Zeitschrift: IABSE reports = Rapports AIPC = IVBH Berichte

Band: 62 (1991)

Rubrik: Bond and tension stiffening effect

Effects of Residual Strength of Cracked Concrete on Bond

Effets sur l'adhérence des contraintes résiduelles du béton éclaté

Einfluss der Restzugfestigkeit gerissenen Betons auf den Verbund

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SUMMARY

The local bond-slip law of an anchored ribbed bar after the complete cracking of the surrounding concrete is studied. The theoretical approach is based on the confining effects due both to the transverse reinforcement and the residual tensile strength of cracked concrete. Experimental confirmations and theoretical results are presented. In particular the confining effects produced by the residual strength of cracked concrete are investigated and discussed.

RÉSUMÉ

On étudie ici la loi localisée d'«adhérence-glissement» apparaissant dans le cas d'une barre nervurée ancrée après éclatement complet du béton d'enrobage. L'approche théorique se base sur des effets confinants dûs conjointement à l'armature transversale ainsi qu'aux contraintes de traction résiduelles du béton éclaté. Des confirmations expérimentales et des résultats théoriques sont présentés; les effets confinants provoqués par la contrainte résiduelle du béton éclaté sont examinés et discutés en détail.

ZUSAMMENFASSUNG

Das örtliche Verbundgesetz für die Verankerung eines gerippten Bewehrungsstabs in vollständig gerissenem Beton wird untersucht. Grundlage für die theoretische Untersuchung sind die Umschnürungseffekte infolge der Querbewehrung und der Restzugfestigkeit des gerissenen Betons. Theoretische Ergebnisse und experimentelle Bestätigungen werden vorgestellt. Insbesondere werden die Einflüsse einer Umschnürung infolge der Restfestigkeit des Betons untersucht und diskutiert.

1. INTRODUCTION

The importance of the tensile strength of concrete on bond was underlined in [1]. In the same paper this phenomenon was theoretically modelled and the bond strength was evaluated taking into consideration the tensile strength of solid concrete that surrounds the split core. Experimental tests [2] showed that the bond stress-slip relationship is influenced both by the amount of stirrups and by the thickness of the concrete cover. A theoretical interpretation and modelling of the phenomena involved around an anchored bar were proposed in [3,4], where the relevance of the confining action produced by the residual tensile strength of the split concrete was underlined. At the beginning, the splitting crack opens near the anchored bar (Fig.1a,b) and propagates both transversally and along the bar. When it is completely propagated throughout the cross-section, bond strength is still locally possible owing to the confining actions produced by the transverse reinforcement and the residual stress transmitted by the crack faces [5,6]. For light or no transverse reinforcement, bond stress decreases as slip increases, so that an unstable local behaviour occurs. When the split zone is limited as in Fig.1, bond stress redistribution along the bar can occur and a ductile global behaviour of the anchorage is still possible. The bigger the concrete cover and bar spacing become, the more relevant the confining contribution of cracked concrete is. This is because a small residual stress acting on a large split surface can produce a considerable confining action. Since the splitting crack opening is variable both across the transverse section and along the anchored bar, the local response of the cracked concrete is also variable. The cracked concrete confining contribution should be evaluated by means of the tensile stress-crack opening law. The well known specific fracture energy $\mathcal{G}_F$, which is the integral of this law, is not sufficient to express the cracked concrete confining capacity, at least in the present theory on bond.

2. ANALYTICAL FUNDAMENTALS

In anchorages with completely propagated splitting cracks, bond is still possible when an adequate transversal confining action is assured. This confinement can be produced both by the transverse reinforcement (secondary bars or stirrups) and by the residual strength of cracked concrete.

The modelling of the local bond behaviour in anchorages when splitting occurs is developed on the basis of the following assumptions:

1. The splitting crack is completely propagated along the bar spacing and cover in influence zone Δz of one transverse bar (Fig.1c).
2. Δz is small and has the same value of stirrup spacing, so that average crack opening w and bond stress τ can be assumed as the local values.
3. All the principal bars have the same diameter ϕ_p and all the transverse bars have the same diameter ϕ_{st}.

According to these assumptions, the following equations were proposed in [3,4]. For bond:

$$\tau = \tau_{m,o}\,(1 - \gamma_1\, w/\phi_p)\,(1-e^{-(\beta_1+\beta_2 w/\phi_p)(s/\phi_p-\gamma_2 w/\phi_p)}) \quad (1)$$

$$\tau = \tau_0(1/(1+K_1 w/\phi_p)) + \tau_1\sigma_n(1/(1+K_2 w/\phi_p)) \quad (2)$$

where $\tau_{m,o}$ = maximum bond stress for w=0; γ_1, γ_2, β_1, β_2, K_1 and K_2= coefficients experimentally determined on the basis of the curves plotted in Fig.2a,b and obtained in [7]; s = principal bar slip and σ_n = radial stress produced by the principal bar. The limitation $\tau = \tau_{m,o}$ when $\tau > \tau_{m,o}$ was adopted. For stirrup stress (1st confining action) equation:

$$\sigma_{st} = E_s\sqrt{a_2(w/(\alpha\,\phi_{st}))^2 + a_1(w/(\alpha\,\phi_{st})) + a_0} \quad (3)$$

plotted in Fig.2c, was assumed according to [8] where E_s = Young's modulus for steel; a_0, a_1 and a_2 = coefficients of the ideal trilateral local bond stress-slip law of the transverse bars and α = factor characterizing the position of the splitting crack (Fig.4d). For tensile stress transmitted by the splitting crack faces (2nd confining action) equation:

$$\sigma_{rc} = f_{ct0}/(\kappa\ w/\phi_a + 1) \qquad (4)$$

plotted in Fig.2d, was adopted according to [6] where f_{ct0} and κ = coefficients experimentally determined and ϕ_a = maximum aggregate size.

Eq.1 is based on the following similitude criterion: both crack opening w and slip s are proportional to bar diameter ϕ_p. In this way, for the same value of ratios s/ϕ_p and w/ϕ_p all the coefficients γ_1, γ_2, β_1 and β_2 should be independent of ϕ_p. Even coefficients τ_0, τ_1, K_1 and K_2 in Eq.2 should be independent of ϕ_p, having adopted ratio w/ϕ_p in the place of w. The first confining action produced by the stirrup legs increases with the splitting crack opening (Eq.3) and this phenomenon is governed by the progressive unsticking of the bar studied in [8]. For the second confining action due to the tensile strength of cracked concrete (Eq.4), a similitude criterion for the relationship between w and the maximum aggregate size ϕ_a was also proposed in [6], so that coefficient κ turned out to be be independent of ϕ_a.

For equilibrium, the global confining action in zone Δz, given by Eqs.3 and 4, is equal to the global radial force produced by the anchored bars, so that:

$$\sigma_n = \Omega\ \sigma_{st} + B\ \sigma_{rc} \qquad (5)$$

where Ω = stirrup index of confinement, defined as the ratio between global cross section area A^*_{st} of the stirrup legs and area A^*_p of the principal bar in the split plane (Fig.4f), B = concrete index of confinement, defined as the ratio between the net area $(b-n_p\phi_p)\Delta z$ of concrete in the split plane and the afore mentioned area A^*_p.

From Eqs.2 and 5, bond stress τ as a function of σ_{st}, σ_{rc} and w can be obtained:

$$\tau = \tau_0(1/(1+ K_1\ w/\phi_p)) + \tau_1(\ \Omega\ \sigma_{st} + B\ \sigma_{rc}\ )\ (1/(1 + K_2\ w/\phi_p))$$

Owing to the nonlinear equations involved, the relationship of bond stress τ as a function of slip s is obtained for the principal bar by means of a numerical approach which is based on the following procedure. Attributing a value w to crack opening, Eqs.3 and 4 give σ_{st} and σ_{rc}. Then bond stress τ can be calculated by means of Eq.6 and finally slip s is obtained from Eq.1.

3. RESULTS

In Fig.3 curves τ-s obtained by the present theory fit the experimental results well. Curves 1-4 (Fig.3a) concern the cases examined in [2] with different transverse reinforcement diameters ϕ_{st}. Fig.3b, referring to a specific test studied carefully in [3] to check this theory, shows a very good agreement also for crack opening and stirrup stress. This agreement still emphasizes the importance of the confining contribution due to the residual tensile strength of split concrete.

Theoretical diagrams of Fig.4 show the role of some significant parameters. Curves τ-s, w-s, σ_{st}-s refer to the following governing parameter values:

- Eq.1: $\tau_{m,0}$ = 18 MPa β_1 = 75 β_2 = 0 γ_1 = 42 γ_2 = 0.8;
- Eq.2: τ_0 = 1.8 MPa τ_1 = 0.8 k_1 = 115 k_2 = 35;
- Eq.3: τ_{02} = 2.5 MPa $\tau_{12}\phi_{st}$ = 500 MPa τ_{12}/τ_{11} = 0.3;
- Eq.4: f_{ct0} = 1.0 MPa κ = 250;
- geometrical and mechanical characteristics:

n_p=2 ϕ_p=20 mm α=2 E_s=206000 MPa Δz=100 mm ϕ_a=15 mm b=200 mm

Different values of the geometrical or mechanical characteristics adopted for each curve are indicated in Tab.1. Figs.4a,b,c show the role of the transversal extension of the concrete split-area dependent on section width b. Three different amounts of confining reinforcement are adopted and expressed through stirrup index of confinement Ω. Figs.4g,h,i show the influence of fracture energy $\mathcal{G}_F$ obtained by integrating the σ_{rc}-w curve (Eq.4) from w=0 to $w=w_u$. High values of $\mathcal{G}_F$, correspondent to an appreciable residual strength of cracked concrete, and large values of width b both increase the value of bond stress τ.

