Bond Model for Punching Strength of Slab-Column Connections

Modèle de liaison pour résistance au poinçonnement dans les têtes de colonnes en béton armé

Physikalisches Modell für das Durchstanzen von Flachdecken

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SUMMARY

A physical model that explains the mechanism of punching failure in reinforced concrete column-flat plate connections is presented. Load is carried to the column face by arching action in the radial direction. The curvature of the arch is defined by the force gradient (bond) that can be developed in the reinforcement perpendicular to the arch. The validity of the model is demonstrated by comparing predicted capacities with results from tests of such connections.

RÉSUMÉ

Un modèle physique est présenté qui explique le mécanisme de rupture par poinçonnement dans les têtes de colonnes en béton armé, au droit de leur connexion avec une dalle. La charge est transmise au bord de la colonne de façon radiale, sous forme d'une action légèrement arquée. La courbure de cet arc est définie par le gradient de forces pouvant se développer dans l'armature qui se trouve perpendiculaire à l'arc. La validité de ce modèle est démontrée par la compression des capacités prévues de résistance, illustrée par les résultats d'essais effectués sur de tels systèmes de connexion.

ZUSAMMENFASSUNG

Es wird ein physikalisches Modell für das Durchstanzen von Flachdecken vorgestellt. Die Last wird über eine radiale Bogentragwirkung zum Stützenkopf übertragen. Die Bogenform wird durch den Abbau des Verbundes in der Ringbewehrung bestimmt. Die Gültigkeit des Modells wird durch den Vergleich mit Bruchversuchen bestätigt.

1 Introduction

The truss model[1] describes a slab-column connection as a space truss composed of steel tension ties and straight-line concrete compression struts. Although this model provides an excellent qualitative description for the behavior of slab-column connections, the location of the intersection between the effective centroid of the strut and the top mat reinforcing steel does not agree with the position of the strut as determined from measured bar force profiles at failure[2]. A model that retains the desirable characteristics of the truss model and is consistent with experimental measurements of strain is required.

A curved arch, as shown in Fig. 1, is consistent with measured bar force profiles. In plan, the arch is parallel to the reinforcement. As with the truss model, the horizontal component of the arch is equilibrated by tension in the reinforcement. By limiting the width of the arch to the width of the column support, the arch may be considered a purely radial component within the plate, thereby preserving the truss model concept of shear being carried radially by an inclined concrete compression strut. The geometry of the curved radial arch, however, cannot be determined from the bar force profile of the reinforcement tying the arch.

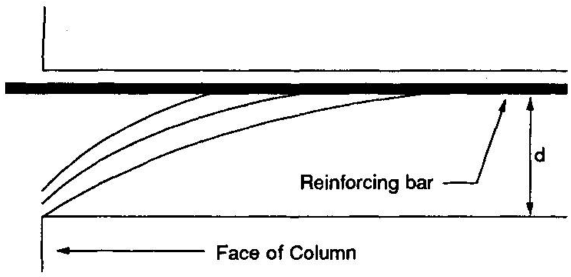

Fig. 1: Radial Arch

If the horizontal force component in the arch is assumed to be constant, then the shear carried by the arch varies from a maximum at the face of the column where the slope of the arch is large to a minimum, or perhaps zero, at the intersection of the arch and the reinforcing steel where the slope is small. The shear that was carried by the arch at the face of the column must be dissipated in a direction perpendicular to the arch at some distance away from the column. The rate at which shear can be dissipated determines the curvature of the arch.

In a reinforced concrete flexural member, moment is calculated as the product of the steel force, T, and an effective moment arm, jd. Moment gradient or shear results wherever the magnitude of the force or moment arm varies along the length of the member (x-axis).

$$V = \frac{d(Tjd)}{dx} = \frac{d(T)}{dx} jd + \frac{d(jd)}{dx} T \qquad [1]$$

Shear that is the result of a gradient in tensile force acting on a constant moment arm is carried by **beam action**. Shear resulting from a constant tensile force acting on a varying moment arm is carried by **arching action**. Whereas beam action at a particular cross-section requires bond forces at that cross-section, arching action requires only remote anchorage of the reinforcement.

Experimental observations suggest that beam action is the only possible mechanism of shear transfer in what amounts to a circumferential direction. For example, Kinnunen and Nylander[3], report that the deformed shape of test specimens under load is essentially conic, with little or no curvature in the radial direction. This requires a linear distribution of circumferential strain through the thickness of the plate, with maximum compressive strain at the slab soffit. This means that the flexural depth, jd, in the circumferential direction is relatively constant, and any shear carried in the circumferential direction must be carried by the two-way plate equivalent of beam action.

