

**Zeitschrift:** IABSE reports = Rapports AIPC = IVBH Berichte  
**Band:** 62 (1991)  
  
**Artikel:** Pull-out test of anchor bolts embedded in concrete  
**Autor:** Valente, Gianfranco  
**DOI:** <https://doi.org/10.5169/seals-47713>

### **Nutzungsbedingungen**

Die ETH-Bibliothek ist die Anbieterin der digitalisierten Zeitschriften auf E-Periodica. Sie besitzt keine Urheberrechte an den Zeitschriften und ist nicht verantwortlich für deren Inhalte. Die Rechte liegen in der Regel bei den Herausgebern beziehungsweise den externen Rechteinhabern. Das Veröffentlichen von Bildern in Print- und Online-Publikationen sowie auf Social Media-Kanälen oder Webseiten ist nur mit vorheriger Genehmigung der Rechteinhaber erlaubt. [Mehr erfahren](#)

### **Conditions d'utilisation**

L'ETH Library est le fournisseur des revues numérisées. Elle ne détient aucun droit d'auteur sur les revues et n'est pas responsable de leur contenu. En règle générale, les droits sont détenus par les éditeurs ou les détenteurs de droits externes. La reproduction d'images dans des publications imprimées ou en ligne ainsi que sur des canaux de médias sociaux ou des sites web n'est autorisée qu'avec l'accord préalable des détenteurs des droits. [En savoir plus](#)

### **Terms of use**

The ETH Library is the provider of the digitised journals. It does not own any copyrights to the journals and is not responsible for their content. The rights usually lie with the publishers or the external rights holders. Publishing images in print and online publications, as well as on social media channels or websites, is only permitted with the prior consent of the rights holders. [Find out more](#)

**Download PDF:** 05.09.2025

**ETH-Bibliothek Zürich, E-Periodica, <https://www.e-periodica.ch>**

## **Pull-Out Test of Anchor Bolts Embedded in Concrete**

Test d'arrachement du boulon d'ancrage dans le béton

Ausziehversuche von Ankerbolzen im Beton

### **Gianfranco VALENTE**

Assoc. Professor  
Univ. de l'Aquila  
L'Aquila, Italy



Gianfranco Valente, born 1943, received his civil engineering degree at the University of Rome in 1967. Assistant Research in Rome until 1986.

### **SUMMARY**

An incremental-iterative procedure for the nonlinear finite element analysis of reinforced concrete structures is adopted here. Solid concrete is modelled as a hypoelastic material, whilst cracked concrete (including fracture and aggregate interlock) is modelled with crack bands, according to the smeared crack approach. The proposed approach is applied to the analysis proposed by RILEM TC-90 FMA regarding pull-out tests of anchor bolts in concrete. Only the plane stress condition is considered in this paper.

### **RÉSUMÉ**

On adopte ici une procédure itérative et incrémentale dans le cas de l'analyse non-linéaire de structures en béton armé. Le béton sain est modélisé par un matériau hypoélastique, tandis que le béton fissuré (incluant fractures et déchaussements des agrégats) est modélisé par des «bandes de fissuration», ceci en accord avec l'aspect-même des fissures apparues. Cette approche est appliquée à l'analyse proposée par RILEM TC-90 FMA, concernant le test d'arrachement d'un boulon ancré dans le béton. On ne considère dans cet article que l'état de contrainte bidimensionnel.

### **ZUSAMMENFASSUNG**

Ein inkrementelles Iterationsverfahren wurde für eine nichtlineare Finite-Element-Analyse von Stahlbetonbauten verwendet. Beton wird als ein hypoelastischer Werkstoff dargestellt, hingegen wird gerissener Beton (Bruchmechanik und Kornverzahnung) mit Rissbändern, nach dem «smeared crack»-Verfahren dargestellt. Das vorgeschlagene Verfahren wird für die vom RILEM TC-90 FMA vorgeschlagenen Analysis, Ausziehversuchen von Betonankerschrauben angewendet. Es wird nur ein ebener Spannungszustand berücksichtigt.



