) = N * n*A_s/A_i$$

$$C_{c,el} = N * (A_c*E_c) / (A_s*E_s + A_c*E_c) = N * A_c/A_i$$

When the concrete creeps, it reduces its stress at the expense of the steel, that is to say, the part $C_{c,creep}$ of the concrete force $C_{c,el}$ is transfered to the steel as a compression force $C_{s,creep}$. The total steel force therefore amounts to

$$T_{s,total} = P - C_{s,el} - C_{s,creep}.$$

This equation preserves all events according to their occurances in reality:

-   the pretensioning of the steel,
-   the application of the prestressing force onto the reinforced concrete member,
-   the redistribution due to creep.

In the extreme case of unlimited creep the compression force C_c in the concrete decrease to zero and the compression force in the steel grows up to $C_s = P$ in the prestressed member (fig. 3c) respectively to $C_s = N$ in the column (fig. 3e). The total steel force then is in the prestressed member

$$T_{s,total} = P - P = 0.$$

This procedure can be applied to the creep problem of the beam in the example (fig. 1). When the concrete creeps in the internally indeterminate structure, the cable force T_{creep} comes into being due to redistribution (fig. 4a).

Fig. 4b shows the statically determinate structure loaded with all its loads, that is the dead load g_1 and g_2 and the force of the cable $T_s = P + T_{g2}$. The loading, which creates creep, depends itself on the earlier created redistribution forces due to creep. For simplicity it is only shown the first short time step $\Delta t = t_1 - t_0$ with $T_{creep} = 0$ as an initial condition. In any chosen statically determinate structure creep only results in displacements and not in any force. The deformations due to creep in every section can be assumed in proportion to the elastic deformations of the concrete under the influence of the loads in fig. 4b. The resulting displacement $\delta_{10} = \delta_{creep}$ of the cable due to creep (fig. 4c) is to be reversed by the cable force X_1 (fig. 4d), which is to be determined as ΔT_{creep}.
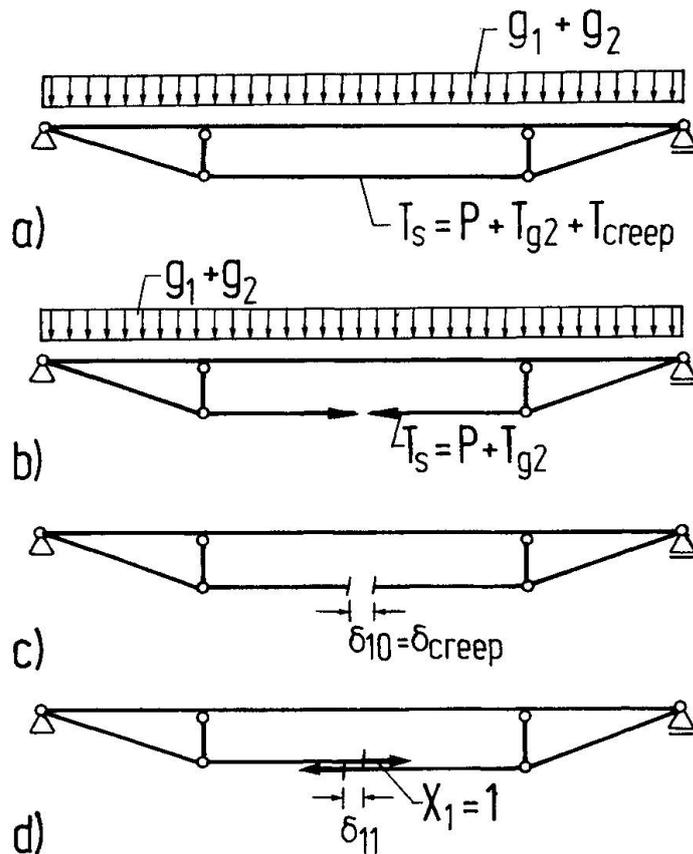
Fig. 4 a) When the concrete creeps, the tie force T_{creep}
comes into being

b) the statically determinate structure and its loads

c) cable displacement $\delta_{10} = \delta_{creep}$ at the statically
determinate structure, due to creep deformations

d) elastic displacement δ_{11} at the statically determinate
structure due to $X_1 = 1$, considering the actual
stiffness

The displacement δ_{11} of the cable due to $X_1 = 1$ depends on the
actual stiffnesses, these are the stiffnesses under the influence
of the loads in fig. 4b. All the internal forces of the structure
due to ΔT_{creep} are redistributation forces. At the end of the
first time step the total force in the cable amounts to

$$T_{total} = P + T_{g2} + \Delta T_{creep}.$$

You see, all things are kept tidy. The facts are not obscured, the
relations between causes and their effects remain transparent. The
procedure follows the facts and therefore it is generally valid.

Partially Prestressed Highway Bridges

Précontrainte partielle sur les ponts routiers

Teilweise vorgespannte Strassenbrücken

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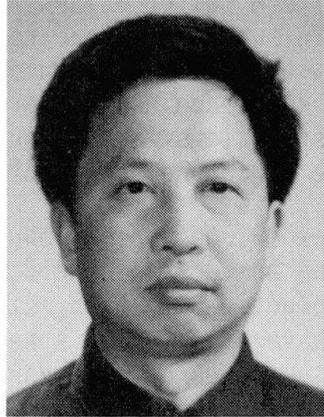
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SUMMARY

Systematic research on basic theories of the partially prestressed concrete bridges has been carried out. About 120 beams have been theories.This contribution presents the brief description of the experimental studies and emphasis put on some research results on flexural design. For partially prestressed concrete highway bridges, the design approach according to the prestressing degree and method to control cracking by means of the stresses of steel, are proposed.

RÉSUMÉ

Une recherche systématique sur les théories de base des ponts à précontrainte partielle a été réalisée en testant environ 120 poutres. Ce rapport présente une brève description des études expérimentales; l'importance est donnée aux résultats de recherche sur le dimensionnement à la flexion. Pour les ponts routiers à précontrainte partielle, ou propose und approche du dimensionnement basée sur le degré de précontrainte et sur le contrôle de le fissuration à travers le limitation des contraintes dans l'acier.

ZUSAMMENFASSUNG

Eine systematische Untersuchung der grundlegenden Theorien für teilweise vorgespannte Brücken wurde durchgeführt. Es wurden 120 Versuche an Balken durchgeführt. Dieser Beitrag enthält eine kurze Beschreibung der Versuche und hebt einige Forschungsergebnisse über Biegebemessung hervor. Für teilweise vorgespannte Brücken wird eine Bemessung entsprechend dem Vorspanngrad und die Rissbreitenbeschränkung über die Kontrolle der Stahl-Spannungen vorgeschlagen.

1. INTRODUCTION

The use of partial prestressing to highway bridges in China was begun in the middle seventies. At that times the design of PPC bridges was helped in the main by the existence of the European Specifications such as CEB, CP-110, FIP, as well as by informations gained from foreign previous experience and research publications. At present, while popularizing futher,the PPC at home is in the condition of that the experimental and practical engineering experience are accumulated. But up to now the PPC is getting not more generally used. Perhaps this is because of that quite a few of Chinese bridge engineers, who do not enough understand the PPC in substance, tend to be conservative and reluctant to take any risk with a new technology. Fundamentally speaking, it is accounted for the occurence that the knowledge about the strength, crack, stiffness etc. gained from tests and investigations are in sufficient, the design codes and analysis methodology on the whole are copies of the experience from abroad.

In oder to gather up more experinces on our own, to provide scientific and technical basis for revising the design code for highway bridges, to develop the design methodology fit to practice of China, since 1984, under direction of the author the CHRI and et.al have carried out a series of experimental studies and theoritical analysises on the fundamental theories of the PPC highway bridges. Based on the achieved results a more complete design recommendations and calculation system for PPC highway bridges has been proposed. In this contribution, some necessary introduction will be given in brief, but emphasis shall be merely put on the studies of crack because of the limit of the paper length.

2.BRIEF DESCRIPTION OF EXPERIMENTAL STUDIES

The whole work of the research is divided into three parts, i.e.the research on basis principles for flexural design, on basic design considerations for shear and on effects of the non-prestressing reinforcement.

The research on basic principles for flexural design of PPC beam can be summaried as follow:
(1) The ultimate flexural capacity
(2) The computing methods for normal stresses
(3) Design approach for crack control and calculation method of crack widths
(4) Calculation methods of deflection and stiffness
(5) The fatigue strength , cracking and deflection under cyclar loading
Above mentioned study tasks were brought to fruition through the rupture tests on 52 specimen beams, among them were 46 static loading and 6 cyclar loading. There were three types of tested beams. Their cross section forms involved conventional rectangular , T and I section. The forms respectively simulated the T-beams and the hollow plate beams of highway bridges. The I beams were 40cm high,prestressing by cold-strentched formed bars, but the others were 45cm high, prestressing by post-tensing high tensile strength wire tendons.The span of the specimens was 450cm long. The points of load application were symmetrically located at the one-third of span.

The studies on basic design considerations on shear includes following topics:
(1) The mechanisms of shear failure of PPC beams, the flexural capacity of inclined sections.
(2) The diagonal cracking and the computing methods for diagonal cracks.
(3) The diagonal cracking under repeated loading.
The basic data for the research on shear have been gained by the shear failure

tests on 44 beam specimens conducted in two batches. The first batch of test beams amounted to 24. Their steel percentages were just the same, but prestressing degrees, shear span ratios and stirrup percentages were distinctive. There were on 6 beams fatigue tests for inclinding sections have been accomplished.

The experimental studies on influences of non-prestresssing steel upon PPC beams were on two sides: creep of concrete and deflection. The studies have been carried out through the static tests on 20 specimen beams, among them eight have been observed over along term (above 900 days). In addition, the creep tests over 20 months on 23 concrete specimens have been made.The specimens and test beams were grouped by the grades of concrete and the ages at loading or the prestressing dergees and the steel percentages.

3. SOME RESEARCH RESULTS ON FAILURAL DESIGN

To designing the test beams the method according to prestressing degree was used.The method according to prestressing degree to design the PPC beams is a more simple, convenient, clear on idea and easy to use. Only basic checking calculations are need for designing. As the PD bring about a continuous transition from RC to PC, designing accordiing to PD is a practical and desirable unified design method for all RC, PPC and PC beams. The tests show that the tested PPC beams,designed like this (in acordance with PD), have higher ductility. The collapse of all the test beams have displaied plastic behaviour. The beam in brittle rupture have not emerged.

The measured limit flexural capacities of the tested beams of various beams having different PD, including RC beams whom PD is zero, have non-important to PD. In accordance with plasticity theory and the hypothesis of that the compressed region of concrete is a rectangle, the computing stresses are very close to the measured results. The average of the ratios of the measured stress to the computed is equal to 1.012, the standard deviation, $\sigma = 0.0645$, the coefficient of the deviation, $\delta = 0.052$.

The measured deflections, strains and cracking on the varied tested beams have analogous characters. Therefore, whatever beams of RC, PPC or PC may be, a unified design approach and basic calculation methods can be used.

The strains measured on the beam specimens, prototype beams and tested bridges are better agreement with the plane section hypothesis.The average strains of concrete and steel along the depth of the tested beams is distributed as a straight line. Even to the failure moment the deformed sections all are still nearly plane.

Before cracking between the strains and loads a linear relation is kept better. After cracking, along with addition of the tested loads the increase of the strains of steel speed up,but after a short interval a relation near straight line is renewed. It is analogous to the pattern of variation in the intertia moments of the cracked sections. The stresses of concrete and steel, computing based upon elasticity theory, show very litter difference with the tested results.It can be proposed that the calculation approach like this way is dependable and exact enough.

As the modulus of rupture of concrete is not easy to define with addition of that the prestressing losses is often estimated not exactly,to estimate the cracking load of a beam accurately is not easy too. From our test results,it is

can recommened that in practic the following formulae can be used to estimate the cracks moment of beams.

$$Mf=(\sigma_c +R_L^*)W_\bullet \qquad (1\text{-}1)$$

or $\quad Mf=(\sigma_c +\gamma)W_\bullet \qquad (1\text{-}2)$

where σ_c = effective prestressing stress of concrete on tensile edge,

$\quad$ R_L^* = concrete tensile strength for designing,

$\quad$ $W_\bullet$ = resistance moment of the section to tensile edge,

$\quad$ γ = plastic coefficient.