Note that fracture energy $\mathcal{G}_F$ could be assumed as one of the governing parameters of the present bond stress-slip relationship, but some specifications and remarks are necessary. In reality, the bond stress-slip relationship obtained in [4] showed the importance of the parameters f_{ct0}, κ and ϕ_a, characterizing the σ_{rc}-w relationship. These results were independent of the ultimate crack opening w_u (correspondent to stress-free crack surface). In fact, the maximum value of the splitting crack opening involved was 0.2-0.3mm, which was remarkably less than values w_u=0.4-0.7 mm indicated by experiments [6]. Fracture energy $\mathcal{G}_F$ depends on the same governing parameters f_{cto}, κ and ϕ_a, but also on w_u. This ultimate crack opening seems to be only variable with maximum aggregate size ϕ_a, according to both the similitude criterion introduced in Eq.4 and to some experiments in progress, so that the ratio w_u/ϕ_a could be assumed as a constant for every type of concrete, as well as coefficient κ. In this way $\mathcal{G}_F$ and ϕ_a can become the only governing parameters involved in σ_{rc}-w relationship. Diagrams of Fig.4g,h,i refer to an aggregate size ϕ_a=15mm (w_u/ϕ_a=0.05) and three values of $\mathcal{G}_F$ (50,100,150 J/m^2) correspondent to low, medium and high residual strength.

4. CONCLUDING REMARKS

The analytical model here proposed for the local bond stress-slip relationship after concrete splitting gives results which have a good agreement with the experimental tests (Figs.3a,b). The theoretical curves considerably depend on the residual tensile strength of cracked concrete especially when light or no transverse reinforcement is present. This residual tensile strength of split concrete is here introduced by means of two governing parameters which are fracture energy $\mathcal{G}_F$ and maximum aggregate size ϕ_a. In the present theory the single parameter $\mathcal{G}_F$ is not sufficient to describe this confining action due to the split concrete.

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Diag.	ϕ_{st} [mm]	n_{st}	f_{ct0} [MPa]	$\mathcal{G}_F$ [J/m²]
4,a	0	0	1.00	150
4,b	6	1	1.00	150
4,c	8	2	1.00	150
4,g	0	0	1.00 0.66 0.33	150 100 50
4,h	6	1	1.00 0.66 0.33	150 100 50
4,i	8	2	1.00 0.66 0.33	150 100 50

Table 1

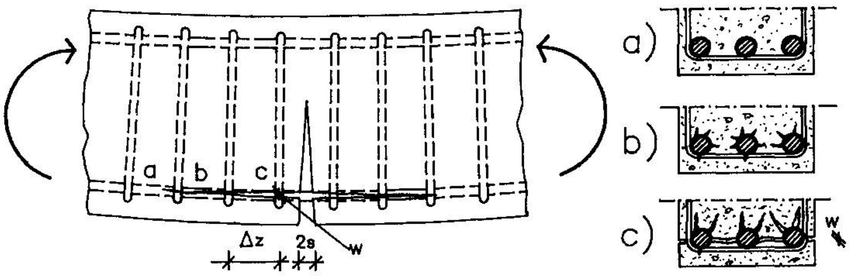
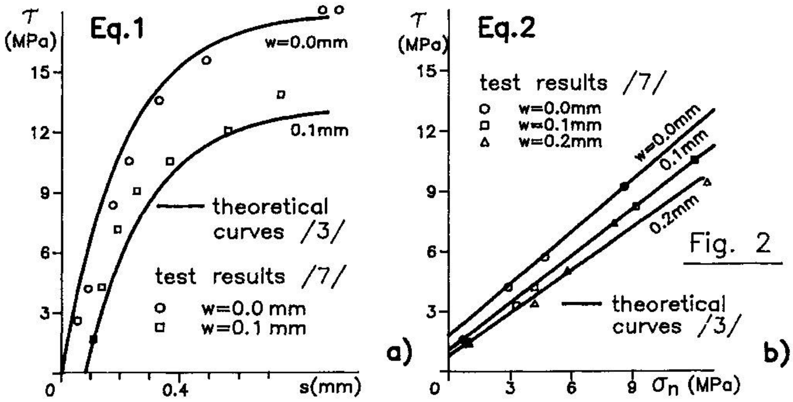
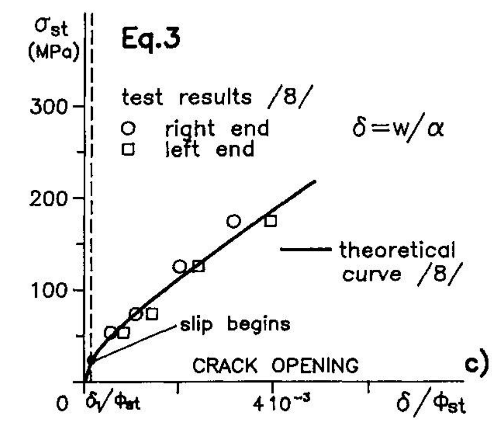


Fig. 1 – Splitting crack propagation in anchorages: a)no splitting crack; b)partially propagated splitting crack; c)completely propagated splitting crack.





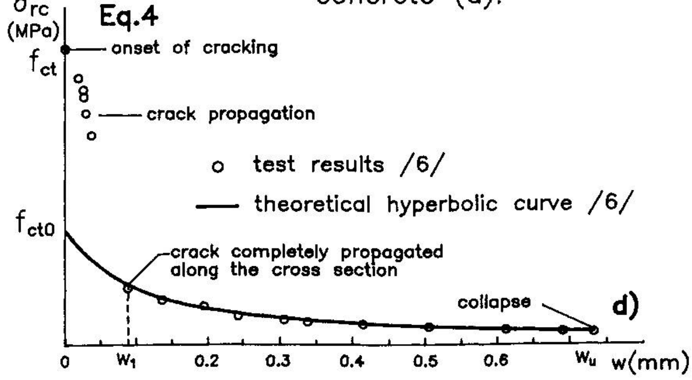
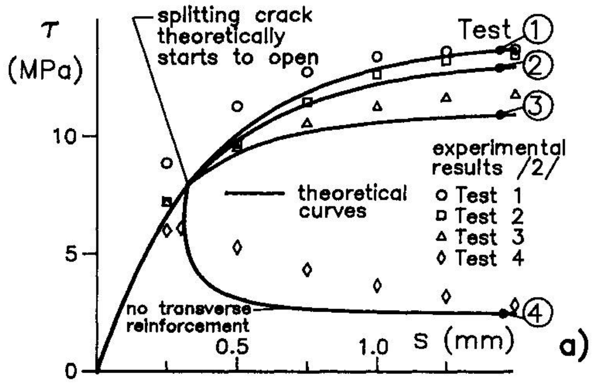


Fig. 2 – Analytical modelling of bond stress τ (a), radial stress σ_n (b), confining stress σ_{st} of stirrups (c) and of residual tensile stress σ_{rc} of cracked concrete (d).



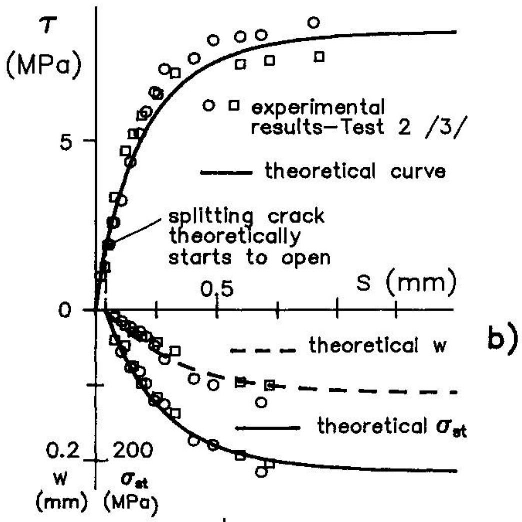


Fig. 3 – Comparison between theoretical and experimental results.

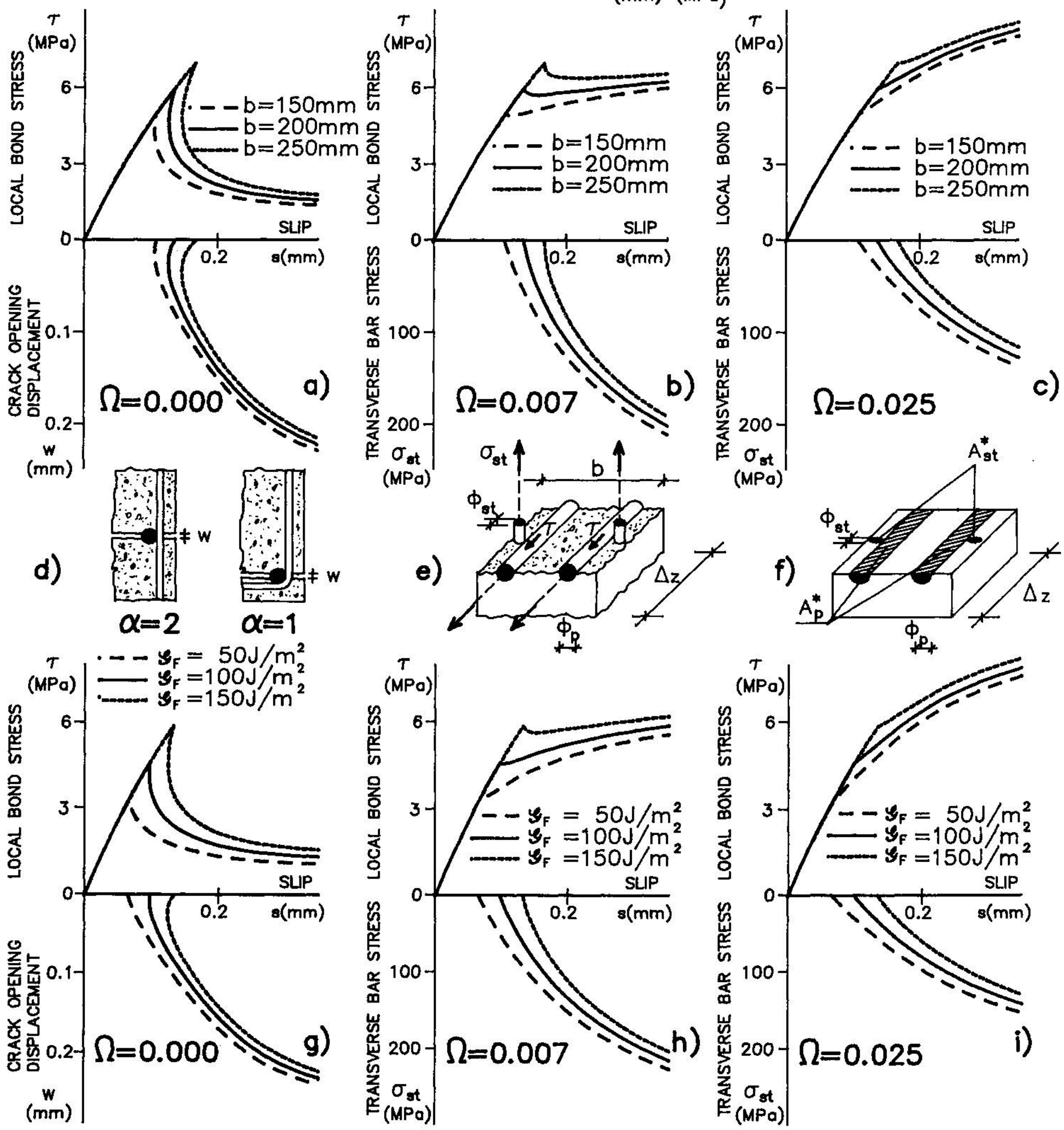


Fig. 4 – Influence of cross section width b and fracture energy $\mathcal{G}_F$ on bond after splitting.

