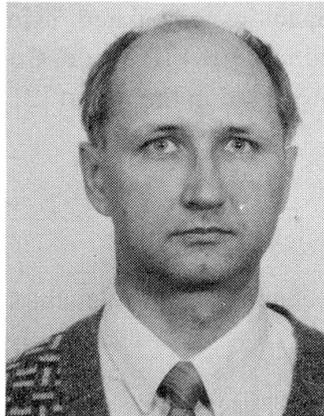# Cracking and Shear Capacity of Prestressed Hollow Core Slabs

Résistance à la fissuration et à l'effort tranchant des dalles alvéolées précontraintes

Kapazität der vorgespannten Hohlplatten

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SUMMARY

Predicted and observed cracking and shear capacities of prestressed hollow core slabs are compared. The flexural tensile strength of the concrete seems to be independent of the thickness of the slab. The shear compression failure is not needed to predict the observed shear capacities. A reliable criterion for the shear tension failure is obtained by setting the maximum principal stress in the web equal to the direct tensile strength of the concrete.

RÉSUMÉ

On compare ici les valeurs prévues et observées de la résistance à la fissuration et à l'effort tranchant de dalles alvéolées précontraintes. La résistance à la traction du béton fléchi semble être indépendante de l'épaisseur de la dalle. La connaissance de la rupture par cisaillement de la dalle comprimée n'est pas nécessaire pour prévoir la capacité de résistance à l'effort tranchant observées. Un critère fiable pour évaluer la ruine par effort tranchant de la dalle en traction peut être obtenu en introduisant dans la section une contrainte principale maximale égale à la résistance à la traction du béton.

ZUSAMMENFASSUNG

Berechnete und gemessene Rissmomente- und Schubtragfähigkeiten von vorgespannten Hohlplatten werden verglichen. Die Biegezugfestigkeit des Betons scheint unabhängig von der Dicke der Hohlplatte zu sein. Es ist nicht nötig, den Schubdruckbruch bei der Berechnung der Schubtragfähigkeit zu berücksichtigen. Ein zuverlässiges Kriterium für den Schubzugbruch ist es, die grösste Hauptspannung im Steg gleich der Zugfestigkeit des Betons zu setzen.

1. INTRODUCTION

The production method of extruded prestressed hollow core slabs does not allow the use of crosswise reinforcement. For this reason, the shear stresses and the local tensile stresses due to the transfer of the prestressing force must be carried by the concrete itself. The same is true also for the positive bending stresses due to the prestressing moment. In the following, discussion is limited to slabs that can be modelled by a simply supported beam without torsion. Possible failure or cracking at the release of the prestressing force or during transportation or installation are also excluded. The purpose is to show the correspondence between results of full scale tests and theoretical load-bearing capacities calculated using basic structural mechanics and the material properties closely related to CEB-FIP Model Code [1]. The theory and the results of the loading tests are discussed in more detail in the report [2].

2. CRITICAL LIMIT STATES

Fig. 1 shows possible failure mechanisms and relevant serviceability limit states for prestressed hollow core slabs.

