Zeitschrift: IABSE reports = Rapports AIPC = IVBH Berichte

Band: 62 (1991)

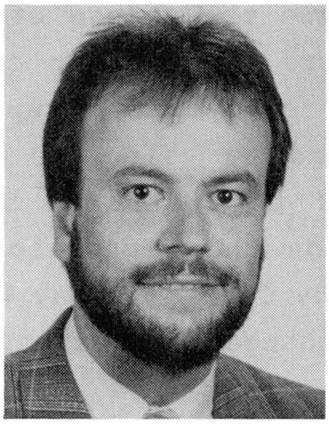
Rubrik: Time dependent effects

Creep Effects on Structural Concrete

Effets différés dans les structures en béton

Auswirkungen des zeitabhängigen Materialverhaltens auf Betonverbundtragwerke

Benno BLESSENOHL
Dr.-ing.
RWTH Aachen
Aachen, Germany



Benno Blessenohl, born 1955, obtained his civil engineering degree at the RWTH University Aachen. He was engaged in a bridge building consulting firm before returning to RWTH University for his doctorate.

SUMMARY

Starting with an analytical stress-strain-time relation of concrete, the numerical analysis of time-dependent effects on structural concrete using an equivalent-stiffness method and an iterative method are explained. If several abrupt loadings have to be superimposed these methods can be used with the help of an incremental constitutive law.

RÉSUMÉ

La relation algébrique contenant contrainte, déformation et temps intervient dans l'analyse numérique des effets différés du béton, et ceci en tant qu'élément de base; l'utilisation de la rigidité équivalente est expliquée, conjointement à la méthode itérative utilisée. Une relation constitutive est introduite afin de tenir compte de la superposition de plusieurs sauts de contrainte.

ZUSAMMENFASSUNG

Ausgehend von der algebraischen Spannungs-Dehnungs-Zeit Beziehung für Beton wird die numerische Berechnung der Auswirkungen des zeitabhängigen Betonverhaltens sowohl für die Methode der äquivalenten Steifigkeiten als auch für die iterative Methode erläutert. Anschliessend wird eine bei Anwendung dieser Methoden für die Superposition mehrerer Spannungssprünge vorteilhafte inkrementelle konstitutive Beziehung vorgestellt.

1. INTRODUCTION

The analysis of structural concrete at serviceability limit state regarding deflections, stress distribution and cracking due to creep and shrinkage is essential for a good performance. Nowadays two groups of methods are used for the numerical analysis of time-dependent effects on structural concrete. The first of them is the step-by-step method which is most often used for computer programs and allows to calculate the change of stress in short time steps. According to the nature of creep in concrete the change of stress has to be stored for every time step and every cross section or point of the structure to calculate the creep strain in further time steps. To avoid this storage of huge numbers of data the second method based on the algebraic stress-strain-time relation which was introduced by Trost [1] may be used. This relation allows the calculation of changes of stress as a result of creep and shrinkage in only one time step.

2. QUASI-ELASTIC METHODS OF CALCULATION OF TIME-DEPENDENT EFFECTS

Creep problems are generally solved by the incremental step-by-step analysis of structural concrete as a sequence of elasticity problems. Then the stress-strain relation within a time step is described by a linear function using the rectangle or the trapezoidal rule for time integration. These linear functions may be used to formulate an incremental elastic modulus according to the type of time integration (see [2]). The incremental method with short time steps based on the history integral is associated with the disadvantage that every preceding value of all stress components for each finite element must be stored. This may be reduced using differential-type formulations for the storage of stress history. But then computing time is still very long because only creep functions composed of e-functions can be used and short time steps are conditional.

The key idea to overcome these disadvantages and to obtain an efficient algorithm was to formulate an incremental constitutive law for long time steps in analogy to the stress-strain-time relation formulated by Trost [1]. Starting with

$$\varepsilon(t) = \frac{\sigma_c(t_0)}{E_c}\,[1 + \varphi(t,t_0)] + \frac{\sigma_c(t) - \sigma_c(t_0)}{E_c}\,[1 + \chi(t,t_0)\,\varphi(t,t_0)],$$

subtracting the elastic strain at loading age and replacing the change of strain $\varepsilon(t) - \varepsilon(t_0)$ by $\Delta\varepsilon(t,t_0)$ respectively the change of stress by $\Delta\sigma(t,t_0)$ results in

$$\Delta\varepsilon(t,t_0) = \frac{\sigma_c(t_0)}{E_c}\,\varphi(t,t_0) + \frac{\Delta\sigma_c(t,t_0)}{E_c}\,[1 + \chi(t,t_0)\,\varphi(t,t_0)] \qquad (1)$$

This equation describes the physically measurable change of strain in the time interval from t_0 to t as a sum of three fictitious strains: First the unrestrained creep strain caused by the abrupt stress change at t_0 which is furthermore called primary creep. Second the elastic strain and third the unrestrained creep strain caused by the steady change of stress in the time interval from t_0 to t. The third part as well as the steady stress change itself is caused by primary creep and is therefore called secondary creep.

Two different ways of time dependent analysis of structural concrete may be adopted simply by transforming eq. (1). Furthermore the first of them is called equivalent-stiffness method. It requires the following conversion of eq. (1):

$$\Delta\sigma_c(t,t_0) = \frac{E_c}{1 + \chi(t,t_0)\,\varphi(t,t_0)} \left[\Delta\varepsilon(t,t_0) - \frac{\sigma_c(t_0)}{E_c}\,\varphi(t,t_0) \right]$$

This equation may be written as

$$\Delta\sigma_c(t,t_0) = E_{AAEM}\,\Delta\varepsilon(t,t_0) - E_{AAEM}\,\frac{\sigma_c(t_0)}{E_c}\,\varphi(t,t_0) \qquad (2)$$

with the age adjusted effective modulus (AAEM, see [2])

$$E_{AAEM} = \frac{E_c}{1 + \chi(t,t_0)\,\varphi(t,t_0)} \qquad (3)$$

The first part of eq. (2) corresponds entirely with Hooke's law and the second part contains the primary creep strain due to $\sigma_b(t_0)$ which can be treated like an imposed strain due to change of temperature. Thus the change of stress and strain in the time interval from t_0 to t may be calculated by the equivalent-stiffness method by considering primary creep as an imposed load and by modification of either the elastic modulus or more generally of the stiffness matrix of the structure in a way that secondary creep is enclosed. Then it is possible to calculate the change of stress and strain with conventional methods based on elastic theory.

The second method is called iterative method and may be explained with equation

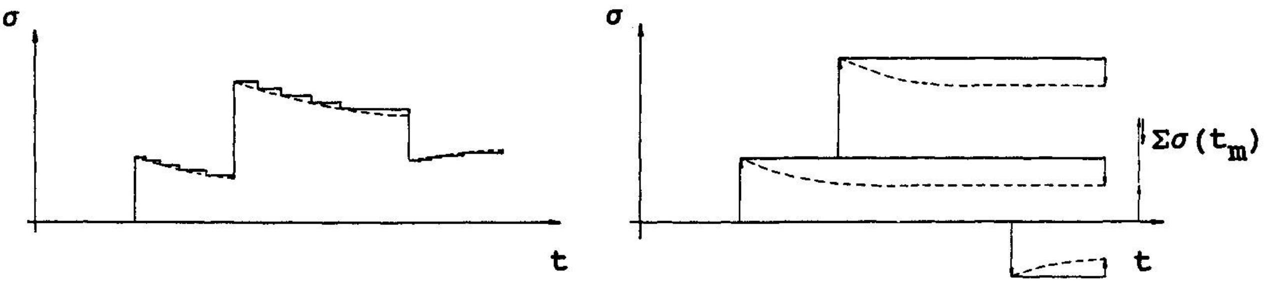
$$\Delta\sigma_c(t,t_0) = E_c\left[\Delta\varepsilon(t,t_0) - \frac{\sigma_c(t_0)}{E_c}\varphi(t,t_0) - \frac{\Delta\sigma_c(t,t_0)}{E_c}\chi(t,t_0)\varphi(t,t_0)\right] \tag{4}$$

which can easily be obtained from eq. (1). The first part of eq. (4) is identical with Hooke's law, the second part contains primary creep and the third part secondary creep.

The first iteration may be done setting $\Delta\sigma_c(t,t_0) = 0$ on the right side of eq. (4). Thus changes of stress and strain caused by primary creep as external load in the time step from t_0 to t can approximately be calculated with conventional methods based on elastic theory. Then the third part of eq. (4) can be estimated with $\Delta\sigma_c(t,t_0)$ taken from the results of the first iteration step. The second iteration step follows with an improved external load containing now the sum of primary and secondary creep. Thus it is possible to improve the change of stress and strain and the load including secondary creep iteratively. Zienkiewicz [3] described this method already but he didn't take $\chi(t,t_0)$ into consideration. Therefore he had to limit the method to short time intervals with $\chi(t,t_0) \approx 1$ [4] what is not generally necessary.