Among the above two formulae the former is more conservative.

After decompressing the regular cracking patterns of PPC beams is similar to that of RC beams, thus the crack control for PPC beams can be considerated as for RC beams. The stable crack spacings of the specimens have assumed normal distribution. The aviations in the mean crack spacing are as a linear function of the d/μ or $d/\mu e$ (where d--diameter of steel, μ--steel percentage, μe-steel percentage in effective region of the steel). By means of the linear regression, the mean crack spacing can be expressed as

$$Lf=3.1+0.078d/\mu \text{ (cm)} \qquad (2\text{-}1)$$

or $\quad Lf=2.6c+0.18d/\mu e \text{(cm)} \qquad (2\text{-}2)$

where c-cover of outer row bars

The checking calculations show: the ratios of the maximum crack width to the mean width are always 1.4 2, the average of the ratios for the tested beams is about 1.67.

the dominant factor exerting influence on crack width is steel stress. In the service range, the variation of the crack widths with the steel stresses is linear. From the test data, it has been found that the relationship between steel stress and maximum crack width can be taken as following form:

$$Wmax=a+b\sigma s \text{ (mm)} \qquad (3)$$

where σs -steel stress.

This expression is tenable on varied beams, having various section forms or different PD. Based on the test data of 46 beams and used the linear regression analysis, the achieved static results are $a=0.0032, b=0.599\times-3$. While a unit of σs is 1MPa, the correlativity coefficient $R=0.8$, the standard deviation, $\sigma =$ 0.0652. The tests show the effect of PD upon the value of the a and the b is not distinctive. It can be seen that along with the higher PD, in a certain limit, the a trend towerds a desrease in value, but b towards a incease. The statistic results for 24 tested beams are:

$$a=a-0.07146(Md/Mu)\times 0.564 \qquad (4\text{-}1)$$

and $\quad b=0.6543+0.2873(Md/Mu) \qquad (4\text{-}2)$

Because of the litter effect of PD on the a and b, it is reputed that the steel stresses already reflect the effect of PD. Therefore, when practic designing, to calculate the crack width the formulae (3) can be used, but the PD need not to be considerated once more. In accordance with the statistic analysis of the test data, the formula of the maximum width of crack (less than 0.3mm) is gained as following:

$$Wmax=0.1131+0.599\times-3\sigma \text{(mm)} \qquad (5)$$

The guarantee percentage of this formula is 95%. Using this formula, the cracks can be controled through control to steel stresses. Based upon recent crack theories and the test data a formula for calculating maximum crack width can be easy written down as follows:

$$Wmax=1.4\sigma sLf\psi/Es \qquad (6)$$

where $\quad \psi$ =non-uniformity factor of steel strains, to be computed from

$\psi =1.2(1-(Mf/M)) \qquad (6\text{-}1)$

Or $\quad \psi =1.1-0.65R /(\mu e \sigma s) \qquad (6\text{-}2)$

R-standard tensile strength of concrete,
Es-elastic modulus of steel.
Numerous checking computations show the agreement of the calculations by above
mentioned formulae with test results are better.The compartions of our formulae
with other formulae at home and abroad indicate that above mentioned formulae
are not only reliable but also practical in designing PPC bridges.

The tests present the factitious tensile stresses of concrete bear obvious
relation with the crack widths.Thus using factitious tensile stresses of
concrete to control cracks is reasonable. But the tests also show:
(1) There do not exist the relationship in one by one between the factitious
stresses and the crack widths.
(2) Corresponding with same factitious stresses, there may be exist large
different beams.
(3) The relationships of the crack widths with the factitious stresses are
different in different beams.
In recent years, using the allowable factitions tensile stresses, corresponding
to the allowable crack widths,to control cracks is a usual approach in designing
highway bridges. The allowable factitious stresses are stipulated in Codes, ex.
JTJ 023-85(1) .The tests have discovered the allowable factitious stress in the
Code JTJ 023-85 may be proper for the certain beams, but may be conservative in
excess for some beams or may not on the safe side for another beams. It should
be point out that a futher investigation and accumulation of experiment data
must be continued. For the sake of to gain the reliable allowable factitious
stresses possessed a sure guarantee percentage, the clear relatinships of fact-
itious stresses with section forms, beam depths, prestressing type and PD must
explored. The more proper calculation method for allowable factitious also must
be sought.

472 measured data on 46 specimens showed both of the bilinear method and concept
of effective moment of inertia (Ie) can reflect the variations in stiffness
of the cracking PPC beam.

By the bilinear method the deflection of beam after cracking can be estimated
from:

$$f=a \quad *2(Mf/(a_1 EcIo)+(M-Mf)/(a_2 EcIo_1)) \tag{7}$$

where Io and Io_1 are respectively the moment of inertia of non-cracking
and cracking section. From the statistic results of the test data, the mean
values of a1 and a2 are about 0.9, the standard deviation, $\sigma=0.15$, the linear
correlativety coefficient $R=0.95$. Provided the guarantee percentage is adopted
of 95%, then a1=a2=0.85, coinsided with of the Code JTJ 023-85.

A number of checking calculations indicate that, if the effective moment of
inertia takes the following form:

$$Ie=Io+(Io-Io_1)(Mf/M)*4$$

the computed deflections agree with measured on tested beams.

The fatigue tests on the cracking PPC specimens have showed, all failures due to
fatigue occurred in the non-prestresssing steel, even through the prestressed
steel wires are thiner. Therefore the fatigue of PPC beam can be consideed as RC
beam.the fatigue tests also have showed there is not a beam occur fatigue
failure after 2 million cycles of load. If the range of cyclic stresses is
simulated the stresses under the deaded loads and the maximum service loads
calaculated according to the Chinese Code JTJ 021-85(2). Therefore, at the
moment in designing PPC highway bridges the effect due to fatigue usually

need not be considered.

4.BRIEF INTRODUCTION OF THE TRAIL BRIDGES

In order to examine the reliability of the bridges designed by use of aforeci-tied research results, a few of trial bridges were designed and built.There are three trial bridges tested by us. The briefs of these bridges are summaried in the following table.

Table 1. The briefs of the trail bridges

Bridge Name		Red Flag Gully		ChenJiaZhang	NandaZang
Length of bridge(m)		2*20.5+30+5*20.5		2*16	15*13
Span length of beam(m)		20.5	30	16	13
Type of section form		T	T	T	hollow plate
Beam depth(m)		175	120	110	50
Prestressing degree		0.684	0.699	0.655	0.568
prestressing steel		5×24φ5 wires		4φ25 high tensile strength formed bars	
Non-prestesssing steel		20φ14		5φ16	20φ14
Computed crack width(mm)	by CEB-FIP Model Code(3)	0.0423	0.0320	0.0462	0.0480
	by ours	0.0434	0.0379	0.0399	0.0273
Factitious tensile stess(MP)		5.75 (5.03)	5.02 (5.91)	5.83 (4.00)	4.21 (6.38)
Tensile stress in non_ prestressing steel(MPa)		68.73	54.44	64.39	49.00

note:In brackets are the allowable factitious stresses defined by Code JTJ 023_85.

These trail bridge have already been opened to traffic in succession in recent years. While constructing the Red Flag Gully Bridge the static loading test on a beam spanning 20.5m have been carried out. After put into service on the Nan-dazhang Bridge and the Chengjiazhuang Bridge extensive load tests under heavier vechicle loads were performed. In addition two prototype beams,which are alike of the Nandazhang Bridge, were tested. The test results prove the actual state of the beams under traffic loads is better conformable to the designed.

The success of the trial bridges led to wider recognition of the both technical and economic benifits of application of partial prestrssing in bridges.

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Recommendations on Reinforcement in Flexural and Compression Members

Recommendations concernant l'armature des structures en béton armé

Empfehlungen für die Bewehrung von Betonbauteilen

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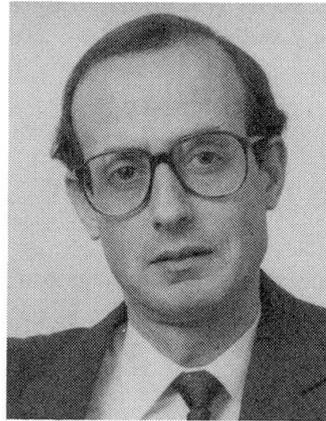
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SUMMARY

A set of recommendations related to the reinforcement in structural concrete flexural and compression members are presented. They address minimum and maximum levels of reinforcement, the percent of moment redistribution in continuous members, and the ultimate stress in the prestressing steel for bonded or unbonded tendons. The recommendations are tuned to lead to numerical results in accordance with the ACI Building Code; however, they are non-dimensional and can be applied to any code.

RÉSUMÉ

Sont présentées ici une série de recommendations concernant des structures en béton armé fléchies et comprimées. Elles concernent la qualité minimale et maximale d'armature, le pourcentage de la distribution des moments à considérer dans une structure continue, ainsi que la contrainte ultime à prendre en compte dans l'acier des câbles d'une précontrainte adhérente ou non. Les recommendations sont orientées dans le but d'obtenir des résultats numériques en accord avec le code de construction ACI; cependant, comme elles sont en fait adimensionnelles, elles peuvent s'appliquer à n'importe quel type de norme ou de code.

ZUSAMMENFASSUNG

Es werden einige Empfehlungen für die Bewehrung von Betonbauteilen unter Biege- und Normalkraftbeanspruchung dargestellt. Sie betreffen Minimal- und Maximalbewehrungsgrade, den Prozentsatz der Lastumlagerung von durchlaufenden Tragwerken und die Maximalspannung in den Spanngliedern bei Vorspannung mit oder ohne Verbund. Die Empfehlungen sollen numerische Ergebnisse in Übereinstimmung mit den ACI-Bauvorschriften geben, aber sie sind dimensionsfrei und können für jede Norm benutzt werden.

1. SCOPE

Unifying code recommendations to accommodate Structural Concrete (i.e. reinforced, prestressed, and partially prestressed concrete) in a simple and rational manner that does not violate the fundamental principles on which the provisions are based, should be an essential goal of future editions of any code of practice.

The recommendations proposed in this paper are related to the reinforcement of structural concrete members reinforced with conventional reinforcing bars, prestressing tendons, or any combination of them.   The numerical values derived from these recommendations are tuned to reflect, as a reference base, the current provisions of the American Concrete Institute's Building Code Requirements (ACI 318 - 1989). However, they are written in a non-dimensionalized form and could be easily adapted to any code of practice. Some related background information can be found in   [1-9].

2. FLEXURAL MEMBERS

2.1   Definition

The depth d_e from the extreme compression fiber to the centroid of the tensile force in the tensile reinforcement at nominal resistance of the section is given by the following expression (Fig. 1):

$$d_e = \frac{A_{ps}f_{ps}d_p + A_s f_y d_s}{A_{ps}f_{ps} + A_s f_y} \qquad (1)$$

where:

A_{ps}	=	Area of prestressing reinforcement in the tensile zone
f_{ps}	=	stress in the prestressing steel at nominal flexural resistance of the section (see Sections 2.5 and 5).
d_{ps}	=	distance from extreme compression fiber to centroid of prestressing steel
A_s	=	area of non-prestressed tension reinforcement
f_y	=	specified yield strength of non-prestressed tensile reinforcement
d_s	=	distance from extreme compression fiber to centroid of nonprestressed tensile reinforcement

The definition of d_e could also be easily extended to multi-layered systems, such as columns, having different layers of prestressing reinforcement and/or conventional reinforcing bars.

Note that while it is generally assumed that the reinforcing steel yields at ultimate behavior of the member, the stress, f_{ps}, in the prestressing steel is unknown and must be estimated separately (see Sections 2.5 and 5).

2.2   Maximum Reinforcement

The amount of prestressed and non-prestressed reinforcement, used for computation of moment strength of a member, shall be such that:

$$c/d_e \leq 0.42. \qquad (2)$$

where c is the depth to the neutral axis at nominal resistance in bending, and d_e is as defined in Eq. 1.

