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Performance Requirements for Structural Concrete

Exigences concernant les performances du béton

Verhaltensanforderungen am Konstruktionsbeton

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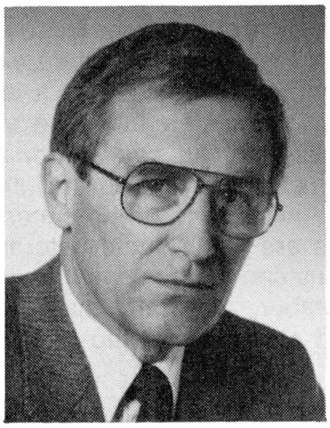

Manfred Wicke, born 1933, graduated in Civil Engineering and took his Dr. techn. degree at Vienna Technical University. For 12 years he firstly worked in, and later on headed, the design office of a firm. He was mainly involved in design work for buildings, bridges and power plants.Since 1971 he is full professor for concrete structures at Innsbruck University. Since 1977 he has inspected more than one hundred bridges.

SUMMARY

The performance requirements are recognized as boundary conditions applied to structural concrete by the associated systems. It is unimportant whether structural concrete is assigned to a system deliberately or unintentionally. Performance requirements are assigned to different areas. Using those applied to the relationships under service load conditions, measures which can be applied to fulfil the performance requirements will be explained. Here, it is shown that the construction material, structural concrete, can react very flexibly to a wide range of requirements.

RÉSUMÉ

Ce type d'exigences est reconnu en tant que condition marginale d'une structure en béton considérée comme un «système supérieur». Il n'est pas important de savoir si le béton est destiné à constituer un système supérieur par une manœuvre intentionelle ou aléatoire; en effet, la notion d'exigence sur la performance s'applique à divers domaines: dans ce cas, on s'efforcera d'aborder celle traitant des conditions régnant sous charges de service afin d'expliquer comment satisfaire pleinement ce type d'exigences. Ici, on montre particulièrement que le béton est un matériau de construction répondant de façon très flexible à une large catégorie d'exigences.

ZUSAMMENFASSUNG

Die Verhaltensanforderungen werden als die aus den übergeordneten Systemen auf den Konstruktionsbeton wirkenden Randbedingungen erkannt. Dabei ist es unerheblich, ob der Konstruktionsbeton planmässig oder ungewollt einem Obersystem zugeordnet ist. Die Verhaltensanforderungen werden verschiedenen Bereichen zugeordnet. Am Beispiel jener für die Verhältnisse unter Gebrauchslast werden die Massnahmen erläutert, die zur Erfüllung der Verhaltensanforderungen ergriffen werden können. Dabei zeigt sich, dass der Konstruktionsbeton sehr flexibel auf die unterschiedlichsten Anforderungen reagieren kann.

1. GENERAL REMARKS

The "performance requirements" establish conditions for structural concrete. These are, however, not the only terms which are made and which must be fulfilled. To serve the better understanding of their significance, one can apply the four different causality terms already formulated by Aristotle: the *causa finalis* (the final cause), the *causa formalis* (the formal cause), the *causa efficiens* (the effective cause) and the *causa materialis* (the material cause).

1.1 Causes and their Effects

Using the example of the construction of a house, these different causes become much clearer. The *causa finalis* is the intention of the builder to create a new residence for himself and his family. The reason for the house determines its function as a one-family dwelling. The house is constructed according to building plans. These determine the shape of the house and the application of structural materials, thus representing the *causa formalis* of the house. Financing is necessary for the realization of this construction project, so that the relevant activities, such as the engagement of an architect and the commissioning of the general contractor and additional specialists can take place. The financial means are the ultimate determinants of whether the construction project will be carried out or not, and thus can be labeled its *causa efficiens*. The completed house is made up of bricks, concrete, steel, timber, glass and many other materials which go into its construction. These materials, arranged according to the building plans, represent the structure's *causa materialis* (Fig. 1).

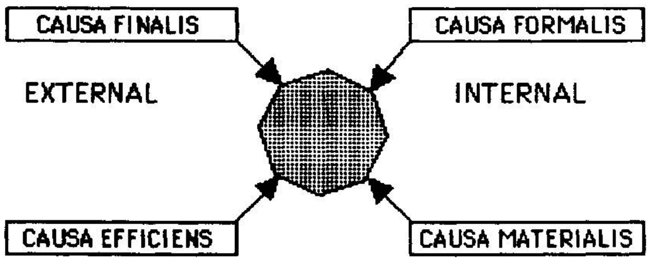

Fig. 1

1.2 System Structures

It is also possible to gain a perspective by thinking according to hierarchically arranged systems. The house is also part of a superordinate system, a city or village, along with other houses. It can simultaneously belong to additional superordinate systems, for example the environment or energy consumption systems. The system under observation, the house, is also made up of subordinate systems, the rooms. Just as it is possible to locate the house in diverse superordinate systems, it is conceivable that numerous subordinate systems can be applied as well. Another subdivision, for example, could be made up of the subsystems: brick, finishing work and housing technology (Fig. 2).

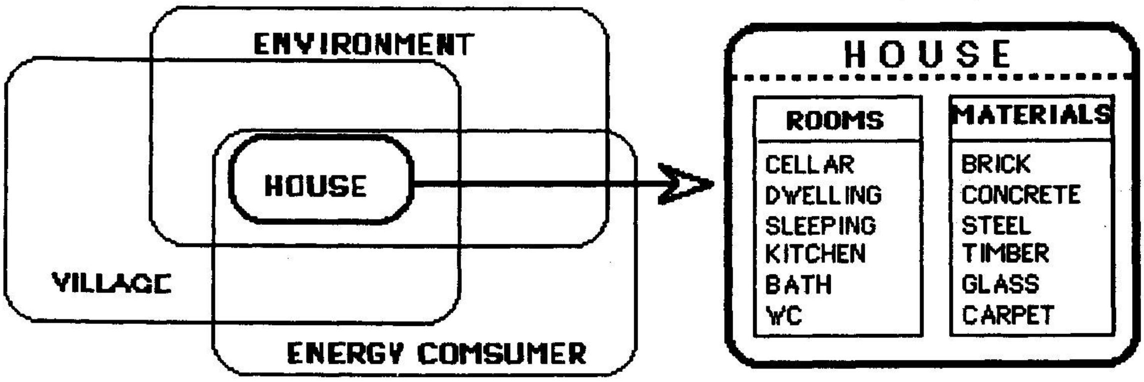

Fig. 2

Whether the system under observation is a sub- or superordinate system depends solely on the momentary perspective. Every system serves as a superordinate system for its subsystems, and simultaneously, a subordinate system for its supersystems.

1.3 Effects in a System Structure

One can also create a synopsis of both considerations discussed in 1.1 and 1.2 and combine them with one another. Here, the *causa finalis* and the *causa formalis* can be viewed as the influences of the supersystems on the subsystems, while the *causa efficiens* and the *causa materialis* function in the opposite manner, i.e. the subsystems affect the supersystems. Aristotle already deemed the *causa finalis* and the *causa efficiens* external causes, as opposed to the internal causes, *causa formalis* and *causa materialis.* Within a system structure comprised of super- and subsystems, it is therefore the internal causes which are particularly influential and clearly recognizable. In the example discussed above, the construction of a house, the building plans of the house establish the arrangement of the rooms, as well as the arrangement of the building materials, and thus act as the *causa formalis* with regard to the supersystem "house", on the subsystems of the individual rooms and construction materials. The latter, themselves to be viewed as subsystems, determine the supersystem, house, and thus represent the *causa materialis* of the house. The same holds true for the subsystems of the individual rooms. The dimensions and arrangement of the rooms, in turn, are determined by the superordinate system, "house".

The influences imposed by external causes cannot be followed to such an extent as those from internal causes. The decision to build a house to improve the living conditions of the family is ultimately the initiating factor - and thus the *causa finalis* - at the base of all further activities which are necessary for the realization of this decision. On the other hand, the decision has an external influence on the "room-house-city" system structure and is not founded in this itself. The same could be stated for the *causa efficiens*, which we can trace back to financial means in our economy-oriented world.

1.4 Evolution

The term "evolution" was originally introduced by Charles Darwin in the field of biology. According to its classic definition, which is adequate for our further considerations, evolution is the interplay of mutation and selection in the origin of species. This concept could also be applied with success in other fields, so that today, one recognizes chemical and physical evolution, social and cultural evolution, in addition to the original biological type of evolution, to only mention the most significant types. The field of engineering, and with it, structural engineering, are generally considered as belonging to cultural evolution. Evolution as a general process can be observed in any system built up in a hierarchical fashion, in that the diverse subsystems on one level compete with one another. In this case, the system develops its own dynamics, which we designate "evolution". Among competing systems, those which fulfill the demands of the superordinate system best will survive. These demands are, however, the *causa formalis*, which thus becomes the selective factor for competing subsystems.

2. STRUCTURAL CONCRETE

The following is an attempt to describe how the system "structural concrete" is integrated into the appropriate supersystem, as well as what subsystems are involved. Moreover, competitive construction materials will also be taken into consideration in this discussion.