Local Bond between Reinforcing Steel and Concrete

Adhérence localisée acier-béton

Lokaler Verbund zwischen Bewehrungsstahl und Beton

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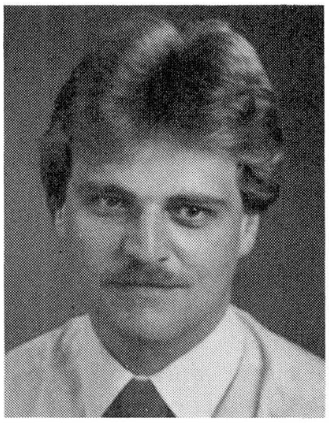

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SUMMARY

Experimental and analytical investigations on the local bond behaviour are presented. The permanent magnet – Hall sensor measuring system was developed for this purpose. This system allows local relative displacements in axial and radial directions between steel and concrete inside the test specimen to be determined. The test results yielded basic knowledge for a better understanding of bond behaviour. Furthermore, a material model was developed which is especially suitable for investigations with the aid of the Finite Element Method.

RÉSUMÉ

L'adhérence localisée acier-béton est présentée à la lumière des recherches expérimentales et analytiques. Grâce à l'emploi de la sonde Hall munie d'un aimant permanent, il a été possible de déterminer les déplacements relatifs locaux dans les directives axiales et radiales apparaissant entre béton et armature, et ceci à l'intérieur-même des échantillons testés. Les résultats du test ont permis d'établir des connaissances fondamentales pour une meilleure compréhension du mécanisme d'adhérence. Un modèle a d'ailleurs été mis au point qui permet d'effectuer des recherches assistées par éléments finis.

ZUSAMMENFASSUNG

In dem Beitrag werden experimentelle und analytische Untersuchungen zum lokalen Verbundverhalten vorgestellt. Mit dem hierfür entwickelten Permanentmagnet-Hallsonden-Messverfahren war es möglich, die lokalen Relativverschiebungen in axialer und radialer Richtung zwischen Stahl und Beton im Inneren der Versuchskörper zu bestimmen. Aus den Versuchsergebnissen wurden grundlegende Erkenntnisse zum besseren Verständnis des Verbundverhaltens gewonnen. Ferner konnte ein Materialmodell erstellt werden, das besonders für Untersuchungen mit der Finite Elemente Methode geeignet ist.

1. INTRODUCTION

Cracks form in reinforced concrete structures when the concrete tensile strength is exceeded. Besides the visible cracks on the surface, inner cracks form at the reinforcing bar ribs. The more the crack formation has progressed, the smaller are the bond forces between steel and concrete. As the crack development depends directly on the concrete strength, the bond stiffness is also influenced by this factor.

The analytical consideration of bond behavior of steel and concrete gains considerable importance. Therefore, the knowledge of realistic models of material behavior is necessary. With these models realistic deformation analyses and economical load-carrying capacity analyses can be executed.

2. GENERAL REMARKS ON THE EXPERIMENTAL INVESTIGATIONS

A detailed description of both experiments and results can be found in [1]. The tension specimens were made of Portland cement PZ35F and natural, unbroken aggregate with a maximum grain size of 16 mm and were centrically reinforced. The following parameters were varied:

- specimen length (short tension specimens without separating cracks of different lengths, long tension specimens with intended separating cracks of different lengths),
- concrete (cover and strength),
- reinforcements (related rib area and diameter),
- loading (number of repeated loads).

For the measurement of the local relative displacements between steel and concrete a small permanent magnet is fixed in the groove of an austenitic reinforcing steel. A Hall sensor is fastened to the concrete at a certain distance to the magnet (see Fig. 1). The magnetic induction influencing the Hall sensor changes with the distance between the measuring system elements. For the determination of the relative displacements a relation between distance and Hall voltage dependent on the magnetic induction is established by calibration before casting. The magnetic induction decreases with increasing distance to the permanent magnet. The relative displacements between steel and concrete at the edge of the specimen were determined using LVDTs.

The forces in the reinforcing steel were determined with the aid of strain gauges glued into two opposite grooves. The bond stress related to one relative displacement measuring point (permanent magnet at reinforcing steel) was analyzed applying the force difference of the strain gauges arranged on both sides of the measuring point.

3. RESULTS

Fig. 1 shows the bond stress - axial displacement relations of two experiments. It can be seen that these relations depend considerably on the concrete strength.

The measuring points arranged at a distance of 35 mm to the edge of the specimen mostly reach the maximum bond stress or even exceed it at the maximum external load of 100 kN. For the measuring points nearer to the center of the specimen (at a distance of 70 mm to the edge of the specimen) the maximum bond strength was reached only with those specimens with a concrete compressive strength of less than 35 N/mm^2 at the maximum external load of 100 kN.

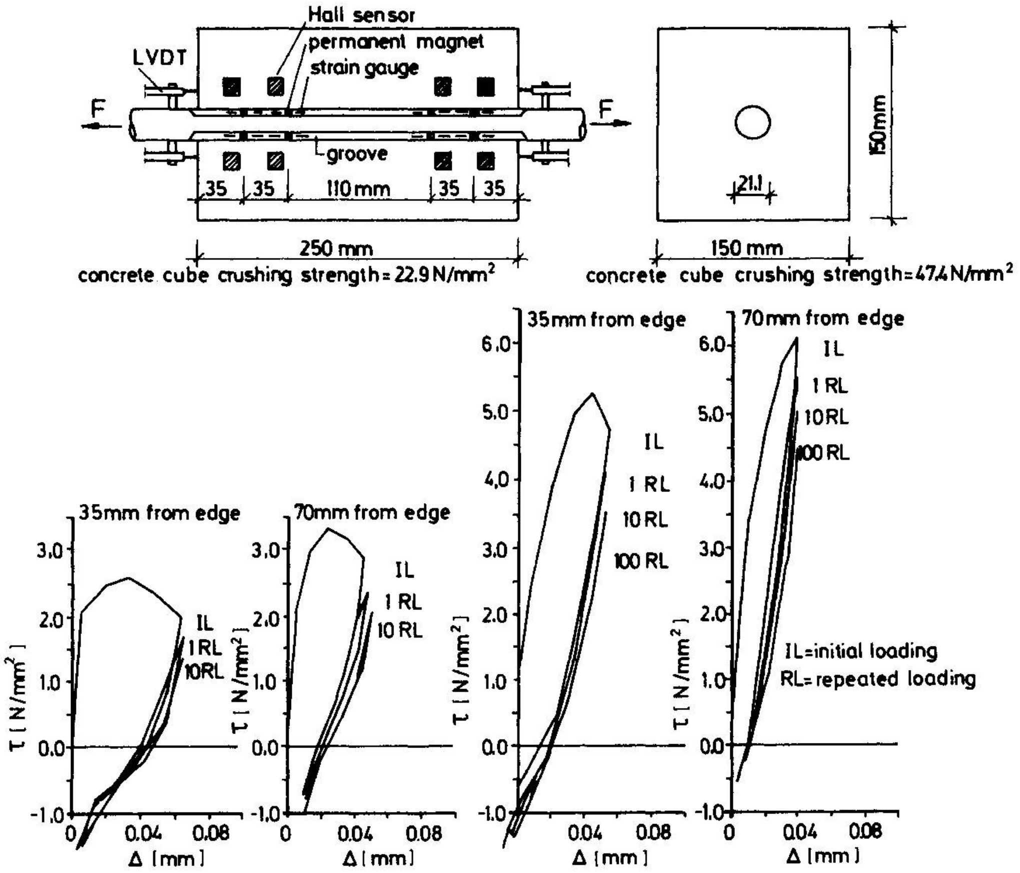

Fig.1 Bond stress - axial displacement relations dependent on concrete strength

An influence of the related rib area on the bond stress - axial displacement relations could not be found. Obviously a certain minimum size of rib area is necessary for the introduction of bond stresses into the concrete. This miminum size at the concrete ribs must be large enough to prevent them from deforming. A deformation of the concrete ribs was not observed with any of the concrete specimens which were split after the tests.

For the determination of the influence of the specimen length specimens which only differed in their lengths (175, 250, and 400 mm) were tested. At equal concrete strength and equal distance to the edge of the specimen smaller bond strengths resulted with increasing specimen length.

Smaller negative or no negative bond stresses occur with specimens with intended cracks contrary to short tension specimens with free edges. This can be traced back to the fact that at a certain unloading of the long specimens an almost constant course of steel stress results. When no external load affects the tension specimens with intended cracks, a nearly constant tensile stress remains in the steel owing to the cracks which do not close completely. Consequently, the concrete is in compression at this point. The intended cracks in the long specimens were arranged in a way that they corresponded to the average crack spacing. This is because the bond stress - axial displacement relations depend on the specimen lengths.