## 1. INTRODUCTION

Tensile failure in concrete occurs when the tensile stress in one principal direction exceeds the tensile strength. In this case the usual assumption that a plane of failure develops at right angle to the previous principal direction is introduced. Tensile cracking is identified using failure surfaces fixed at the onset of the cracking. The smeared crack concept for fracture and aggregate interlock is adopted. As regards fracture, it is easier to use compliance rather than stiffness matrices, and it suffices to adjust some terms of the compliance matrix. Instead, for aggregate interlock it is easier to use a stiffness matrix approach and it suffices to adjust four terms of this matrix, which is nonsymmetric and does not yield coincident principal axes for stress and strain increments because shear and normal components are coupled. As a general rule, in concrete models, after tensile cracking, the normal stress is released completely (ADINA [1]) and some coefficients of the concrete stiffness matrix are reduced with two constant factors (ADINA, DIANA [1], [5]). Isoparametric elements with a maximum of eight nodes and four integration Gauss points in each direction are used. For concrete, we must take into account three different angles at each integration point for: failure plane, principal strains, principal stresses [9]. The tangent stiffness matrix is referred to:

- principal stress directions, before tensile failure;
- failure coordinate system (axes parallel and transverse to the crack planes) after tensile failure.

The tensile failure envelope given in Fig.1 is employed. It may be observed that compressive stress change this tensile strength. The nominal stress at failure decreases as the size increases. This is caused by the fact that in the presence of the softening the failure cannot be simultaneous but must occur through propagation of a failure across the structure. In a larger structure, this nonsimultaneous nature of failure is more pronounced. In pullout failure, the existence of the size effect must clearly be expected, due to the brittle nature of these failures.

## 2. RELATIONSHIP BETWEEN THE STRESSES AND THE CRACK OPENING IN FRACTURE

The simplifying assumption that the descending branch is a straight line is adopted here. The analytical curve is shown in Fig.2a). The material behaves in the nonlinear way shown in Fig 2b), where:

- $E_0$  is the Young's modulus of concrete;
- the quantity of energy absorbed per unit crack area when the crack widens from zero up to or beyond  $\delta_0$  is represented by the area lying between the curve and the  $\epsilon$  axis:

$$w \int \sigma(\epsilon) d\epsilon = G_F \quad (1)$$

Then, from Fig.2b):

$$\epsilon_1 = \sigma/E_0 + \delta/w \quad (2)$$

$$\epsilon_p = \sigma_t/E_0 \quad (3)$$

$$1/C_f + 1/E_0 = 1/E_t \quad (4)$$

$$G_F = (2.72 + 3.10 \sigma_t) \sigma_t^2 d_s/E_0 \quad (\text{from [2]}) \quad (5)$$

## 3. AGGREGATE INTERLOCK CONSTITUTIVE LAWS

The "Rough Crack Model" initially proposed by Bažant and Gambarova [3], and later improved ([4], [8], [9], [10]), is here adopted:

$$\sigma_{nm} = a_{12} r \sqrt{\delta_n} \sigma_{nt}/h \quad (6)$$

$$\sigma_{nt} = \tau_0 (1 - \sqrt{2} \delta_n / d_a) r (f/g) \quad (7)$$

where:

$$f = a_3 + a_4 |r|^3 \quad (8)$$

$$g = 1 + a_4 r^4 \quad (9)$$

$$h = (1 + r^2)^{1/4} \quad (10)$$

$$r = \delta_t / \delta_n, \quad a_{12} = -0.62, \quad a_3 = 2.45 / \tau_0, \quad a_4 = 2.44 (1 - 4 / \tau_0), \quad \tau_0 = 0.25 \tilde{\sigma}_c,$$

$d_a$  - maximum aggregate size (3.5mm in the present paper), from (5).

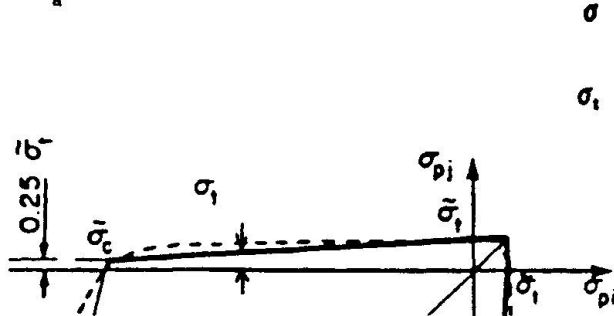


FIG.1-Plane Tensile failure Envelope of Model. (—)Code, (---)Kupfer et al.