2.1 Supersystems and their Requirements

Structural concrete is used for the construction of structures, which in turn comprise construction works. The individual construction works do not stand alone; they, too, are components of super-ordinate systems. In our example, the house was a component of the systems "city" and "environment." Similarly, a bridge is a component of the superordinate systems "transport route" and "environment", a pressure pipeline, a component of the systems "power plant" and "environment", and finally, a television tower, a component of the systems "environment", "city", and "information transmission". Innumerable similar examples could be found, but these should be sufficient in providing a good idea of what is meant here. The supersystem directly above structural concrete is always the structures itself. This is a component of a construction work and assumes a load-carrying function. The structure itself belongs to several superordinate systems, of which one is always the environment.

We have recognized the influences of closely interrelated supersystems as the *causa formalis.* The supersystem directly above structural concrete, the structure, has the greatest influence. The requirements which arise from the loads-carrying function are basically similar for various types of structures. We can therefore expect to find very similar requirements, ones which bear great significance. Such requirements would be high resistance to fracture, ductility and economy. In fulfilling these requirements, however, structural concrete must compete with other construction materials such as steel, timber, masonry, composite structures, etc., whereby the requirements mentioned above are also criteria for the selection of these materials.

The supersystems of the structure, the construction work, the environment, etc., may also make additional demands on the structure and could make themselves known as far down as the construction material level. An example here would be a water tank, which requires not only that the walls and floor slab support the load, but also that these be waterproof. This also means that additional require-ments must be fulfilled by the structural concrete, such as "uncracked state" or "limitation of crack width". In general, it can be stated that the requirements established as a result of the supersystems can be extremely varied. Nevertheless, it is possible that the same requirements be imposed on different structures when the superordinate system is the same, as is commonly the case with the super-system, "environment".

The influences imposed on structural concrete by the supersystems described above, which we have recognized as the *causa formalis*, represent the performance requirements for structural concrete mentioned in the title. In addition, these provide the selection criteria for the construction materials. The performance requirements, therefore, also play a significant role in the evolution of construction materials, and thus in the types of construction.

2.2 The Subsystems

The simplest way to determine which subsystems are involved is by asking which components comprise a system. When this question is posed about structural concrete, the answer would be: concrete, reinforcing steel, bond behavior, tendons and grout (Fig. 3).

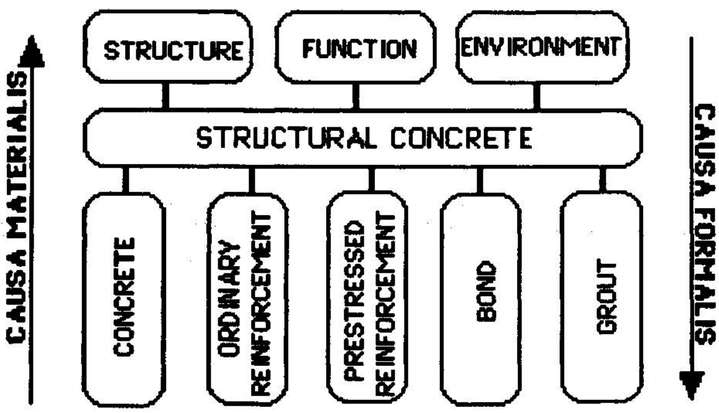

Fig. 3

It is not necessary that all components be represented, however, since we also want to include reinforced concrete and plain concrete, as well as unbonded tendons, in the term structural concrete. By combining the mentioned components appropriately, it is possible to make a very wide range of construction materials, for example: plain concrete, lightly reinforced concrete, reinforced concrete and prestressed concrete with partial, limited or full prestressing and with unbonded tendons, or with pre- or posttensioned tendons (Fig. 4).

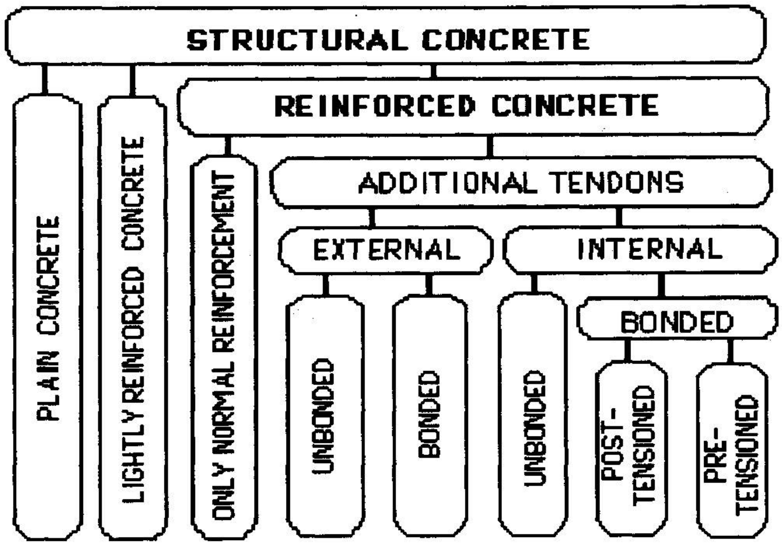

Fig. 4

By applying the criteria for selection, it is possible to choose the most applicable construction material for a specified task. As mentioned above, the selection cannot be made only in the area of structural concrete, but must also include other construction materials such as steel, timber, masonry, etc.

The range of choices expands even further when one regards each of the five components mentioned above as a system as well, and divides each into its subsystems. For example, concrete is comprised of aggregate, cement, admixtures, and water. There is a wide range of each of the first three components available. Furthermore, the relationships with which these components can be combined can influence the properties of the concrete. Thus, the construction material structural concrete offers a broad range of mutants. By applying the selection requirements (performance requirements), it is possible to choose the ideal type and combination needed for an actual task. To enable determining the fittest material from the numerous mutants, sometimes with fluid boundaries between, an exact formulation of the relevant performance requirements is necessary.

3. PERFORMANCE REQUIREMENTS

In Section 2.1, we presented a detailed discussion explaining that performance requirements are the conditions imposed on structural concrete by its supersystems. A supersystem is any system to which structural concrete would belong, regardless of whether membership to the superordinate system is planned or whether it happens as necessity. For example, a bridge pier belongs to the planned supersystems "bridge" and "roadway", while at the same time, is an element of the "river" and "environment" systems. It is essential to remember that performance requirements are determined by **all** supersystems.

As shown at the beginning, the intention to build a structure comes ultimately from the builder, who expects the structure to satisfy a particular need. Therefore, the builder is primarily interested in the function of the structure. Since he usually is not an expert, he is only able to present his ideas in a very general fashion, i.e. with such formulations as:

> The structure should enable the intended use, whereby the construction costs as well as maintenance expenses for the required life of the structure should be reasonable.

Deriving concrete information about the demands on the structure's design and construction materials requires the assistance of an engineer. This remains true in for the compilation of generally applicable rules, such as those found in technical handbooks, as well as determining the detailed requirements for an actual construction project. Since structural concrete is always used in structures, many of the demands placed on it can be directly derived from the requirements applicable to the entire structure itself. In addition, other superordinate systems impose further requirements. Because of the dominant position of the supporting function, the danger exists that these other requirements might be overseen, a reason to pay them increased attention.

3.1 Requirements made by the Supporting Function

It is advantageous to separate the requirements made by the supporting function into those for the ultimate limit state and those for the serviceability limit state. When applying the ultimate limit state, it is helpful to consider an extremely improbable but nevertheless possible condition. With the serviceability limit state, on the other hand, the daily situations which can occur in the supporting structure are considered. With this information, it is possible to determine the performance requirements:

For the ultimate limit state: sufficient strength, ductility, clear fracture warnings, ductile fracture, possibility of redristribution, etc., and

for the serviceability limit state: sufficient rigidity, controlled crack formation, controlled long-term deformation, etc.

Each of these verbally designated requirements must first be quantitatively determined to justify its functioning as a selection criterion. In the very recent past, much research has been conducted regarding this matter, though a much more in-depth consideration of this question would be desirable. The verbal requirements are as old as the reinforced concrete itself: to provide an example, however, it has only been with the quantification of ductility requirements and their relationship to the rotation capacity on the narrow sector of bending moments, which has first been dealt with very recently in connection with discussions about the MC 90, that progress has been made.

3.2 Additional Requirements made by Function

The planned function could place further requirements on the behavior under service load. In particular, liquid or gas impermeability could be named, when structural concrete must provide a sealing function in addition to its supporting function. Using the example of water impermeability, it becomes especially clear that the performance requirements need to be precisely defined to ensure that the desired behavior is obtained. Financial requirements differ significantly, depending on whether the requirement states that the structure must be "completely dry" or "mostly dry" or whether occasional damp spots will not cause complaint.

3.3 Durability

Here, the basic requirement made is that within the estimated life time, no destructive events occur which could interfere with the planned use of the structure. For example, concrete cover dimensions can be derived for the anti-corrosives needed for the reinforcement under normal environmental conditions. When harmful chemical substances such as the chloride ions of road salt are involved, it has not yet been possible to provide a reliable and final statement; the allowable crack width is still under discussion.

The requirements with regard to other harmful chemical substances also require additional quantitative clarification and which go beyond today's very general considerations. This also applies to planned attacks by harmful substances, such as those in the chemical industry, as well as to unintentional attacks by the ever increasing amounts of harmful agents in air, water and soil. In the field of storage and treatment of wastes, including special wastes, better data are urgently required.

