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Deformations of T-shaped Beams under Sustained Loads

Déformations des poutres en T sous charge de longue durée

Durchbiegungen von Plattenbalken unter andauernder Belastung

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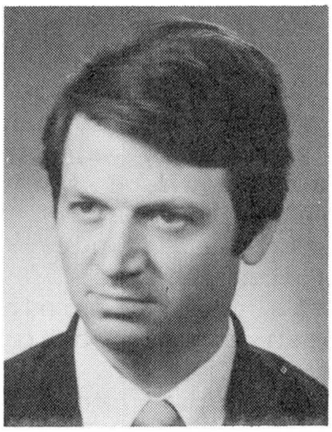


Mariusz Szechinski, born 1947, graduated in Civil Engineering and took his Dr. techn. degree at Wroclaw Technical University. Since 1970 he worked as Assistant and later as an Assistant Professor at the Institute of Building Science. He has published 30 papers on the theory and practice of Concrete Structures and was involved in more than 140 works for industry.

SUMMARY
Experiments show that a considerable growth of strains occurs in long-term loaded reinforced concrete structures, as result of time-dependent changes of properties of concrete. Results of calculations were compared with both results from a finite element model and with experimental values.

RÉSUMÉ
Un modèle de calcul décrivant les poutres en T en béton armé sous charges de longue durée est présenté. Les résultats expérimentaux provenant d'un modèle spécialement développé ont été comparés à ceux obtenus analytiquement grâce à la méthode des éléments finis.

ZUSAMMENFASSUNG
In der Arbeit wird ein Berechnungsmodell für Stahlbetonplattenbalken unter dauernd wirkender Belastung formuliert. Die Messergebnisse werden mit Ergebnissen aus FEM-Berechnungen und aus Berechnungen anhand eines eigenen Modells verglichen.

1. INTRODUCTION

Experiments indicate that in the long term loaded reinforced concrete constructions, as a result of changes in time of properties of concrete, there occurs a considerable growth of strains in concrete.
The increase of the strains in concrete, leads to the deformation of the whole element and to the increase in deflection or additionally to the increase in the width of the opening of the cracks. The stresses also vary and therefore a strain in the materials may increase.
These phenomena must be predicted and taken into account during planning stage, and therefore it is necessary to express them analytically in order to make practical calculations possible.
In particular this situation applies to the flexed T-shape reinforced concrete beams, where the concrete closely cooperates with the steel during load transfer. Calculations of deformations for reinforced concrete beams under a long-term loading may be divided into two essentioal stages.
The first stage consists of determining the state of stresses and strains in freely-chosen cross-section of concrete beam. In this case the problem reduces itself to allocating the stresses and strains at any point of the cross section that,s consider at any point of time with regard to the most heterogeneous factors.
The second stage consists of observing the element as a whole. Here we may take into account rigidity, deflection, spacing as well as the width of the opening of the cracks. There we make use of considerations carried out at the first stage and therefore the assumptions made in the first stage will influence the obtained results obtained in an important way. It makes that correct solution of the problem of defining the changes of stresses and strains in any cross-section of concrete operating under long-term loading is the basic task having aa influence on all considerations.
In the present paper it has been decided to make an analysis of this problem on the basis of some new assumptions and existing solutions and to provide some means of formulation it comprehensively.
The proposed standardization of the method of presenting equilibrium equations in different phases and generalization of the method of presenting the dependence between creep stresses and strains allowed us to build the analytical model of problem and examine exactly the state of stresses and strains in reinforced concrete beam depending on the time duration of the load. To get a practical results of calculations there was made the computer program, which allowed to verify an influence of many different factors on the state of stresses and strains in the T-shape reinforced concrete beam.
The results of upper proposed traditional analytical calculations were compared with the result of calculations of the model built of structural concrete.