The bond stress - axial displacement relations of long specimens with average crack spacings are decisively dependent on the concrete cover. The larger the concrete cover, the larger the transfer of bond stress between steel and concrete (see Fig. 2).

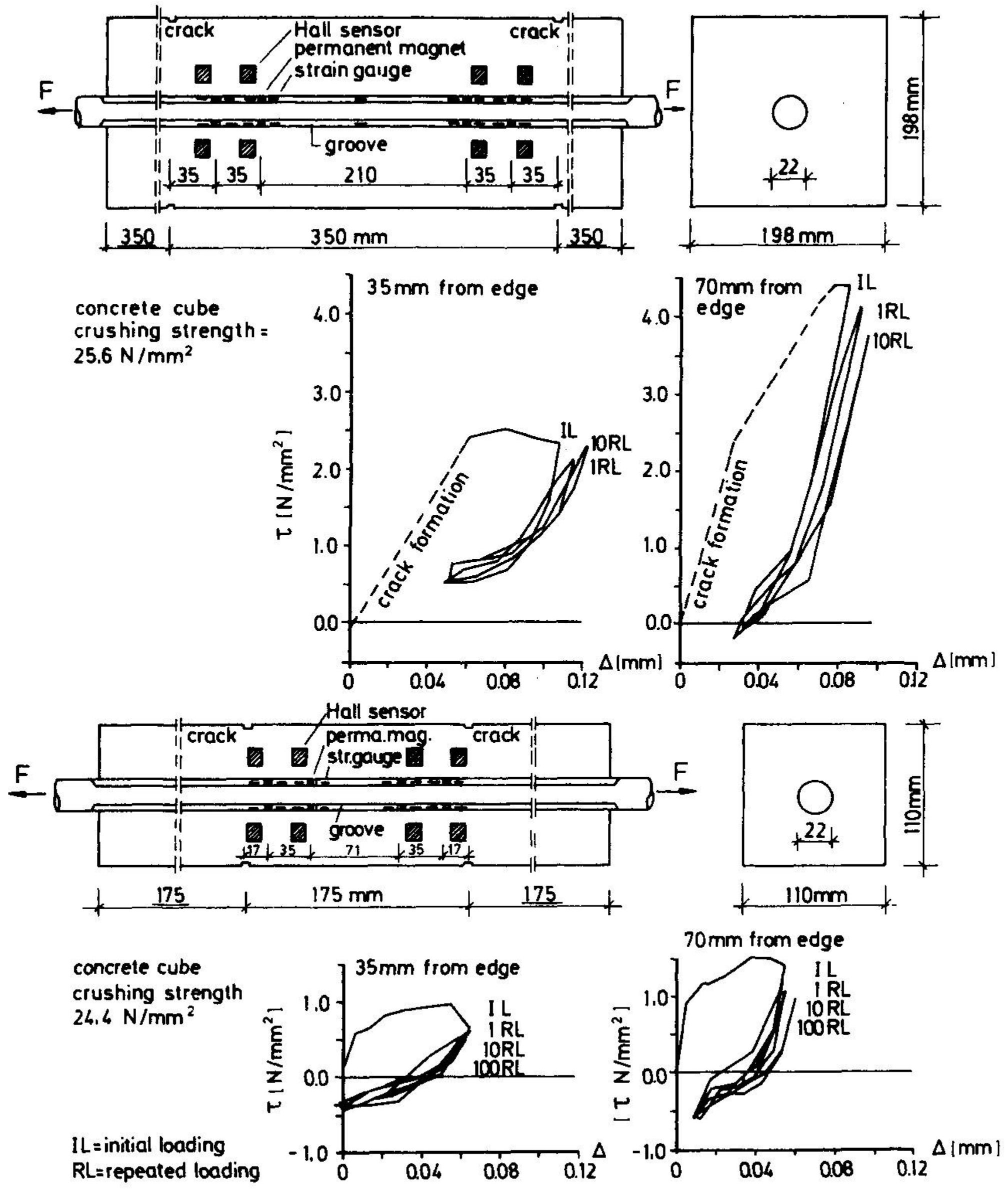

Fig.2 Bond stress - axial displacement relations dependent on concrete cover

For specimens with average crack spacings and a concrete cover of the triple size of differing reinforcing bar diameters the differences of bond stress - axial displacement relations at equal distance to the crack were small. This can be explained by the fact that the relations of concrete cross sectional area and steel surface over which the bond forces are introduced into the concrete have a similar amount.

The courses of relative displacements in radial direction which can be seen in Fig. 3 show that the relative displacements in radial direction at the edge of the specimen increase with increasing specimen length. The relative displacements in radial direction at the edge of a short specimen, however, are larger than those of a long specimen at equal distance to the specimen center. Particularly with the specimens with a length of 400 mm longitudinal cracks form during loading owing to which relatively large relative displacements in radial direction resulted. This explains the lower bond stiffness of edge areas compared to inner areas and the worse bond strength transfer of long specimens compared to short ones at equal distance of the measuring points to the specimen edge.

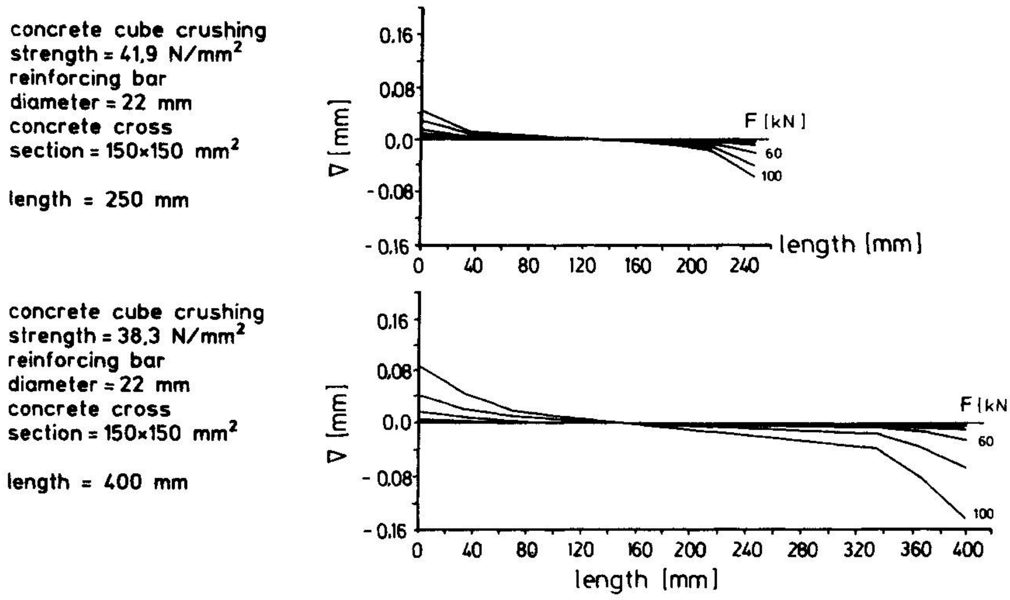

Fig.3 Relative displacements in radial direction dependent on specimen lengths

Fig. 4 shows the relation between relative displacements in axial and radial direction at initial loading. It can be seen that the relative displacements in radial direction are smaller than the relative displacements in axial direction. A distinct increase of relative displacements in radial direction is present only at relative displacements in axial direction of about 0.03 mm or more.

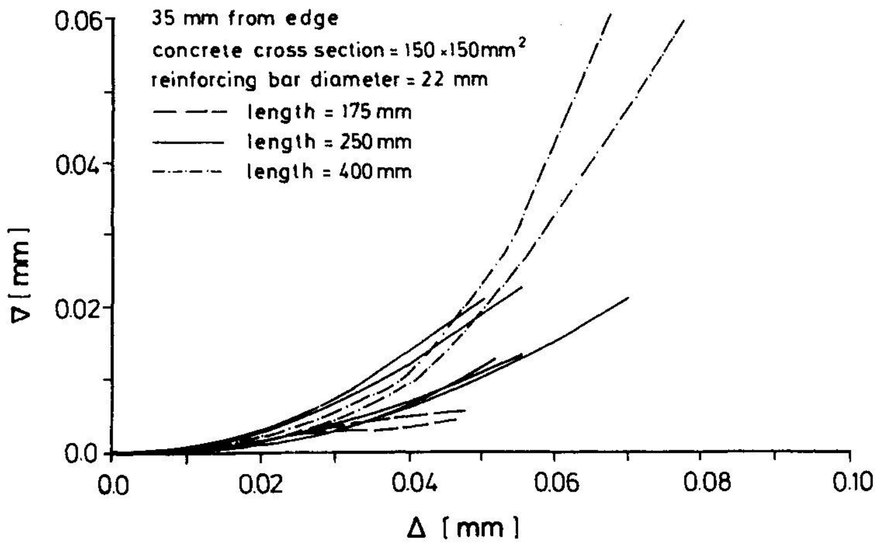

Fig.4 Relation between relative displacements in axial and radial direction for initial loading

4. MATERIAL MODEL

The material model shown in Fig. 5 was determined on the basis of the specimens with average crack spacing. It describes approximately the average bond stress - axial displacement relation between two cracks. The place dependence of bond was not considered as this requires a great deal of additional work in Finite Element analyses. More detailed information for special analyses can be taken from [1].