3. SUPERIMPOSING OF SEVERAL ABRUPT LOADINGS

The superposition of several abrupt loads is easily possible if any step-by-step method is used. In connection with the algebraic stress-strain relation of Trost [1] or equations (2) or (4) several abrupt loads have to be treated separately according figure 1b and summed up for time t in question. If stress or strain of any other time is needed the whole calculation has to be carried out again.
These difficulties may be overcome by a combination of both methods using the incremental structure of the step-by-step method and the long time steps of Trost's method. Incremental structure means that the time axis is subdivided in intervals of any length and that the stress history is evaluated step after step from the beginning.



a) step-by-step method b) algebraic relation (Trost)
Fig.1 Superposition of several loads according to different methods

Limits of the time intervals should be set when abrupt loadings are implemented, when the cross section or the restraint conditions are modified and when values of stress or strain are needed. This means that the length of time intervals is optional - short or long.

4. INCREMENTAL METHOD WITH LONG TIME STEPS

An algebraic stress-strain-relation for the time interval from t_{m-1} to t_m which is needed for the incremental method was defined in [4]. It can be obtained from the integral equation (see [1]) by formulating the strain at t_m and subtracting the strain at t_{m-1} from it. Introducing the coefficient $\chi(t_m,t_{m-1})$ (exactly conform to the relaxation coefficient of Trost $\chi(t,t_0)$), the aging coefficient for the elastic modulus $k_E(t_m,t_{m-1})$ and the incremental aging coefficient for primary creep $\chi(t_m-t_{m-1},t_j)$ the remaining integrals can be transformed to algebraic expressions in analogy to Trost's algebraic relation.
The resulting constitutive law for uniaxial stress runs as follows

$$\Delta\varepsilon_C(t_m,t_{m-1}) = \Delta\varepsilon_{0,\varphi}(t_m,t_{m-1}) + \Delta\varepsilon_{0,S}(t_m,t_{m-1}) + \Delta\varepsilon_{0,T}(t_m,t_{m-1})$$
$$+ \frac{\Delta\sigma_{C\varphi}(t_m,t_{m-1})}{E_C} \cdot \left[\frac{E_C\ k_E(t_m,t_{m-1})}{E_C(t_{m-1})} + \rho(t_m,t_{m-1})\ \varphi(t_m,t_{m-1}) \right] \quad (5)$$

with the primary creep

$$\Delta\varepsilon_{0,\varphi}(t_m,t_{m-1}) = \sum_{j=1}^{m-1} \frac{\Delta\sigma_{cL}(t_j)}{E_C} \cdot [\varphi(t_m,t_j) - \varphi(t_{m-1},t_j)]$$
$$+ \sum_{j=1}^{m-2} \frac{\Delta\sigma_{C\varphi}(t_{j+1},t_j)}{E_C}\ \rho(t_m-t_{m-1},t_j)\ [\varphi(t_m,t_j) - \varphi(t_{m-1},t_j)] \quad (6)$$

and the unrestrained changes of shrinkage strain $\Delta\varepsilon_{0,S}(t_m,t_{m-1})$ and temperature strain $\Delta\varepsilon_{0,T}(t_m,t_{m-1})$ in the time interval from t_{m-1} to

t_m. Eq. (6) describes the unrestrained creep strain caused by changes of stress in the past, subdivided into abrupt changes $\Delta\sigma_{cL}(t_j)$ and steady changes $\Delta\sigma_{c\varphi}(t_{j+1},t_j)$. These steady changes took place during the time interval from t_j to t_{j+1} and do not exactly fit to the loading age t_j in eq. (6), but the incremental aging coefficient $\chi(t_m-t_{m-1},t_j)$ acts as a correction factor in this case. It's approximate value is 1,0 [4], differing from the well known average value of $\chi(t_m,t_{m-1})$ = 0,8 for the aging coefficient defined by Trost [1].

In consequence of the analogy between equations (5) and (1) the equivalent-stiffness and the iterative method may both be used in connection with the incremental constitutive law for the analysis of time-dependent effects in structural concrete. The first condition is that primary and secondary creep are strictly separated like in eq. (5) and treated properly as set forth in chapter 2.

5. CONCLUSIONS

Any algorithm or computer program based on elastic theory may be used for the analysis of time-dependent effects in structural concrete if quasi-elastic methods in connection with a suitable constitutive law are used. An incremental stress-strain-time relation like eq. (5) renders long time steps possible and reduces the necessity to store a lot of values in connection with short computing time. Some practical examples dealing with loss of prestress, stress redistribution in an inhomogenous cross section and bridges build in sections can be found in [4].

REFERENCES

1. TROST, H., Auswirkungen des Superpositionsprinzips auf Kriech- und Relaxationsprobleme bei Beton und Spannbeton. Beton- und Stahlbetonbau Vol. 62, no. 10 and no. 11, October 1967.

2. BAZANT, Z. P., Editor, Creep and Shrinkage of Concrete: Mathematical Modeling. Fourth RILEM International Symposium, Northwestern University, Preprints, Evanston 1986.

3. ZIENKIEWICZ, O. C., The Finite Element Method. Maidenhead 1977.

4. BLESSENOHL, B., Zur numerischen Berechnung der Auswirkungen des Kriechens und Schwindens auf Betonverbundtragwerke - Grundlagen und Algorithmen für die EDV -. Dissertation, RWTH Aachen 1990.

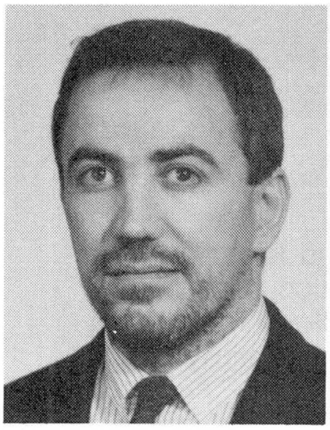
Time-Dependent Behaviour of Prestressed Concrete Structures

Comportement différé des structures précontraintes

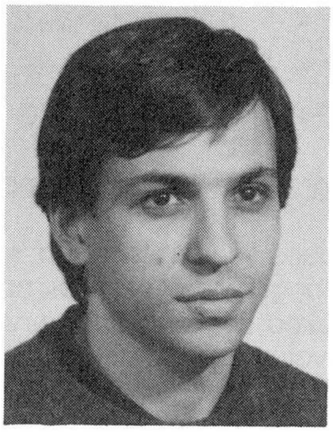
Zeitabhängiges Verhalten von Spannbetonkonstruktionen

Hugo CORRES PEIRETTI
Prof.
Polytechn. Univ. Madrid
Madrid, Spain

Alejandro PEREZ CALDENTEY
Civil Eng.
Polytechn. Univ. Madrid
Madrid, Spain



Hugo Corres Peiretti, born in 1954, obtained his degree in Civil Engineering in 1975 and his Ph.D. in 1979 at the Polytechnical University of Madrid. At present he is a Professor of Reinforced and Prestressed Concrete at the E.T.S.I. Caminos Canales y Puertos. He has been involved in research in structural concrete since 1976.



Alejandro Perez Caldentey, born in 1964, graduated as a Civil Engineer from the Polytechnical University of Madrid in 1989 and is now a Ph.D. candidate.

SUMMARY

This paper describes a model which permits the analysis of the time-dependent behaviour non-cracked prestressed concrete structures under service loads. The influence of the quality and reliability of the input data on the accuracy of the results obtained is stressed. Results obtained with the model are then compared to experimental data. Finally, from the comparison with experimental results it is concluded that such a model is a valuable computational instrument once its limitations are recognized.

RÉSUMÉ

Le modèle décrit permet l'analyse du comportement différé de structures en béton précontraint non-fissurées à l'état de service. L'accent est particulièrement mis sur la qualité des données introduites et sur la fiabilité des résultats obtenus, qui sont ensuite comparés aux données expérimentales. Un modèle comme celui présenté ne peut être utile qu'à condition de tenir compte de ses limites.

ZUSAMMENFASSUNG

Dieser Bericht beschreibt ein Modell für die Analyse des zeitabhängigen Verhaltens ungerissener Spannbetonkonstruktionen unter Gebrauchslast. Der Einfluss der Qualität und der Zuverlässigkeit der Daten auf die Präzision der Ergebnisse wird besonders hervorgehoben. Die Ergebnisse aus der Theorie und den Versuchen werden dann verglichen. Dieser Vergleich führt zu dem Ergebnis, dass dieses Modell, unter Berücksichtigung seiner Einschränkungen, gültig ist.