2. ASSUMPTIONS

The assumptions, which is necessary to make for the examination of the present problem, may be divided into 3 groups.
The first group is concerned with the method of making

allowances in the calculations for the rheological properties of concrete; shrinkage, creep or relaxation, under the influence of which changes in the strain of concrete are in time brought about. Customarily, making allowances for such influences has become generally accepted by means creep function, which in the general case may be shown as follows:

$$\varepsilon_c(t,\tau) = \frac{\sigma_c(\tau)}{E_c(\tau)} + F[\sigma_c(\tau)\delta(t,\tau)] + \int_\tau^t \frac{1}{E_c(\tau)}\frac{\partial\sigma_c(\tau)}{\partial\tau}\,d\tau + \int_\tau^t \frac{\partial F[\sigma_c(\tau)]}{\partial\tau}\,C(t,\tau)\,d\tau \qquad (1)$$

In the presented paper thedependence (1) shown abovewas adopted as basic to the consideration. The second group of assumptions consists of assumptions concerning the stress-deformation dependence. The analysis of this problem carried out by Szechiński [1,2] allows us to state that until now the problem has been limited to examinations of cross-sections of concrete at different phases of operation, accepting the most varying shapes of the stress diagram. All these cases may be generalized so as to obtain the shape of the diagram as proposed by Szechiński [1,2] in the figure 1.

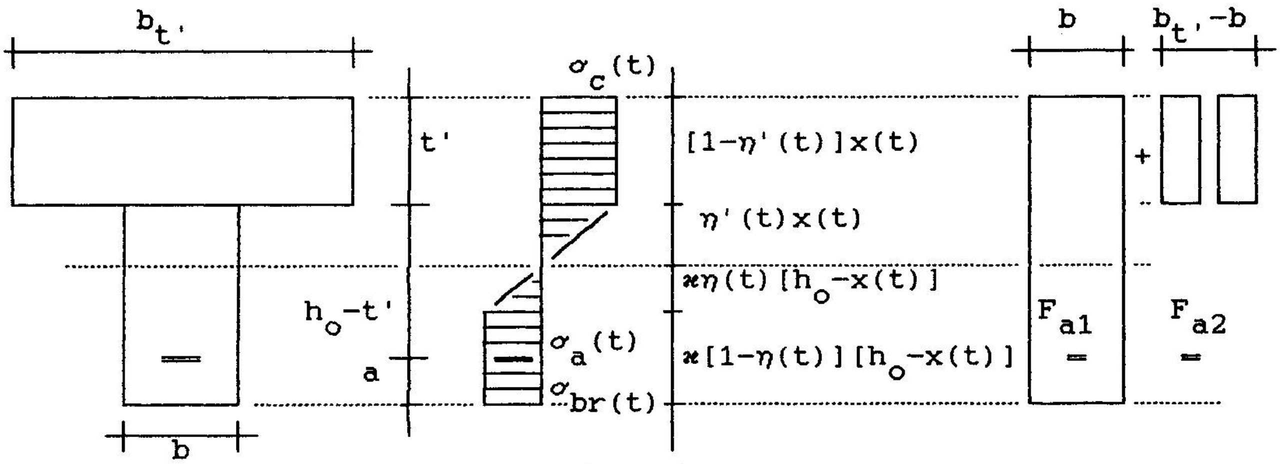


fig. 1
Assumptions about the concrete cross-section.

Acceptance of this generalization allows a completely universal notation of equilibrium equations,valid for every phase of operation of the concrete cross-section.
The third group of assumptions is concerned with deformations in the cross-section of the concrete beam. Customarily, it is accepted that the cross-section before and after deformation remains flat.

3 ANALYSIS OF THE CONCRETE CROSS-SECTION.

Accepting the assumptions given in previous section, fig.1, we get the following equilibrium equations.

$$\Sigma X = 0,\ -\frac{\sigma_c(t)}{2}\eta_o'(t)\ x(t)\ b + b\ \sigma_c(t)\ x(t) = \sigma_{ct}(t)\ [h_o - x(t)]b +$$

$$+ \varkappa\eta(t)\ \frac{h_o - x(t)}{\;}\ \sigma_{ct}(t)\ b + \sigma_a(t)\ F_{a1},$$

$$\sigma_c(t)\ t'\ (b_{t'} - b) = \sigma_a\ (t)\ F_{a1}. \qquad (2)$$

$\Sigma M = M_1+M_2,$

$$M_1= \sigma_c(t)bx[(h_o-x(t)/2] -\sigma_b(t)\eta'(t)x(t)\frac{b}{2}[h_o- x(t) +\eta'(t)\frac{x(t)}{3}] +$$

$$-\sigma_{ct}(t)[h_o-x(t)]^2 b/2 +\sigma_{ct}(t)\varkappa\eta(t)[h_o-x(t)b/2[h_o-x(t)-\varkappa\eta(t)x(t)/3],$$

$$M_2 = (b_t'-b)\ \sigma_c(t)t'(h_o - t'/2). \tag{3}$$

Taking into consideration dependence (1) and geometrical dependence, that is assumption 3 from the previous section, the integral equation (4) was received.

