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**Autor:** Bljucer, Fiodor  
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## Cracking Analysis of Concrete Structures

Analyse de la fissuration des structures en béton

Rissbildungsanalyse von Betonbauten

### Fiodor BLJUGER

Sen. Researcher  
Nat. Build. Res. Inst.  
Haifa, Israel



F. Eph. Bljugar, born 1929, has been Researcher and Adjunct Lecturer at the Faculty of Civil Engineering, Technion since 1974. Prior to this he was Senior Scientist and Head of Laboratory at Moscow Institutes, USSR since 1963. He is the author of many articles on reinforced concrete and of the book «Design of Precast Concrete Structures», UK, 1988.

### SUMMARY

A probabilistic approach based on an experimental model and on the variation characteristics of concrete strength and load, is presented. Traditional estimation of serviceability in terms of cracking is shown to yield significantly different reliabilities for concrete structures. The situation may be improved with the aid of suitable analysis; in any event, the traditional approach should be revised.

### RÉSUMÉ

Une approche probabiliste basée sur un modèle expérimental est mise en évidence; elle tient compte de la variation des caractéristiques de résistance du béton, ainsi que de la charge. En effet, l'approche traditionnelle de l'état de service basée sur la fissuration est présentée, afin de montrer qu'elle peut mener à des sécurités différentes. Cette situation peut et doit être améliorée à l'aide d'une analyse appropriée.

### ZUSAMMENFASSUNG

Eine probabilistische Methode, basierend auf einem experimentellen Modell und auf Variationscharakteristiken von Betonfestigkeit und Belastung, wird dargestellt. Die bisherige Erfassung des Gebrauchszustandes in Form von Rissbreitennachweisen liefert deutlich andere Aussagen für die Zuverlässigkeit von Betonkonstruktionen. Die Situation kann mit Hilfe einer geeigneten Analyse verbessert werden; auf jeden Fall sollten die traditionellen Verfahren überprüft werden.



## 1. INTRODUCTION

The cracking resistance of structures in bending defines their serviceability in terms of premature crack appearance and excessive deflections, depending on the cracking moment [1-4]. It is of prime importance in modern structures made of high-grade concrete [7]. In practice, this serviceability is assured by semi-probabilistic design methods, using deterministic values of loads, material strengths and criteria for the limit state. In reality, crack appearance in a structure complies with a particular probability. Probabilistic cracking analysis of some concrete structures has shown that the main influence on the probability of crack appearance is due to the lengthwise variation of concrete strength over the structure and to the variability of mean concrete strength of the members within their general population, as well as to the load variability [5-6].

In this paper the reliability of particular structures designed under the traditional approach are analysed on the basis of an experimentally obtained model and of the variation characteristics of tensile strength of concrete.

## 2. STRUCTURAL MODEL FOR ANALYSIS

Consider a simply-supported member of rectangular section under uniformly distributed load (Fig. 1).

Potential cracks in the member in bending may appear at  $\ell_s$  - 40mm spacing (determined experimentally [4] in its middle part -  $\ell_o$  (Fig. 2).

According to [5], the probability of crack appearance in the x-section is:

$$P_x = \frac{\Delta_i}{\sqrt{2\pi}} \sum_{i=-3}^j \exp(-i^2/2) \quad (1)$$

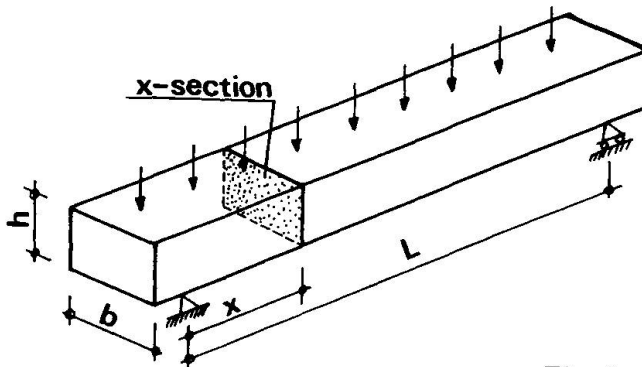


Fig. 1. Member for cracking analysis

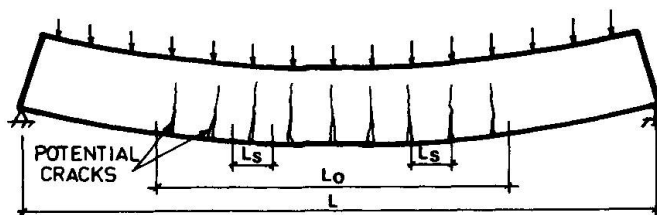


Fig. 2. Model of member with potential cracks.