1. INTRODUCTION

When dealing with singular structures (cable-stayes bridges, cantilever construction, bridges built by phases) it is important to evaluate with good accuracy the evolution with time of strains and stresses. This sort of problems cannot usually be tackled in a simplified manner and therefore requires the use of more complete models which take into account all the phenomena involved.

This paper describes a model which permits the analysis of the behaviour with time of evolutive non-cracked prestressed concrete structures for service loads. The influence of the quality and reliability of the input data on the accuracy of the results obtained is stressed. Results obtained with the model are then compared to experimental data. Finally, from the comparison with experimental results it is concluded that such a model is a valuable computational instrument once its limitations are taken into account.

2. DESCRIPTION OF THE PROPOSED MODEL

2.1 Stress-Strain Diagram and Time-Dependent Behaviour of Concrete

For instantaneous loads, concrete is considered as an elastic and linear material characterised by its longitudinal modulus of elasticity. This assumption is justified by considering non-cracked sections and service loads. The model allows for different formulations of the evolution with time of the longitudinal modulus of elasticity.

Time dependent behaviour of concrete due to creep and shrinkage is handled according to the general approach proposed by CEB [1]. In order to avoid storing the whole stress history of the structure an alternate method, based on a Dirichlet series aproximation of the creep coefficient curves as proposed by Zienckiewicz and Watson [2] has also been implemented.

Several formulations for both creep coefficient and shrinkage strain are supported, including methods proposed by ACI[3] and CEB[4]. Experimental data may also be used through a Dirichlet series approximation.

2.2 Stress-Strain Diagram and Relaxation of Steel

Both prestressed and non-prestressed steels are considered as linear and elastic materials for both tension and compressive instantaneous loads. Again this assumption is valid while considering service loads only.

Relaxation of steel in prestressed concrete members occurs at variable length because of interaction with creep and shrinkage of concrete. The model uses a formulation explained in Fig. 1 and reference [5] which takes this into account. This method proposes an analytical formulation for relaxation at constant length at different initial stress levels and a procedure to represent the evolution of stress with time.

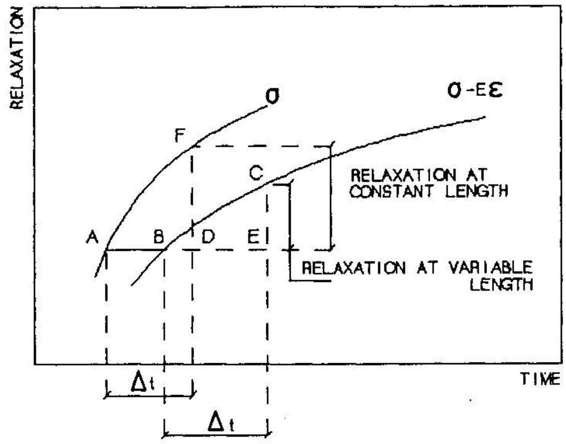


Fig. 1 Relaxation at variable length

2.3 Concrete-Steel Bond

Perfect bond between concrete and steel bars is assumed, both for prestressed and non-prestressed steel.

2.4 Analysis

The model allows changes in the geometry of both the structure (new nodes and bars) and of the different sections (composite sections), and can therefore simulate an evolutive construction process.

For each construction phase a different structure is considered. The structures are divided into a sufficient amount of sections. At each time interval incremental strains due to creep and shrinkage of concrete and relaxation of steel are determined for each section. These strains are then introduced on the structure and new bending moments and axial forces due to these time-dependent phenomena are obtained.

Strains, curvatures, bending moments and axial forces are referred to the same fiber throughout the analysis. This fiber does not necesarily coincide with the center of gravity of the sections which is itself subject to change with time. Using only one reference fiber makes stresses and strains which occur at different times additive without any need for transformation.

2.5 About the Data Required by the Proposed Model

As pointed out before the major drawback of a general model like the one proposed in this paper is the large amount of data required. Furthermore the results obtained will be no more precise than the data entered. It is therefore important to be aware of what data is necessary in order to measure the appropriate variables at the proper time if possible and, if not, to evaluate the possible errors due to this lack of precise data.

In reference to concrete, values are needed for:

- Evolution with time of the longitudinal modulus of elasticity. Analysis is specially sensible to this variable and care should be taken in determining its value.
- Creep coefficient.
- Shrinkage strain.

In reference to steel, data required includes:

- Modulus of elasticity of the different steel types used.
- Parameters defining the relaxation of prestessing steel.

3. COMPARISON WITH EXPERIMENTAL RESULTS

In this section reference is made to experimental results obtained at the E.T.S.I. Caminos, Canales y Puertos of the Polytechnical University of Madrid. These results are detailed in reference [6]. A more detailed comparison can be found in reference [7] including other experimental data dealing with a composite section and a two span continuous beam.

3.1 Available Experimental Data vs Required Data

As mentioned in paragraph 2.5, it is important to take into account the quality of the available data. The Corres-Rodríguez tests were very careful in this sense since they were designed with the purpose of testing an analytical model for determined prestressed concrete structures. All required data is therefore available.

Parallel to measuring strains on the beams themselves, a series of complementary tests were carried out. For concrete these included compression and tension strength, evolution of the modulus of elasticity as well as creep and shrinkage tests. Tension strength tests were also carried out for both prestressing and non-prestressing steels. Finally, relaxation of prestessed steel at constant length was measured for three different initial stresses.

3.2 Data for the Model

3.2.1 Evolution with time of the Modulus of Elasticity of Concrete

Because of the large amount of data available a particular model adjusted by means of the least squares method was used (see Equation (1)).

$$E_c = E_{c28}(2.5t/(t+42))^{0.10} \tag{1}$$

In this equation E_{c28} is the modulus of elasticity of concrete at 28 days and t is the age of concrete in days.

3.2.2 Creep Coefficient and Shrinkage Strain

Experimental results from creep and shrinkage tests were compared to analytical results given by ACI[3] and the former CEB[4] models. For the creep coefficient excellent agreement was found between experimental data and the model proposed by ACI while the 1978 CEB model was found to consistently overestimate creep.

For shrinkage strain neither ACI nor CEB models provided a close fit to experimental results although differences were not truly significant.

In view of these results the ACI models were used to represent both phenomena.

3.2.3 Relaxation of Steel

From the constant length relaxation tests enough information was provided in order to adjust by the least squares method the parameters required by the Elices-Sanchez-Gálvez model [5] mentioned in paragraph 2.2.

3.3 Discussion of Results

Figures 2 and 3 compare respectively the experimental and theoretical evolution of the curvature and deflection obtained at midspan of beams 1 and 2. As can be seen good agreement is obtained by application of the analytical model in both cases.