$$\varepsilon_c(t,\tau)= \\ = \sigma_c(\tau)\delta(t,\tau)+\beta'\sigma_c^2(\tau)c(t,\tau)+\int_\tau^t\frac{1}{E(\tau)}\frac{\partial\sigma_c(\tau)}{\partial\tau}d\tau+\int_\tau^t c(t,\tau)\frac{\partial[\sigma_c(\tau)+\beta'\sigma_c(\tau)]}{\partial\tau}d\tau. \tag{4}$$

The eq.(4) may be simplified to linear form assuming $\beta'= 0$. Furthermore eq. (4) was simplified to the algebraic form, from whihch the values of x(t) were calculated and further from equilibrium equations (2) values $\sigma(t)$, $\sigma_{ct}(t)$, $\sigma_a(t)$, $\varepsilon_c(t)$, $\varepsilon_a(t)$.

4. CALCULATING THE DEFLECTIONS.

From the geometrical dependences, for intermediate loads we may receive

$$\frac{M}{B_o} = \frac{\varepsilon_{co} + \dfrac{\varepsilon_{co}(1-\xi_o)}{\xi_o}}{h_o} = \frac{\varepsilon_{co}}{\xi_o h_o} = \frac{\sigma_{co}}{E_{co}\xi_o h_o} = \frac{\sigma_{ao}}{E_a\ (1-\xi_o)h_o}, \tag{5}$$

It means that

$$B_o= B(t=0)= \frac{M\ E_a(1-\xi o)h_o}{\sigma_{ao}} \quad \text{or} \quad B(t) = \frac{M\ E_a[1-\xi(t)]h_o}{\sigma_a(t)}, \tag{6}$$

From the equilibrium equations and after solving eq.(4) we may receive σ (t) that means we may get the cross-section stiffnes B(t).Furthermore taking into consideration the fact that stiffness is changing on the beam length thre was assumed according to [3] following stiffness function

$$B_b(t,\zeta) = f[B_{Ii}(t),m_{cr}] + h\ [B_{Ii}(t),B_{IIi}(t),m_{cr}], \tag{7}$$

where

- m_{cr}= M/Mcr – relation between the moment on beam and the cracking moment,
- B_{Ii}, B_{IIi} - stiffness of beam cross-sections in different phases of work,
- B_b- stiffness function on the beam length.

Using the curvature equation and dependence (7) allows to calculate displacements of the beam due to the time flow.

5. FINITE ELEMENT MODEL OF THE BEAM.

Using the FEM program system [4], the computer model of the beam shown in fig. 2, was built, as it is shown in fig.3a. The three-dimensional solid elements, as shown in fig 3b, were used to describe concrete and reinforcement. Concrete and reinforcement were represented by different values of the modulus of the elasticity, weight and the coefficient ν.