The above provision requires the determination of c, which could be obtained from writing the two equations of equilibrium of the critical section at nominal bending resistance.
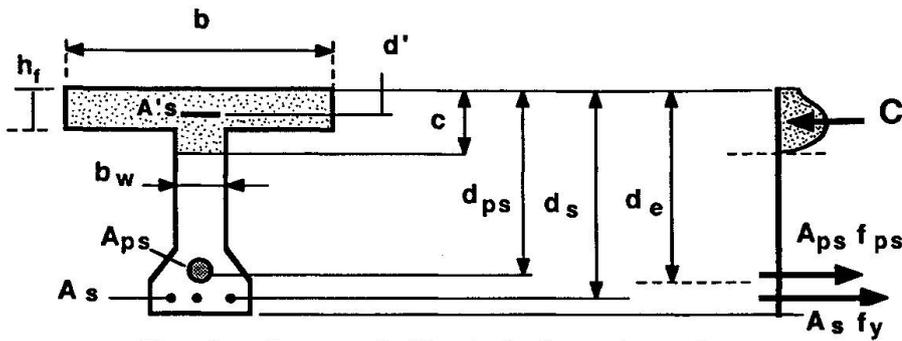
Fig. 1 Forces at ultimate in flexural members

2.3 Minimum Reinforcement

At any section of a flexural member, except where positive reinforcement is required by analysis, the amount of reinforcement shall be adequate to develop a design factored load, ϕP_n, at least 1.2 times the cracking load, P_{cr}, computed on the basis of the modulus of rupture f_r of the concrete material. Thus:

$$\phi \; P_n \;\; \geq \;\; 1.2 \; P_{cr} \tag{3}$$

For concrete members reinforced with conventional reinforcing bars only, this provision can be satisfied by providing a minimum reinforcement ratio given by:

$$\rho_{min} \;\; \geq \;\; 0.03 \; f'c/f_y \tag{4}$$

where:

$$\rho \;\; = \;\; A_s/bd_s \tag{5}$$

in which $f'c$ is the compressive strength of concrete obtained from cylinder tests and other terms are as defined earlier. Note that b is taken equal b_w (Fig. 1) for T sections and joists where the web is in tension.

2.4 Moment Redistribution

Where bonded reinforcement is provided at supports in accordance with Section 18.9 of the ACI Code, negative moments calculated by elastic theory for any assumed loading arrangement may be increased or decreased by not more than

$$20(1 \; - \; 2.36 \; c/d_e) \qquad \text{in percent} \tag{6}$$

provided the value of c/d_e obtained from the design of the section at ultimate is such that:

$$c/d_e \;\; \leq \;\; 0.28 \tag{7}$$

2.5 Stress in Prestressing Steel at Ultimate

In lieu of a more accurate determination of f_{ps} based on strain compatibility, the following approximate values of f_{ps} shall be used if f_{pe} is not less than 0.5 f_{pu}

(a) Members with bonded tendons:

$$f_{ps} \; = \; f_{pu}(1 \; - \; k \; \frac{c}{d_p}) \tag{8}$$

where k is given by:

$$k = 2(1.04 - \frac{f_{py}}{f_{pu}}) \qquad\qquad (9)$$

If any compression reinforcement is taken into account when calculating f_{ps}, the value of c should be larger than or equal to 3d' to insure yielding of the compressive reinforcement. d' is defined as the depth from the extreme compression fiber to the centroid of the compressive reinforcement. If c is lesser than 3d', the contribution of the compressive reinforcement may be neglected. The basis for Eqs. 8 and 9 can be found in [1,6,7].

(b)   Members with Unbonded Tendons

$$f_{ps} = f_{pe} + \Omega_u \, E_{ps} \, \varepsilon_{cu} \, (d_{ps}/c - 1) \, L_1/L_2 \;\leq\; 0.94 \; f_{py} \qquad (1\,0)$$

where:

E_{ps}	= elastic modulus of prestressing steel
ε_{cu}	= assumed failure strain of concrete as per code used (i.e. 0.003 for ACI Code)
L	= span length
L_1	= length of loaded span or spans affected by the same tendon
L_2	= length of tendon between anchorages
Ω_u	= 3 / (L/d_{ps}) for uniform or third point loading
Ω_u	= 1.5 / (L/d_{ps}) for one point midspan loading

In order to solve for the value of fps in Eqs (9,10), the equation of force equilibrium at ultimate is needed. Thus two equations with two unknowns (f_{ps} and c) need to be solved simultaneously to achieve a numerical solution. The background and basis for Eq. 10 can be found in [8,9].

3. COMPRESSION MEMBERS

3.1 Maximum Reinforcement in Compression Members

The areas of prestressed and nonprestressed longitudinal reinforcement for non-composite compression members shall satisfy the following two conditions simultaneously (Fig. 2):

$$\frac{A_s}{A_g} + \frac{A_{ps}}{A_g} \times \frac{f_{pu}}{f_y} \;\leq\; 0.08 \qquad\qquad (1\,1)$$

and:

$$\frac{A_{ps} f_{pe}}{A_g \, f'_c} \;\leq\; 0.3 \qquad\qquad (1\,2)$$

Equation 11 limits the percentage of total reinforcement in the section, while Equation 12 limits the allowable uniform compressive stress in the concrete due to prestressing, if any.

Typical Prestressing Tendons
A_{ps}

Typical Reinforcing Steel
A_s
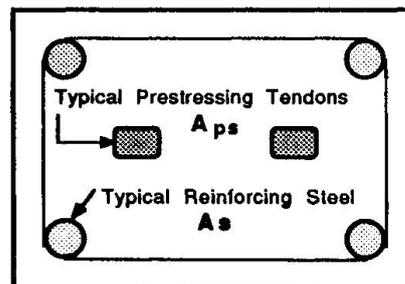
Fig. 2 Typical section of compression members

3.2 Minimum Reinforcement for Compression Members

The areas of prestressed and nonprestressed longitudinal reinforcement for non-composite compression members shall satisfy the following condition (Fig. 2):

$$\frac{A_s f_y}{A_g f'_c} + \frac{A_{ps} f_{pu}}{A_g f'_c} \geq 0.12 \qquad\qquad (13)$$

where A_g is the gross area of the compression member, f_{pu} is the ultimate strength of the prestressing tendons and other terms are as defined above.

4. PRESTRESS LOSSES - STRUCTURAL CONCRETE?

This is a subject where the general term "structural concrete" may have to be broken down into three groups, namely, reinforced, prestressed and partially prestressed concrete. Prestress losses affects only the last two groups. The accurate determination of prestress losses in prestressed and partially prestressed concrete should be based on a time step analysis. However, lump sum estimates can be used for partially prestressed as well as for fully prestressed concrete. The following remarks may be in order for partially prestressed concrete:

1. The average stress in the concrete in a partially prestressed member is generally smaller than that in a fully prestressed member. Thus the loss of prestress due to creep is also expected to be smaller.

2. If the prestressing steel is tensioned to the same initial tensile stress as in the case of fully prestressed concrete, the intrinsic relaxation loss would be the same. However, since prestress loss due to creep is smaller in a partially prestressed member, and since loss due to creep influences that due to relaxation, the relaxation loss in partially prestressed concrete members is expected to be slightly higher than in fully prestressed concrete members.

3. Everything else being equal, the loss of prestress due to shrinkage of the concrete should be the same for prestressed and partially prestressed concrete members.

4. Other instantaneous prestress losses such as friction, anchorage set, and elastic shortening can be computed in the same manner as in prestressed members.

5. The presence of a substantial amount of non-prestressed reinforcement (conventional reinforcing bars) such as in partially prestressed concrete, influences stress redistribution along the section due to creep of concrete with time, and generally leads to smaller prestress losses.

6. It is advisable to estimate creep loss on the basis of the ratio of average stress in the concrete to its compressive strength.

5. STRESS IN PRESTRESSING STEEL AT ULTIMATE - SIMPLIFIED APPROACH

In the above Section 2.5, the latest developments known to the author regarding prediction of the stress at ultimate in prestressed flexural members have been described in Eqs. 8 to 10. Such equations, combined with the equations of equilibrium at ultimate, allow for the computation of nominal bending resistance. This is as close in accuracy to a strain compatibility analysis as can be achieved to date. In an analysis or investigation situation, the combination of Eqs. 1, 8 or 10, with the two equations of force and moment equilibrium at ultimate, leads to solving four equations with four unknowns. In a design situation where, for instance, the non-prestressed steel is to be determined, an additional unknown is present. The solution becomes unnecessarily messy (involved) and its accuracy may not be needed in many design cases.

Thus, it is tempting to suggest very simplified and safe recommendations to estimate the stress at ultimate in prestressed and partially prestressed concrete. The following approach is proposed:

(a)   Members with bonded tendons:

$$f_{ps} = f_{py} \qquad\qquad (14)$$

This equation is always on the safe side since the limit on maximum reinforcement (Eq. 2) does not allow for the design of overreinforced members; thus actual f_{ps} will always be larger than f_{py}.

(b)   Members with unbonded tendons:

$$f_{ps} = f_{pe} + 70 \qquad MPa \qquad\qquad (15)$$

This is generally on the safe side as observed for most of the 143 beams analyzed in [10].

Thus Eqs. 14 and 15 may be used in a first step analysis and, only if additional accuracy is needed to satisfy the design, one may revert to Eqs. 8 and 10, or to a non-linear analysis procedure.

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Strut-and-Tie Modelling of Structural Concrete

Analogie du treillis dans les structures en béton

Stabwerksmodelle für Konstruktionsbeton

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SUMMARY

Strut-and-tie models can illustrate very well the internal flow of forces in structural concrete and thereby, provide valuable assistance to the designer who is striving for an appropriate and functional conceptual design. Moreover, such models are good enough to serve as a basis for the design and dimensioning of the modelled structure or structural detail for the cracked state. After some introductory statements this will be demonstrated by the example of a box girder, cantilevering from two supporting walls. Different arrangements of diaphragm walls and the effect of prestressing will be discussed.

RÉSUMÉ

L'analogie du treillis permet de visualiser de façon très claire le cheminement des forces dans les structures en béton; ceci constitue une aide précieuse pour l'ingénieur lors du dimensionnement approprié d'éléments porteurs ainsi que des détails de construction. De plus, de tels modèles représentent le fondement du calcul des constructions en béton armé et précontraint à l'état fissuré. Après quelques remarques introductives, ces faits sont démontrés à l'aide d'un exemple décrivant une poutre-caisson, dont deux parois constituent les appuis. Diverses positions d'un diaphragme intérieur ainsi que l'effet de la précontrainte dans le caisson sont successivement discutées.

ZUSAMMENFASSUNG

Mit Stabwerksmodellen kann der innere Kraftfluss sehr anschaulich dargestellt werden. Dadurch sind sie auch eine wertvolle Hilfe für den Entwurf zweckmässiger Tragwerke und Details. Sie sind ausserdem eine geeignete Grundlage für die Bemessung von Stahl- und Spannbetonkonstruktionen im gerissenen Zustand II. Dies wird im vorliegenden Beitrag am Beispiel eines auskragenden Hohlkastenträgers gezeigt. Dabei werden verschiedene Varianten von Querschotts und die Wirkung der Vorspannung diskutiert.

1. INTRODUCTION

Breen [4] proclaimed as a key topic of this conference "useful and transparent models, which can enhance the designer's realization of structural action" and he emphasized several times strut-and-tie models (STM) as such a tool. Schlaich, in his Introductory Report to "Modelling", integrated the strut-and-tie method into the framework of the design process of structural concrete [7]. Beginning with Ritter's truss model for beams such models were used for the visualization of forces in some specific cracked reinforced concrete elements and for proportioning their reinforcement. Thürlimann and his Zürich School developed a more general design concept using stress fields on the basis of theory of plasticity. More recently Schlaich and his co-workers proposed to generalize the strut-and-tie method for the application to all kinds of structural concrete elements or structural details and to compliment the method by a unified concept for the dimensioning of cracked structural concrete, including the node regions of struts and ties [1, 2, 3].

Such a concept is urgently needed considering the Codes of Practise, which give design rules only for elements with linear strain distribution (B-regions) but neglect all others, more complicated ones, where damage most frequently occurs. The lack of a consistent methodical approach for the design and dimensioning of such discontinuity regions (D-regions) was felt particularly bad when they were taught to the students. Considering the importance of the D-regions for the safety and endurance of structures their design cannot be left to the draftsman's skill or the engineer's good guess. Any rational approximate method is better than this state of dimensioning.