In (1)  $\Delta_i$  is the summation step,  $i$  is an independent parameter,  $j$  in the general case (including prestressed members) is given by:

$$j = [(\frac{M_x}{W} - f_p)/f_{cm} - 1]/C_{v1} \quad (2)$$

and the Moment  $M_x$  in the  $x$ -section for a particular uniformly distributed load  $p$  is:

$$M_x = \frac{p}{2} (\ell x - x^2) \quad (3)$$

$f_p$  - residual prestress in most stressed fibre (for non-prestressed member  $f_p=0$ ).

$f_{cm}$  - mean concrete flexural-tensile strength in member.

$C_{v1}$  - coefficient of lengthwise strength variation.

The probability of crack appearance in the considered member with particular mean concrete strength is:

$$P_{rm} = 1 - \prod_{x_1}^{x_2} (1 - P_x) \quad (4)$$

According to [1], the mean flexural-tensile strength of concrete is:

$$f_{cmm} = f_{ctm} (.6 + .4h^{-.25}) \quad (5)$$

and the mean tensile strength in the general population is:

$$f_{ctm} = .3 f_{ck}^{2/3} \quad (6)$$

As may be inferred from [1], the overall variation coefficient of tensile strength  $C_v = 18.3\%$ . The variation coefficient of the mean concrete strength in members is:

$$C_{v0} = \sqrt{C_v^2 - C_{v1}^2} \quad (7)$$

### 3. PROBABILITY ESTIMATION OF CRACK APPEARANCE IN A MEMBER IN THE STRUCTURE POPULATION

The probability of crack appearance in a member (Fig. 2) with a given grade of concrete under a particular load is evaluated by:

$$P_r = \sum P_{rm} P_m \quad (8)$$

where  $P_m$  -probability of occurrence of the specific mean concrete strength, namely:

$$P_m = \frac{\Delta_m}{\sqrt{2\pi}} \exp (-m^2/2) \quad (9)$$

The mean concrete strength in a member is defined as:  $f_{cm} = f_{cmm} (1 + m C_{v0})$ .

The overall probability of crack appearance in a member within the general population is evaluated by:



$$P = \sum_n P_r P_q \quad (10)$$

$$P_q = \frac{\Delta_n}{\sqrt{2\pi}} \exp(-n^2/2) \quad \text{for } q = q_m(1 + n C_q), \quad (11)$$

where:  $n$  - independent parameter,  $q_m$  - mean load,  $C_q$  - coefficient of load variation.

Fig. 3 shows the cracking probability for beams made of C-30 concrete under a characteristic load .0022 MN/m, versus  $C_q$  [5].

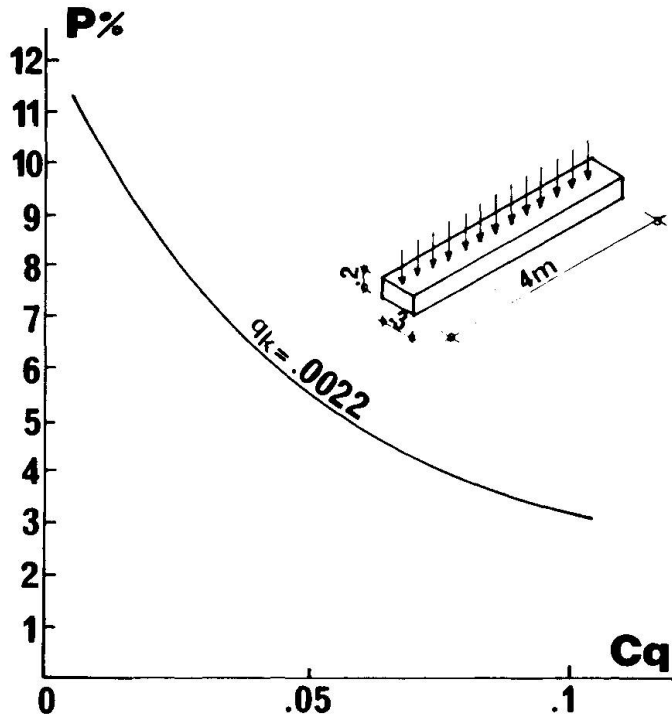


Fig. 3. Probabilities of crack appearance in beam made of C-30 concrete under 0.0022 MN/m characteristic load - versus coefficient of load variation.