Beam action shear requires a force gradient in the reinforcement. Force gradient may be limited by either yielding of the reinforcement or by bond failure. For those connections that fail prior to widespread yielding, bond strength is the most important limitation on force gradient, hence, the term **bond model.**

2 Development of Bond Model

The orthogonally reinforced slab-column connection is modelled as a rectangular grillage, as shown in Fig. 2. Four strips, called radial strips, extend from the column parallel to the reinforcement. Any load reaching the column must pass through one of these four radial strips. The width of each strip is defined by the column width. The end of the strip farthest from the column support, called the remote end, is placed at a position of zero shear. Thus, radial strips are loaded in shear on their side faces only. The total length of a strip is designated as L. The strength of the connection is determined by assessing both the flexural strength of each radial strip and the ability of the adjacent quadrants of two-way plate to load each radial strip.

Consider the free body diagram of one-half of a radial strip shown in Fig. 3. The half-strip must support the combined effect of any load applied directly to the strip (q), including the self-weight of the strip, and the internal shears and moments developed on the side faces of the strip by the adjacent quadrants of two-way plate. The near side face of the half-strip is loaded in shear (v), torsion (m_t) and bending (m_n) by the adjacent quadrant of two-way plate. The far side face lies on an axis of symmetry for the plate. Under concentric loading, both shear and torsion on this face are zero. The bending moment applied to the far side face of the half-strip is equal and opposite to the bending moment on the near side face.

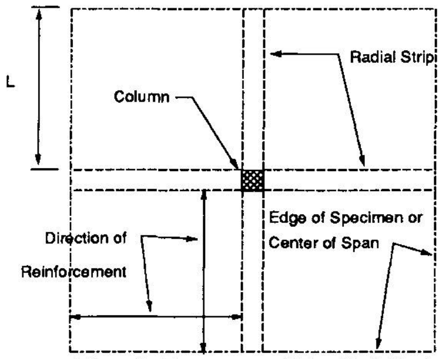

Fig. 2: Layout of Radial Strips

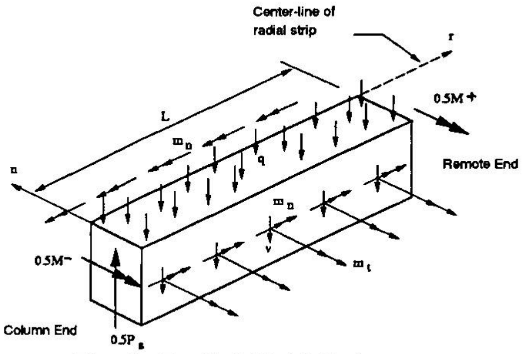

Fig. 3: Radial Half-Strip

The combination of shear and torsion on the side face of the radial half-strip is replaced by a statically equivalent line load, $\bar{v}$, acting on the strip. The term $\bar{v}$, referred to as the Kirchhoff shear, has its origins in elastic plate theory as a way of satisfying equilibrium at a free or simply supported edge of a plate.

$$\bar{v} = \frac{\partial m_n}{\partial n} + 2 \times \frac{\partial m_t}{\partial r} \qquad [2]$$

The Kirchhoff shear has two components. The first is a shear resulting from the gradient in bending moment perpendicular to the radial strip. This is referred to as **primary shear**. The second is a shear resulting from twisting moment gradient along the side face of the radial strip, called **torsional shear**.

Primary shear results from the gradient in bending moment perpendicular to the radial strip. If bending moment gradient perpendicular to the radial strip is the result of beam action alone, then:

$$\frac{\partial m_n}{\partial n} = \frac{jd}{s} \times F_b{}' = \tau \times jd \qquad [3]$$

where $F_b{}'$ is the force gradient in the reinforcing bars perpendicular to the radial strip, s is the spacing and jd is the flexural moment arm. If $F_b{}'$ is averaged over the bar spacing, the resulting term is the horizontal shear stress required for moment gradient, τ.

In a region dominated by beam action, a limiting value of force gradient in the reinforcement is the equivalent of a limiting shear stress. Any limit to the force gradient that can be sustained at the boundary between steel and concrete is a bond limitation on the quantity $\partial m_n / \partial n$. Alternatively, for very lightly reinforced plate-column connections, force gradient at the edge of the radial strip may be limited by the spread of yielding.

Torsional shear is the result of gradient in the torsional moment on the side face of the radial strip, in a direction parallel to the radial strip. The factors governing the magnitude of the torsional moment and the torsional moment gradient are not known, nor is it clear how these quantities might be measured. However, it is possible to outline some of the effects of torsional shear on the basis of equilibrium conditions.