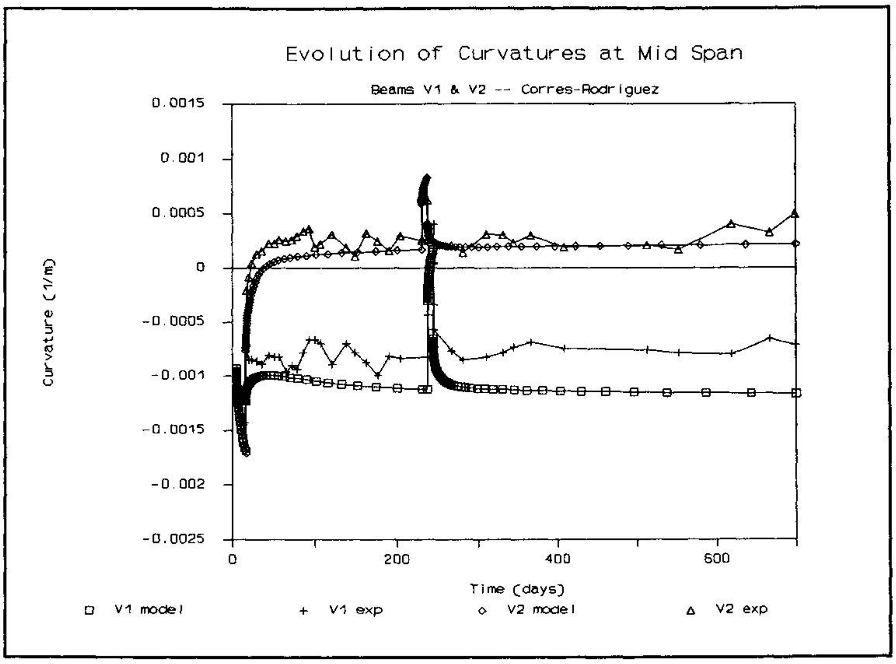


Fig. 2 Evolution of mid-span curvatures

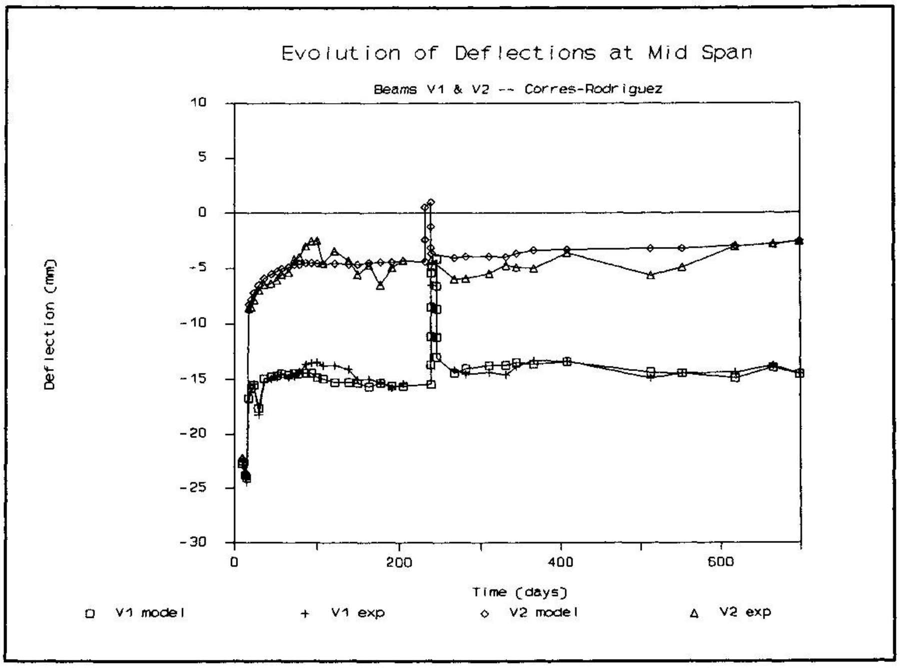


Fig. 3 Evolution of mid-span defelections

4. CONCLUSION

In this paper a flexible and general analytical model for the study of evolutive non-cracked prestressed concrete structures subject to service loads has been outlined. This model implements, as an alternative to Dirichlet series approximation of the creep coefficient, the general method proposed by CEB [1] and uses special procedure to represent relaxation at variable length.

Results obtained with the model have been compared with experimental tests obtaining good agreement.

As discussed above, this sort of model requires a large amount of data which in most cases cannot be accurately determined through analytical formulae and which greatly influence final results. It is therefore important to define the degree of precision required.

In this line it can be concluded that such a model is an ideal tool for the analysis in the service limit state of prestressed concrete structures if accurate data is available. In design, the use of such a model requires realistic estimates of creep coefficients, shrinkage strains and modulus of elasticity of concrete. In such conditions the accuracy provided by the model should be sufficient in most cases to point out the main aspects of the behaviour of the structures. Finally, in construction, experimental tests must be carried out to measure "in situ" the more important variables used by the model in order to obtain more accurate results.

5. ACKNOWLEDGEMENTS

This experimental and theoretical program was financed by the Dirección General de Carreteras of the Ministerio de Obras Públicas y Urbanismo of Spain.

REFERENCES

1. CEB, Structural Effects of Time-Dependent Behaviour of Concrete. Bulletin d'Information CEB Nº54, 1976.

2. ZIENCKIEWICZ, O. & WATSON, M., Some Creep Effects in Stress Analysis with Particular References to Concrete Pressure Vessels. Nuclear Engineering and Design Nº4, 1966.

3. ACI COMMITEE 209, Prediction of Creep, Shrinkage and Temperature Effects in Concrete Structures. SP 76-10, 1982.

4. CEB-FIP, Model Code for Concrete Structures. Bulletin d'Information CEB Nº124-125, 1978.

5. SANCHEZ-GALVEZ, V. & ELICES M., Pérdidas de pretensado por fluencia y relajación: 1. Teoría y 2. Comprobación experimental. Hormigón y Acero Nº153, 1984.

6. RODRIGUEZ ,R., Estudio teórico y experimental de deformaciones en vigas pretensadas de puentes. PhD thesis. E.T.S.I. Caminos, Canales y Puertos. Polytechnic University of Madrid. 1989.

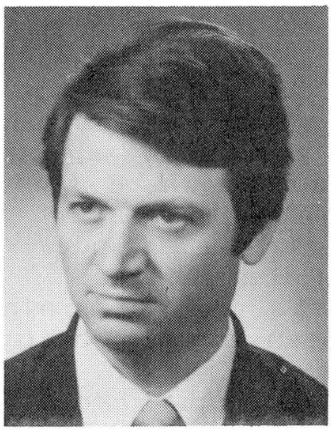
7. CORRES, H. & PEREZ, A., Comportamiento en servicio de estructuras de hormigón pretensado. Modelo teórico y contrastación experimental. Hormigón y Acero Nº176, 1990.

Deformations of T-shaped Beams under Sustained Loads

Déformations des poutres en T sous charge de longue durée

Durchbiegungen von Plattenbalken unter andauernder Belastung

Mariusz SZECHINSKI
Assist. Prof.
Wroclaw Techn. Univ.
Wroclaw, Poland



Mariusz Szechinski, born 1947, graduated in Civil Engineering and took his Dr. techn. degree at Wroclaw Technical University. Since 1970 he worked as Assistant and later as an Assistant Professor at the Institute of Building Science. He has published 30 papers on the theory and practice of Concrete Structures and was involved in more than 140 works for industry.

SUMMARY
Experiments show that a considerable growth of strains occurs in long-term loaded reinforced concrete structures, as result of time-dependent changes of properties of concrete. Results of calculations were compared with both results from a finite element model and with experimental values.

RÉSUMÉ
Un modèle de calcul décrivant les poutres en T en béton armé sous charges de longue durée est présenté. Les résultats expérimentaux provenant d'un modèle spécialement développé ont été comparés à ceux obtenus analytiquement grâce à la méthode des éléments finis.

ZUSAMMENFASSUNG
In der Arbeit wird ein Berechnungsmodell für Stahlbetonplattenbalken unter dauernd wirkender Belastung formuliert. Die Messergebnisse werden mit Ergebnissen aus FEM-Berechnungen und aus Berechnungen anhand eines eigenen Modells verglichen.

1. INTRODUCTION

Experiments indicate that in the long term loaded reinforced concrete constructions, as a result of changes in time of properties of concrete, there occurs a considerable growth of strains in concrete.
The increase of the strains in concrete,leads to the deformation of the whole element and to the increase in deflection or additionally to the increase in the width of the opening of the cracks. The stresses also vary and therefore a strain in the materials may increase.
These phenomena must be predicted and taken into account during planning stage, and therefore it is necessary to express them analytically in order to make practical calculations possible.
In particular this situation applies to the flexed T-shape reinforced concrete beams, where the concrete closely cooperates with the steel during load transfer. Calculations of deformations for reinforced concrete beams under a long-term loading may be divided into two essentioal stages.
The first stage consists of determining the state of stresses and strains in freely-chosen cross-section of concrete beam. In this case the problem reduces itself to allocating the stresses and strains at any point of the cross section that,s consider at any point of time with regard to the most heterogeneous factors.
The second stage consists of observing the element as a whole. Here we may take into account rigidity, deflection, spacing as well as the width of the opening of the cracks. There we make use of considerations carried out at the first stage and therefore the assumptions made in the first stage will influence the obtained results obtained in an important way. It makes that correct solution of the problem of defining the changes of stresses and strains in any cross-section of concrete operating under long-term loading is the basic task having aa influence on all considerations.
In the present paper it has been decided to make an analysis of this problem on the basis of some new assumptions and existing solutions and to provide some means of formulation it comprehensively.
The proposed standardization of the method of presenting equilibrium equations in different phases and generalization of the method of presenting the dependence between creep stresses and strains allowed us to build the analytical model of problem and examine exactly the state of stresses and strains in reinforced concrete beam depending on the time duration of the load. To get a practical results of calculations there was made the computer program, which allowed to verify an influence of many different factors on the state of stresses and strains in the T-shape reinforced concrete beam.
The results of upper proposed traditional analytical calculations were compared with the result of calculations of the model built of structural concrete.