2. MODELLING METHODS

Basically three methods are available, which also may be combined:

- Orientation of the model, in particular the struts, at the linear theory of elasticity. Stress trajectories from FEM or stress diagrams in typical sections can be used for locating major struts and ties. A rough orientation at the elastic behaviour is necessary anyhow for compatibility and serviceability reasons.

- Analogy of the stresses with that of a fluid. This analogy - though mechanically not perfect - helps to find the "flow of forces" through the structure by the "load path method".

- Adaptation of known typical models to the specific case. This is facilitated by the fact that certain types of models repeat very frequently in different structures.

The general methods of finding and judging strut-and-tie models are published in some detail in [2, 3] and therefore will not be repeated here. Instead, an example will be presented.

3. SUPPORT OF A CANTILEVERING BOX GIRDER ON TWO WALLS

3.1 General Layout

How to carry the forces in the connection of the members shown in Fig. 1a? Normally diaphragm walls are introduced in the box girder, either two directly above the two supporting walls (Fig. 1b), or - because the inner one is difficult to construct - just one at the end of the box girder (Fig. 1c). The best solution, a diagonal wall, is not obvious in the beginning.

3.2 Frame Corner with Diaphragm Wall at the Box Girder's End only

The diagonal struts C_3 (Fig. 2a) which balance the chord forces T_1 (from the box girder's tensile flange) and T_2 (from the tensile wall) with the compressive forces C_1 and C_2 from the respective compression chords of the frame type structure can only be transferred within the two webs. Therefore, all the chord forces which in the adjacent B-regions are well distributed over the whole widths of the flanges have to be deviated and bundled into the small width of the webs.

First of all, this requires considerable transverse reinforcements in the four chord members according to the strut models given in Figs. 2b-e. The models are all of one type, which appears very frequently in D-regions of very different structures. The internal lever arm z of the transverse forces in Figs. 2c and 2e, oriented at theory of elasticity is approximately 0,6 b. In Figs. 2b and d the corresponding lever arm depends also on the length of overlap of longitudinal reinforcement. Standard lap lengths in Codes do not apply for this situation where the lapped bars are arranged at some distance from each other. A more detailed model (Fig. 2f) of this typical problem shows different transverse tie forces in different places.

Looking again at the struts C_1 and C_2 in plan and sections of Fig. 2 we realize that the corresponding stress fields must be squeezed through the bottleneck of the singular node 2 whose dimensions are restraint by the
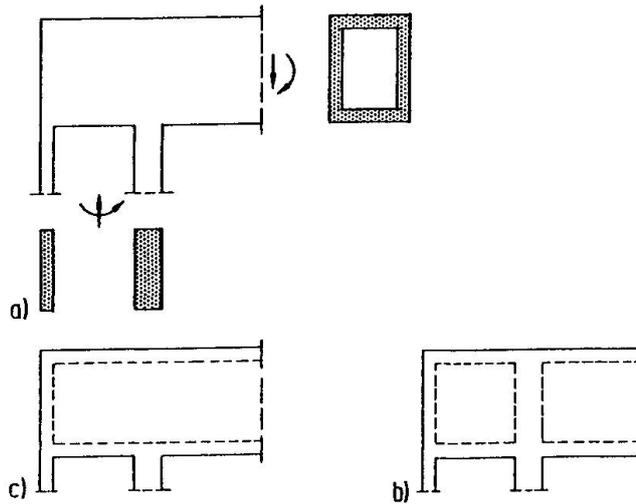
Fig. 1: a) View and cross-section of the box girder and support walls. b) Longitudinal section with interior diaphragm wall. c) Longitudinal section without interior diaphragm wall
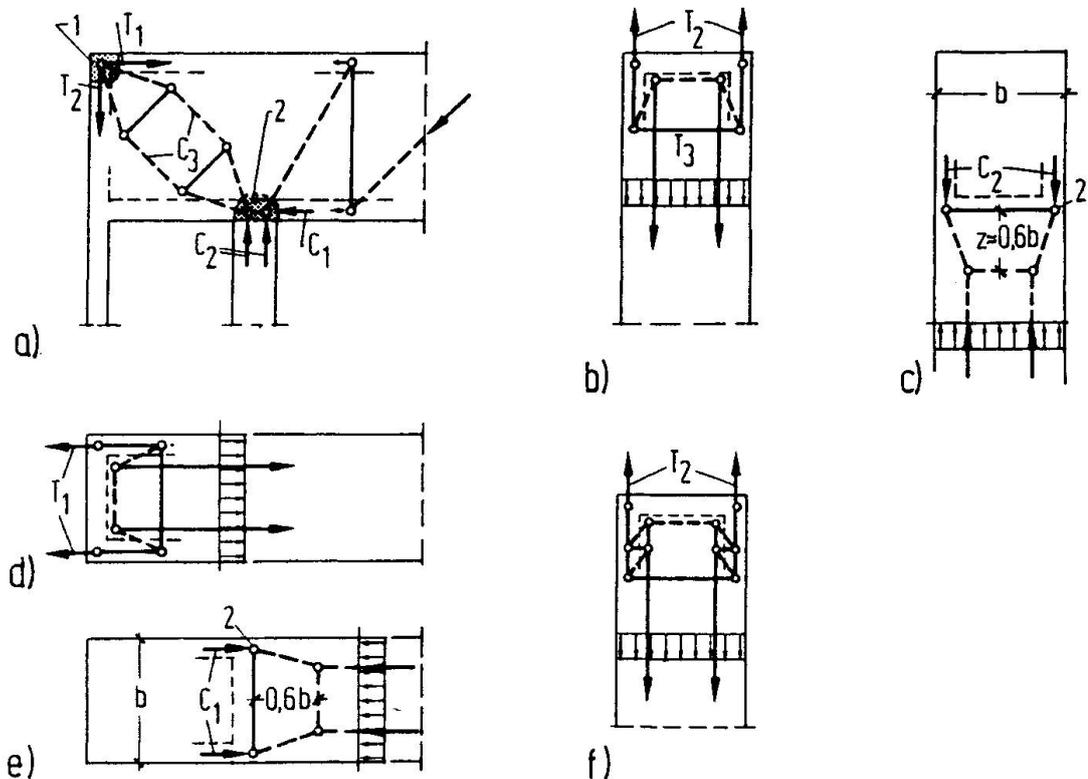

Fig. 2: Strut-and-tie models for the structure without internal diaphragm wall. a) Web; b) tensile wall; c) compression wall; d) top flange; e) bottom flange. f) Refined model for the lap problem characterized in Figs. b and d

thicknesses of the compression wall, web and bottom slab of the box girder. This node will dictate the concrete dimensions, the large width b of the boxgirder slab cannot be used as compression zone. What an unreasonable structure! Who would have recognized this, applying the usual design rules?

A similar problem may arise in node 1, where tensile bars for the total chord forces T_1 resp. T_2 must be arranged within the thickness of the web or at least very close to it in order to avoid large "slab moments" in

the deck and wall. Also this node will become a singular node if reinforcing bars were bent sharply around the corner as shown in Fig. 2a. Consequently, the diagonal strut force C_3 in the web will spread out between nodes 1 and 2, thereby creating transverse tensile forces as indicated in Fig. 2a. Therefore it is much better to bend the chord reinforcement using a mandrel which is adapted to the dimensions of the frame corner (Fig. 3).
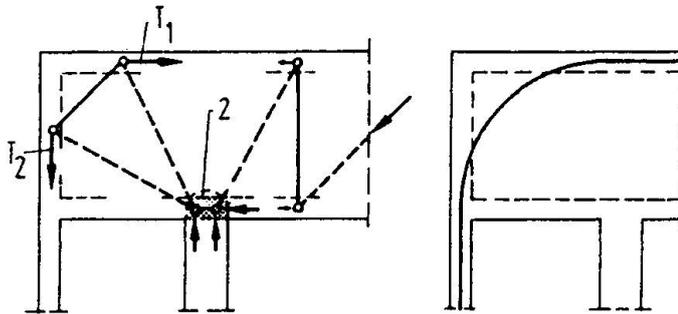
Fig. 3: Model and corresponding chord reinforcement with a well adapted mandrel diameter

By the way, omitting the lower diaphragm plate between the two walls would do no additional harm to the (poor) structural solution. Either none or two orthogonal disphragms are needed, as will be shown hereafter.

3.3 Frame Corner with Two Diaphragm Walls

The necessary transverse reinforcement in the boxgirder plates and the supporting walls is the same as before; but the singular nodes are avoided since the chord forces T_1, T_2, C_1, C_2 now enter the web reasonably well distributed over the whole length of the diaphragms (Fig. 4a). In other words: Each chord plate is no longer supported on two points only but rather along two lines (Fig. 4b). As a consequence the load bearing capacity of the frame corner is essentially increased by the additional diaphragm wall.
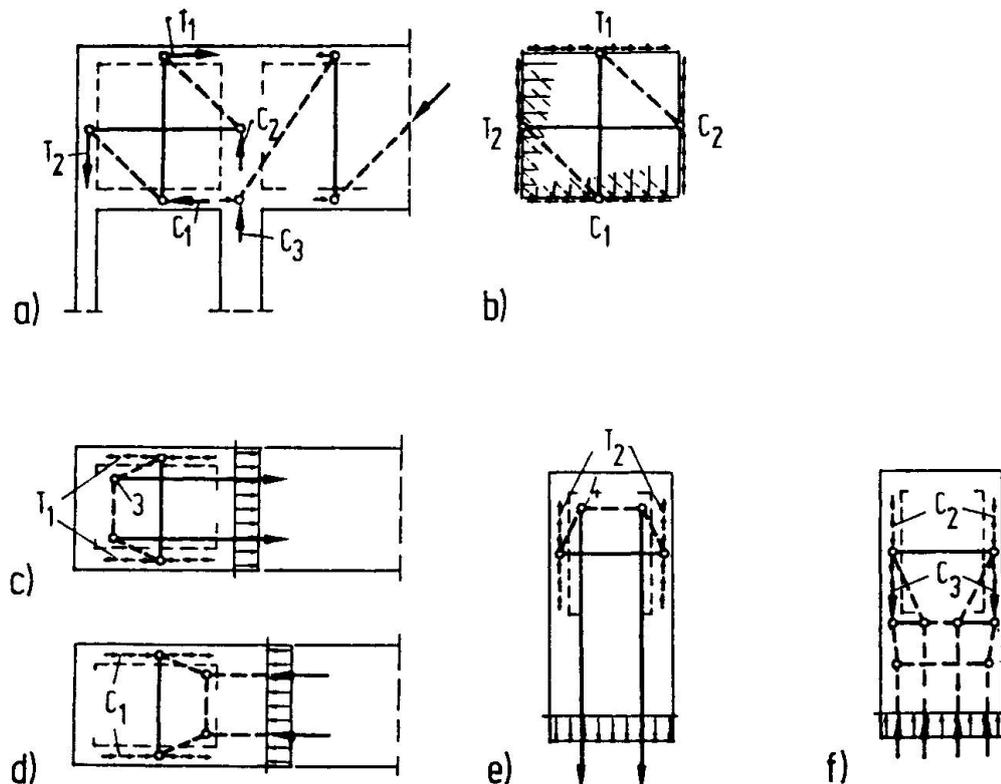
Fig. 4: Strut-and-tie models for the structure with internal diaphragm wall. a) Basic model for the web; b) refined model for the web with smeared forces. c) Top flange. d) Bottom flange. e) Tensile wall. f) Compression wall

3.4 Frame Corner with Diagonal Diaphragms

The best structural solution for the discussed problem is the diagonal diaphragm which follows the load path T_2 in Fig. 5a. This model avoids not only the singular nodes but also the transverse reinforcements in the flanges and walls. Only the spreading out of the support forces C_3, resulting from the shear forces of the webs, require some transverse reinforcement T_3 near the top of the compression wall (Fig. 5b).
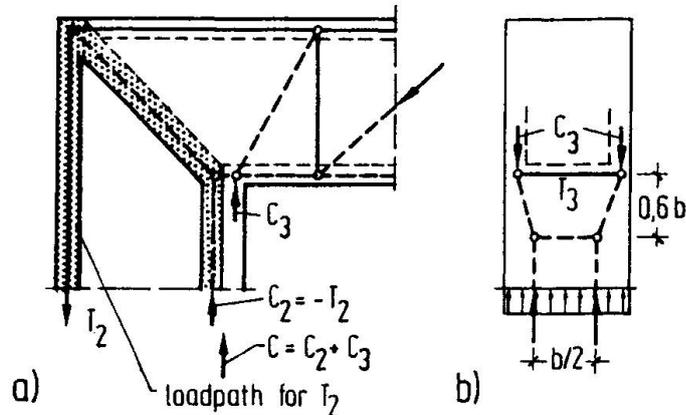
Fig. 5: a) Diagonal diaphragm following the load path for T_2 of the strut-and-tie model. b) Model for the compression wall.