For a concentrically loaded column, the torsional moment must approach zero at the column support as a result of symmetry. Because of the way the radial strip is defined, the magnitude of the torsional moment at the remote end must also be zero, either by symmetry or boundary condition. Since the torsional moment is zero at both ends of the radial strip, the total contribution to Kirchhoff shear made by torsional moment gradient must be zero. Hence, torsional shear affects only the distribution of Kirchhoff shear along the length of the radial strip, leaving primary shear as the root source of all shear load on the side face of a radial strip. The effective centroid of the shear load is moved closer to the column by the action of torsional shear.

Fig. 4 shows a radial half-strip of an interior slab-column connection, excluding the bending moments about the n-axis, with the torsions and shears on the side faces replaced by the Kirchhoff shear. The term $\overline{q}$ is a line load equivalent to the directly applied load, q. The radial strip supports all the loads by acting as a cantilever beam. The total flexural strength of the cantilever is the sum of M- and M+. Flexural equilibrium of the radial strip leads to Eq. 4 and vertical equilibrium produces Eq. 5. In each equation, the factor two accounts for the fact that both $\overline{v}$ and $\overline{q}$ are defined for a radial half-strip. The quantity P_s is the total load carried by one radial strip of an interior column-slab connection.

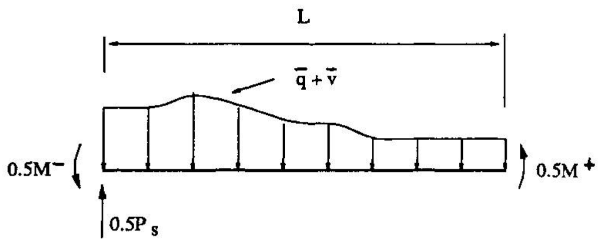

Fig. 4 Loading of Radial Half-Strip

$$M^{+}+M^{-}=M_{s}=2\times\int_{0}^{L}(\overline{v}+\overline{q})r\,dr \qquad [4]$$

$$P_{s}=2\times\int_{0}^{L}(\overline{v}+\overline{q})dr \qquad [5]$$

Expressions for the negative moment capacity, M-, at the column end of the strip and the positive moment capacity, M+, at the remote end of the strip, are given in Eq. 6.

$$M^{-}=\rho^{-}\times f_{y}jd^{2}\times c_{2} \qquad M^{+}=k_{r}\times\rho^{+}\times f_{y}jd^{2}\times c_{2} \qquad [6]$$

The factor, k_r, accounts for the proportion of bottom steel that can be developed by the rotational restraint provided at the remote end of the radial strip. It is reasonable to assume that any steel which passes through the column contributes to M_s. There is, however, a problem in dealing with reinforcing bars which are close to the column. Furthermore, for uniformly spaced reinforcement, the values of ρ- and ρ+ should not depend on whether the mat is bar centered or space centered. These problems are overcome by defining ρ- and ρ+ as:

$$\rho^{-}=\frac{A_{sT}}{b\times d} \qquad \rho^{+}=\frac{A_{sB}}{b\times d} \qquad [7]$$

A_{sT} and A_{sB} are the total cross-sectional area of top and bottom steel, respectively, passing through the column plus one half the area of the first top or bottom bar on either side of the column. The term b is the column dimension plus the distance to the first reinforcing bar on either side of the column.

To evaluate the integrals of Eqs. 4 and 5, assumptions are made regarding the distribution of line load along the length of the radial strip. These assumptions both simplify and optimize the loading of the radial strip in a manner generally consistent with a lower bound approach.

- All Kirchhoff shear is assumed to be the result of primary shear. The torsional shear contribution is considered negligible because the deformations of a plate-column connection that fails in brittle punching are not consistent with the development of large torsional moments.
- At a distance l from the column end of the radial strip, the Kirchhoff shear decreases from the maximum value permitted by primary shear ($(\partial m_n/\partial n)_{max}$) to a value of zero. The length l is referred to as the loaded length of the radial strip.
- The direct load on the radial strip is assumed small compared to $\bar{v}$ and is neglected ($\bar{q} = 0$).

Based on these assumptions, the optimized loading of a radial half-strip is shown in Fig. 5. The loading term w is defined as:

$$w = \left(\frac{\partial m_n}{\partial n}\right)_{max} = F_b{'}_{max} \times \frac{jd}{s} = \tau_{max} \times jd \qquad [8]$$

It is convenient to consider w as the simplified, optimized Kirchhoff shear load acting on one side of a radial strip by an adjacent quadrant of two-way plate. Because a radial strip of an interior column-slab connection has two adjacent quadrants of two-way plate, the total line load on a radial strip of an interior column-slab connection is $2w$. For an edge or corner column, however, there will be radial strips that are parallel to the free edge of the plate and have only one adjacent quadrant of two-way plate. For these strips, the total line load is w.