2. ASSUMPTIONS

The assumptions, which is necessary to make for the examination of the present problem, may be divided into 3 groups.
The first group is concerned with the method of making

allowances in the calculations for the rheological properties of concrete; shrinkage, creep or relaxation, under the influence of which changes in the strain of concrete are in time brought about. Customarily, making allowances for such influences has become generally accepted by means creep function, which in the general case may be shown as follows:

$$\varepsilon_c(t,\tau)=\frac{\sigma_c(\tau)}{E_c(\tau)}+F[\sigma_c(\tau)\delta(t,\tau)]+\int_\tau^t\frac{1}{E_c(\tau)}\frac{\partial\sigma_c(\tau)}{\partial\tau}d\tau+\int_\tau^t\frac{\partial F[\sigma_c(\tau)]}{\partial\tau}C(t,\tau)d\tau \qquad (1)$$

In the presented paper thedependence (1) shown abovewas adopted as basic to the consideration. The second group of assumptions consists of assumptions concerning the stress-deformation dependence. The analysis of this problem carried out by Szechiński [1,2] allows us to state that until now the problem has been limited to examinations of cross-sections of concrete at different phases of operation, accepting the most varying shapes of the stress diagram. All these cases may be generalized so as to obtain the shape of the diagram as proposed by Szechiński [1,2] in the figure 1.

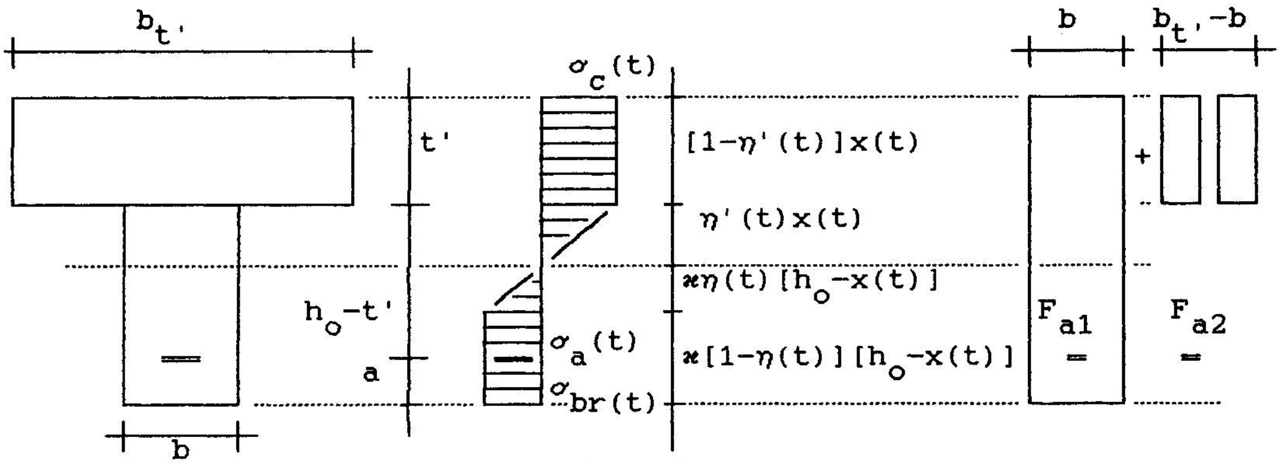


fig. 1
Assumptions about the concrete cross-section.

Acceptance of this generalization allows a completely universal notation of equilibrium equations,valid for every phase of operation of the concrete cross-section.
The third group of assumptions is concerned with deformations in the cross-section of the concrete beam. Customarily, it is accepted that the cross-section before and after deformation remains flat.

3 ANALYSIS OF THE CONCRETE CROSS-SECTION.

Accepting the assumptions given in previous section, fig.1, we get the following equilibrium equations.

$$\Sigma X = 0,\ -\frac{\sigma_c(t)}{2}\eta_o'(t)\ x(t)\ b + b\ \sigma_c(t)\ x(t) = \sigma_{ct}(t)\ [h_o - x(t)]b +$$

$$+\ \varkappa\eta(t)\ \frac{h_o - x(t)}{\;}\ \sigma_{ct}(t)\ b + \sigma_a(t)\ F_{a1},$$

$$\sigma_c(t)\ t'\ (b_{t'}-b) = \sigma_a\ (t)\ F_{a1}. \qquad (2)$$

$$\Sigma M = M_1 + M_2,$$

$$M_1 = \sigma_c(t)bx[(h_o - x(t)/2] - \sigma_b(t)\eta'(t)x(t)\frac{b}{2}[h_o - x(t) + \eta'(t)\frac{x(t)}{3}] +$$

$$-\sigma_{ct}(t)[h_o - x(t)]^2 b/2 + \sigma_{ct}(t)\varkappa\eta(t)[h_o - x(t)b/2[h_o - x(t) - \varkappa\eta(t)x(t)/3],$$

$$M_2 = (b_t' - b)\ \sigma_c(t)t'(h_o - t'/2). \tag{3}$$

Taking into consideration dependence (1) and geometrical dependence, that is assumption 3 from the previous section, the integral equation (4) was received.

$$\varepsilon_c(t,\tau) = \sigma_c(\tau)\delta(t,\tau) + \beta'\sigma_c^2(\tau)c(t,\tau) + \int_\tau^t \frac{1}{E(\tau)}\frac{\partial\sigma_c(\tau)}{\partial\tau}d\tau + \int_\tau^t c(t,\tau)\frac{\partial[\sigma_c(\tau) + \beta'\sigma_c(\tau)]}{\partial\tau}d\tau. \tag{4}$$

The eq.(4) may be simplified to linear form assuming $\beta' = 0$. Furthermore eq. (4) was simplified to the algebraic form, from whihch the values of $x(t)$ were calculated and further from equilibrium equations (2) values $\sigma(t)$, $\sigma_{ct}(t)$, $\sigma_a(t)$, $\varepsilon_c(t)$, $\varepsilon_a(t)$.

4. CALCULATING THE DEFLECTIONS.

From the geometrical dependences, for intermediate loads we may receive

$$\frac{M}{B_o} = \frac{\varepsilon_{co} + \dfrac{\varepsilon_{co}(1-\xi_o)}{\xi_o}}{h_o} = \frac{\varepsilon_{co}}{\xi_o h_o} = \frac{\sigma_{co}}{E_{co}\xi_o h_o} = \frac{\sigma_{ao}}{E_a(1-\xi_o)h_o}, \tag{5}$$

It means that

$$B_o = B(t=0) = \frac{M E_a(1-\xi o)h_o}{\sigma_{ao}} \quad \text{or} \quad B(t) = \frac{M E_a[1-\xi(t)]h_o}{\sigma_a(t)}, \tag{6}$$

From the equilibrium equations and after solving eq.(4) we may receive σ (t) that means we may get the cross-section stiffnes B(t).Furthermore taking into consideration the fact that stiffness is changing on the beam length thre was assumed according to [3] following stiffness function

$$B_b(t,\zeta) = f[B_{Ii}(t), m_{cr}] + h[B_{Ii}(t), B_{IIi}(t), m_{cr}], \tag{7}$$

where

- m_{cr} = M/Mcr – relation between the moment on beam and the cracking moment,
- B_{Ii}, B_{IIi} – stiffness of beam cross-sections in different phases of work,
- B_b – stiffness function on the beam length.

Using the curvature equation and dependence (7) allows to calculate displacements of the beam due to the time flow.

5. FINITE ELEMENT MODEL OF THE BEAM.

Using the FEM program system [4], the computer model of the beam shown in fig. 2, was built, as it is shown in fig.3a. The three-dimensional solid elements, as shown in fig 3b, were used to describe concrete and reinforcement. Concrete and reinforcement were represented by different values of the modulus of the elasticity, weight and the coefficient ν.