We can conclude from this example that strut-and-tie models are not only suitable for dimensioning but are also very helpful for the conceptional design of good structural solutions.

3.5 The same Structure with Prestress

Let's prestress now the top plate and tension wall of the structure without inner diaphragm wall (see Fig. 3.2) with prestressing forces $P_1 = T_1$, $P_2=T_2$ just large enough to balance the concrete tensile forces T_1, T_2 due to dead load (Fig. 6a). At first sight one could think that thereby also the problem of transverse forces in the frame corner is cancelled, at least in the prestressed members. But it isn't at all! The model with prestress applied as external forces [4, 5, 6] discloses that the load paths of the compression forces C_1, C_2 have to squeeze as before (see section 3.2) through node 2 into webs in order to arrive at their "supports" provided by the anchor forces P_1, P_2 of the tendons. In order to avoid further detours of the load paths (see Fig. 2b and d), the tendons in the corner should be arranged within the web, either similar to Fig.3 or Fig.6b, thus balancing the compression strut in the web directly.

If the load is increased after prestressing, e.g. due to live loads or a safety factor for ultimate conditions, the tendons react like non-prestressed reinforcement with additional tendon forces ΔT_1, ΔT_2. These are anchored by bond according to the model shown in Fig. 6c.

In the structure with inner diaphragm wall (acc. to Fig.4) the tendons may be distributed over the whole width b of the structure and anchored near the edge, if transverse forces are carried according to the models given in Fig.4c and e. However, the position of the model nodes 3, 4, which in Fig.4c and e represent the centroid of bond forces, have to be reconsidered for the prestressing tendons (see Fig. 6c). The prestress force P is always applied at the tendon anchor. Only that part of the tendon force ΔT which exceeds the initial prestress force P is anchored by bond, and these bond stresses may develop at a considerable distance from the anchor.

Separating the prestressing loads from the additional tendon forces after prestressing as suggested by Breen, Bruggeling and Jennewein [4, 5, 6] is reasonable also for the application of strut-and-tie models to prestressed D-regions and leads to a clear understanding of structural behaviour.

4. OUTLOOK

Finding an adequate strut-and-tie model is not always as simple as in the examples shown above. It implies to have learned and practised the method for some time, like any other engineering skill or method of analysis. To develop an individual model still takes considerable time. But Rückert's contribution shows that in the future the computer can assist the design engineer also in this work [8]. And with an increasing number of published examples it will be easier to find a model which only has to be adapted to one's specific problem.
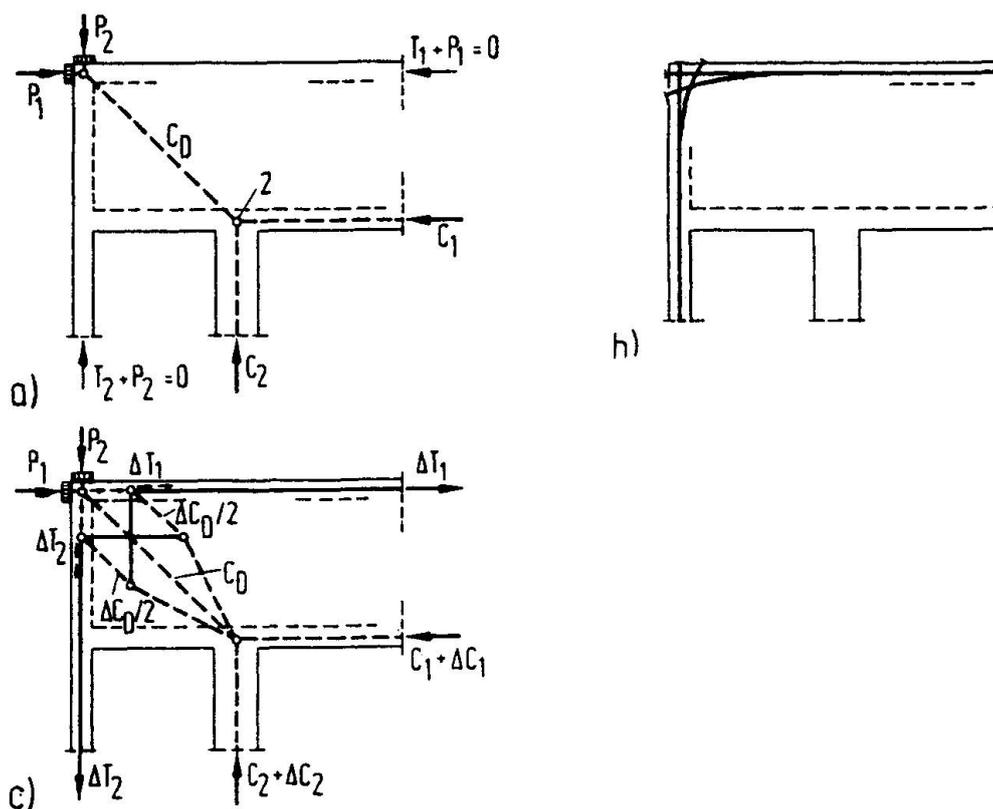
Fig. 6: Prestressed structure without internal diaphragm wall a) Top slab and tension wall prestressed under dead load to give zero concrete stresses. Model showing the load paths. b) Practical reinforcement layout c) Strut-and-tie model for increased loads, prestress as before.

At the moment standard models and procedures for dimensioning certain types of D-regions are being prepared by the authors. These include frame corners, beam-column connections, corbels and beams with openings.

In the future emphasis should be shifted from modelling techniques to a consistent design of node regions. Necessary anchorage lengths of reinforcement and permissible concrete stresses depend considerably on the type and geometry of nodes. Though the node problem is not specific for the strut-and-tie method but rather a problem of structural concrete, the authors' experience shows that strut-and-tie models help to understand and explain also the nodes' intrinsic behaviour.

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Design of the Support Regions of Concrete Box Girders

Dimensionnement des zones d'appui des poutres-caisson en béton

Bemessung der Auflagerbereiche von Beton-Hohlkästen

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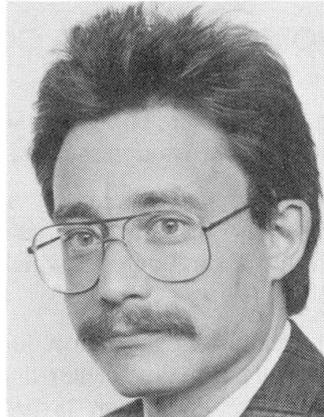
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SUMMARY

For dimensioning and detailing the support regions of concrete box girders, strut-and-tie-models are a valuable tool. In this paper it will be shown how a strut-and-tie-model for the transverse diaphragm can be developed and how in the model the interaction between the longitudinal and cross directions must be considered.

RÉSUMÉ

L'analogie du treillis faisant intervenir tirants et bielles constitue un outil très appréciable lors du dimensionnement des zones d'appui des poutres-caisson. Dans cet article, un exemple présente l'analyse d'un diaphragme à l'aide de cette analogie, ainsi que la nécessité d'une modélisation cohérente dans le cadre d'un calcul effectué dans les directions longitudinale et transversale.

ZUSAMMENFASSUNG

Für die Bemessung und konstruktive Durchbildung der Auflagerbereiche von Beton-Hohlkasten-trägern sind Stabwerksmodelle ein wertvolles Hilfsmittel. In diesem Aufsatz wird anhand eines Beispiels die Entwicklung eines Stabwerksmodells für das Querschott und die Notwendigkeit konsistenter Modelle für die Bemessung in der Längs- und Querrichtung gezeigt.

1. INTRODUCTION

Support regions of concrete box girders are important and highly stressed parts of the structure. For dimensioning and detailing these D-regions strut-and-tie-models are a valuable tool. They allow to follow up the forces consistently from the B-regions (webs, flanges) to the supports as strictly required by Schlaich et. al. in /1,3,4/. Examples of basic strut-and-tie-models for these regions are shown in /3,4/.

This paper shows how a strut-and-tie-model for the complicated diaphragm of a concrete box girder can be developed. The interaction between the load bearing behavior in the longitudinal and the cross direction is thereby considered.

2. DESIGN OF THE END-SUPPORT REGION OF A CONCRETE BOX GIRDER USING STRUT-AND-TIE-MODELS

Fig. 1 shows the end-support region of a concrete box girder subjected to shear and torsion. The bearings are spaced apart by means of "corbels" to avoid tension in the bearings under the given loading condition.

It is common use to treat "shear" and "torsion" separately and to superimpose the results later on. This, however, is unsatisfactory, as generally in structural concrete design the design-models cannot be superimposed. It is necessary to develop a single model which takes into account all the forces at the same time, as required by Breen /2/.

According to the basic assumption of a constant shear-flow for circulatory torsion, the in-plane forces due to shear and torsion are distributed uniformly along the center-lines of the individual webs V'$_w$ and flanges V'$_f$. The following strut-and-tie-model for the diaphragm is based on the assumption that these forces are transfered evenly to the diaphragm. A corresponding model for the webs is shown in chapter 2.8.
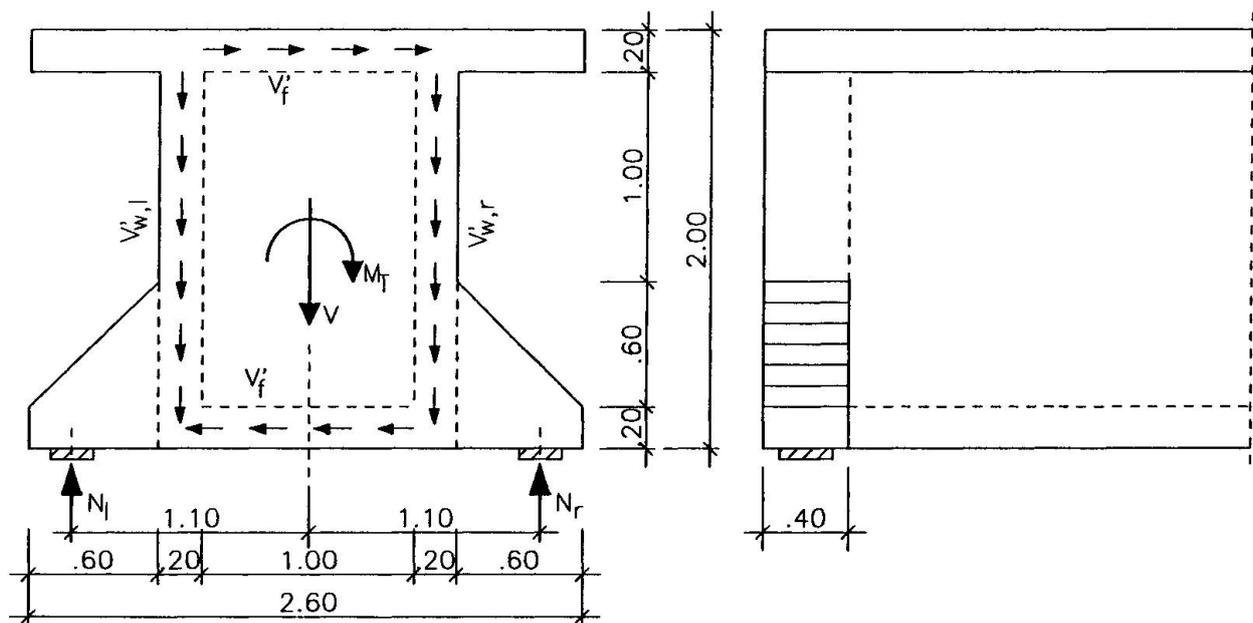
Forces at support:

M_T = 960 kNm

V = 1200 kN

Support reactions:

N_{left} = 1200/2 - 960/2.2 = 164 kN

N_{right} = 1200/2 + 960/2.2 = 1036 kN

Distributed forces in the webs and flanges due to shear and torsion:

V'_f = 960/(2·1.8·1.2) = 222 kN/m

$V'_{w,left}$ = 1200/(2·1.8) - 222 = 111 kN/m

$V'_{w,right}$ = 1200/(2·1.8) + 222 = 555 kN/m

Fig. 1: End-support region of the box girder
Support reactions and forces in the webs and flanges due to shear and torsion

2.1 Modelling technique

To develop the strut-and-tie-model of the whole D-region it is sometimes helpful to compose this model of already wellknown and established "sub-models". Thus parts of the structure can be treated separately. However, at the intersections of the different, parts the sub-models have to correspond with each other and must compose to the overall consistent model. Equilibrium must be satisfied for each of the sub-models as well as for the resulting overall model. In the following a strut-and-tie-model for the diaphragm will be developed in such a way.