3.4 Environmental Tolerability

This carries over o the field of active environmental protection. Structural concrete must meet the requirements of not introducing any harmful substances into the environment. This must hold true for the structure's planned functions as well as under exceptional conditions, such as fire. Even though structural concrete possesses comparably significant advantages, involvement with this aspect must be increased, particularly in view of the sensitivity to environmental awareness on the part of the public.

One could also include the aesthetic suitability of the structure into its surroundings as a part of environmental tolerability. This aspect, however, is more applicable to the structure and construction work, and less to the construction material structural concrete. It must not be overlooked that at least in German-speaking countries, severe animosities are expressed with regard to concrete and cement. There is therefore a tremendous need for informational and educational efforts.

3.5 Economy

It is also important that the economic side of these requirements not be ignored, since it provides the regulatory factor which helps bring requirements which have gotten out of hand, back down to reality. Economic requirements can lead to new production procedures, combinations and the use of new components in the construction material, structural concrete.

4. MEASURES TO MEET THE PERFORMANCE REQUIREMENTS OF SERVICEABILITY LIMIT STATE

The requirements imposed by structural safety are for the most part independent of the construction material used. The requirements imposed by environmental acceptance, durability and economy provide very diverse measures for the construction materials structural concrete, structural steel, timber and masonry. The same holds true for requirements arising out of utility, with the addendum that a structure's function can produce additional differentiation of requirements.

The utilization requirements are decisive in establishing which measures are necessary to ensure serviceability. They play a role in determining how structural concrete will be applied - as plain or lightly reinforced concrete, or as reinforced concrete with or without prestressing. In the latter case, a decision must be reached as to whether the tendons are to be bonded or not, and whether they are better placed inside or outside the cross-section. There are numerous and diverse requirements imposed by utility, which certainly cannot be listed without overlooking one or the other. An attempt is therefore made to compare the advantages and disadvantages of the different types of structural concrete with regard to the essential and repetative performance requirements.

4.1 Deformations

The determining deformations could be deflection, inclination a bearings and in some cases, curvatures, which are associated with one another through the differential equation of the bending line.

In plain and lightly reinforced concrete, deformations do not generally play a decisive role since dimensions are usually very massive, which means that the rigidity is very high.

In reinforced concrete, bends are essentially influenced by the slenderness ratio (l/h) or by the prestressing tendons. Other measures, such as increasing the ordinary reinforcement or raising the strength class of the concrete, have theoretical influence, which in fact is subordinate and usually covered by the diversity of the construction material characteristics. A reduction of the slenderness ratio by enlarging the depth of a member will in part be compensated by the associated higher selfweight. On the other hand, even a small percentage of prestressed reinforcement produces a significant reduction in deflection. Furthermore, the confidence value of the predicting calculation is increased, since the affect of the dispersion of the tensile strengths of the concrete is reduced.

The cracking situation of the structural concrete has a significant influence on the extent of deformation. Deformation in cracked state can be a multiple of that in uncracked state. The ratio between the deformations in these two states are reduced, however, under long-term loading, since the influence of creep is less in a cracked section that in an uncracked one. In the performance requirements for structural concrete, it must be determined whether these are to apply to short-term or long-term loads. It could also be reasonable to limit the increase in deflection from a specified point in time on. As an example, the long-term deflection from the time of construction is decisive in the occurrence of cracks in partitions. The question of relevant load combinations is also to be considered in individual cases and used as a basis for the performance requirements.

4.2 Crack Control

Crack control means the avoidance of cracks as well as the limitation of crack widths. The original demand that prestressed concrete be crack-free can no longer be maintained in this form. Therefore, controlling crack widths plays a significant role in all types of structural concrete. Here, emphasis is placed on ordinary reinforcement, which should, for this purpose, possess high bond quality. Prestressed reinforcement only has a subordinate significance; it can contribute absolutely nothing when unbonded tendons are used, while its influence is minor with pretensioning or post-tensioning, since its bond strength is low in comparison to that of normal reinforcement.

The measures to be applied are also influenced by the purpose of crack control. Crack control can be applied for reasons of appearance, corrosion protection or tightness with regard to gas or liquid permeability. For appearance and corrosion control, only crack width at the surface are of importance. When discussing impermeability, a distinction must be made between cracks caused by bending and those from separation. Flexion cracks which do not extend over the entire section are tight, as long as the ccompression zone has an adequate thickness of about 20 cm. When dealing with separation cracks, it is also necessary to determine tolerated leakage rates. This is best accomplished by defining such requirements as *completely dry* or *damp spots* or *small puddles tolerated* . Fulfilling such requirements is, on the one hand, dependent on such nominal dimensions as water pressure, wall thickness and crack width, and on the other, by the evaporation rate on the exposed side. To fulfill highest demands, the structure is to be forseen with joints or prestressing tendons, whereby limitation of crack width may also be taken into account for tolerated water penetration.

Separation cracks are often caused by imposed or hindered deformations which are induced by cooling of hydration heat alone, or in conjunction with subsequent shrinkage. Tension forces, however, only occur when shortening caused by the affects mentioned above are partially or totally hindered. For example, in a slab located on the foundation, tension forces can only occur to such an extent as can be introduced by friction along the base. This condition makes it possible to derive the necessary joint distances required to avoid separation cracks. Separation cracks can also occur as flexion cracks when the bending moments change their signs, which means that cracks originating at both surfaces grow together. This occurs, for example, on the inner walls of silos placed in groups. Should separation cracks be hindered by pretensioning, care must be taken to determine the tensile stresses resulting from all influences, regardless of whether these are induced by loads or imposed or hindered deformations.

4.3 Vibrations

Vibrations can have numerous detrimental effects on the serviceability of structures. Their causes are in a close relationship with the function of the structure, and can be induced, for example, by the following variable actions:

- Jumping and dancing
- Machines
- Waves due to wind and water
- Rail or road traffic
- Construction work such as the driving in of piles
- Blasting work

The vibrational behavior or structures can be influenced by the following measures:

- Changing the dynamic actions
- Changing the natural frequencies by changing the structure's stiffness or the vibrating mass
- Increasing the damping

A comparison between the frequency of the action (excitation frequency) and the structure's natural frequencies can be used to assess the situation. It should be taken into consideration that when periodic vibrations are involved, a substantial amount of dynamic stress can be induced when a natural freqency of the structure (the fundamental or a higher frequency), is an integral multiple or an integral fraction of the freqency of the action.

5. SUMMARIZING COMMENTARY

The construction material, structural concrete, covers a broad range - from plain and lightly reinforced concrete to reinforced concrete with and without tendons. The great wealth of variations allows this construction material to be adapted to the relevant performance requirements as no other. To be able to do fulfill these possibilities, it is necessary that the performance requirements be exactly defined. It is hoped that the new attitude toward structural concrete will accelerate the evolution of this construction material.

Cracking and Deformation in Structural Concrete

Risse und Verformungen in Konstruktionsbeton

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SUMMARY

A modern conception of a consistent theory of serviceability limit state of structural concrete is reported. The common mechanical basis for cracking and deformation is the bond-stress-slip relationship and the according displacements within the transmission length. Formulae are given preferably for monotonic instantaneous loading and ribbed reinforcing bars. The extension to long-term or repeated loading is demonstrated.

ZUSAMMENFASSUNG

Es werden die neueren Auffassungen einer konsistenten Theorie für den Grenzzustand der Gebrauchstauglichkeit von Konstruktionsbeton mitgeteilt. Gemeinsame Grundlage für Rissebildung und Verformungen bildet dabei das Verbundgesetz sowie das daraus abgeleitete Verformungsverhalten in der Einleitungszone. Es werden die Zusammenhänge für den Fall monotoner Kurzzeitbelastung für gerippte Bewehrungsstäbe angegeben und die Möglichkeit der Erweiterung für Langzeitbelastungen oder wiederholter Belastungen dargestellt.

1. INTRODUCTION

The serviceability limit states (SLS) of structural concrete comprise the limitations of stresses, cracking, deformation and vibration. Cracking and deformation, however, are the most significant SLS for reinforced and prestressed members. Structures made of plain or slightly reinforced concrete are usually very rigid and therefore, deformation is not significant for design. Cracking in such structures should be considered under specific requirements, which are different from those in reinforced concrete: After cracking, the equilibrium state in concrete should be verified and no uncontrolled crack propagation should be permitted.

Deformation in reinforced or prestressed concrete is considerably increased when cracking occurs. Therefore, the calculation of deformation in the cracked state (State II) is of predominant interest. Cracking and deformation are thus closely linked to another and should be approached within a consistent theory of structural concrete on a sound mechanical basis.