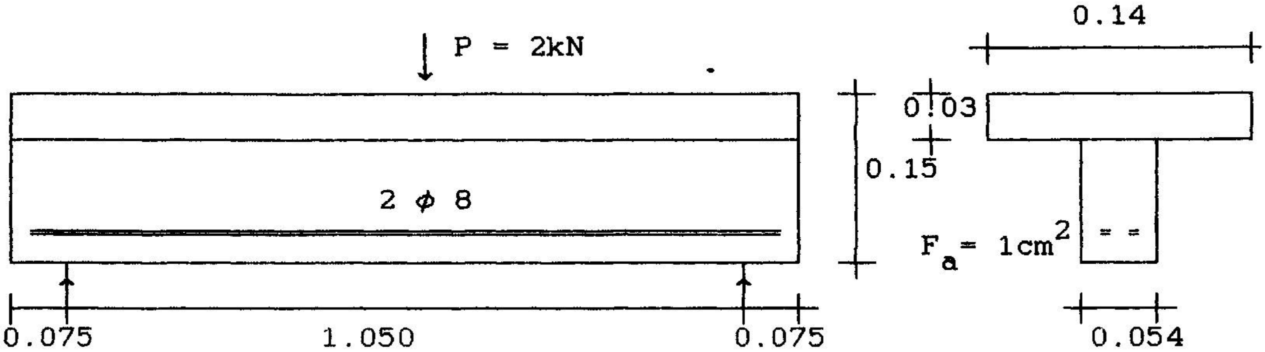
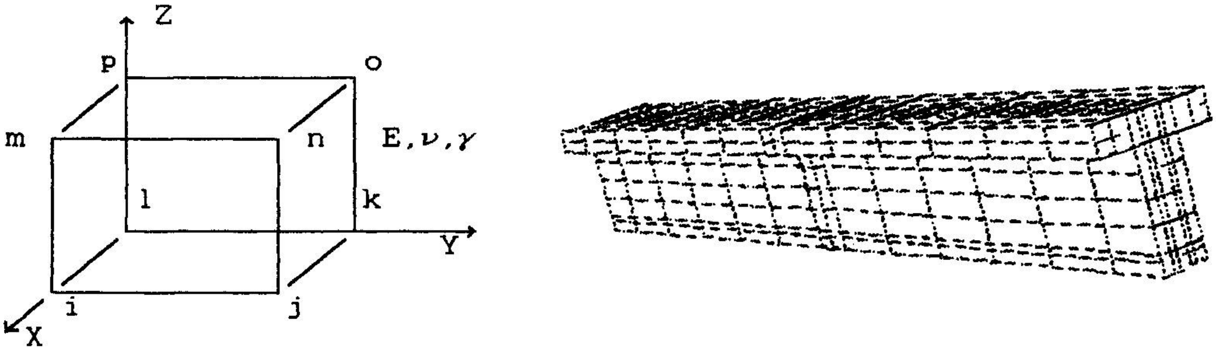


fig.2

Construction of the sample beam



a.Three-dimensional isotropic brick element

b. Nodal mesh on the beam

fig.3 a,b,

Finite Element Method model of the beam

This model, because of program limitations, was examined only under the intermedaite load (t=0). The results compared with results of traditional calculations and experimental results are shown in fig. 4,5.

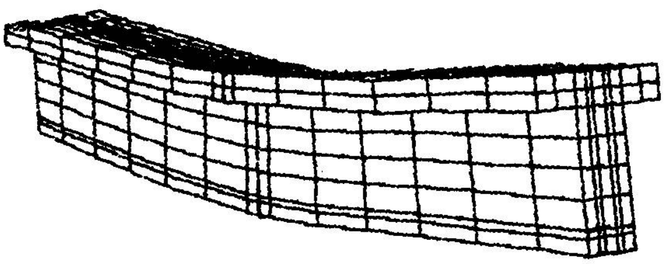
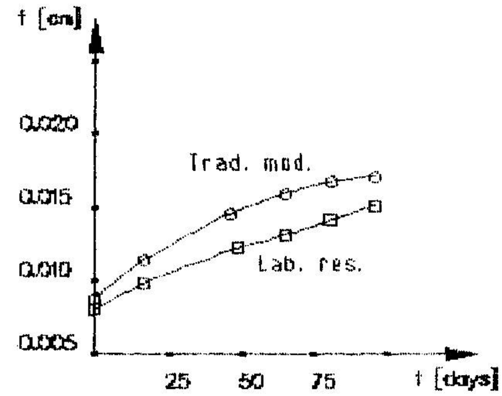


fig.4

Displacements of the beam. FEM model.

Time [days]	Maximum displacements		
	Trad. mod.	FEM mod.	Lab. res.
0	0.00824	0.00817	0.0078
16	0.01180		0.0095
32	0.01410		0.0110
48	0.01540		0.0124
64	0.01581		0.0128
80	0.01638		0.0134
96	0.01700		0.0144



Changes of displacements in time.
Traditional model. Laboratory research.

fig.5
Comparison of the results

6.CONCLUSIONS.

The results of structure analysis presented above show advantages and disadvantages of the used methods. Traditional method based on differential equations allows to find displacements at every point of beam length and at every time, but it doesn'tgive possibility to describe influence of cracks. There is no problem in modeling cracks in FEM model but the lack of time elements precluded to make analysis of time effects. Results of calculations in both methods were similar and were comparable with the results of the labaratory research.
During the shown process of the structural analysis it is possible to take into account many new factors and observe a structure response. Moreover it is possible to analyse the influence of a range of features of structure, e.g. material, shape or ways of reinforcing.

REFERENCES

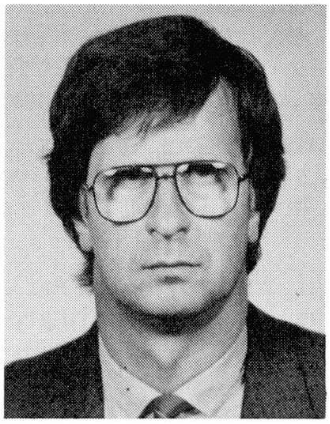
1. SZECHIŃSKI M., Stresses and Strains in the RC Beams under the Sustained Load (pl).Archive of Civil Engineering (Polish Academy of Science). No.1, Warsaw, 1988.
2. SZECHIŃSKI M., Changes in the Shape of the Stress Diagram in the RC Beams. Materials and Structures, vol.15, No 86, Paris 1982,
3. SZECHIŃSKI M., The Flexed RC Beams under the SUstained Loads, In Mathematical Models in Approach to Concrete Structures, Commitee of Civil Engineering of Polish Academy of Science, Zielona Góra 1988,
4. WILSON E., HABIBULAH A.; SAP-86 -A Series of Computer Programs for Static and Dynamic Finite Element Analysis of Structures. Computers and Structures Inc. Berkeley, California, 1988.

Long-Term Strains of Compression Elements in Tall Buildings

Contraintes à long terme apparaissant dans les éléments comprimés des bâtiments élevés

Langzeitverformungen gedrückter Elemente in Hochhäusern

Stefan GRAMBLIČKA
Reader
Slovak Technical Univ.
Bratislava, Czechoslovakia



Stefan Gramblička, born 1948, received his M.Sc. and Ph.D. from the Slovak TU, Bratislava. His experience includes design, teaching and research of reinforced concrete and composite steel-concrete structures. He is the author of a number of original research works on effects of creep and shrinkage of concrete.

SUMMARY

This paper presents a theoretical solution for the determination of strains due to creep and shrinkage of concrete, in compression elements of tall buildings. The analysis of the creep and shrinkage effects takes into account the variability in percentages of cross-section reinforcement as well as the actual variation of the increasing normal forces. On the site of the Press Centre in Bratislava, selected compression elements were measured over a two year period. At the same time, measurements were made of unloaded specimens. The author presents an analysis of the measured strain values.

RÉSUMÉ

La solution théorique concernant la détermination des contraintes dues au fluage et au retrait est présentée, dans le cas des éléments comprimés de bâtiments élevés. Cette analyse tient compte des différents pourcentages d'armature dans les sections comprimées, ainsi que de l'augmentation de l'effort normal dans le temps. Pendant plus de deux ans, des mesures ont été effectuées sur des éléments comprimés sélectionnés du centre de presse de Bratislava. D'autres tests étaient menés conjointement sur des éléments non-chargés. Une analyse des résultats obtenus est présentée.

ZUSAMMENFASSUNG

Der Beitrag behandelt die theoretische Ermittlung von Betonverformungen infolge Kriechens und Schwindens in gedrückten Elementen von Hochhäusern. Bei der Analyse der Kriech- und Schwindauswirkungen werden verschiedene Bewehrungsgrade der gedrückten Elemente sowie verschiedene Annahmen über die zeitliche Belastungserhöhung untersucht. Beim Bau des Pressezentrums in Bratislava wurden im Zeitraum von 2 Jahren Langzeitverformungen ausgewählter gedrückter Elemente gemessen. Zum Vergleich wurden im gleichen Zeitraum Verformungen an unbelasteten Elementen ermittelt. Der Autor behandelt die Analyse der gemessenen Verformungen.

1. INTRODUCTION

Within the last years a large number of tall buildings exceeding 30 stories have been built. Such buildings of great height are very sensitive to cumulative differential length changes of their vertical elements. One of the influences affecting these changes in reinforced concrete structures are long-term strains due to volume changes, namely to creep and shrinkage of concrete which depend on a considerable number of influances. The overall contraction of the vertical load-carrying elements is the sum of a number of partial changes. The determination of elastic strains due to load does not present any difficulties. Therefore, in the paper we shall concentrate on the analysis and possibilities of determination of long-term strains which, under certain conditions, may exceed elastic strains several times. The extent of the creep and shrinkage of concrete is influenced by a variety of factors such as environmental effects, age of concrete during the exposure of the member to the load, concrete grade, reinforcement percentage, etc.