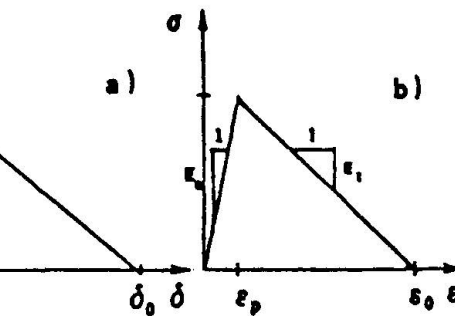


FIG.2-Stress-Strain Diagrams for Fracture.

#### 4. ANALYSIS OF ANCHOR BOLTS IN CONCRETE

The material properties are as follows:

$E_0 = 30000 \text{ N/mm}^2$  initial tangent modulus;

$\nu = 0.2$  Poisson's ratio;

$\tilde{\sigma}_t = 3.0 \text{ N/mm}^2$  uniaxial tensile strength;

$\tilde{\sigma}_c = -40.0 \text{ N/mm}^2$  uniaxial compressive strength;

$\epsilon_c = -.0022$  uniaxial crushing strain;

$\epsilon_u = -.0031$  uniaxial ultimate strain;

$\beta = 0.6$  stress ratio for failure surface input;

$\gamma = 1.0$  strain scaling factor for multiaxiality;

$k = 0.6$  control for iso/orthotropic material law;

$\alpha = .01$  control for loading/unloading criterion;

$l_{ch} = E_0 G_F / \sigma_t = 333.3 \text{ mm}$  characteristic length.

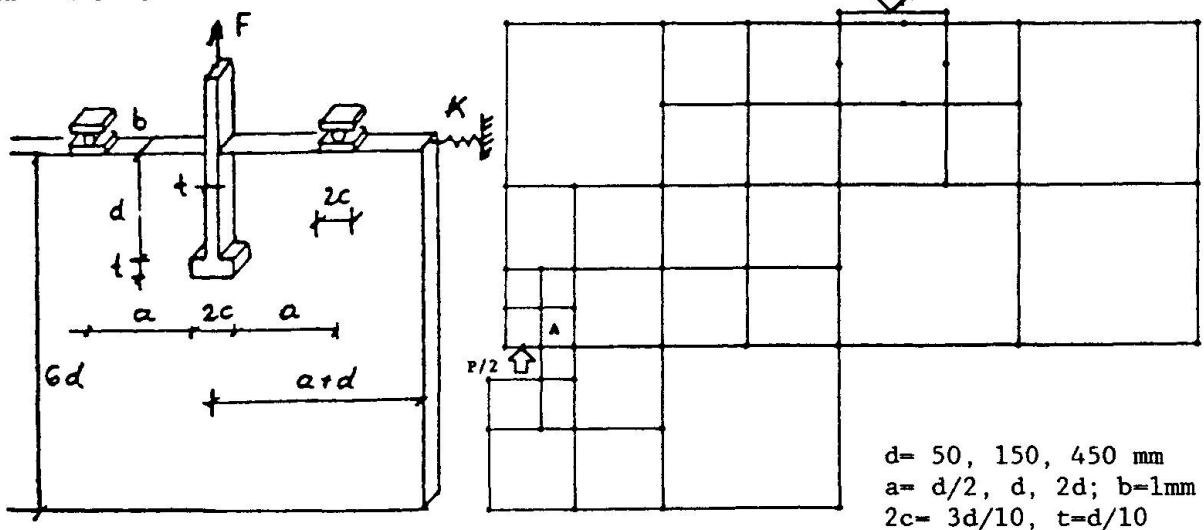


FIG.3-Model by Round Robin and F.E.Mesh.

$d = 50, 150, 450 \text{ mm}$   
 $a = d/2, d, 2d; b = 1 \text{ mm}$   
 $2c = 3d/10, t = d/10$



The finite element mesh is shown in Fig.3. The load is applied at point A. For the unit width  $b=1$  mm, the load-displacement curves of the full slab are shown in Fig.4 and have the maximum values  $P_u$ ,  $\delta_u$  as in Table I. Within the deadline of the Round Robin [11], the Author performed a case alone for  $d=150$  mm,  $a=d$ ,  $K=0$ . But the final results gathered in [11] compelled himself to extend the analysis to all the six cases for  $a=d$  in order to complete the comparisons, especially with Cervenka and Ozbolt.