Those parts of the structure in which the internal forces are reasonably well known are :
- the upper part of the diaphragm
- the corbels.

In the lower part of the diaphragm the state of stress is primarily unknown due to the geometric discontinuity.

2.2 The upper part of the diaphragm

The upper part of the diaphragm is shown in fig. 2 . Equilibrium requires that the horizontal shear force along line a-a has to balance the total shear force in the upper flange. The shear force along line a-a is also assumed evenly distributed.

The state of stress in this part of the diaphragm and the appropriate model is that of an ordinary rectangular diaphragm loaded by shear forces from the webs and top flange and directly supported under the webs /4/. The forces from the flanges V'_f have to be diverted by inclined struts C'_1 and vertical ties T'_1. For simplicity the inclination of the strut is assumed to be $45°$. To balance the horizontal components of the strut forces additional horizontal ties T'_2 are necessary. The vertical components of the strut forces and the forces of the webs $V'_{w,l}$, $V'_{w,r}$ sum up to give T_3 and C_3.

Note, that the struts and ties crossing the diaphragm represent stress-fields, therefore the reinforcement covering the tie forces has to be distributed accordingly.

Tie forces per unit length in the diaphragm
(for a $45°$ inclination of the compression field):
$T'_1 = T'_2 = 222$ kN/m

Forces in the webs:
$C_3 = 555 \cdot 1.2 + 222 \cdot 1.2 = 933$ kN
$T_3 = -111 \cdot 1.2 + 222 \cdot 1.2 = 133$ kN
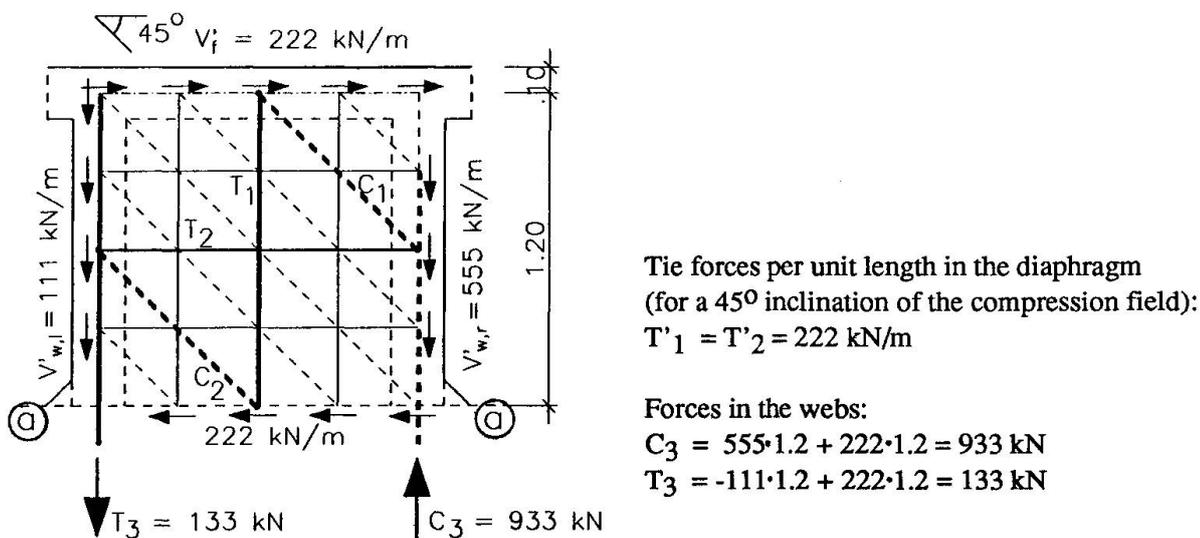
Fig. 2: Upper part of the diaphragm with forces and model

2.3 The corbels

Fig. 3 shows the corbels loaded by the bearing forces and by the vertical forces from the webs necessary to obtain equilibrium in the vertical direction. To ensure overall equilibrium the models require inclined struts C_6 resp. C_7, horizontal ties T_4 resp. T_5 and horizontal compression forces C_4 resp. C_5.

The concentrated node at the bearing plate is shown at the right corbel. The dimensions of this node are determined by the width of the bearing plate and by the reinforcement layout (node K6 acc. /4/). This critical concentrated node should already be checked at this early design state with respect to bearing pressure and anchorage of the reinforcement according to /4/.
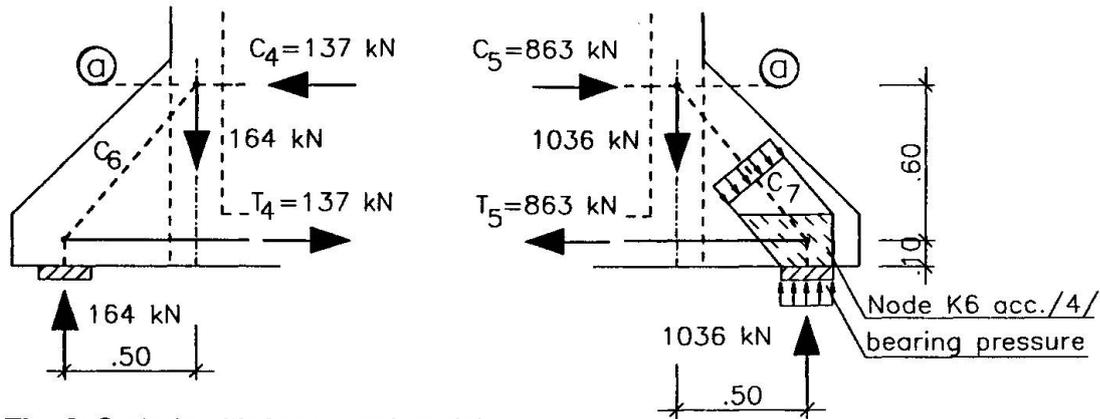
Fig. 3: Corbels with forces and models

2.4 The lower part of the diaphragm

The lower part of the diaphragm (fig. 4a) is loaded by the shear forces from the webs and bottom flange, by the forces from the upper part of the diaphragm and by the forces from the corbels.

A horizontal strut at the top C_8 and a horizontal tie at the bottom T_8 balance parts of the horizontal forces and further introduce evenly distributed forces along the horizontal edges of the lower part of the diaphragm. The concentrated vertical tension forces in the axis of the webs T_9, T_{10} are anchored in the diaphragm by reinforcement, and thus introduce distributed forces along the vertical edges of the diaphragm (fig. 4b).

What remains from all the forces (acc. to figs. 4a and 4b) is a "shear-wall" loaded along its edges (fig. 4c). The model for this shear-wall is of the same type as for the upper part of the diaphragm. For the inclination of the struts again 45° was assumed. Note, that the tie forces per unit length in the lower part of the diaphragm T'_{11}, T'_{12} are much higher than in the upper part $(T'_{11} = 1.73\ T'_1)$!
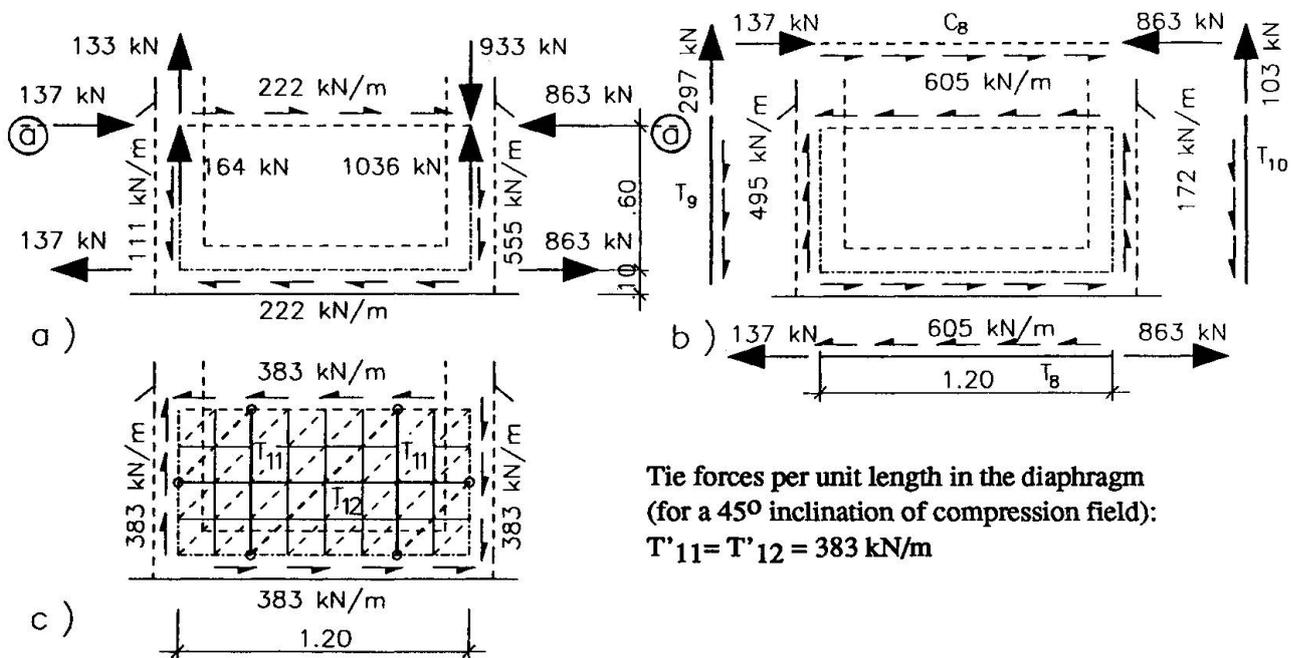
Tie forces per unit length in the diaphragm
(for a 45° inclination of compression field):
$T'_{11} = T'_{12} = 383\ \text{kN/m}$

Fig. 4: Lower part of the diaphragm
   a) with forces acting on it
   b) additional shear forces onto the diaphragm
   c) remaining shear-wall with appropriate model

2.5 Complete model

Fig. 5 presents the overall model of the end-section resulting from the previous sub-models. The inclined struts in the corbels and the horizontal strut in the diaphragm along line a-a also require a certain width as indicated for the strut C_7 in the corbel (fig. 3).

2.6 Principal reinforcement layout

Fig. 6 shows the principal reinforcement layout for the end-section. Note again the additional horizontal and vertical reinforcement necessary in the lower part.
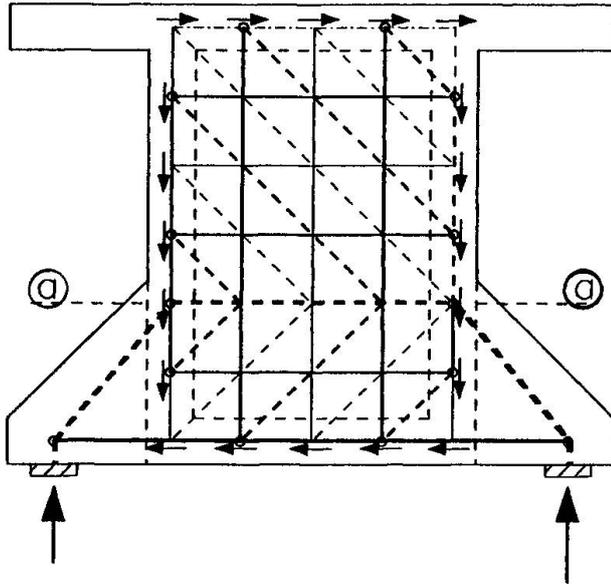
Fig. 5: Complete model
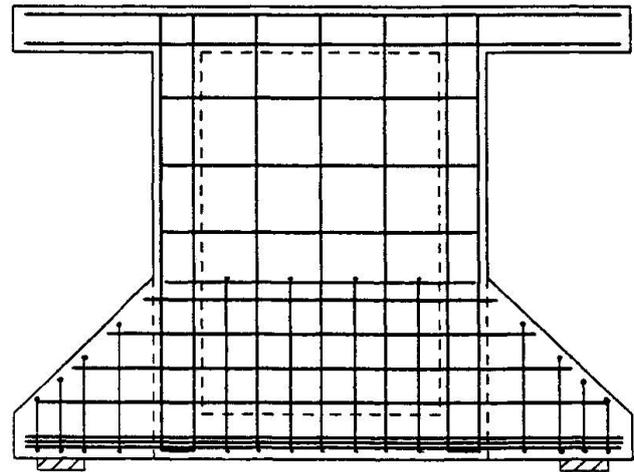
Fig. 6: Principal reinforcement layout

2.7 Check of concrete compression stresses

The highly stressed and rather concentrated struts C_6 resp. C_7 in the corbels have to be checked at the nodes at the bearing plates. This can be done according to regulations given by Schlaich et. al. in /4/ or by Sundermann in /6/. Within the remaining parts of the diaphragm only stress-fields occur in which the concrete stresses are not critical.