2. CONSTITUTIVE LAWS IN TRANSMISSION LENGTH

The constitutive laws of cracking and deformation can be found by considering the forces and displacements between the reinforcement and the adjacent concrete near the crack. As a simple model, a member under pure tension may be suitable in indicating the relevant relationships (Fig. 1). In an uncracked state, concrete strain ε_{c1} and the steel strain ε_{s1} are equal due to the compatibility conditions of full bond. After a crack has occurred, tensile force F acting in the reinforcing steel is only ($F_s = F$). At a distance equal or greater than transmission length l_t, either side of the crack, the compatibility condition of the uncracked state is maintained. The forces acting in the concrete (F_c) and the steel (F_s) may be calculated using rigidity values as follows:

$$F_c = F.\ A_c/(A_c + \alpha_e A_s) = F/(1+\alpha_e.\rho) \qquad [1]$$

$$F_s = F.\ \alpha_e A_s/(A_c + \alpha_e A_s) = F.\alpha_e.\rho/(1+\alpha_e.\rho) \qquad [2]$$

$$F_c + F_s = F \qquad [3]$$

Concrete force F_c is transmitted from the steel to the concrete by bond forces for the distance given by transmission length l_t. As shown in Fig. 1, in this length steel and concrete strain diverge from each other within that distance. The difference between steel and concrete strain is the first derivative of local slip, i.e. the differential displacement between the steel and the concrete (u_s - u_c).

$$ds/dx = \varepsilon_s - \varepsilon_c \qquad [4]$$

$$s = \int(\varepsilon_s - \varepsilon_c)dx = u_s - u_c \qquad [5]$$

Due to the slip s, bond stresses are generated according to the bond stress slip relationship, $\tau = \tau(s)$. This relationship depends predominantly on the surface of the steel, concrete strength fc, the position of the reinforcing steel during concreting (bond condition), the type of loading and whether the concrete is confined or not.

Various bond stress-slip-relationships have been proposed by different researchers. A synthesis has been provided in the very general law given in CEB Model Code 1990. This general law may be specified when one keeps in mind that ribbed reinforcing bars are preferable for crack control. In the case of unconfined concrete and for monotonic loading, the diagram shown in Fig. 2 may be applied.

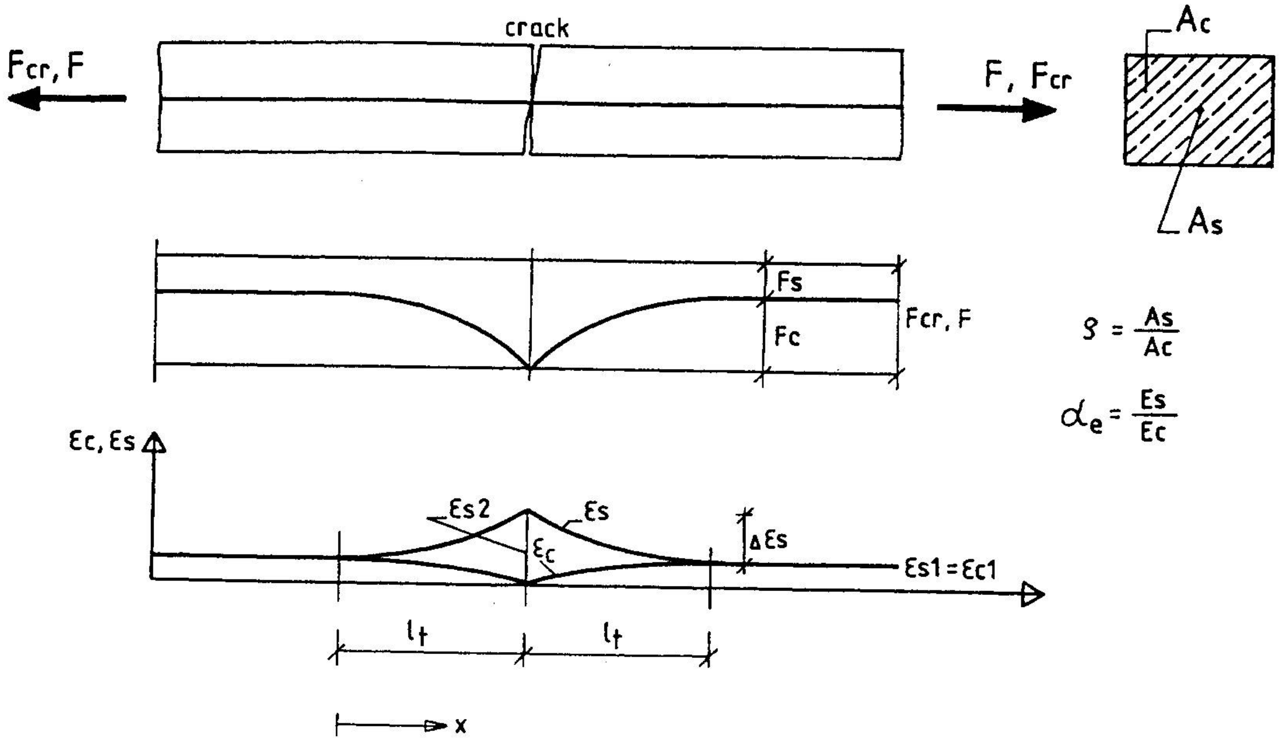

Fig. 1: Pure Tension, Forces and Strains near a Crack

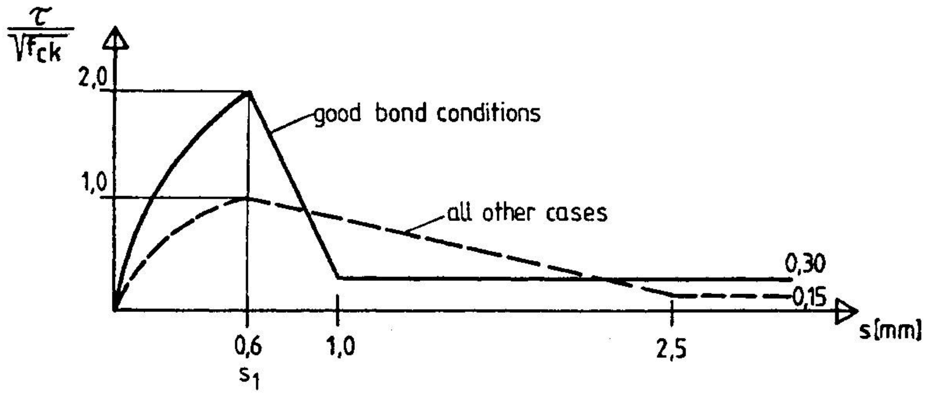

Fig. 2: Bond-Stress-Slip-Relationship of Ribbed Reinforcing Bars under Monotonic Short-Term-Loading

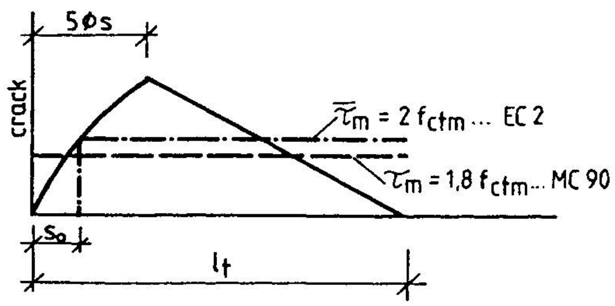

Fig. 3: Distribution of Bond Stress in the Transmission Length

Under service load conditions, crack width should be limited by a value of approximately $w_k = 2s \leq 0.5$ mm. Obviously, all calculations of the serviceability limit state use the first branch of the diagram only. Similarly, this may be predicted for confined concrete as the limit for the first branch (s_1) is increased from 0.6 to 1.0 mm. The constitutive law of this branch may be given by

$$\tau = \tau_{max} \, (s/s_1)^{\alpha} \qquad [6]$$

In the literature, actual values for exponent α are given in a range from 0.22 to 0.40.

In the vicinity of a transverse crack, a zone of reduced bond between the reinforcement bar and the surrounding concrete can be observed. For ribbed bars, the length of this zone depends on bar diameter Φ_s, concrete cover c, and the spacing of the ribs. Using formula [6] it is suggested that bond stress τ and slip s be reduced to within a distance of $x \leq 5\Phi_s$ from that crack by a factor λ, where

$$\lambda = x/5\Phi_s \leq 1 \qquad [7]$$

The differential equation for sliding bond,

$$d^2s/dx^2 = k.\,\tau \qquad [8]$$

with the bond-slip-relationship [6] reads as follows:

$$d^2s/dx^2 = k.\tau_{max}.\,(s/s_1)^{\alpha} \qquad [9]$$

This is a homogeneous non-linear differential equation of the second order, where the solution for the slip s(x) can be given as

$$s(x) = k_s \cdot x^{\frac{2}{1-\alpha}} \qquad [10a]$$

From this solution for slip the distribution of steel stress σ_s and bond stress τ can be found with the following derivatives:

$$\sigma_s(x) = k_\sigma \cdot x^{\frac{1+\alpha}{1-\alpha}} \qquad [10b]$$

$$\tau(x) = k_\tau \cdot x^{\frac{2\alpha}{1-\alpha}} \qquad [10c]$$

Equation [10] describes the distribution of the forces and displacements along transmission length l_t and hereby fulfills the requirements stated at the beginning of this chapter.

The previous considerations are only valid for monotonic short-term loading. Under long term loading (t) or repeated loading (n) the slip will increase. A simple way to describe this increased slip $s_{n,t}$ is by using Equation [11]

$$s_{n,t} = s.\,(1+k_{n,t}) \qquad [11a]$$

The displacement factor k_t for a permanent load can be calculated according to equation [11b]

$$k_t = (1+10t)^{0.080} - 1 \qquad [11b]$$

where t is load duration in hours.

For repeated loading the displacement factor k_n can be determined with Equation [11c]

$$k_n = (1+n)^{0.107} - 1 \qquad [11c]$$

where n is the number of load cycles.