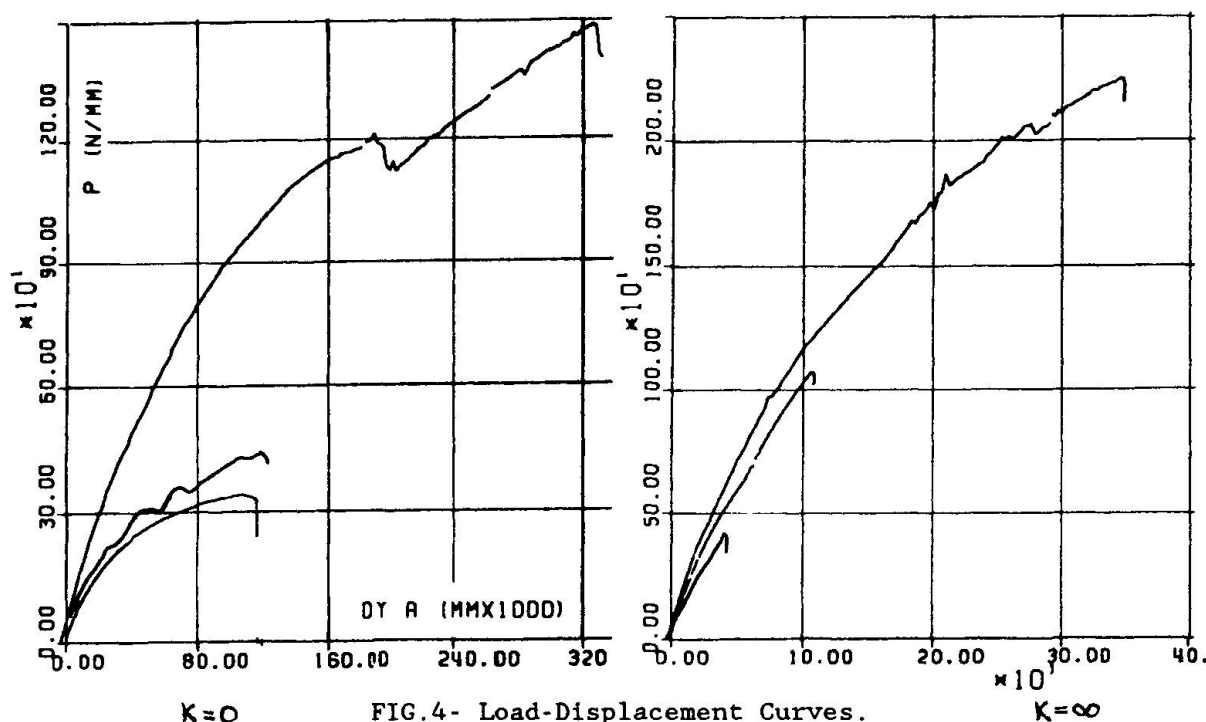


TABLE I-Maximum Loads and Displacements  $P_u/\delta_u$  [N/ $\mu$ ], ([11]).

K=	0			$\infty$		
d [mm]	50	150	450	50	150	450
Ozbolt	175/35	427/90	934/185	328/60	804/200	1790/400
Author	338/107	440/116	1469/327	414/40	1063/110	2232/346
Cervenka(f)	318/100	472/115	1090/340	690/120	1307/310	2549/600

Some of the features of the solution process are:

- both material and geometrical nonlinearities are considered;
- only prescribed displacements are used;
- the load steps are performed in such a way that the peaks of the stress-strain curves for tensile stresses are matched within +2% (Fig.2b);
- the strain-softening range runs for at least 2 steps;
- the stiffness matrix is updated at each step;
- the equilibrium iterations are performed during each step, with the following tolerance on convergence:

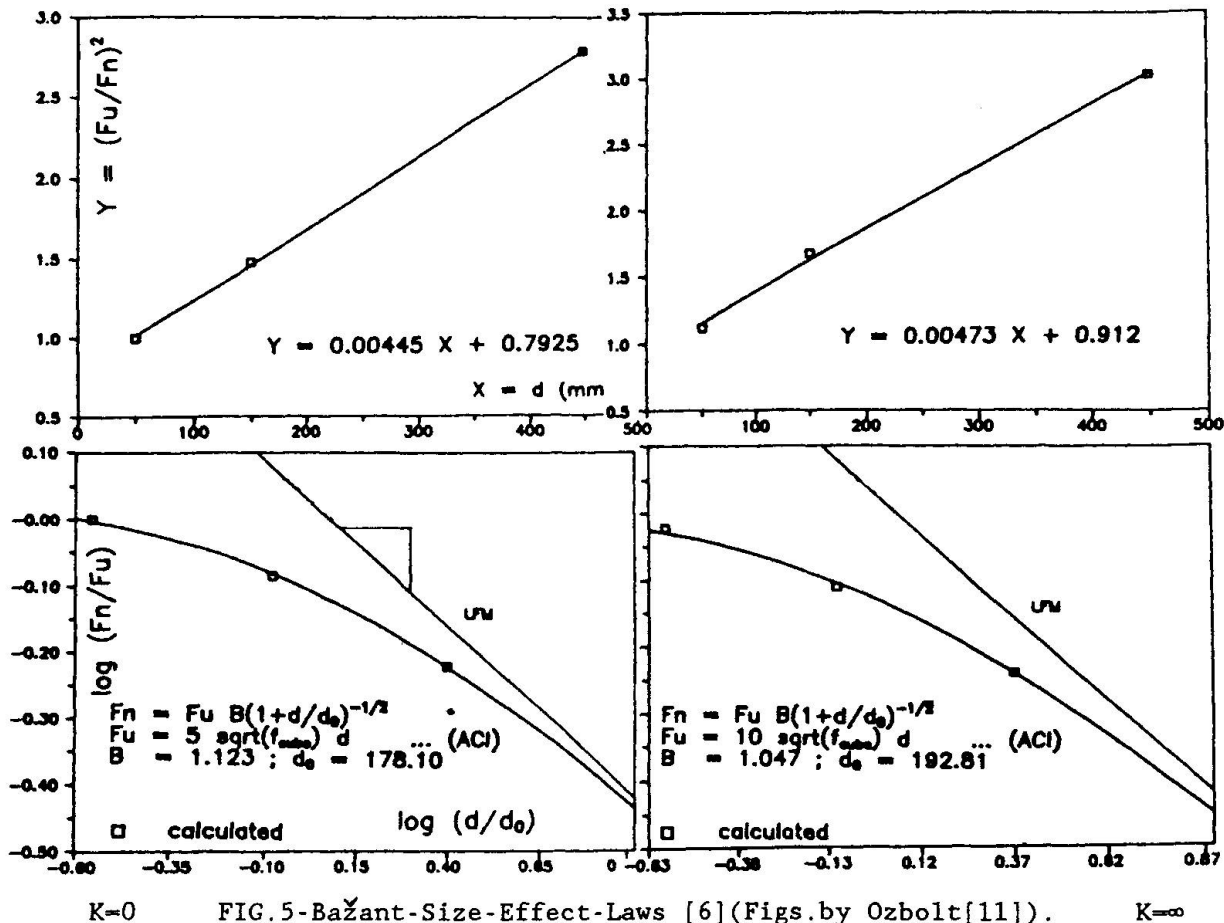
(0.5% $P_u$ ) maximum allowed unbalanced load norm,

1% for the norm of unbalanced incremental energy.

## 5. BAŽANT-SIZE-EFFECT-LAW

In concrete structures, the size effect [6] is intermediate between the linear elastic fracture mechanics, for which is much too strong, and the plastic

limit analysis, for which it is absent. These laws are represented in Fig.5. B is the negative slope of the tangent to the curve in Figs.5c,d. This slope approaches that of linear fracture mechanics for increasing d. In terms of sensitivity, it seems that in these cases the value of B is close to 0.5. One consequence of this is also that the formal stress associated with pull out of a bolt is very size-dependent.



K=0 FIG.5-Bažant-Size-Effect-Laws [6](Figs.by Ozbolt[11]).