a) flexural cracking

e) anchorage failure

b) flexural tension failure

f) shear tension failure

c) flexural compression failure

g) excessive crack width

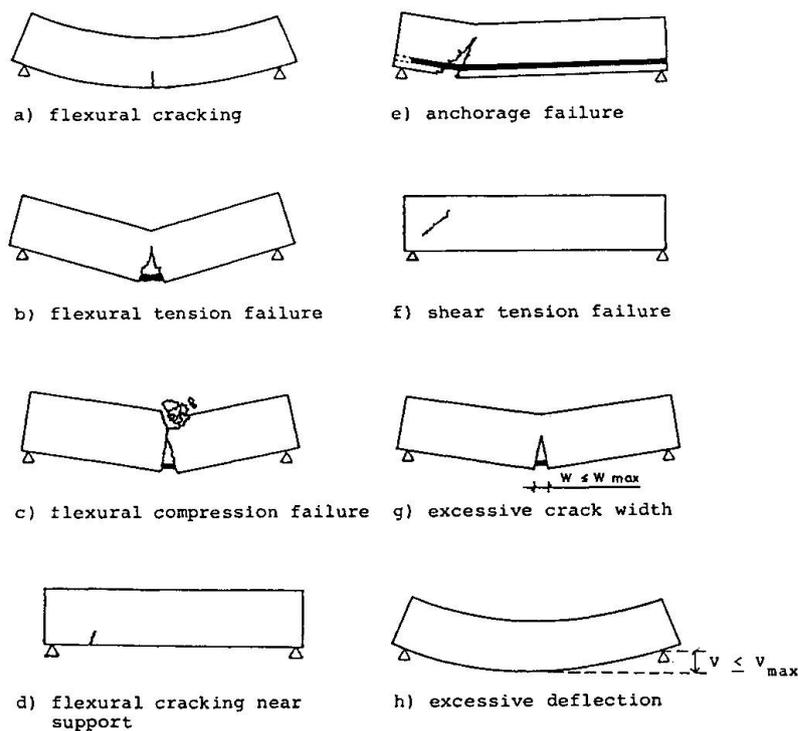d) flexural cracking near
support

h) excessive deflection

Fig. 1 Relevant limit states.

The limit states where the tensile strength of concrete is of minor importance are not discussed here. To these belong flexural tension failure, flexural compression failure, excessive crack width and excessive deflection.

2.1 Flexural cracking

The horizontal bending stress σ is obtained from

$$\sigma = \frac{-P}{A_r} + \frac{-Pe + M_g + M_q}{I_r / y} \tag{1}$$

where P is the prestressing force of strands with due regard to the losses of prestressing, A_r the reduced area of cross-section (steel reduced to concrete), y the vertical coordinate with origin at the centroid of the reduced cross-section and positive direction downwards, I_r the reduced second moment of area of the cross-section, e is y coordinate of the strands, M_g the bending moment due to the self-weight of the slab and M_p bending moment due to the imposed load. By setting σ equal to the flexural tensile strength of concrete f_{ctf} and y equal to the y coordinate of the bottom fibre of the slab, cracking moment capacity can be solved from Eq. 1.

2.2 Anchorage failure and shear compression failure

If the strands are anchored firmly enough, the presence of an inclined crack does not cause the structure to fail. Instead, the crack width increases with increasing load until the strands yield or break (flexural tension failure, Fig. 1b), slip (anchorage failure, Fig. 1e) or the concrete is crushed at the top of the slab (shear compression failure). For concrete to be crushed at the top surface, the crack width and strand slip at the bottom surface must be so great that a test which seems to end in compression failure, might equally well be regarded as having ended in anchorage failure. It was assumed that shear compression failure need not be examined separately. The anchorage capacity was checked according to the free body diagram shown in Fig. 2. Only cross-sections cracked in flexure were considered.
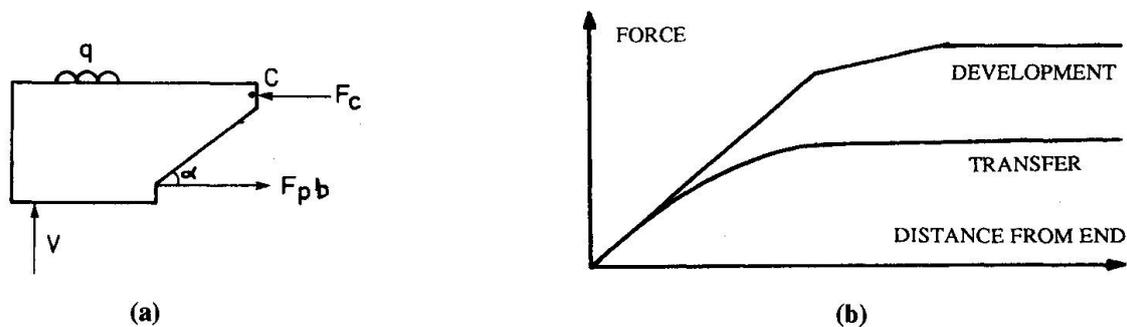

Fig. 2 (a) Free body diagram for anchorage failure.