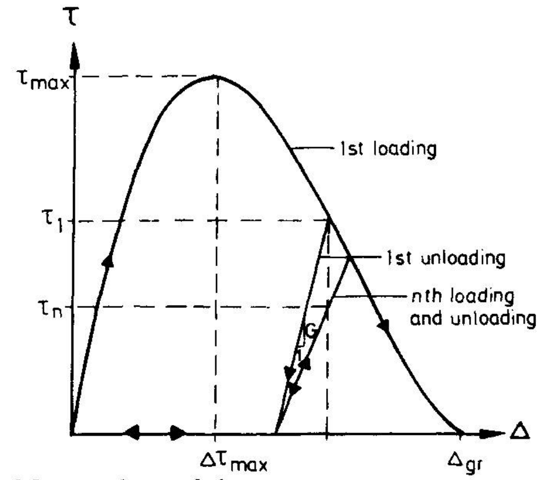

Fig.5 Material model

1st loading

$\tau_{max} = \frac{f_{cube}}{k}$ with

k	c
16	$2 \times d_b$
8	$3 \times d_b$
6	$4 \times d_b$

$\tau = \tau_{max}(4300\Delta^3 - 1000\Delta^2 + 58\Delta)$; Δ[mm]

$\Delta\tau_{max} = 0.04$mm

$\Delta_{gr} = 0.11$ mm

1st unloading

$\tau = G \cdot \Delta$; $G = 200$ N/mm³

nth loading and unloading

$\tau_n = 0.04\tau_1^2 + c \cdot \tau_1$; τ [N/mm²]

c = 0.65 10th loading and unloading

c = 0.50 100th loading and unloading

5. EXAMPLE FOR THE USE OF THE MATERIAL MODEL IN FINITE ELEMENT ANALYSES

A tension specimen was chosen to demonstrate the use of the material model (see Fig. 6). The calculation was made with the Finite Element Program ADINA which has been enlarged by the contact element [2]. Axisymmetric elements were used for the idealization of the reinforcement and the concrete. The nodes of a concrete element are connected to the nodes of a reinforcement element by a contact element which has no physical dimension in transverse direction. Fig. 6 shows that steel stresses, crack spacings, deformations etc. can be calculate if there are realistic material models.

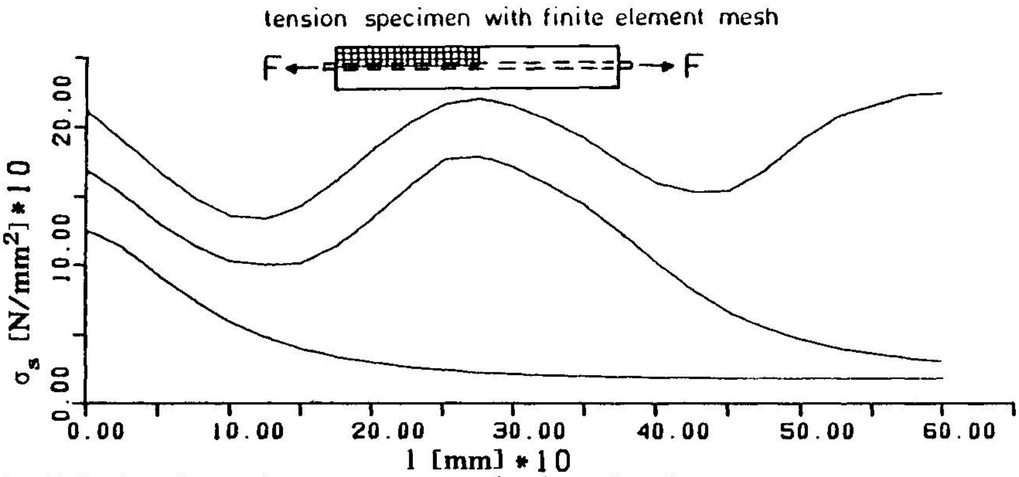

Fig. 6 Calculated steel stresses over the length of a tension specimen

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Tension Stiffening in Reinforced Concrete Elements

Contribution du béton tendu dans des éléments en béton armé fissurés

«Tension-Stiffening» in Stahlbeton-Strukturen

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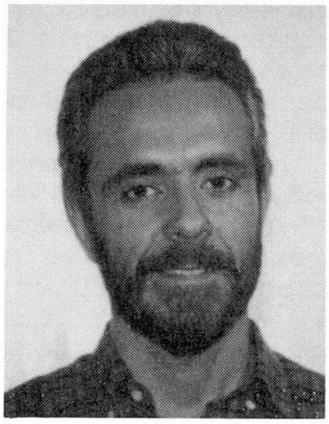

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SUMMARY

The methods to evaluate tension stiffening effects in reinforced concrete structures, are generally based on empirical formulations derived from experimental tests. Recently, an analytical study was presented for modelling the post-cracking behaviour of reinforced concrete bars and membrane elements in a general manner, without a priori hypotheses. The aim of the present paper is to develop an analytical model for reinforced concrete beams subject to bending and axial load, whose formulation is based solely on the classical hypothesis of cross sections remaining plane using suitable constitutive laws for materials and bond characteristics.

RÉSUMÉ

L'évaluation de la contribution du béton tendu dans des éléments en béton armé fissurés se base généralement sur des formules empiriques dérivées d'essais expérimentaux. Une étude analytique a été récemment présentée afin de modéliser le comportement après fissuration de tirants, ainsi que d'éléments-membrane dans la phase suivant la fissuration, mais sans introduire des hypothèses à priori. Le modèle analytique est donc développé pour des poutres en béton armé soumises à des charges axiales et flexionnelles; leur formulation repose uniquement sur l'hypothése classique de la conservation des sections planes, tout en adoptant certaines lois constitutives ainsi que des mécanismes d'adhérence appropriés.

ZUSAMMENFASSUNG

Die Beurteilung der «tension stiffening»-Auswirkungen in Stahlbeton-Strukturen stützt sich im allgemeinen auf empirische Formulierungen, die aus Versuchen abgeleitet wurden. Vor kurzem wurde eine analytische Untersuchung zur allgemeinen Bestimmung des Verhaltens von Zugstangen und Membranstrukturen nach dem Auftreten von Rissen bekannt, die sich nicht von vornherein auf irgendwelche Annahmen stützt. Das Ziel dieser Arbeit ist, ein analytisches Modell für Stahlbeton-Träger zu entwickeln, die Biege- und Normalkraftbeanspruchung unterworfen sind und dessen Formulierung auf der klassischen Hypothese des Ebenbleibens der Querschnitte und der Verwendung geeigneter Material- und Verbundgesetze basiert.

1. INTRODUCTION

Generally the studies to evaluate the tension stiffening effects in reinforced concrete cracked elements are mainly based on semiempirical formulations established to fit test data.[1÷9]
These studies, even if show a good agreement between the theoretical model and experimental results, don't fulfil the requirement of a rational approach to the phenomena [10].
Recently Gupta and al. [11÷12], referring to reinforced concrete bars and membrane elements, presented a theoretical model for the tension stiffening, without resorting to any empirical hypotesis.
The aim of the present paper is similar to the Gupta's one.
A mathematical model for r.c. beams subject to bending with and without axial load is developed, on the basis of the hypotheses generally agreed in studying the behaviour of r.c. cross sections in uncracked and cracked stages.
The basic assumptions concern a linear relationship between concrete compressive strains and steel tensile strains and a linear behaviour of concrete in tension between cracks; no further restrictive hypothesis, of empiric type, is introduced.
In the paper a linear stress-strain behaviour for concrete and steel was adopted, as well as a linear bond stress-slip relationship at the steel-concrete interface.
These crude approximations are used only to reduce the complexity that would be associated with more general relationships, but still keeping a general validity to the formulation.
The proposed model gives reasonably good results, allowing to improve the mathematical formulation by adopting non linear constitutive laws for materials and bond behaviour.
The paper is mainly intended to run an unified approach to describe the post-cracking behaviour of elements in bending in more general way, according to a comprehensive theoretical treatment of the complex phenomena.

2. THE ANALYTICAL MODEL

Fig. 1 shows a part of a reinforced concrete beam between two cracks. The total length of this part is the same as crack spacing 2a and the origin of the x axis is taken midway between the cracks.
The beam has a rectangular, singly reinforced cross section and is subjected to combined axial force and bending moment.

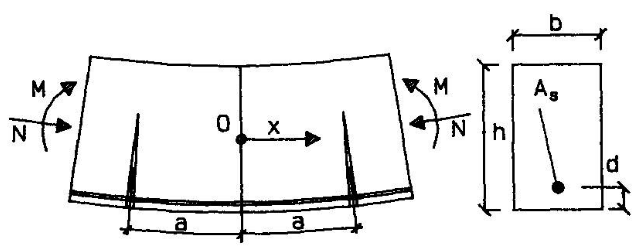

Fig. 1 A part of r.c. beam between cracks and cross section

Let us consider an infinitesimal element of length dx, at distance x from the origin. Fig. 2 shows the stress distributions on the opposite sides of this element. The force F_{bx} is the resultant of the steel-concrete interface stresses in the element and, under the hypothesis of linear bond relationship, is given as

$$F_{bx} = E_b \cdot s \cdot p \cdot dx \qquad (1)$$

where E_b = the slope of the linear bond stress-slip curve, generally called the slip modulus and having unit of FL^{-3}, $s = u_s - u_c$ = the slip between steel and concrete, defined by axial displacements u, p = the sum of the perimeters of the steel bars.

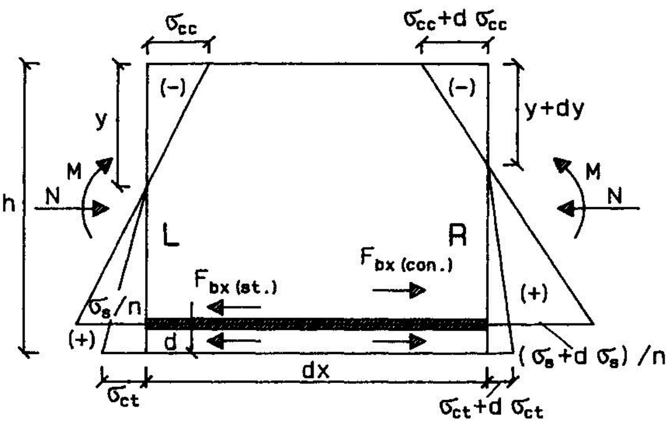

Fig. 2 Free body diagrams for cracked element

The governing parameters of the problem are the stress state and the slip. If the numerical values of these parameters on one side section of the element are fixed, the unknown quantities are the corresponding values on the other side or their variations : e.g. of the concrete stress in compression $d\sigma_{cc}$ and in tension $d\sigma_{ct}$, of the neutral axis depth dy and of the slip ds. So a set of four equations, relating these four variations of governing parameters, are necessary to solve the problem.
The equilibrium of axial forces leads to the first equation:

$$(\sigma_{ct}-\sigma_{cc})\frac{dy}{dx} - (h-y)\frac{d\sigma_{ct}}{dx} - y\frac{d\sigma_{cc}}{dx} - \left[\frac{2E_b p}{b}\right] s = 0 \qquad (2)$$