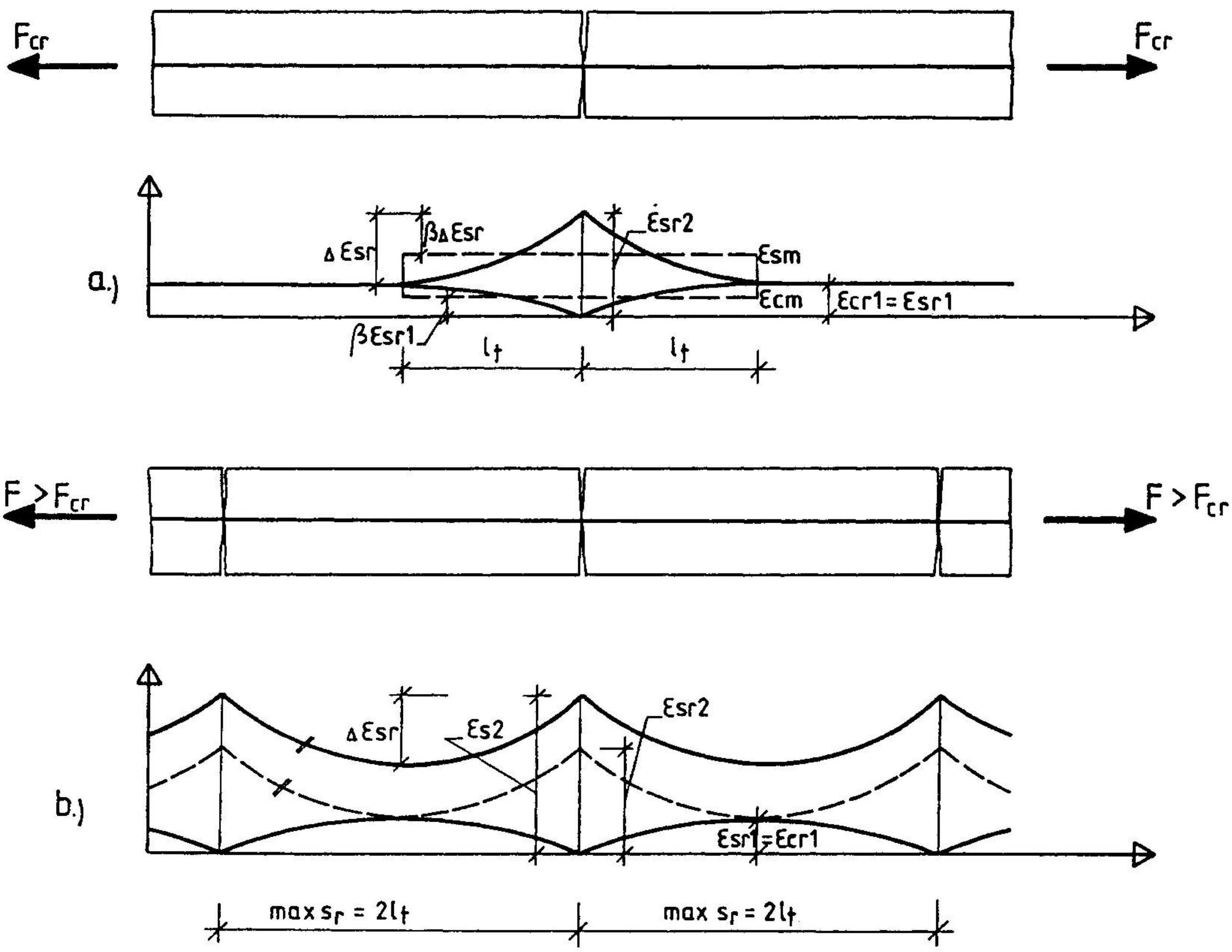

Fig. 4: Strains a) Near a Single Crack
b) For maximum Crack Spacing max s_r

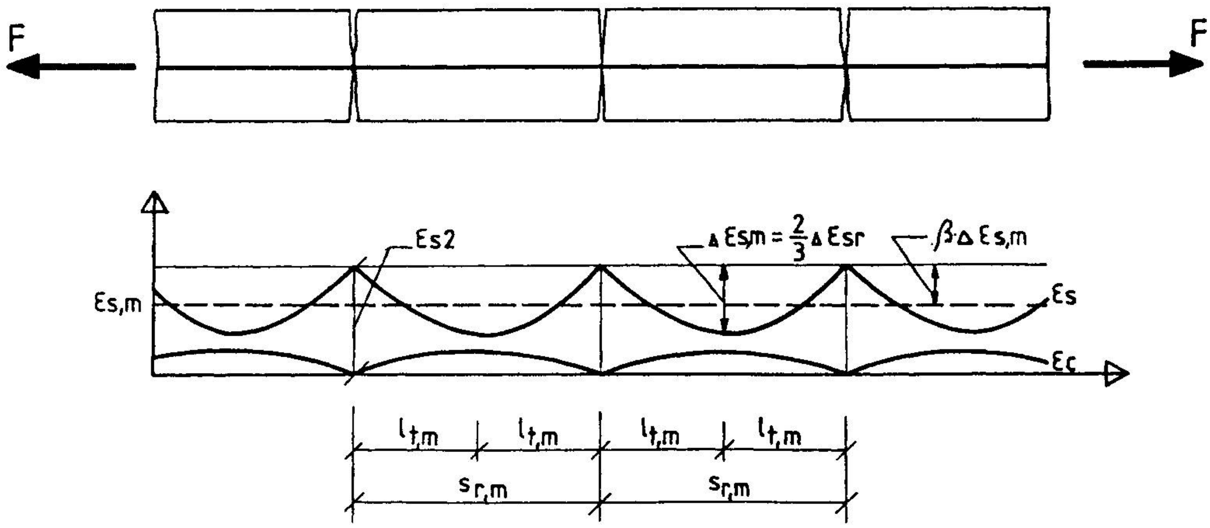

Fig. 5: Strains for average Crack Spacing $s_{r,m}$

3. APPROPRIATE SIMPLIFICATIONS UNDER SERVICE LOAD CONDITIONS

The distribution of bond stress over transmission length under short-term monotonic loading is plotted as a solid line in Fig. 3. For practical application, it is suitable to use mean bond stress τ_m over transmission length. The zone of reduced bond close to the crack may be approximated by bond less length s_o as it is done in EC 2 or by reducing mean bond stress along the MC 90 line. The values for mean bond stress in these two codes can be derived from the mean tensile stress of concrete f_{ctm} and equal to 2.0 f_{ctm} and 1.8 f_{ctm} respectively.

Using uniformly distributed bond stress simplified equations for transmission length, crack width and mean steel strain can be established.

4. STAGES OF CRACKING

For serviceability limit problems, the distinction between the uncracked concrete, crack formation phase and stabilized cracking stages is helpful in estimating crack width and deformation.

The prerequisite for the uncracked stage is that no crack has occurred. This will normally be fulfilled where concrete tensile stress σ_c is limited to the lower fractile of the tensile strength. A more conservative criterion is not to allow any tensile stresses.

$$\sigma_c \leq f_{ctk,min} \text{ or } \leq 0 \qquad [12]$$

When calculating the tensile stress, all actions, loads, and imposed or hindered deformation should be considered.

After the first crack has appeared, one crack after another occurs during the crack formation phase. In this phase, single cracks play an important role (Fig. 1). The distance between single cracks is greater than twice the transmission length, and no transmission length overlapping of adjacent cracks takes place. There is no influence of one crack on the crack width of the next.

At the end of the crack formation phase, the final primary crack pattern has been estabilished. The distance between the cracks is greater than the transmission length but less than the doubled value. The remaining tensile force in the concrete is too low for further cracks to occur, except for sporadic secondary cracking. Increasing load or imposed deformation will induce an increase in crack width, but no further cracks will appear.

5. CALCULATION OF CRACK WIDTH

The process of cracking discussed above allows calculating crack width for both stages of cracking on a uniform basis. Crack width w can be estimated as w = 2s where s is slip on both sides of the crack according to Equation [5]. By introducing average steel and concrete strain to transmission length ε_{sm} and ε_{cm} according to Fig. 4a, crack width can be calculated as

$$w = 2l_t.(\varepsilon_{sm} - \varepsilon_{cm}) \qquad [13]$$

Transmission length can be estimated with the simplifications presented in Chapter 3, using the equilibrium between steel force and bond force:

$$F\text{-}F_s = F_c = T = l_t.\Phi_s.\Pi.\tau_m$$

$$A_s.\sigma_{s2}/(1+\alpha_e.\rho) = l_t.\Phi_s.\Pi.\tau_m$$

$$l_t = (\Phi_s/4).(\sigma_{s2}/\tau_m).[1/(1+\alpha_e.\rho)] \qquad [14]$$

where σ_{s2} is steel stress at the crack and ε_{s2}, the corresponding strain.

Average steel strain and concrete strain can be given as:

$\varepsilon_{sm} = \varepsilon_{s2} - \beta\Delta\varepsilon_{sr}$ and [15a]

$\varepsilon_{cm} = \beta\varepsilon_{cr1} = \beta\varepsilon_{sr1}$ [15b]

β is an integration factor for strain distribution in transmission length, taking into account the simplification of the uniformly distributed bond stress. Here, it could be considered with a value: $\beta = 0.6$.