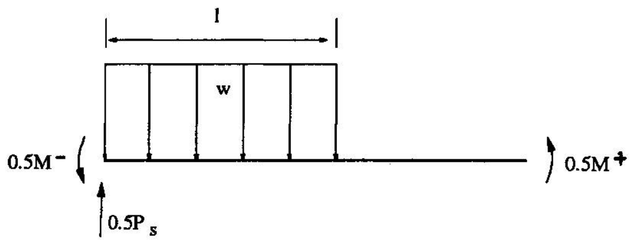

Fig. 5 Simplified Loading of Radial Half-Strip

Eq. 8 suggests that w may be based on either the maximum force gradient, $F_b{'}_{max}$, in the reinforcement perpendicular to the radial strip or a maximum critical shear stress, τ_{max}, applied on the side faces of the radial strips. $F_b{'}_{max}$ may be obtained from an estimate of the bond strength as governed by splitting failure. τ_{max} may be obtained from the critical value of shear stress as defined by building codes for one-way flexural members. Since most treatments of splitting bond failure focus on either lap splices or anchorage zones rather than locations at some distance from the ends of the bars, τ_{max} provides a more attractive basis for a predictive model for punching shear.

For simplicity, building codes usually replace $\tau_{max} \times j$ with v_c. The critical value of v_c according to the ACI Standard[4] and the corresponding value of w are:

$$v_c = 0.166 \times \sqrt{f'_c} \qquad w_{ACI} = d \times 0.166 \times \sqrt{f'_c} \qquad [9]$$

The assumptions made regarding the distribution of line load along the length of a radial strip and the method chosen for estimating w result in the simplified free body diagram shown in Fig. 5. Using this figure, Eq. 4 may be rewritten and solved for l as follows:

$$M_s = 2 \times \int_0^L (w) r\,dr = 2 \times \int_0^l (w) r\,dr = 2 \times \frac{wl^2}{2} \quad ; \quad l = \sqrt{M_s/w} \qquad [10]$$

Substituting l into Eq. 5 yields:

$$P_s = 2 \times \int_0^L (w)\,dr = 2 \times \int_0^l (w)\,dr = 2 \times wl = 2 \times \sqrt{M_s \times w} \qquad [11]$$

The punching capacity of a slab-column connection is obtained by summing the contribution of each radial strip.

3 Discussion

The bond model, using w_{ACI}, was applied to 115 tests reported in the literature. The ratios of test load to predicted load are shown in Fig. 5. The average ratio of test to predicted punching load is 1.29 with a coefficient of variation of 12.3 per cent. Using the ACI[4] and BS 8110[5] code procedures on the same body of data, the average test to predicted values are 1.59 and 1.06 and the coefficients of variation are 26.5 and 15.1 per cent, respectively.

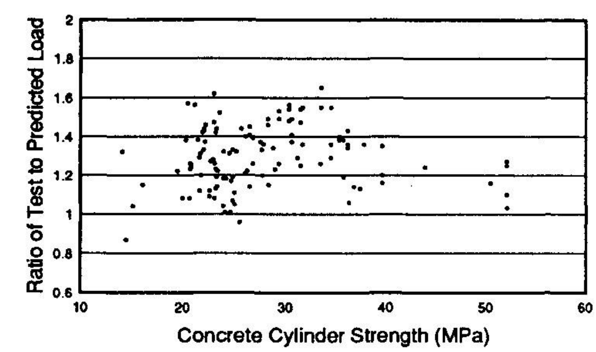

Fig. 6 Comparison of Test Results with Bond Model Predictions

The difference between the average test to predicted ratio for the ACI and BS 8110 codes can be attributed to differing design philosophies. The quality of these models should be judged on the consistency of their predictions rather than the magnitude of the average test to predicted ratio, since this can be adjusted simply by multiplying by a constant.

It is considered that the principal reason for the bond model consistently underestimating punching strengths is that the contribution of torsional shear is neglected. Despite this approximation, the bond model produces results that are more reliable than most building codes currently in use. Furthermore, the model is simple to apply and does not resort to the definition of artificial critical sections for shear.

The curved arch of the bond model is a natural progression from the straight-line compression strut of the truss model. In this way, the bond model is consistent with the truss model. The curvature of the arch requires a tension field within the concrete. The magnitude of this tension field is governed indirectly by the beam action shear perpendicular to the arch. As shown above, beam action shear as limited by bond can be represented as a critical shear stress. Thus, the bond model shows why the code approaches for estimating punching strength based on a critical shear stress acting on a critical shear section give satisfactory results.

The bond model links shear strength and bond strength at locations other than anchorage or splice locations. The importance of bond in the vicinity of the column has not been fully appreciated. The link between force gradient in the reinforcement and punching strength also explains why excessive yielding of the reinforcement in the vicinity of the column produces lower punching strengths.

4 References

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