2.8 The strut-and-tie-model for the webs

An appropriate model for the webs for the above assumed evenly distributed vertical forces at the connection to the diaphragm V'_w is shown in fig. 7. This model has been developed on the basis of stress-fields and is explained in detail by Reineck et. al. in /5/. Here only the results are presented.

According to this model distributed longitudinal reinforcement T'_1 is necessary over the full height of the web. Furthermore increased vertical reinforcement T'_w is required in the D-region. Locally the concrete compression stresses in the inclined stress-fields are twice as high as in the B-region.

The flanges are loaded by the in-plane shear forces V'_f due to torsion as well as by the longitudinal forces $V_w \cot \vartheta$ resp. $V'_w \cot \vartheta$ from the webs. Developing the complete model for the flanges would exceed the limits of this paper. Separate models for either the shear force or the longitudinal forces are shown in /4/.
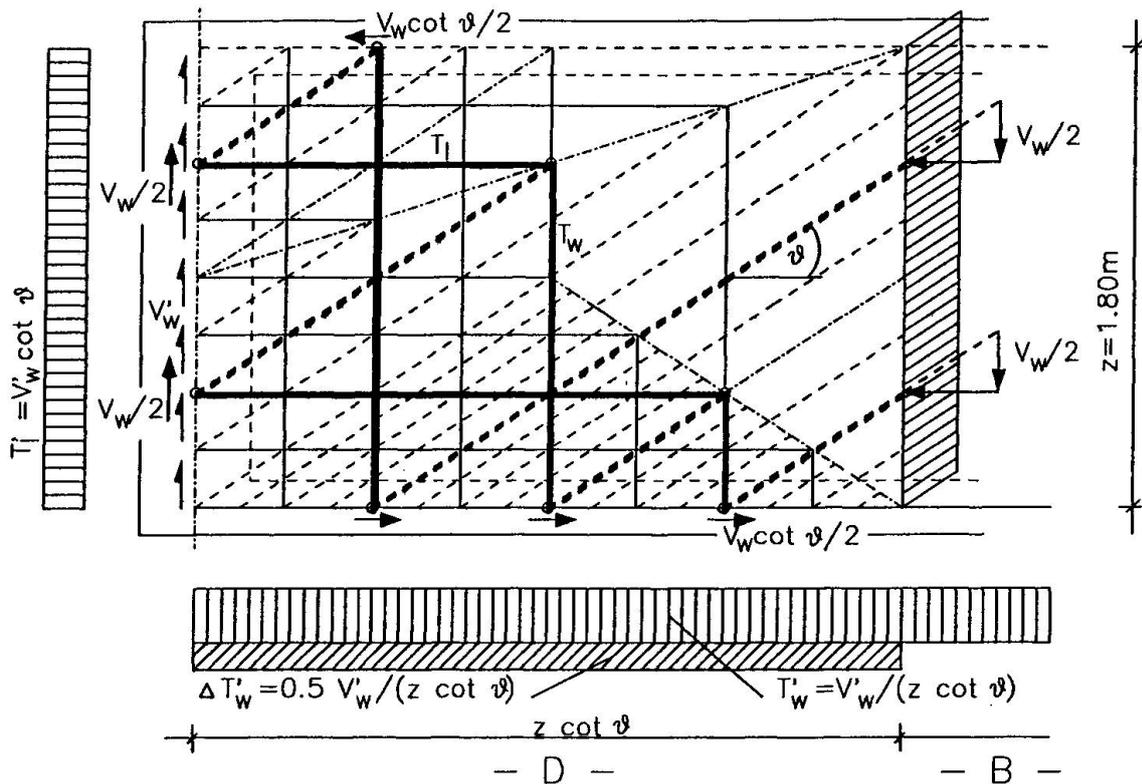
Fig. 7: Strut-and-tie-model for the webs acc. to /5/

3. CONCLUSIONS

This example demonstrates how a consistent model for the complicated diaphragm of a concrete box girder can be developed. For proper modelling and detailing of the D-region it is necessary to consider distributed forces by using stress-fields, in order to distribute the reinforcement accordingly. The complete model for the diaphragm can be composed of established and wellknown "sub-models". The models for the webs and flanges must be consistent with the model for the diaphragm. With this modelling-technique a safe design and a proper detailing of this complicated D-region can be guaranteed.

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Practical Experience with Modelling of Structural Concrete Members

Expérience pratique avec des modèles de barres pour des constructions en béton

Praktische Erfahrung bei der Anwendung von Stabmodellen im Betonbau

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SUMMARY

This report deals with examples of three actual projects where strut models helped to clarify the global flow of forces, to simplify the calculation and to develop structural details.

RÉSUMÉ

Trois projets vus à la lumière de différents modèles de barres permettent d'aider à comprendre le cheminement des efforts, de simplifier les calculs et de développer la conception des détails constructifs.

ZUSAMMENFASSUNG

Dieser Bericht zeigt am Beispiel von drei ausgeführten Projekten, wie bei der praktischen Arbeit das Modellieren von Stabwerken zum Verständnis des Gesamttragverhaltens, zur Vereinfachung der Bemessung und bei der Detailausbildung helfen kann.

1. INTRODUCTION

In the CEB-Bulletin d'Information 150: Detailing of concrete structures [1] a method was proposed how to perform methodically the practical design and detailing of concrete structures. Its main idea is expressed there: a deep understanding of the flow of forces is needed to design good structures, that means materially sound as well as beautiful.

This idea is still valid and over the last 10 years has caused further activities [2] ./. [5] to transfer this way of thinking into practical work.

Did it succeed? Is it accepted and does it help? Three examples will demonstrate that strut models can indeed help to understand the global flow of forces, to replace sometimes extensive calculation and to develop structural details.

2. MODELLING OF THE GLOBAL FLOW OF FORCES

Project: University of Kassel, Technik III [6]

The new building "Technik III" of the University of Kassel is divided in two: The southern part consists of halls for engineering tests and the northern part serves as an institute- and laboratory building (Fig. 1). It is about 170 m long and 6 storeys high. Its northern side exists of columns, which are inclined between level 0 and + 2. At the kinks high forces occur due to change of direction, and the effects of these forces have to be followed carefully (Fig. 2).

According to Fig. 3 the vertical force V, which results from the column load at level + 2 creates an equally distributed groundpressure σ_b below level - 1.

To transfer the load from the top to the bottom the following members are available (for one half of the building): The slab I at level + 2 collects the horizontal forces due to change of direction at the kink of the columns and transfers them concentrated into the vertical wall II. There they are taken to slab III, creating equilibrium with the horizontal forces at the kink of the columns at level ± 0. The vertical forces at that point are transferred continuously into the wall IV and form there into the bottomplate VI. In the walls V the horizontal forces are transferred into vertical ones, creating a moment at the edges of the bottomplate VI. This moment together with the vertical forces from the columns causes an equally distributed groundpressure σ_b, provided that the bending stiffness of the plate VI is big enough.

All the members, which are necessary to carry the loads, are shown in Fig. 4. They each can now be designed and detailed separately according to their geometry and loadings. Joined together the single members form a 3-dimensional strut model, which shows the global flow of forces of the whole structure in a transparent and demonstrative way (Fig. 5).

3. STRUT MODELS AS A REPLACEMENT FOR EXTENSIVE CALCULATIONS

Project: Extension of the Casino building of the Bayerische Rückversicherung [7]

The Casino building of the Bayerische Rückversicherung in Munich was extended in 1989 by three additional storeys. It is a cylindrical suspension house with slabs, which are suspended by 6 steel bars equally distributed along their edges. The suspension elements hang straight down from the edge of the ceiling of the last storey, above which they are inclined to the top of the concrete core (Fig. 6). During the erection stage this part existed only of radial girders between a concrete ring, which was formed polygonally at its interior edge.

The critical loading case for this member was an eccentric load, which created two horizontal forces of about 1100 kN each at the edge of the concrete ring. A strut model according to Fig. 8 was taken as a basis and yielded a reinforcement 13 $\varnothing$ 20, which was to be arranged polygonally from one support to the next one.

Unfortunately this result was just accepted as a predesign, but for the official calculations some more efforts were expected!

Therefore a computer calculation for the ring with radial girders and the geometric according to Fig. 7 was carried out. The resulting diagrams for the bending moments, normal- and shear forces in fact were quite impressive (Fig. 9) and stimulated to do a proper design calculation for support-, span- and especially shear reinforcement.

But the ambition was even stronger to show that strut models are competitive. Therefore the direction and the value of the resultant forces out of N and Q were determined and their location calculated from M and N. The result is shown in Fig. 10 and is now satisfying the strut model designer too.

4. DETAILING

Project: Ice Skating Hall, Munich

In the Munich Olympia area a further hall was built for the Skating-World-Championship in 1991 (Fig. 11). The hall is standing on concrete columns above a parking place with a raster of 10.80 m x 5.40 m. The slab is made out of prestressed concrete, with cantilevers on both ends of 6.75 m length, which are formed as T-beams (Fig. 12). Its webs are 0.60 m wide and 0.40 m high with a slab on top, which is 0.20 m thick. The distance of the webs amounts to 5.40 m. The cantilever is inclined to the horizontal level by 21° with the kink in a distance to the last column row of 1.35 m. There the value of the bending moment amounts to M = - 1800 kNm per T-Beam. With an internal lever arm z = 0,45 m the horizontal forces become 4000 kN each. Due to the change of direction at the kink, vertical forces occur, which are named $U_{(T)}$ and $U_{(C)}$ and are 1440 kN big. They are to be connected by stirrup reinforcement (1) according to Fig. 13.
The effective width of the tension area in the slab was determined to 1.40 m and therefore a part of the bending reinforcement ($\approx$ 1/3) is arranged there. The reinforcement is shown in Fig. 14 and the expert, who already possesses long-term experience with reinforcing structures, which are - at least at the first glance - similar, e.g.: framecorners and staircases, will normally judge the reinforcement as correct and complete straight away.
But the careful study of the flow of forces in cross direction shows that important reinforcement is missing: the forces due to change of direction $U_{(T2)}$ in the side parts of the slab, are not taken over by any reinforcement (Fig. 15).
With the stirrup reinforcement (3) - (Fig. 16) - in the slab, which amounts to 65 cm^2/m^2 in this example and with the bending reinforcement (4) in cross direction as bottom reinforcement 13 cm^2/m the design is complete and equilibrium is now installed between the forces due to change of direction $U_{(T)}$ and $U_{(C)}$.

5. CONCLUSION

I am convinced that the method of strut modelling is able to produce the necessary knowledge to design good and harmonious structures. Unfortunately the method is constantly being underestimated: even the inexperienced designer expects quick results with a solution, which he can represent. That does not fit together! Confidence in the solution develops only after studying the problem intensively. This method needs its time for application too!

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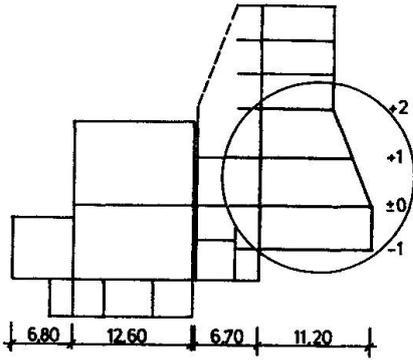   Aufstockung eines Casinogebäudes. Stahlbau 59 (1990), H. 12.