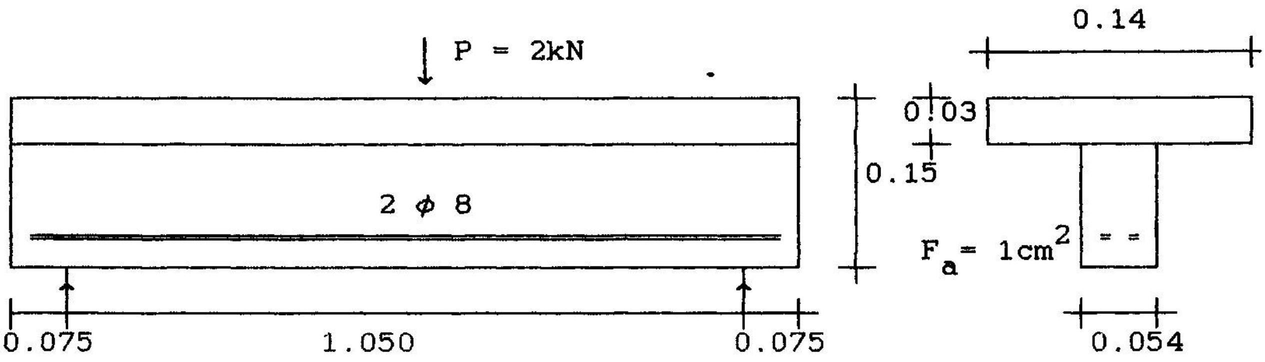
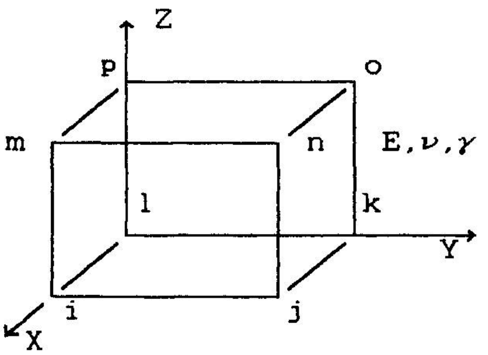
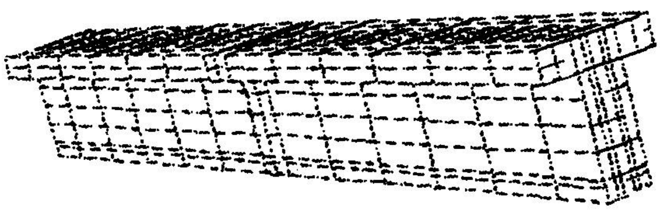


fig.2

Construction of the sample beam



a. Three-dimensional isotropic brick element



b. Nodal mesh on the beam

fig.3 a,b,

Finite Element Method model of the beam

This model, because of program limitations, was examined only under the intermedaite load (t=0). The results compared with results of traditional calculations and experimental results are shown in fig. 4,5.

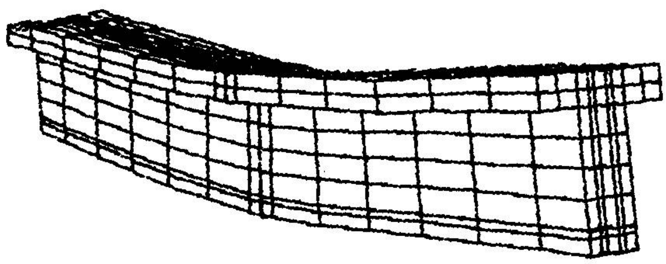
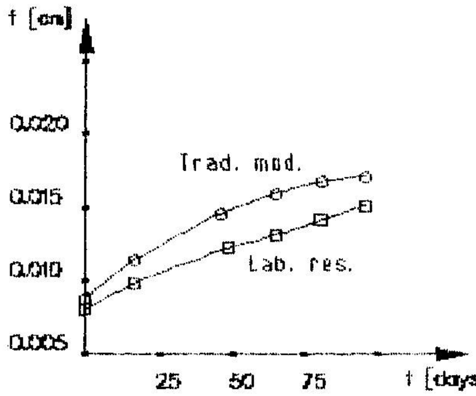


fig.4

Displacements of the beam. FEM model.

Time (days)	Maximum displacements		
	Trad. mod.	FEM mod.	Lab. res.
0	0.00824	0.00817	0.0078
16	0.01180		0.0095
32	0.01410		0.0110
48	0.01540		0.0124
64	0.01581		0.0128
80	0.01638		0.0134
96	0.01700		0.0144



Changes of displacements in time. Traditional model. Laboratory research.

fig.5
Comparison of the results

6.CONCLUSIONS.

The results of structure analysis presented above show advantages and disadvantages of the used methods. Traditional method based on differential equations allows to find displacements at every point of beam length and at every time, but it doesn'tgive possibility to describe influence of cracks. There is no problem in modeling cracks in FEM model but the lack of time elements precluded to make analysis of time effects. Results of calculations in both methods were similar and were comparable with the results of the labaratory research.

During the shown process of the structural analysis it is possible to take into account many new factors and observe a structure response. Moreover it is possible to analyse the influence of a range of features of structure, e.g. material, shape or ways of reinforcing.

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