From $\Delta\varepsilon_{sr} = \varepsilon_{sr2} - \varepsilon_{sr1}$, it follows that

$\varepsilon_{sm} - \varepsilon_{cm} = \varepsilon_{s2} - \beta\varepsilon_{sr2}$ [16]

When tensile force is equal to cracking force as plotted in Fig 4a, ε_{s2} in Equation [16] reads ε_{sr2}. Consequently, Equation [16] can be transformed to

$\varepsilon_{sm} - \varepsilon_{cm} = (1-\beta)\ \varepsilon_{sr2} = 0.4\ \varepsilon_{sr2}$ [17]

Maximum crack width, max w in the final crack pattern can be correlated to maximum crack spacing being twice the transmission length.

$\max s_r = 2.l_t$ [18]

The greatest crack width can be found when crack spacing on both sides of the crack under consideration is equal to maximum crack spacing (Fig 4b). In general, tensile force is higher than cracking force and the differential average strain values for steel and concrete can by calculated according to Equation [16].

A general formula for max w can be derived from Equations [13], [14] and [16] introducing tensile force F and cracking force F_{cr} as indicated in Fig 4.

$\max w = 2.(\Phi_s/4).[F_{cr}/(A_s.\tau_m)].[1/(1+\alpha_e.\rho)](F - \beta F_{cr})/A_s.E_s$ [19]

which can be transformed into

$$A_s = \sqrt{\frac{\Phi_s \cdot F_{cr}\ (F - \beta \cdot F_{cr})}{2\tau_m \cdot E_s \cdot \max w \cdot (1+\alpha_e\rho)}} \qquad [20]$$

From Equation [20], the necessary amount of reinforcement required for crack width max w and a selected bar diameter Φ_s may be calculated directly.

The calculation of crack width under long-term or repeated loading may be based on equivalent considerations taking into account the relevant losses of bond stress. This can be achieved by using the appropriate values for average bond stress τ_m and integration factor β.

Crack control in thick members needs specific consideration. When calculating the cracking force due to imposed deformation, it should be taken into account that concrete stresses are not on the thickness of the cross section. When the first crack has appeared, cracking force at the crack is acting in the reinforcing bars near the surface. The cracking force is transmitted to the adjacent concrete by bond forces. Further cracks will appear, when tensile strength in the effective concrete zone $A_{c,ef}$ near the surface is exceeded. This might occur before full cracking force is transmitted to the concrete. Thus, "brush cracking" can occur. $A_{c,ef}$ depends on bar diameter and thickness of concrete cover c and may be estimated as

$A_{c,ef} = 2.5\ (c+\Phi_s/2)$ [21]

Equation [20] may be applied to thick members by replacing ρ with ρ_{ef}, which is the reinforced percentage of the effective concrete area.

$\rho_{ef} = A_s / A_{c,ef}$ [22]

6. CRACK WIDTH CONTROL WITHOUT CALCULATION

For specific values of max w, a correlation between steel stress σ_s and necessary bar diameter Φ_s can be found. Such relationships are shown on Table 7.4.3 in MC 90, where the values for reinforced concrete are based on a crack width of w = 0.3 mm, and these for prestressed concrete of w = 0.2 mm respectively.

Steel stress [MPa]	Maximum bar diameter [mm]	
	Reinforced sections	Prestressed sections
160	32	25
200	25	16
240	20	12
280	14	8
320	10	6
360	8	5

7. DEFORMATION DUE TO TENSION

The elongation of a member under tension, a tensile chord of a beam or a tie can calculated with

$$\Delta l = \varepsilon_{s,m}.l \qquad [23]$$

Average steel strain, $\varepsilon_{s,m}$, indicates the average elongation in the full member. this value differs from ε_{sm} in the transmission length of a single crack or between two cracks with maximum crack spacing expressed in Equation [15a]. The difference between these two values is caused by differential crack spacing. Mean steel strain ε_{sm} is associated with the maximum crack spacing. Actual crack spacing s_r in a member with a greater total length, however, varies in transmission length to twice that value

$$l_t \le s_r \le 2.l_t \qquad [24]$$

Crack spacing s_r along the axis of the member is arbitrarily distributed according to the probabilistic distribution of cracking resistance $A_c.f_{ct}$ or $A_{c,ef}.f_{ct}$ respectively. Average crack spacing $s_{r,m}$ can be calculated for an assumed probability distribution. For various realistic distributions, average crack spacing can is found to be close to 4/3 l_t. Thus, this value would be a reasonable approximation.

$$s_{r,m} = 4/3\ l_t \qquad [25]$$

Accordingly, transmission length on both sides of the crack is reduced to

$$l_{t,m} = 0.5\ s_{rm} = 2/3\ l_t \qquad [26]$$

Then, transferred bond force is reduced corresponding to the reduced transmission length

$$T = 2/3.l_t.\Phi_s.\Pi.\tau_m = 2/3\ A_s.E_s.\ \Delta\varepsilon_{sr} \qquad [27]$$

The reduction of steel strain at the crack to the midpoint between the cracks is then given by

$$\Delta\varepsilon_{s,m} = 2/3\ \Delta\varepsilon_{sr} \qquad [28]$$

The mean strain throughout the entire member may thus be taken as:

$$\varepsilon_{s,m} = \varepsilon_{s2} - \beta.\ \Delta\varepsilon_{s,m} = \varepsilon_{s2} - \beta.2/3.\ \Delta\varepsilon_{sr} \qquad [29]$$

For instantaneous loading, the integration factor may be assumed as being $\beta = 0.6$, as demonstrated in Chapter 5. Thus, average steel strain in the total member can be given as

$$\varepsilon_{s,m} = \varepsilon_{s2} - 0.6.2/3.\ \Delta\varepsilon_{sr} = \varepsilon_{s2} - 0.4.\ \Delta\varepsilon_{sr} \quad [30]$$

The simplified stress-strain-diagram of a member under pure tension thorughout the full cracking stage range relevant for serviceability limit states is shown in Fig 6.

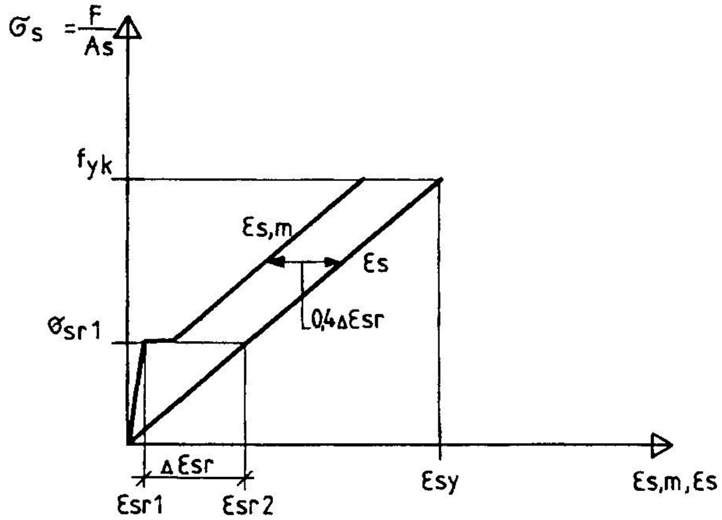

Fig. 6: Simplified Stress-Strain-Relationship of Embedded Reinforcing Steel under Monotonic Instanteneous Loading

8. RANGE OF APPLICATION

The principles and application rules reported above may be applied to any type of structural concrete with reinforcement, i.e. normally reinforced or prestressed concrete. Prestressing may be applied with any type of tendon: internal or external tendons, bonded or unbonded prestressing steel. For serviceability limit states, prestressing force is taken into account as an external force acting on the reinforced member. Stresses due to direct and indirect actions are combined with prestressing stresses in accordance with the relevant combination rule. Thus, the combination of action effects which cause cracking can be analysed.

When crack control is required, it is preferable that ribbed reinforcing steel has applied. Prestressing steel and smooth bars are of minor advantage with respect to crack control. Excessive deformation, in particular, can be controlled with prestressed tendons.

As stated above the relationships shown are valid for service load conditions. The parameters are predominantly reported for instantaneous loading. They may be adapted to long-term and repeated loading. Especially mean bond strength τ_m and integration factor β should be modified accordingly.

The analysis of a structure at ultimate limit state may be carried out on the basis of a strut and tie model. If this model is orientated on a stress field according a linear analysis it may also be applied to serviceability limit state. Cracking and deformation of ties can be calculated with the models reported above.

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Creep, Relaxation and Shrinkage of Structural Concrete

Fluage, relaxation et retrait du béton de construction

Kriechen, Relaxation und Schwinden von Konstruktionsbeton

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SUMMARY

The effects of time-dependent material properties of structural concrete are analysed on a common basis. Starting with the principle of superposition an algebraic σ-ε-τ relation containing a relaxation parameter or ageing coefficient χ is formulated and its application to the analysis and design of reinforced and prestressed concrete structures is illustrated: changes of forces due to imposed deformations and modifications of restraint conditions, stress redistributions between concrete and steel, time-dependent deformations.

RÉSUMÉ

Les effets différés affectant le béton de construction sont analysés sur une base commune: basée sur le principe de superposition, une relation σ-ε-τ contenant le paramètre de vieillissement χ est formulée. Son application dans l'analyse et la conception des structures armées et/ou précontraintes est illustrée dans le cas de la variation des efforts provenant de déformations imposées, de la modification des conditions de contrainte, des redistributions des efforts entre béton et acier et des déformations dues aux effets différés.