#### 4. PROBABILITY ANALYSIS OF MEMBERS WITH VARIABLE CONCRETE STRENGTH UNDER VARIABLE LOADS

The characteristic load for analysis is determined as the load causing crack appearance in the most stressed section of a member:

$$q_k = 8 f_{cmk} W/l^2 \quad (12)$$

Numerical analysis results for the members, as shown in Fig. 1 ( $h=.2m$ ,  $b=.3m$ , grade C-50), with different lengthwise strength variations ( $C_{v1}$ ) - are presented in Fig. 4.

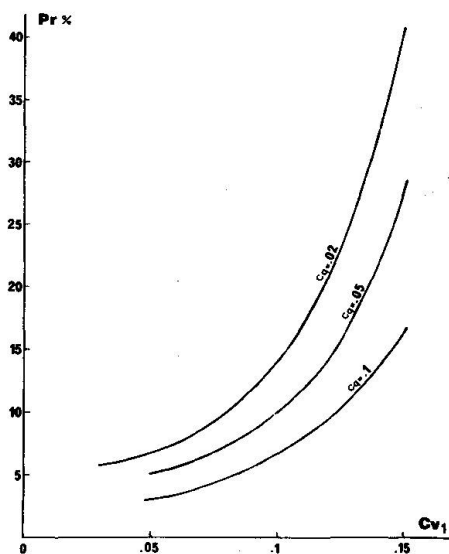


Fig. 4. Probabilities of crack appearance in members vs. lengthwise strength variation ( $C_{v1}$ ) under the same characteristic load with different variations ( $C_q$ ).

It is felt that present codes favor design with a wide range of reliability in terms of cracking.

As shown in [6] for composite slabs, the probability of crack appearance in the spans of statically-indeterminate structures should be significantly lower than in simply-supported ones, and such structures may be more reliable. Their reliability depends on the variability of concrete strength in certain parts of the structure, and combinations of low strength make for very low probabilities.

Narrow variation of concrete strength may drastically reduce the probability of crack appearance in a structure. Analysis of cracking probability in members with narrower strength variability under the same mean load (Fig. 5) shows that the reliability of a code-designed structure may be significantly improved through improved homogeneity of the concrete in terms of tensile strength.

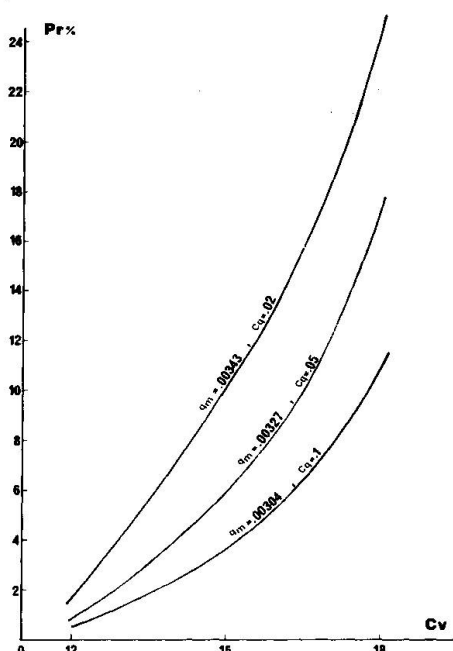


Fig. 5. Probabilities of crack appearance in members under the same mean loads appropriate to their variations - vs. real coefficient of tensile strength variation of concrete.

## 5. CONCLUSIONS

The analysis shows that traditional estimation of structure serviceability in terms of cracking, yields significantly different reliabilities for concrete structures in bending. Based on probabilistic criteria and statistical initial data, the above situation may be improved with the aid of suitable analysis. In any event the traditional estimation should be corrected by behaviour factors, taking into account the variabilities of concrete strength and load combinations.

The proposed design approach calls for supplementation of the codes by suitable statistical data and by probabilistic restrictions.

As the reliability of a real structure depends on the variability of concrete strength in practice, gradation and strength control of the concrete on the basis of its tensile strength are of extreme importance.



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