The equilibrium of moments of the same forces, referred to the steel in tension, leads to the second equation:

$$\left[-\sigma_{cc}(-h+\frac{2}{3}y+d) - \sigma_{ct}\left[\frac{2(h-y)}{3}-d\right]\right]\frac{dy}{dx} + (h-y)\left[\frac{h-y}{3}-d\right]\frac{d\sigma_{ct}}{dx} + y\left[h-\frac{y}{3}-d\right]\frac{d\sigma_{cc}}{dx} = 0 \qquad (3)$$

The third equation is related to the hypothesis of a linear relationship between concrete compressive strains and steel tensile ones.
From the geometric relation $\epsilon_s = -\epsilon_{cc}\frac{(h-y-d)}{y}$ it follows

$$\sigma_s = -\sigma_{cc}\frac{E_s}{E_c}\frac{(h-y-d)}{y} = -n\sigma_{cc}\frac{(h-y-d)}{y}$$ and by differentiating

$$\frac{d\sigma_s}{dx} = -n\frac{(h-y-d)}{y}\frac{d\sigma_{cc}}{dx} + n\sigma_{cc}\frac{(h-d)}{y^2}\frac{dy}{dx} \qquad (4)$$

The equilibrium for the steel axial forces gives:

$$\frac{d\sigma_s}{dx} = \frac{(E_b p)}{A_s} s \qquad (5)$$

By introducing eq. (5) in eqs. (2),(4), the final form of third equation can be worked out:

$$\left[\sigma_{ct} - \left[1+\frac{2nA_s}{b}\frac{(h-d)}{y^2}\right]\sigma_{cc}\right]\frac{dy}{dx} - (h-y)\frac{d\sigma_{ct}}{dx} + \left[\frac{2nA_s}{b}\frac{(h-y-d)}{y} - y\right]\frac{d\sigma_{cc}}{dx} = 0 \qquad (6)$$

The last equation refers to the slip between concrete and steel, both in tension. The basic differential equation is:

$$\frac{d^2 u_s}{dx^2} - \frac{d^2 \bar{u}_{ct}}{dx^2} = \frac{d^2 s}{dx^2} = \frac{1}{E_s}\frac{d\sigma_s}{dx} - \frac{1}{E_c}\frac{d\bar{\sigma}_{ct}}{dx} \tag{7}$$

where $d\bar{\sigma}_{ct}$ and $d^2\bar{u}_{ct}$ are referred to concrete at steel level.
From the relation between tensile stresses at different levels

$\bar{\sigma}_{ct} = \sigma_{ct}\dfrac{(h-y-d)}{(h-y)}$, by differentiating it follows:

$$\frac{d\bar{\sigma}_{ct}}{dx} = \left[\frac{h-y-d}{h-y}\right]\frac{d\sigma_{ct}}{dx} - \sigma_{ct}\frac{d}{(h-y)^2}\frac{dy}{dx} \tag{8}$$

By introducing the relationships (5),(8) in the slip equation (7) the last equation required to define the problem becomes:

$$-\left[\frac{\sigma_{ct}\, d}{E_c(h-y)^2}\right]\frac{dy}{dx} + \left[\frac{(h-y-d)}{E_c(h-y)}\right]\frac{d\sigma_{ct}}{dx} - \left[\frac{E_b\, p}{E_s\, As}\right] s + \frac{d^2 s}{dx^2} = 0 \tag{9}$$

3. BOUNDARY CONDITIONS AND NUMERICAL SOLUTIONS

At the onset of a new crack between the existing ones of Fig. 1 the section at x=0 (midway of the element) is still uncracked and the end section at x=a is fully cracked, as in the classical theory for r.c. elements in the II stage.
The boundary conditions at x=0 are quite known because $\sigma_{ct}=f_{ct}$, being f_{ct}=concrete strength in tension, s=0 for symmetry condition, σ_{cc}, σ_s and y have to fulfil equilibrium conditions for fixed values of axial load N and bending moment M.
In the section at x=a $\sigma_{ct}=0$, because of cracking, the slip value s(a) is unknown and is determined together with the distance "a" from the origin.
For solving equations (2),(3),(6),(9) a finite differences technique is used. By substituting the derivatives with the corresponding increment ratios, the set of four differential equations is transformed into multiple sets of four linear algebraic equations.
Each set allows to determine the values at section $x=x^{i+1}=x^i+\Delta x$ if the corrisponding ones are known at section $x=x^i$. So if we start from section x=0 (section L in Fig. 2), where all the boundary conditions are known, the stress state and slip in the opposite section (R) can be easily calculated and from these, in turn, the values in other sections, by proceeding in the same manner.
The distance "a" is unknown, so a value has to be guessed and an iterative numerical procedure is used which stops when in a section the fully cracked condition $\sigma_{ct}=0$ is fulfilled with a fixed degree of accuracy.
The sum of Δx gives half-length between the first cracks and the slip value in this section is the final one: s=s(a).
From the stress state it is possible to calculate u_{cc} and u_s displacements, the rotations and from these the mean cross section curvature in the element betweeen two cracks.
If one starts the procedure from a value of $M=M_{cr}$, being M_{cr}=the cracking moment, and continues until steel yields, a complete bending moment versus mean curvature diagram can be evaluate and, as a consequence, the bending deformability changes for increasing ap-

plied moments.

4. NUMERICAL RESULTS

To test the model a beam is examined having the following characteristics : h=500 mm, b=300 mm, d=30 mm, σ_{ct}=1.6 MPa, E_b=20,40,80 MN/mm^3, E_c=2000 MPa, E_s=20000 MPa. The beam is subjected to bending moment only, without axial load and the steel reinforcement consists of n. 3 bars, placed in tension zone; three different diameters were considered ϕ=10,14,20 mm, leading to the following:

Case A	3 ϕ 10	A_s=235.6 mm²	p= 94.25 mm	A_s/A_c = 0.157%
B	3 ϕ 14	A_s=461.8 mm²	p=131.95 mm	A_s/A_c = 0.308%
C	3 ϕ 20	A_s=942.5 mm²	p=188.50 mm	A_s/A_c = 0.628%

The "a" values when E_b=40 MN/mm^3 and cracking moments are applied, result

A M_{cr}=21 MN·m a=269 mm
B M_{cr}=22 MN·m a=235 mm
C M_{cr}=24 MN·m a=193 mm

and well agree, within the crude approximations adopted, with existing formulae.
In Fig. 3 the moment-mean curvature relationships for the examined cases in uncracked and cracked stage are plotted. When tension stiffening effects are considered, according to the hypoteses of linear constitutive laws for materials and bond, the curves shift parallel from the fully cracked curves (II stage) at a distance depending on the geometrical percentage of steel reinforcement.

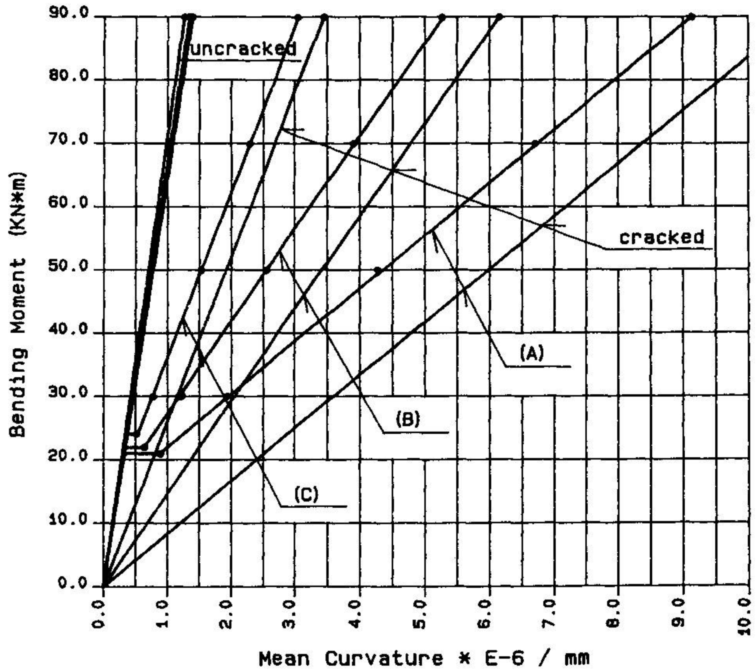

Fig. 3 moment-mean curvature relationships

The model shows that the bond parameter doesn't affect the slope of the curve but only the crack spacing, just as in the most common formulations. If E_b is halved or doubled the following values are obtained:

E_b=20 MN/mm^3	case A a= 380 mm	E_b=80 MN/mm^3	case A a= 190 mm
	B a= 332 mm		B a= 166 mm
	C a= 272 mm		C a= 136 mm

while the moment-curvature diagrams remain the same, whithin the numerical approximations.

5. CONCLUDING REMARKS

The proposed analytical model, which is based only on the classical hypothesis of cross section remaining plane, seems to be a good starting-point for a rational approach to tension stiffening phenomena. Owing to its quite general formulation the model may be improved by adopting more realistic non linear constitutive laws for materials and bond behaviour. From this point of wiew it's well suited for studying the tension stiffening effects even after steel yields.

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Influence of Tension Stiffening on Behaviour of Structures

Influence de la contribution du béton tendu sur le comportement des structures

Der Einfluss des Tension-Stiffening-Effekts auf das Verhalten von Tragwerken

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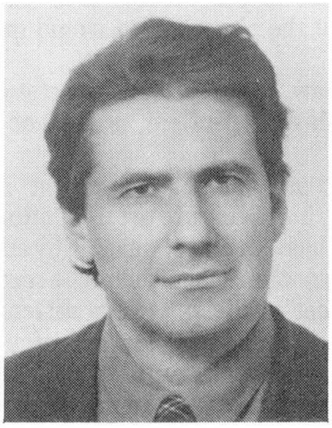

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SUMMARY

The problem of a simplified simulation of the so-called «Tension Stiffening» effect in the nonlinear analysis of reinforced concrete structures is treated. Its importance on the behaviour of Structural Concrete is emphasized, with reference to different kinds of loads and limit states. The uncertainties in the evaluation of the parameters which influence cracking of concrete and tension stiffening are discussed, with reference to the necessity to perform a safe design using simplified methods of nonlinear analysis.