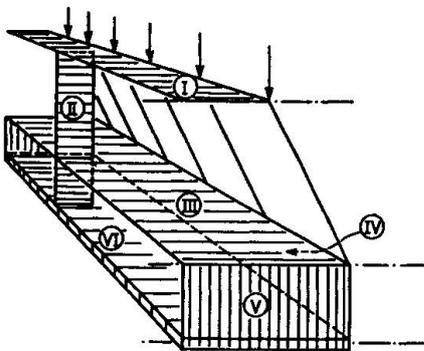**Fig. 1**   Project: University of Kassel
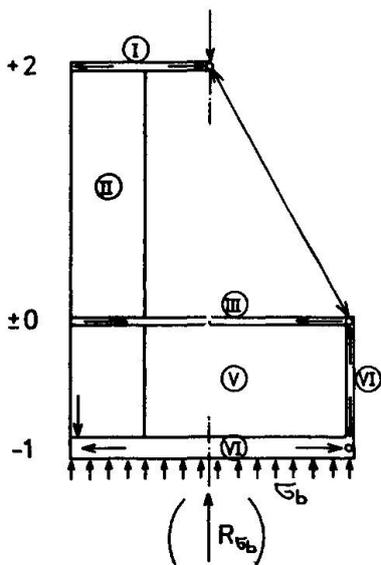Technik III

Fig. 2   Part of the laboratory building

Fig. 3   Section through the
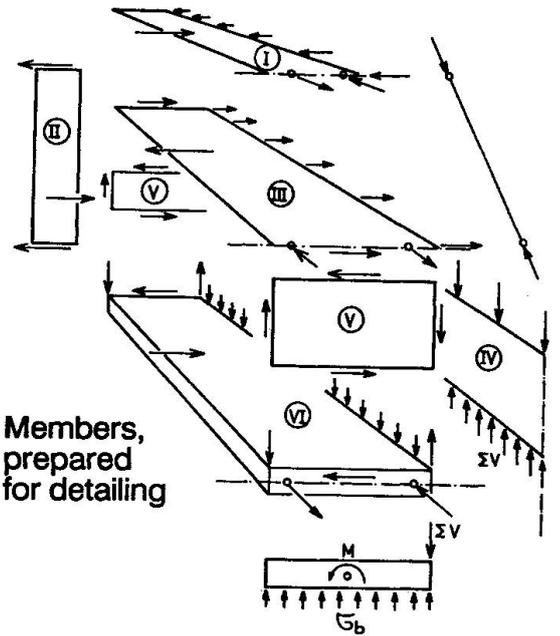laboratory building

Fig. 4   Members,
prepared
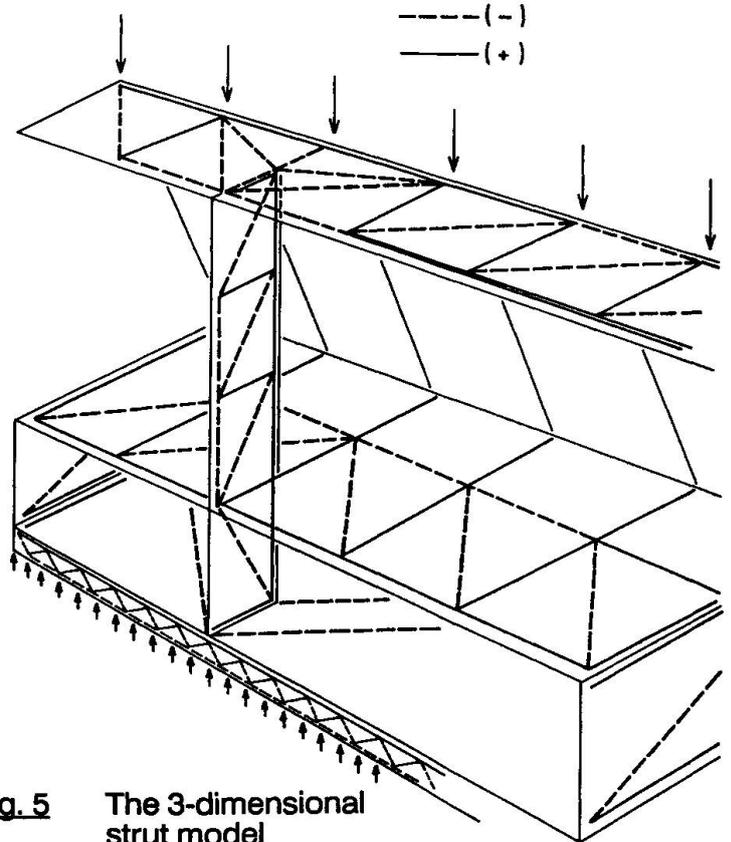for detailing

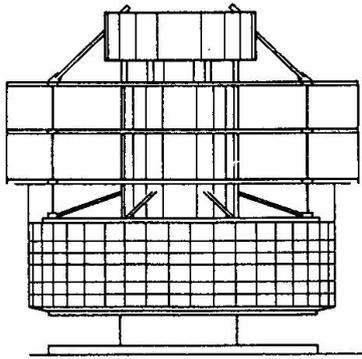**Fig. 5**   The 3-dimensional
strut model

Fig. 6   Project: suspension building
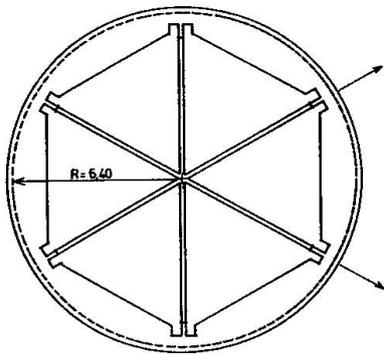of the Bayerische Rückversicherung

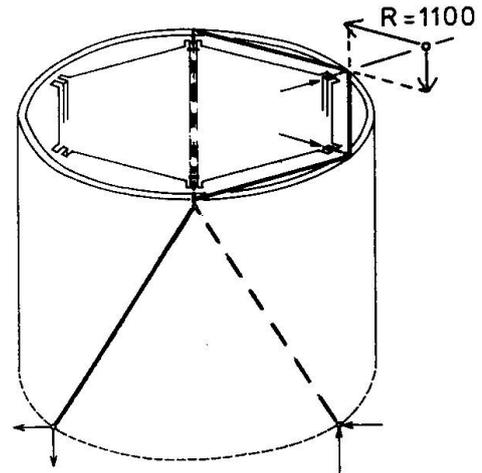**Fig. 7**   Top slab at erection state
with eccentric loading
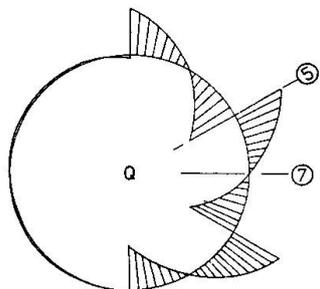

Fig. 8   Strut model for
loading case acc. to Fig. 7
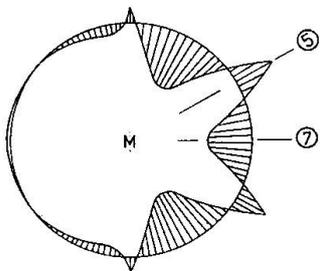
Fig. 9   Action effects acc. to Computer
calculations
- Bending moments M
- Shear forces Q
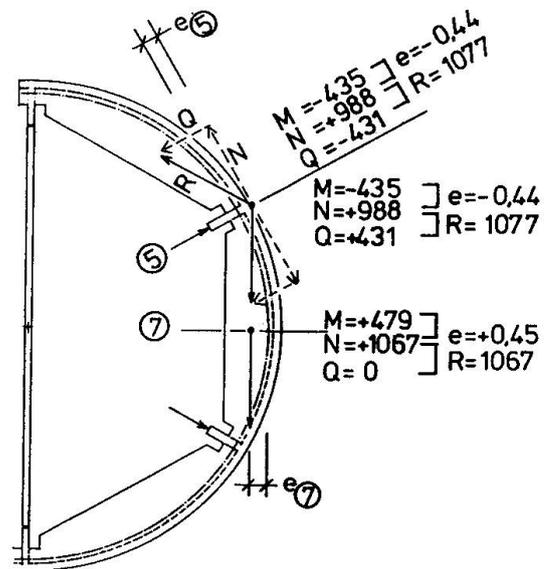
Fig. 10   The resultant forces and
eccentricities.
Compare with Fig. 8!
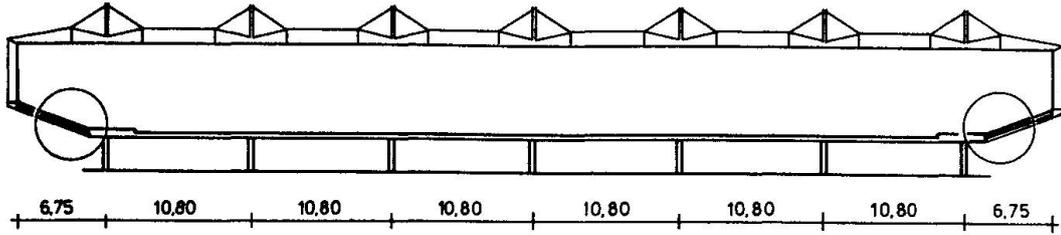
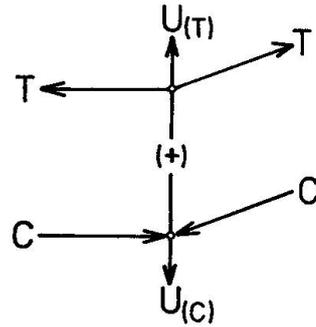Fig. 11 Project: Skating hall in Munich
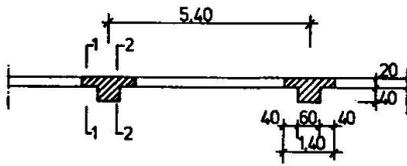
Fig. 12 T-beams at the cantilevers

Fig. 13 Forces U due to change of direction
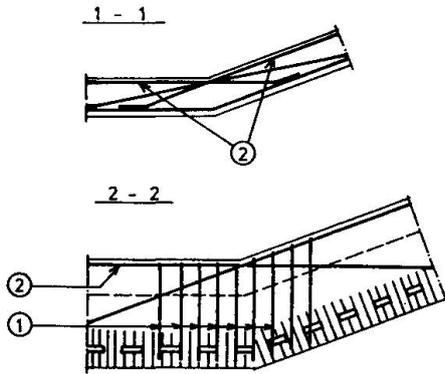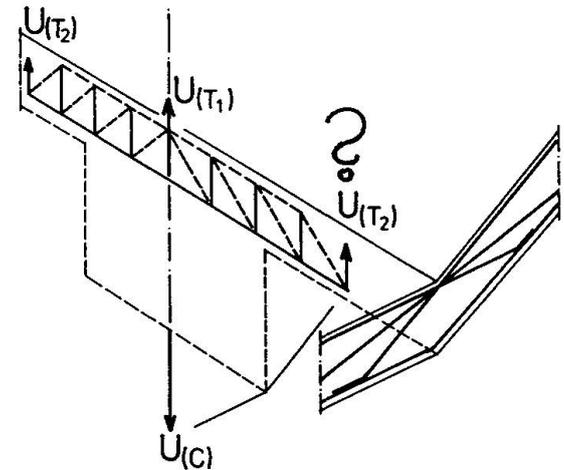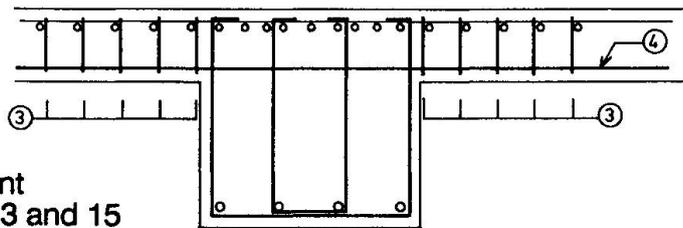

Fig. 14 Reinforcement acc. to Fig. 13

Fig. 15 A 3-dimensional effect!

Fig. 16 Reinforcement acc. to Fig. 13 and 15