Prof. Bruggeling [1] presents the basic introduction of time-dependent effects. We would like to support his note that it is very important to understand when and why time-dependent effects are of importance for the behaviour and on the durability of a structure.

Traditionally, the effect of creep, shrinkage and temperature is considered in horizontal structures, such as long span bridges. These effects are usually neglected in multistory concrete buildings since, in the past, such structures seldom exceeded 20 stories. A number of recent ultra high rise buildings built without consideration of creep, shrinkage, and temperature effect in the vertical elements have developed partition distress, as well as structural overstress in horizontal elements. It is necessary for the structural engineer to consider the various differential movements and to develop acceptable structural, as well as architectural details, for the satisfactory performance of the building. We shall try to solve the specific problem of deformations which will complete the part 4.1 of the paper [2] .

2. ANALYSIS OF CONCRETE CREEP

In view of the calculation of effects of concrete creep on volume changes, the vertical elements in tall buildings can be characterized by several main factors:
- variability of reinforcement of the cross-sections of vertical elements,
- curve of the increase of load over time, depending on the progress in the construction of the building,
- stress due primarily to compressive forces.

Besides considering the effects of volume changes on length changes (shortening of the vertical elements in tall buildings), it is also necessary to account for these changes with respect to the distribution of the stresses over the cross-section of the member. The higher percentage of reinforcement reduces the increment of compressive stress acting on the reinforcement and increases the increment of tensile stress acting on concrete, adversely affecting the load-carrying capacity of the materials used.

This effect in combination with varying curves of possible load increases demonstrates Fig.1. In view of the extent of strain due to concrete creep it may be noted that increased percentage of reinforcement reduces the extent of deformation due to the creep of concrete (Fig.2).

The real course of load increasing over time can, in essence, be assumed for each element of the tall building already at the design stage. In general, this course can be modelled as shown in Fig.3. In the calculation of the effects of concrete creep, this course must be replaced by an appropriate function. The simplest solution is the one assuming a constant load increase, immediately from the onset of loading. However, such calculation is very unrealistic and can only provide very distorted date.

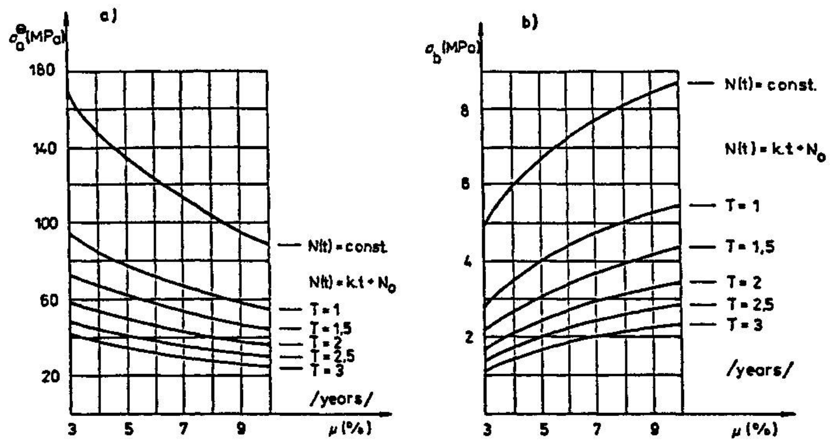


Fig. 1 Stress in the reinforcement (a) and stress in the concrete (b) due to concrete creep at varying percentages of cross-section reinforcement, accounting for the differences in the duration of building construction time

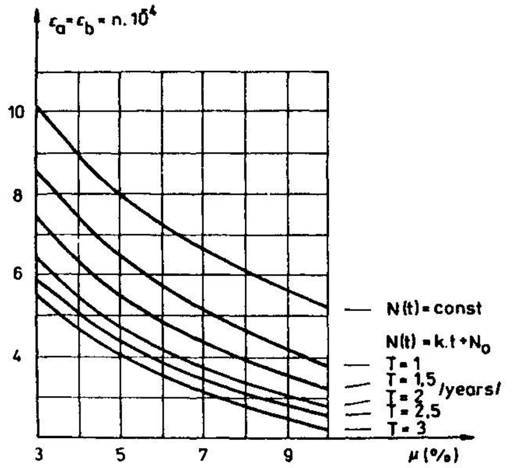


Fig. 2 Dependence of the extent of deformations due to creep on the percentage of cross-section reinforcement, accounting for the differences in the duration of building construction time

The most suitable is the possibility of substituting a bilinear relationships for the course of load increase on the member where, in the first part, the course of loading is a linear function of time and, in the second part, it is constant. This is a relatively most accurate expression of the real course of the load. The decisive factor is the time of construction T which it is relatively easy to determine in practice. The following conclusions can be drawn from the solutions of concrete creep effect at varying substitutions for the course of loading (Fig.1):

- the longer is the construction time T , the lower are the increments of compressive stress from creep on the reinforcement, and the increments of tensile stress from creep on the concrete. A similar dependence also applies to the strain generated by the creep of concrete.

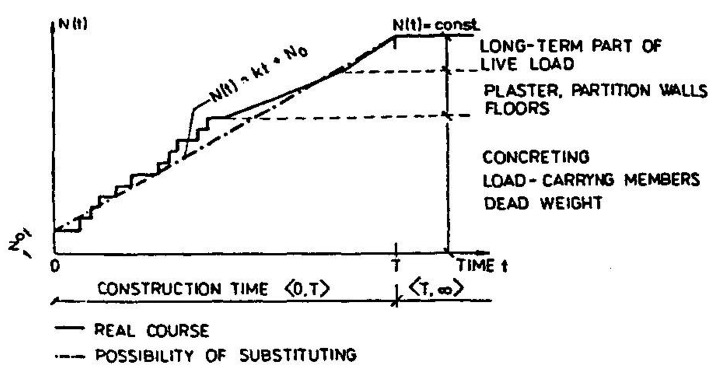


Fig. 3 The curve of normal force increase in the column of tall building

3. ANALYSIS OF SHRINKAGE EFFECTS

In relation to the effects of shrinkage, a higher percentage of reinforcement results in reducing the increment of compressive stress from shrinkage acting on the reinforcement and in increasing the increment of tensile stress acting on the concrete (Fig.4). Because the shrinkage is independent on the stress, it is also independent on the course of load increase. With respect to the strain, a higher percentage of reinforcement reduces the extent of strain from the shrinkage.

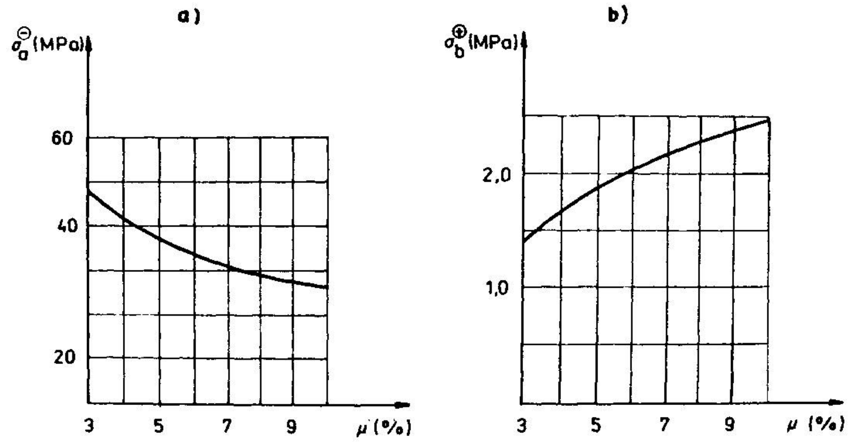


Fig. 4 Dependence of the stress in the reinforcement (a) and in the concrete (b) due to concrete shrinkage on the percentage of cross-section reinforcement

4. TALL BUILDING AND MEASUREMENT POINTS

The measurements of long-term strains of compression elements in the Press Centre tall building, Bratislava, may be listed, with regard to the investigated results, in the group of measurements whose conclusions offer a picture about the course of creep and shrinkage of concrete in a specific structure type. Due to this fact our attention was given to two structurally and statically important compression elements of the structure, i. e. load-carrying columns with a high percentage of reinforcement and gable walls with a low percentage of reinforcement.