RÉSUMÉ

L'article concerne la simulation par une méthode simple de l'effet de la contribution du béton tendu dans l'analyse non-linéaire des structures en béton armé fissurées. Son importance est mise en évidence, et ceci par rapport aux différentes situations de charge et d'appui. Les incertitudes dans l'évaluation des paramètres influençant la fissuration du béton et la contribution du béton tendu sont prises en compte par rapport à la nécessité de réaliser un projet qui respecte les conditions de sécurité, tout en utilisant des méthodes simplifiées d'analyse non-linéaire.

ZUSAMMENFASSUNG

Es wird das Problem der vereinfachten Erfassung des sogenannten «Tension stiffening» Effekts bei der Analyse von Stahlbetontragwerken behandelt. Die Wichtigkeit dieses Effekts für die Reaktion des Konstruktionsbetons wird unter Bezugnahme auf Belastungen und Grenzzustände hervorgehoben. Die Unsicherheit bei der Einschätzung der Parameter, die die Rissbildung des Betons beeinflussen, und das «tension stiffening» werden im Zusammenhang mit der Notwendigkeit betrachtet, ein sicheres Bemessungskonzept zu erarbeiten, das vereinfachte Methoden der nichtlinearen Analyse verwendet.

1. INTRODUCTION

As known concrete in tension between cracks has a stiffening effect on cracked reinforced concrete members.
As experience in nonlinear analysis of r.c. practical structures has demonstrated this phenomenon, known as "tension stiffening", may have a considerable influence on results both in terms of displacements and action effects.
On the other hand "tension stiffening" is not easy to simulate not only because it is in itself a complicated mechanism of interaction between the two materials, but also because it is necessarily based upon the cracking pattern which in turn depends on tensile strength of concrete (variable whose dispersion is very high), distribution and size of tensile reinforcement, and also the so called "size effect", according to the recent developments of fracture mechanics.
It is not true that disregarding or underevaluating tension stiffening will always lead to conservative results: certainly the opposite is true in some cases such as the evaluation of stresses due to thermal variations, where the disregarding of the phenomenon would inevitably produce grossly underevalued action effects.
In this discussion we intend to draw some relevant conclusion on the subject, basing on the accumulated experience, recent investigations in this specific field, and the objectives of this Colloquium[1][2] on which the author is partially in agreement.
As one of the basic objectives is to individuate a "trasparent" model for the design of structural concrete, it is necessary to choose, among the many approaches to the simulation of structural behaviour, the ones which best fit the need for both simplicity and adequate accuracy, meaning by adequate a degree of accuracy suitable for design.
Therefore the possible approaches to nonlinear analysis in the cracked stage will be first examined and classified.
Secondly, as an example, a "trasparent" model of tension stiffening simulation will be briefly described, which seems to meet the objectives of simplicity and sufficient accuracy for design purposes.
Thirdly the importance of an adequate simulation of tension stiffening within the framework of a simplified method of nonlinear analysis is emphasized and an example is given.
At last the uncertainties in the assumption of basic input data are considered and final conclusions are drawn.

2. NONLINEAR ANALYSIS WITH REFERENCE TO CRACKING-SMEARED AND DISCRETE MODELS

As well known, the static-dynamic behaviour of reinforced concrete structures is markedly nonlinear in nature even in the elastic stage and difficult to model with accuracy, due to the micro-anisotropic, quasi brittle nature of concrete and the composite nature of the material. Problems arise mainly from uncertainties in definition of concrete constitutive law, complexities in interaction phenomena between the two materials, extensive cracking in tensile areas.
To describe this behaviour two different basic approaches are possible (fig. 1), using micro or macro-elements.
Only the first approach, coupled with a "discrete" representation of cracking, can (potentially at least) describe accurately the interaction between the two materials, as each element model a very small area of steel or concrete and the discontinuous nature of cracking is described.
This approach however can only be used in simulation of laboratory tests and for very simple structures or structural elements and is unsuited for design and analysis of real structures.
It is therefore necessary to formulate the stiffness of macro-elements of r.c. (and not concrete or steel only) which takes into account as well as possible the previously mentioned local phenomena.
In this formulation a "smeared" approach needs be adopted and therefore tension stiffening must be introduced in a suitable way.
Several approaches are possible[13]. In the following paragraph a simple and "trasparent" method is briefly described. More details can be found in [7], [8] and [12].

3. AN EXAMPLE OF "TRASPARENT" MODEL FOR "TENSION STIFFENING"

The problem of "tension stiffening" is concerned with the behaviour of steel bars embedded in concrete and subject to tensile forces. In a given crack, all the tension is trasmitted by the bars; between one crack and the following part of the tension is transferred to concrete, thus reducing the stresses in the steel.

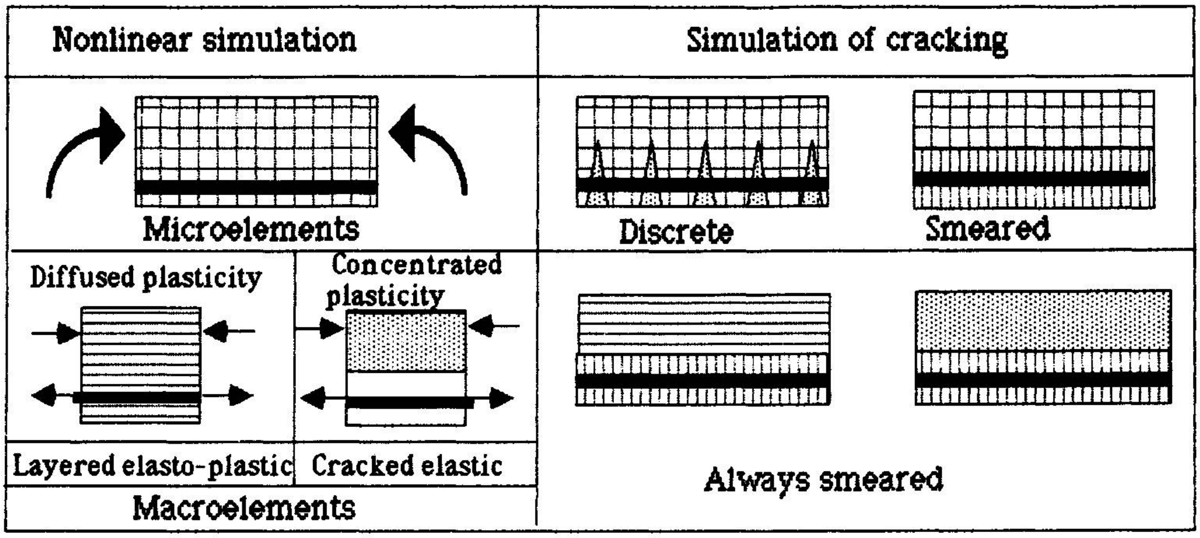


Fig. 1-Simulation of cracking

It is quite natural to think that things behave as if the tensile reacting part of the beam be constituted by the tensile reinforcement and an additional "virtual" reinforcement which represents the contribution of the concrete in tension between cracks. The method can be described as follows:

Each element belonging to the frame is divided in a given number of short macro-elements behavin elastically(as plastic behaviour is considered concentrated in "plastic hinges"). For each element the moment of inertia of the cracked section is introduced when, within the element, the adopted tensile strength of concrete in tension is exceeded. In computing the moment of inertia of the cracked section the influence of "tension stiffening" is simulated by introducing a "virtual" additional steel area AS^* which can resist a constant force $K_1b(a-x)$, K_1 being the mean tensional stress between cracks which will be subsequently called "tension stiffening coefficient" (seefig.2)

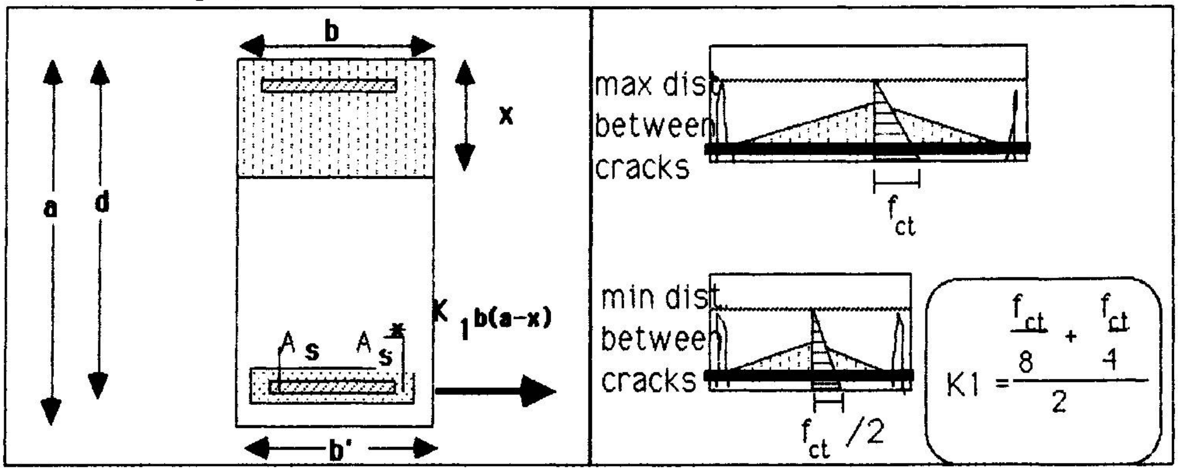


Fig. 2-"Trasparent" simulation of "Tension Stiffening"

A_s^* decreases as loads increase, thus simulating the decreasing influence of" tension stiffening" with increasing loads(which can be verified experimentally).