K=infinity

## 6. CONCLUDING REMARKS

Within this research project, a pre-existing F.E.program for the nonlinear analysis of R.C.elements has been implemented with suitable models for concrete fracture and aggregate interlock.

In principle, the model for tensile fracture can be applied in analysing all the cases where we need to rely on the tensile strength of concrete. We are still far from a sufficient understanding of all types of tensile-induced structural failures. Some of them are very complicated to analyse. A major research effort is needed within this area in order to achieve better guidelines for design rules. Such a research effort may be expected to lead a more even safety factor and large savings. The anchorage of bolts is a case which can be teoretically analysed, although it is rather complicated, because not only tension, but also shear stresses act in the fracture zone, and several fracture zones may have to be taken into account. In order to compare different analytical methods,an invitation was given to a round robin of this common structural detail. Here, comparison with more test results was done. The Code proposed by Cervenka [11] (with fixed cracks) seems very close to that one of the Author. Instead, Ozbolt [11] use a refined nonlocal microplane model. Some of the features of this comparison are:

- the shapes of the load-displacement curve are very similar;



- there is a certain scatter for the values  $P_u/\delta_u$  that can be ascribed to different meshes, different constraint, different shear reduction factors.

For the Bažant-Size-Effect-Law, it may be conclude that:

1. The present analysis results confirm that a size effect is present, i.e. the nominal shear bond stress at failure decreases as the specimen increases.
2. It appears that the results are consistent with Bazant's approximate size effect law for failures due to distributed cracking, as should be theoretically expected according to the known failure mechanism.
3. The analyses indicate that larger specimens tend to fail in a more brittle mode, while smaller specimens tend to fail in a less brittle mode or more plastic shear-pullout mode. This transition in the type of failure as a function of specimen size is in agreement with the physical implications of the size effect law and supports its applicability.

#### ACKNOWLEDGEMENTS

This research is a part of a comprehensive project on the mechanical behavior of concrete elements, which was supported from the Italian Ministry of Education (M.U.R.S.T. 40% 1989).

#### REFERENCES

1. K.J. Bathe and S. Ramaswamy, On Three Dimensional Nonlinear Analysis of Concrete Structures, Nuclear Engineering and Design, Vol.52, 1979, pp.385-409.
2. Z.P. Bažant and B.H. Oh, Crack Theory for Fracture of Concrete, Vol.16, n.93, Matériaux et Constructions.
3. Z.P. Bažant and P.G. Gambarova, Rough Cracks in Reinforced Concrete, Jrn.of the Structural Division, ASCE, Vol.106, No.4, April 1980, pp.819-842.
4. S. Dei Poli, P.G. Gambarova and C. Karacaoğlu Aggregate Interlock Role in R/C Thin-Webbed Beams in Shear, Journal of Structural Engineering, ASCE, Vol.113 No.1, January 1987, pp.1-19.
5. J.G.M. Van Mier, "Examples of non-linear analysis of reinforced concrete structures with DIANA", Heron, Vol.32, 1987, no.3.
6. Z.P. Bažant, S. Şener, "Size Effect in Pull Out Test", ACI Materials Journal, Sep-Oct 1988.
7. L. Elfgren, "Fracture Mechanics of Concrete Structures. From Theory to Applications", a RILEM Report prepared by TC 90-FMA, edited by Elfgren Chapman & Hall, 1989.
8. P.G. Gambarova, G. Valente, "Fracture and Aggregate Interlock Mechanism in R/C SMIRT 10, Div.Q, August 1989, Ca, USA.
9. P.G. Gambarova, G. Valente, "Smeared Crack Analysis for Fracture and Aggregate Interlock in Concrete", Engineering Fracture Mechanics, Vol.35, No.4/5, pp. 651-663, 1990.
10. M. Di Prisco, P.G. Gambarova, G. Valente, "Evolutive vs Limit Analysis in Modelling R/C Thin Webbed Beams failing in Shear", 2nd Int. Conf. on Computer Aided Analysis and Design of Concrete Structures, Austria, April 1990.
11. L. Elfgren, "Round Robin Analysis of Anchor Bolts", RILEM TC-90 FMA, Preliminary Report, September 1990.