(b) Development of anchorage force and transfer of prestress.

2.3 Shear tension failure

Shear tension failure refers to cracking that starts in the web and leads to a brittle failure (Fig. 1f). If diagonal cracking takes place near the support, the development length of the

strands is too short to prevent collapse. The simplest way of assessing such a failure is to compare the greatest principal stress in the web with the tensile strength of concrete. Finite element calculations have shown that in hollow core slabs with circular voids the most critical cross-section for shear tension failure is about at distance H/2 from the support where H is the thickness of the slab [3]. The most critical point at this cross-section is close to the point where the cross-section is narrowest. As a first approximation, it was supposed that this point is critical for all slabs irrespective of the shape of the hollow cores. The greatest principal stress σ_{p1} was calculated by combining the horizontal stress σ obtained from Eq. 1 with the shear stress τ. τ was obtained from $\tau = VS/(I_r b)$ where V is the shear force, S the first moment of area and b the width of the concrete cross-section, width of the voids excluded. I_r is the same as in Eq. 1.

3. MATERIAL PROPERTIES USED WHEN SIMULATING LOADING TESTS

The modulus of elasticity E_c and the characteristic tensile strength of concrete f_{ctk} were obtained from

$$E_c = 4800 \, f_{ck}^{1/2}$$

(2)

$$f_{ctk} = 0.20 \, f_{ck}^{2/3}$$

(3)

where f_{ck} is the characteristic cube strength of concrete (150 mm cubes). The strength of steel used in calculations was 1600 MPa and the modulus of elasticity 195 000 MPa. The parabolic curve used for the transfer of the prestressing force and the trilinear curve used for the development of anchorage force are shown schematically in Fig. 2b.

4. LOADING TESTS

Finnish quality control tests and shear tests carried out by Jonsson [2,3], cf. Fig. 3., were used to verify the proposed calculation method. In both test series, both the geometry of the slab and the strength of the concrete were measured. These were used in the calculations. The prestress was not measured but taken from the documents of the slab producers.
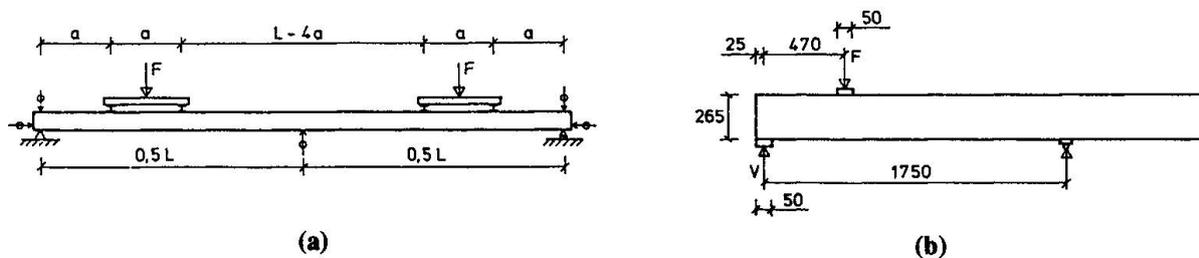

(a)   (b)

Fig. 3 Test layout. (a) quality control tests, (b) Jonsson's tests.

4.1 Cracking capacity

The ratio $\alpha_{cr} = M_{pre}/M_{obs}$, i.e. ratio predicted/observed cracking moment was calculated for each slab by using two different values for the flexural tensile strength f_{ctf}. The results are shown in Table 1. When f_{ctf} was chosen according to Mayer [4], the share of slabs for which α_{cr} was greater than 1.0 was far too big. Furthermore, this share increased with increasing flexural tensile strength. When f_{ctf} was chosen equal to $1.1*f_{ctk}$, the probability for $M_{pre} > M_{obs}$ was of the order 20% for all slab thicknesses. This is still greater that one should expect if $1.1*f_{ctk}$ is regarded as the 5% fractile of the flexural tensile strength. Obviously f_{ctf} should still have been reduced.