The tall building has a height of 104 m. The measurements were conducted on the following floors (Fig.5): 4th floor, the measurement points being S1 to S4 columns and Š1 and Š2 walls, 11th floor, the measurement points being S5 and S6 columns and Š3 wall. S1 to S4 column dimensions were 1 400 x 700 mm (with welded I - section) and reinforcement 16 No 25 mm bars, having an overall reinforcement of 7,44 %. S5 and S6 column dimensions were 1 200 x 600 mm, the overall reinforcement 8,32 %. Wall thickness was 500 mm and the wall was reinforced with steel angles and reinforcing bars, the overall reinforcement being 1,15%. The selected structure points allowed only for overall strain measurements. These strains include a number of partial strains, i. e. strains due to tempe-

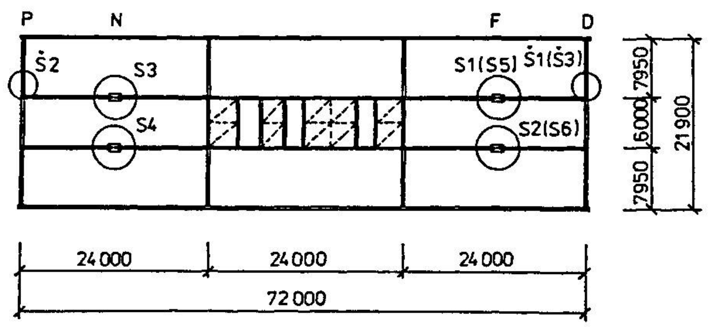


Fig. 5 Scheme of plan layout of tall building of Press Centre

rature changes, shrinkage namely due to changes in humidity, elastic strains due to load on a measured element, and strains due to creep of concrete.

5. THEORETICAL VALUES OF DEFORMATION

The growth of load of particular elements of the measured points in time was calculated according to the actual construction work sequence. Elastic strains were calculated from the assumption of central load of cross-sections of the measurement points. Modulus of elasticity of concrete in particular time was determined on the basic of laboratory results. Theoretical calculations of strains due to creep of concrete was conducted according to the e. g. Dishinger's theory of ageing. Differential equation of the 1st order with the right side will have the following form:

$$\frac{dN_{b\varphi}}{d\varphi} + \alpha N_{b\varphi} = (1-\alpha)\frac{dN(t)}{d\varphi} \qquad \text{where: } \alpha = \frac{A_a}{A_i} = \frac{A_a}{A_a + A_b/n} \tag{1}$$

The solution assumes the variability of the modulus of elasticity of concrete over time [3] :

$$E_{bN} = \frac{E_{b0}}{1+\varphi_t\psi_N} \qquad n_0 = \frac{E_a}{E_{b0}} \qquad n_N = \frac{E_a}{E_{bN}} \qquad \frac{n_N}{n_0} = 1 + \psi_N\varphi_t \tag{2}$$

where: E_{b0} - initial modulus of elasticity of concrete

E_{bN} - modulus of elasticity of concrete in time at axial force load

ψ_N - subsidiary creep coefficient

The differential equation (1) is solved in two intervals. In $\langle 0, T \rangle$ interval on the assumption that $N(t) = k \cdot t + N_0$ (Fig.3). After substitution

$$\frac{dN_{b\varphi}}{d\varphi} + \alpha N_{b\varphi} = (1-\alpha)\frac{0{,}625\ k}{\varphi_\infty - \varphi} \qquad I = \int_0^{\varphi} \frac{e^{\alpha\varphi}}{\varphi_\infty - \varphi}\, d\varphi \tag{3}$$

$$N_{b\varphi} = (1-\alpha)\, e^{-\alpha\varphi} N_0 + e^{-\alpha\varphi}(1-\alpha)\, 0{,}625\ k\ I \tag{4}$$

Subsidiary creep coefficient ψ_N^1 valid for interval $\langle 0, T \rangle$ is

$$\psi_N^1 = \frac{k\,t + N_0}{\varphi\alpha\, e^{-\alpha\varphi}(N_0 + 0{,}625\ k\ I)} - \frac{1}{\alpha\varphi} \tag{5}$$

k - coefficient of erection race, N_0 - initial value of normal force,
φ - creep coefficient in time t ,
ψ_N^2 shall be valid for the interval $\langle T, \infty \rangle$.

$$\psi_N^2 = \left[\frac{N}{(1-\alpha)e^{-\alpha\varphi_\infty}(N_0 + 0{,}625\ k\ I)} - 1\right]\frac{A_b}{A_a\varphi_\infty \cdot n_N^1} - \frac{1}{\varphi_\infty} \tag{6}$$

N - maximum value of normal force, φ_∞ - creep coefficient in time $= \infty$. The following is valid for the resultant value of the elasticity modulus ratios

$$n_{N\infty} = n_N^1 (1 + \psi_N^2\varphi_\infty) \tag{7}$$

The calculation of strains due to creep is

$$\sigma_{b\infty} = \frac{1}{n_{N\infty}}\,\frac{N}{A_{i\infty}} \qquad \sigma_{a\infty} = \frac{N}{A_{i\infty}} \qquad A_{i\infty} = A_a + A_b / n_{N\infty} \tag{8}$$

$$\sigma_b^{creep} = \sigma_{b\infty} - \sigma_{b0} \qquad \sigma_a^{creep} = \sigma_{a\infty} - \sigma_{a0} \tag{9}$$

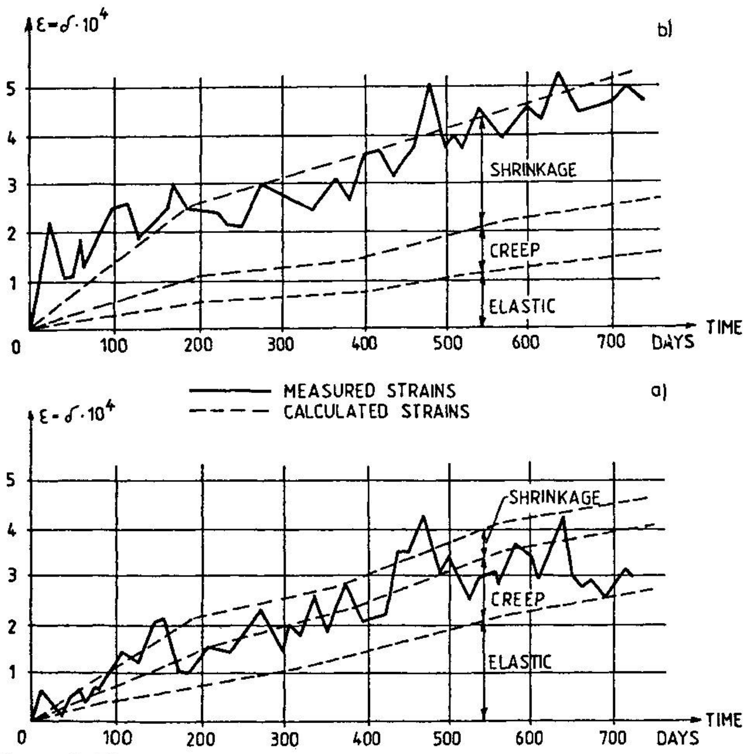
The calculation of strains due to shrinkage (restrained shrinkage) at uniaxial state of stress was conducted according to the following relation

$$\varepsilon_{t\tau_1} = \frac{\varepsilon_{st\tau_1}}{\mu\, n_{\tau_1}\, \varphi_{t\tau_1}} \left(1 - e^{-\alpha \tau_1 \varphi_{t\tau_1}}\right) \tag{10}$$

$\varepsilon_{st\tau_1}$ - relative deformation of concrete at free shrinkage at time t which started to appear at time τ_1 , n_{τ_1} - modular ration of reinforcement for the age of concrete at time τ_1 , $\varphi_{t\tau_1}$ - creep coefficient at time t for the load which started to act at time τ_1.

6. TEST AND MEASUREMENT RESULTS

Concrete compressive strenght and elasticity modulus tests in time were conducted to obtain material characteristics of concrete. Cubes and prisms were fabricated from identical concrete mixture and treated under conditions identical to those of the tall building. Temperature and humidity were measured by thermohydrographs located in the shed at the control non-loaded cubes and at the hole in the wall.



The measured deformations at the non-loaded plain concrete reference blocks include the effects of free shrinkage of concrete and temperature changes. Deformations due to temperature changes were calculated from the measured temperature values. Fig.6 shows plots of mean strains measured at two basic measurement points. A relatively good agreement can be stated when comparing the calculated values of deformations due to creep and shrinkage of concrete and elastic strains. The chosen method of calculation of the theoretical strain values due to creep and shrinkage of concrete for the given type of elements of the structure has been found as suitable and recommentable for use.

Fig. 6 Plots of measured mean strains at measurement points a/S1,S2,S3,S4 b/Š1,Š2

REFERENCES

1. BRUGGELING A:S.G., An engineering model of Structural Concrete. IABSE Colloqium Structural Concrete, April 1991, Stuttgart.
2. WICKE M., Performance Requirements. IABSE Colloqium Structural Concrete, April 1991, Stuttgart.
3. WIPPEL H., Berechnung von Verbundkonstruktionen aus Stahl und Beton, Springer Verlag, 1963 (in german).