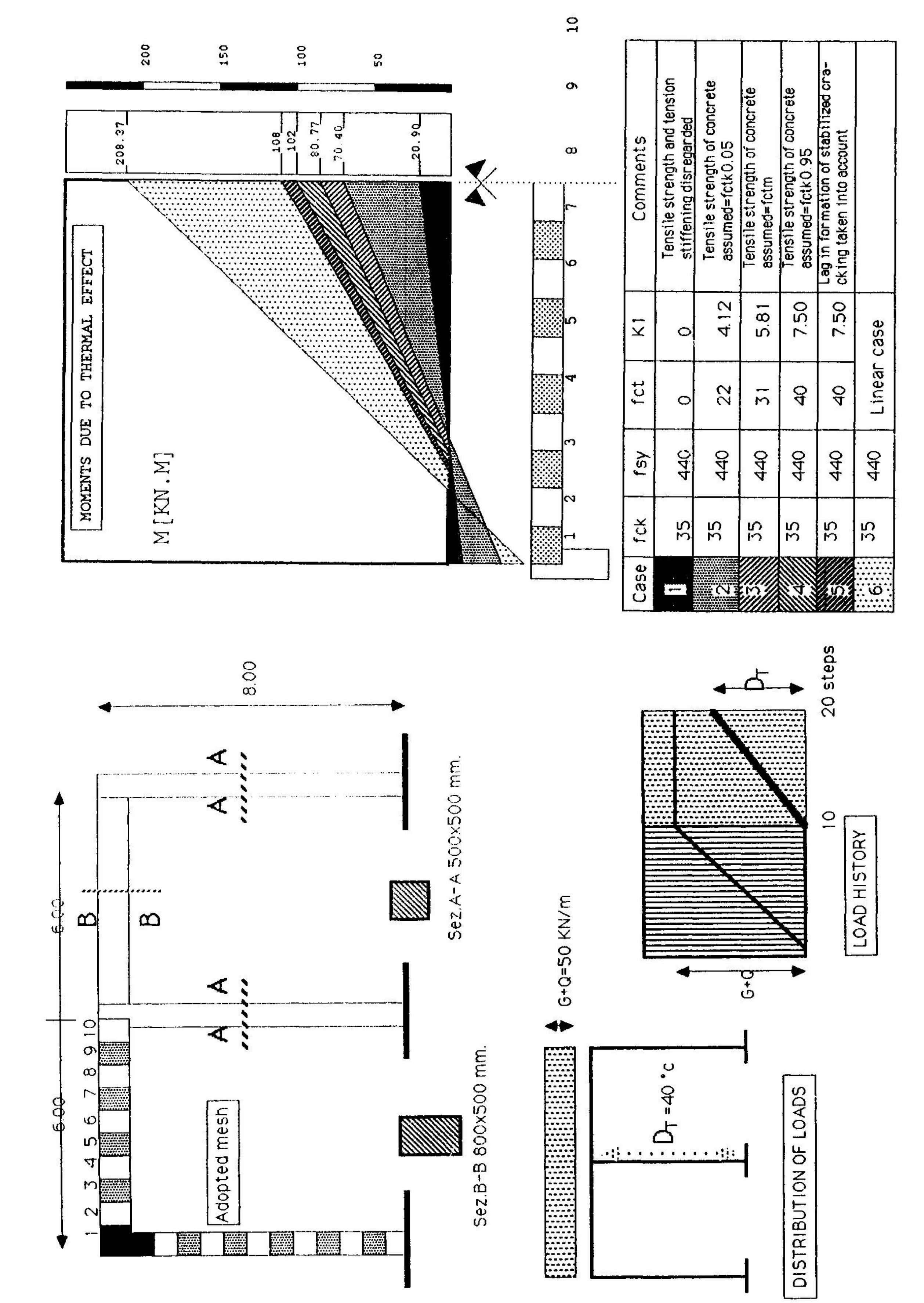
The tension stiffening coefficient can be computed in function of the tensile resistance of concrete, adopting the simplified assumption of uniform distribution of adherence stresses along tensile reinforcement between one crack and the following. If this assumption is made tensile stresses in concrete vary linearly along the element axis. If again the assumption is made that tensile stresses also vary linarly along the section depth, the mean value of these stresses is equal to $f_{ct}/4$ in the case two consecutive cracks form at the maximum possible distance(upperlimit situation) and equal to $f_{ct}/8$ in the lower limit situation(minimum possible distance between cracks). If again a mean value is assumed between these two extremes a value of the tension stiffening coefficient equal to $K1=3/16.f_{ct}$ is obtained.

4. IMPORTANCE OF "TENSION STIFFENING" ON STRUCTURAL BEHAVIOUR

One might wonder whether it is really important,to evaluate the structural behaviour of frames,to model accurately the influence of cracking and the related phenomenon of "tension stiffening",that is to evaluate the nonlinear behaviour in the elastic stage.
In fact experience tells us that these effects are essential to evaluate correctly,among others,the following phenomena:
-redistribution of moments due to cracking in beams, which can influence considerably the behaviour at Ultimate LimitState.
-elastic displacements at service load level
-Influence of cracking on second order effects in slender columns
-Thermal structural effects.
With reference to the latter phenomenon an example taken from [15] is briefly illustrated.
In the frame of fig.3 vertical distributed loads were applied to the beams first and then a thermal variation was applied to the column.
The frame was analyzed both linearly and non linearly and for different values of f_{ct} and corresponding values of K1(tension stiffening coefficient). The 6 cases which were considered are summarized in the table of fig.4.
The lag in formation of stabilized cracking(see following paragraph)was simulated in case n.5.
The load history is represented in fig.3: vertical loads were applied first to the beams in 10 steps so that cracking might take place due to external loads. An uniform thermal field of +40°C was then applied to the central column(again in 10 steps),so that moments of the same order as those produced by external loads(and of the same sign in the central section)were induced in the beams.
In figure 4 the moments due to thermal effect only for the various cases are represented graphically
As may be seen from this diagram the extreme cases(linear analysis of uncracked structure and nonlinear analysis disregarding tensile stress of concrete and tension stiffening)lead,on opposite side,to completely unreliable results. Taking into account these factors according to different evaluation of tensile strength of concrete leads to considerably different results. This consideration leads to the problem of tensile strength evaluation.

5. UNCERTAINTIES IN THE SIMULATION OF CRACKED BEHAVIOUR AND TENSION STIFFENING
ASSUMPTION OF A SUITABLE VALUE FOR TENSILE STRENGTH

Every simulation of the cracking formation process must necessarily be based on an assumed value of concrete tensile strength. On the other hand it is well known that this parameter can only be determined with much uncertainty given the considerable dispersion of test results. Besides other factors influence crack formation and crack propagation[14](size and quantity of reinforcement, cover, size effect, moment gradient)
Therefore we cannot ,in evaluating structural behaviour in the cracked stage,reach the same degree of accuracy that we can obtain in the calculation of yielding moments in critical sections, which are in most cases mainly influenced by steel strength fsy, and to a lesser degree by concrete compressive strength fck.
As we cannot hope, even using sophisticated methods, to obtain very accurate results, the choice of the tensile strength fct must be based on safety probabilistic considerations, which are in turn influenced by the kind of results we need from structural analysis.
Also it is not convenient to adopt a very sophisticated procedure for tension stiffening simulation
If the purpose of analysis is to calculate deflections, the choice on the safe side is to overevaluate them , therefore to underevaluate fct and , as a consequence, tension stiffening. The most logical choice according to CEB Model Code philosophy [5][6]is to adopt a characteristic value with a 5% probability of not being exceeded (fctk.05)
If the purpose is to evaluate redistribution of moments due to cracking, it is not easy (and probably not even possible)to establish which value of fct would yield the safest distribution of action effects. In this case it is most reasonable to look for the "most probable" result and therefore choose a mean value of resistance(fctm)
If at last we need to evaluate thermal effects, the choice on the safe side is of course to overevaluate them and therefore to overevaluate fct(and tension stiffening).
A value of fct with a 95% probability of not being exceeded can be adopted(fct0.95).
This may however not be enough.
The method of simulating stiffness reduction due to cracking which was described in the previous paragraph(as well as most other methods of this kind)are based on the assumption that, as soon that, in a given element, the



Case	fck	fsy	fct	K1	Comments
1	35	440	0	0	Tensile strength and tension stiffening disregarded
2	35	440	22	4.12	Tensile strength of concrete assumed=fctk0.05
3	35	440	31	5.81	Tensile strength of concrete assumed=fctm
4	35	440	40	7.50	Tensile strength of concrete assumed=fctk0.95
5	35	440	40	7.50	Lag in formation of stabilized cracking taken into account
6	35	440	Linear case		

Fig. 3/4-Example: structural effect of thermal variation

tensile strength of concrete is reached in the most stressed fiber, fully opened "stabilized" cracks form in the element and its stiffness can be reduced accordingly.
However this assumption does not correspond to results of experimental tests In fact there is a "lag" between the reaching of the limit value of fct in a zone with constant momentand the formation of stabilized cracking in the same zone. An approximate method for simulating this lag is given in[15].
It may be concluded that a situation of fully open and stabilized cracks takes place gradually in the element and is fully developed for values of maximum tensile stress in concrete that can be much higher then the tensile strength fct.
This fact may be explained intuitively:in fact when fct is reached in the most stressed fiber a crack must begin;however an increase in stress in the adjacent fibers is required for the extension of this crack toward the neutral axis;this can happen only by furtherly increasing external actions also because tensile reinforcement is an obstacle to this propagation.

6. CONCLUSIONS:HOW "TENSION STIFFENING" SHOULD BE TREATED ACCORDING TO THE PHILOSOPHY OF THIS SYMPOSIUM

From the above considerations the following conclusions can be drawn:
-The influence of cracking and "tension stiffening" on structural behaviour must be taken into account in many practicallyimportantcases.
-Given its intrinsic complexity,the phenomenon must be simulated using simplified methods,which,to be accepted and correctly used by the engineer,must be sufficiently "trasparent",as the one which has been,as an example,presented.
-Given the uncertainties related to the prevision of the stress level at which a stabilized crack patterns takse places,the adopted input data and,in particular,the concrete tensile strength should be chosen so that results on the safe side be obtained.This means that values to be adopted must depend on the kind of load to be applied and the kind of situation to be studied.

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