ZUSAMMENFASSUNG

Die Auswirkungen der zeitabhängigen Materialeigenschaften werden bei Konstruktionsbeton auf einer gemeinsamen Grundlage untersucht. Mit dem Superpositionsprinzip wird eine algebrische σ-ε-τ Beziehung unter Verwendung eines Relaxationskennwertes oder Alterungsbeiwertes χ formuliert und ihre Anwendung bei Berechnung und Entwurf von Konstruktionen in Stahl- und Spannbeton erläutert: Schnittgrössenänderungen bei erzwungenen Verformungen und Systemwechseln, Spannungsumlagerungen zwischen Beton und Stahl, zeitabhängige Verformungen.

1. Introduction

The performance of structural concrete at serviceability limit state depends mainly on a good design for quality and durability of the concrete construction. An important tool needed for this quality design is a consistent model and a sophisticated but not too complicated analytical model. This model should include the analysis of stress distribution, deflections and cracking. As creep and shrinkage of concrete have a strong influence on long-term damages of concrete structures, time-dependent effects should be taken into account for serviceability limit states and second order effects.

In order to describe the viscoelastic behaviour of concrete and reinforcement we are talking first of all about creep-problems and mean the time-dependent increase of deformation under known stress history and secondly about relaxation-problems and mean the time-dependent decrease of stresses under specific conditions of deformation; at last we consider shrinkage-problems and mean the shortening of non-loaded concrete members during the natural or artifical draining process.

The structural engineer has to take into consideration the relationship between stress, strain and time of concrete and reinforcement as well as the compatibility condition of the build-up cross-section, which is assumed to remain plain, and, if applicable, the conditions due to statical indeterminacy.

2. Time-dependent stress-strain relation of concrete

The theory of linear creep is based upon the principle of superposition stating that in viscoelastic materials various steps of stresses can be superposed considering their time of duration and their different age τ at loading or maturity of the concrete.

Using the principle of superposition, we may write for the total strain ϵ_t at any time t including shrinkage $\epsilon_s(t)$ according to [1,2]:

$$\epsilon(t) = \frac{\sigma(\tau_0)}{E(\tau_0)}\cdot[1+\phi(t,\tau_0)] + \int_{\tau=\tau_0}^{t}\frac{\partial\sigma(\tau)}{\partial\tau}\cdot\frac{1}{E(\tau)}\cdot[1+\phi(t,\tau)]\cdot d\tau + \epsilon_s(t) \quad (1)$$

and with a constant modulus of elasticity E and the well-known creep coefficient $\phi(t,\tau_0) = \phi_t$

$$\epsilon_t = \frac{\sigma_0}{E}\cdot(1+\phi_t) + \frac{\sigma_t-\sigma_0}{E} + \frac{1}{E}\cdot\int_{\tau=\tau_0}^{t}\frac{\partial\sigma(\tau)}{\partial\tau}\cdot\phi(t,\tau)\cdot d\tau + \epsilon_{s,t}. \quad (2)$$

With this principle of superposition one has to solve creep problems (i.e. calculation of deformations under a known stress history) as well as relaxation problems (i.e. determination of stresses under specified conditions of strain or deformation).

The total strain $\epsilon(t)$ can be determined by using simple quadrature rules in case the stress distribution $\sigma(t)$ is known; on the other hand, when $\epsilon(t)$ is known, equation (1) is an integral equation and the stress distribution $\sigma(t)$ is the unknown function. Under general conditions a complete analytical solution is impossible, because the creep-function $\phi(t,\tau)$ has to be determined with the help of experiments or according to standards and has a complicated mathematical form. To avoid such difficulties, it is possible in order to compute the stress distribution due to external or internal restraints to change the integral equation (1) into a simpler algebraic stress-strain relation and then solve the relaxation-problems using well-known and simple mathematical procedures (see fig.1). The following algebraic equation is obtained by modification of the integral equation (2)

$$\epsilon_t = \frac{\sigma_0}{E}\cdot(1+\phi_t) + \frac{\sigma_t-\sigma_0}{E}\cdot[1+\chi_t\cdot\phi_t] + \epsilon_{s,t} \quad (3)$$

where the parameter χ_t can be calculated and is identified as relaxation parameter [2,3] or aging coefficient [1,4] with

$$\chi_t = \frac{\int_{\tau_0}^{t}\frac{\partial\sigma(\tau)}{\partial\tau}\cdot\phi(t,\tau)\cdot d\tau}{(\sigma_t-\sigma_0)\cdot\phi_t} = \frac{\sum_{\tau_i}\Delta\sigma(\tau_i)\cdot\phi(t,\tau_i)}{(\sigma_t-\sigma_0)\cdot\phi_t} \,. \quad (4)$$

Now introducing the strain condition for relaxation $\epsilon_t=\epsilon_0=\text{const}$, equation (3) leads without ϵ_s to the following relation by using the known aging coefficient χ_t and the condition $E_c \cdot \epsilon_o - \sigma_0 = 0$:

$$0 = \sigma_0 \cdot \phi_t + (\sigma_t - \sigma_0) \cdot (1 + \chi_t \cdot \phi_t) \quad \text{or} \quad \frac{\sigma_t}{\sigma_0} = 1 - \frac{\phi_t}{1 + \chi_t \cdot \phi_t} . \tag{5}$$

After introducing the relaxation coefficient ψ_t analogue to the creep coefficient

$$\phi_t = \frac{\epsilon_t - \epsilon_o}{\epsilon_o} \quad \text{by} \quad \psi_t = - \frac{\sigma_t - \sigma_o}{\sigma_o} = 1 - \frac{\sigma_t}{\sigma_o} \tag{6}$$

one can also compute the exact time-dependent aging coefficient χ_t and proof by experiments [5] with

$$\chi_t = \frac{1}{\psi_t} - \frac{1}{\phi_t} = - \frac{\sigma_0}{\sum_{\tau_i} \Delta\sigma(\tau_i)} - \frac{1}{\phi_t} . \tag{7}$$

Fig. 1 explains the necessary mathematical background as well as some values for the aging coefficient χ_t, which oscillates between 1.0 and 0.6 with a mean value of $\bar{\chi} = 0.8$.

The remaining values of the stresses for $t \to \infty$ are of particular interest, i.e. σ_∞ after completion of creep and relaxation. The result of extensive investigations is the following statement: Assuming a usual load carrying age of 3 days $< \tau_0 <$ 90 days and typical values for the final creep coefficient $1 < \phi_\infty < 4$ of standard concrete one can determine a general aging coefficient of $\chi \approx 0.8$. This leads to the simple form of the stress-relaxation

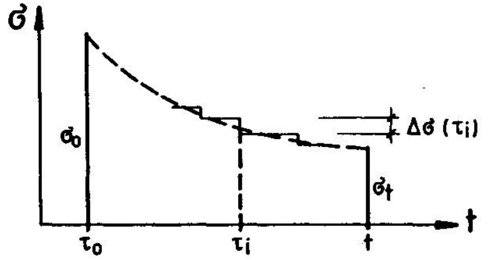
$$\frac{\sigma_t}{\sigma_0} = 1 - \frac{\phi_t}{1+0.8 \cdot \phi_t} . \tag{8}$$

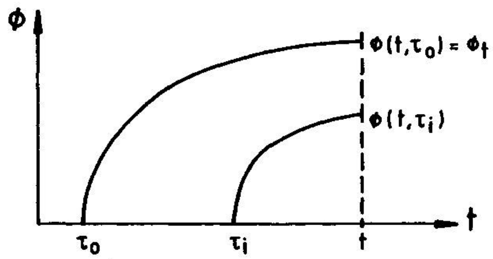
Fig. 2 explains these relations and shows all evaluations for an average final creep coefficient $\phi_\infty = 2.5$.

$$\varepsilon(t) = \frac{\sigma(\tau_0)}{E(\tau_0)}\left[1 + \phi(t,\tau_0)\right] + \int_{\tau=\tau_0}^{t} \frac{\partial\sigma(\tau)}{\partial\tau}\,\frac{1}{E(\tau)}\left[1+\phi(t,\tau)\right] d\tau + \varepsilon_s(t)$$

with $E(\tau) = E_c$; $\phi(t,\tau_0) = \phi_t$ and

aging coefficient $\chi_t = \dfrac{\sum \Delta\sigma(\tau_i)\cdot\phi(t,\tau_i)}{(\sigma_t - \sigma_0)\cdot\phi_t}$