Thickness of slab mm	Number of slabs	f_{ctf}/f_{ct} (Mayer)	Share of slabs with $M_{pre} > M_{obs}$	f_{ctf}/f_{ct}	m	s	Probability for $\alpha_{cr} > 1$ %
150	46	1.5	43	1.1	0.875	0.130	17
200	92	1.4	35	1.1	0.893	0.119	18
265	90	1.3	31	1.1	0.905	0.125	22
400	25	1.2	28	1.1	0.947	0.057	18

Table 1   Comparison of predicted and observed cracking capacities for two choices of f_{ctf}. m is the mean and s the standard deviation for the ratio $\alpha_{cr} = M_{pre}/M_{obs}$.

4.2 Shear capacity

The ratio $\alpha_v = V_{pre}/V_{obs}$, i.e. ratio of predicted/observed shear capacity, was calculated for each slab. Here are discussed only slabs that were subject to an experimental or predicted shear failure. When compared to the observed capacities, the predicted capacities were acceptable for others but for the 400 mm slabs far too high. In fact, the assumption of the most critical point

Thickness of slab mm	Number of slabs	f_{ctf}/f_{ct}	m	s	Probability for $\alpha_v > 1$ %
150(F)	3	1.1	0.898	0.045	1.2
200(F)	24	1.1	0.881	0.140	19.0
265(F)	25	1.1	0.817	0.118	6.2
265(J)	42	1.1	0.767	0.130	3.6
400(F)	27	1.1	0.848	0.112	8.6

Table 2   Comparison of predicted and observed shear capacities for Finnish (F) and Jonsson's tests. m is the mean and s the standard deviation for the ratio $\alpha_v = V_{pre}/V_{obs}$.

stated in paragraph 2.3 is not correct. Finite element calculations show that the assumed stress distribution for the end of the 400 mm slabs with non-circular voids is not correct. The greatest principal stress may be even 40 - 50% higher than that obtained according to 2.3, and it is found lower and closer to the support. Based on this, the greatest principal stress calculated according to 2.3 for 400 mm slabs was multiplied by 1.4. The great value of p for the 200 mm slabs is mainly due to one questionable test result. The results are shown in Table 2.

5. CONCLUSIONS

In prestressed hollow core slabs, the flexural tensile strength of the concrete seems to be independent of the thickness of the slab. This is not in accordance with Hillerborg [5] and König & Duda [6]. The reason for the discrepancy may be due to the large hollow cores, which do not allow exploitation of the strain-softening behaviour of the concrete. It is recommended that the same value is used both for the flexural tensile strength and for the direct tensile strength.

The shear capacity can be predicted accurately enough using the characteristic tensile strength of the concrete and realistic methods to calculate the tensile stresses. A reliable criterion for the shear tension failure is obtained by setting the greatest principal stress in the web equal to the direct tensile strength of the concrete. For slabs with non-circular cross-section, the greatest principal stress in the web cannot be predicted adequately by using the elementary beam theory. The shear compression failure need not be considered.

REFERENCES

[1] CEB-FIP Model Code for concrete structures. Bulletin d'information CEB No. 124-125/ 1978.

[2] PAJARI, M., Design of prestressed hollow core slabs. Technical Research Centre of Finland, Research Reports 657 / 1978. 88 pp. + App.

[3] WALRAVEN, J. C. & MERCX, W.P.M., The bearing capacity of prestressed hollow core slabs. Heron 28(1983) 3, 46 pp.

[4] MAYER, H., Die Berechnung der Durchbiegung von Stahlbetonbauteilen. Deutscher Ausschuss für Stahlbeton, Heft 194 / 1967. 73 pp.

[5] HILLERBORG, A., Reliance upon concrete tensile strength. IABSE Colloqium "Structural Concrete, Stuttgart 1991.

[6] KÖNIG, G. & DUDA, H., Basic concept for using concrete tensile strength. IABSE Colloqium "Structural Concrete", Stuttgart 1991.