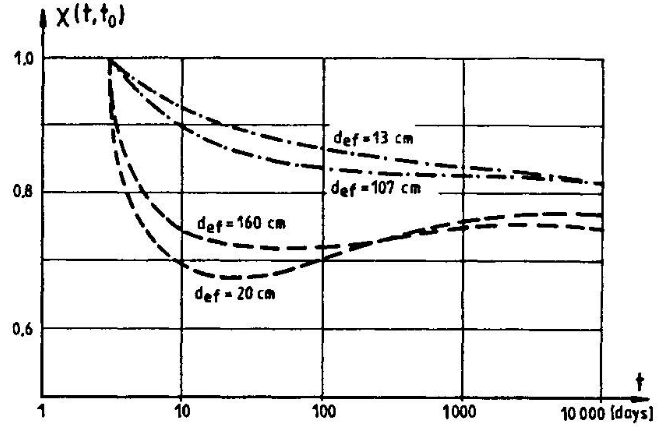




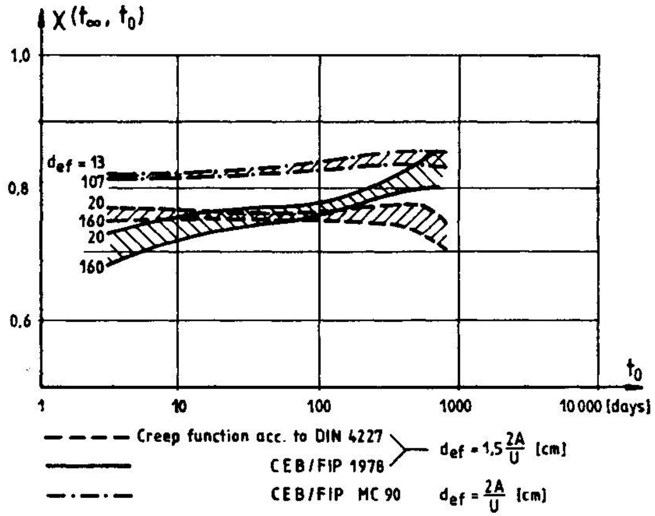
$\sigma_t - \varepsilon_t$ – relation :

$$\varepsilon_t = \frac{\sigma_0}{E_c}(1 + \phi_t) + \frac{\sigma_t - \sigma_0}{E_c}(1 + \chi_t\cdot\phi_t) + \varepsilon_{s,t}$$

Fig. 1: Relation between stress, strain and time due to the application of the principle of superposition and values of the aging coefficient χ (see [5])



development for t_0 = 3 days



end values $\chi_\infty(t_0)$

AGING COEFFICIENT X

$$\varepsilon_t = \frac{\sigma_0}{E_c}\cdot(1+\phi_t) + \frac{\sigma_t-\sigma_0}{E_c}\cdot(1+\chi_t\cdot\phi_t)$$

relaxation test: $\varepsilon_t = \varepsilon_0 = const$

$$E_c\cdot\varepsilon_0 - \sigma_0 = 0 = \sigma_0\cdot\phi_t + (\sigma_t-\sigma_0)\cdot(1+\chi_t\cdot\phi_t)$$

≙ suddenly imposed strain

stress relaxation:

$$\frac{\sigma_t}{\sigma_0} = 1 - \frac{\phi_t}{1+\chi_t\cdot\phi_t} = 1 - \psi_t \qquad \chi_t = \frac{1}{\psi_t} - \frac{1}{\phi_t}$$

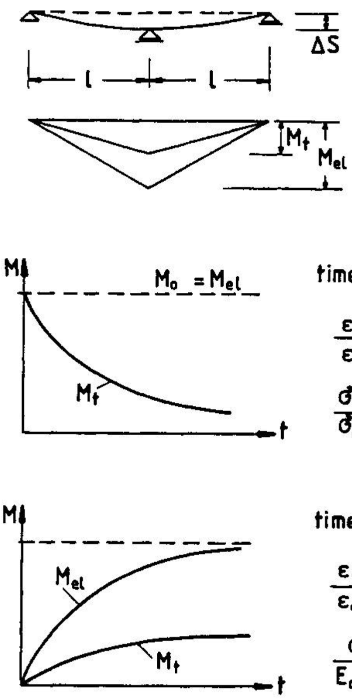
good approximation: $\chi \cong 0,8$

end value for $\phi_\infty = 2,5$: $\frac{\sigma_\infty}{\sigma_0} = 1 - \frac{2,5}{1+2} = \frac{1}{6}$

time dependent restraint $\varepsilon(t)$ for $\sigma_0 = 0$

$$\frac{\sigma_t}{E_c\varepsilon_t} = \frac{\sigma_t}{\sigma_{el}} = \frac{1}{1+\chi_t\cdot\phi_t}$$

for $\phi_\infty = 2,5$: $\frac{\sigma_\infty}{\sigma_{el}} = \frac{1}{1+0,8\cdot 2,5} = \frac{1}{3}$



$$M_{el} = \frac{3E_c I_c}{l^2}\cdot\Delta S(t)$$

time independent restraint

$$\frac{\varepsilon_t}{\varepsilon_0} \mathrel{\hat{=}} \frac{\Delta S(t)}{\Delta S_0} = const. = 1$$

$$\frac{\sigma_t}{\sigma_0} \mathrel{\hat{=}} \frac{M_t}{M_0} = 1 - \frac{\phi_t}{1+0,8\,\phi_t}$$

time dependent restraint

$$\frac{\varepsilon_t}{\varepsilon_\infty} \mathrel{\hat{=}} \frac{\Delta S(t)}{\Delta S_\infty} \cong \frac{\phi_t}{\phi_\infty}$$

$$\frac{\sigma_t}{E_c\,\varepsilon_t} \mathrel{\hat{=}} \frac{M_t}{M_{el}} = \frac{1}{1+0,8\,\phi_t}$$

Fig. 2: Development of stress during relaxation test and sectional forces in continuous beams due to sudden or gradual settlement of support

Fig. 15 and fig. 16 of Journal No.295 of DAfStb [5] proof that the theoretical model can be applied successfully to practical problems. The Journal indicates creep and relaxation tests for very old concrete and shows a good agreement of computed and measured stress relaxation by using the measured creep coefficients.

3. Change of reaction forces and sectional forces

In statical indeterminate systems imposed deformations lead to sectional forces and stresses; those values will decrease according to the relaxation process when considering a sudden restraint and will increase to only a fraction of the elastic values if a gradual time dependent restraint is occuring.

The effects of the time-dependent behaviour of concrete on the sectional forces due to these types of restraints are explained in fig. 2 by means of the settlement of support of a two span beam.

Of course the results are valid for any continuous beam. Using well-known methods of structural analysis one can determine the moment distribution M_{el} due to $\Delta s(t)$ according to the theory of elasticity. Usually one can neglect the contribution of the reinforcement to the stiffness of the build-up cross-section but it is also possible to take this influence into consideration by applying procedures described in [6]. In reality one often observes a combination of sudden and gradual settlement of support.

A typical example for the change of sectional forces is resulting from the modification of the restraint conditions after the application of loads due to a change of the structural system. If precast concrete beams are connected in a continuous manner (see fig. 3) or the whole structure is built in some field stages, the corresponding rotation will be preserved, which is the condition of a constant strain or rotation. According to fig. 3 one can compute the initial internal moment M_A at time t=0 as a sum of the moment M_{el}, which is obtained from an analysis of the structure as a whole, and a restraint moment $M_A - M_{el}$; this illustrates the substantial change of internal forces due to external permanent loads and to the process of creep and relaxation. More investigations about the building of continuous beams in several stages can be found in [7].

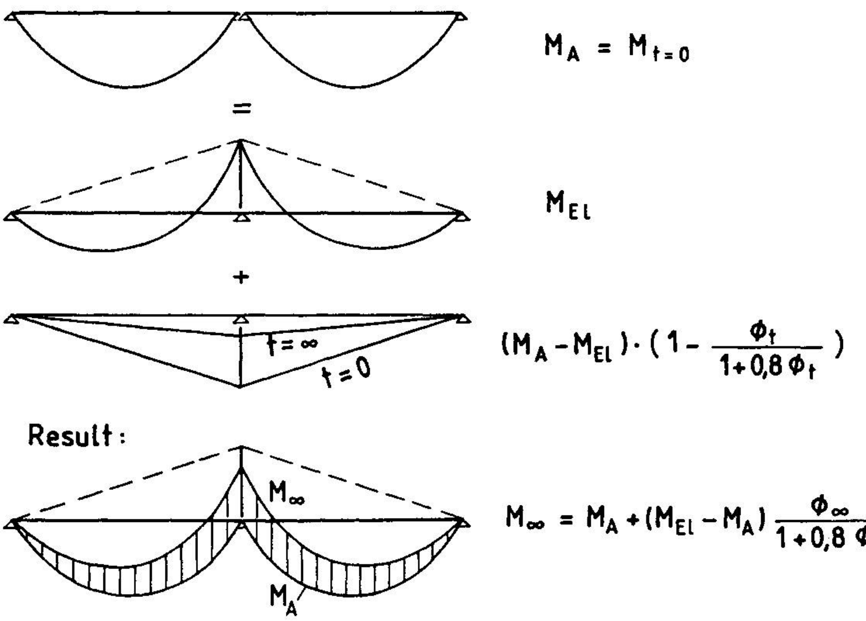


Fig.3: Redistribution of sectional forces due to a change of system

4. Redistribution of internal stresses and forces in cross-sections

This redistribution of internal stresses under external permanent loads and prestressing forces is caused by the amount and distribution of bonded reinforcement in the cross-section - independent if the reinforcement is prestressed or not. This restraining effect of the bonded reinforcement on creep and shrinkage of unreinforced concrete is described with the redistribution-parameter λ as ratio of the real to the free deformation in the steel fiber. These time-dependent stresses in steel and concrete can easily be derived on the basis of equilibrium and compatibility of strains in a section.

Fig. 4 explains the change of stress and deformation in a prestressed member with only one layer of prestressing steel. If the bond between steel and concrete is removed the time-dependent deformation would be the sum of free creep strain $\phi_t \cdot \epsilon_{c,o}$ and shrinkage strain ϵ_s. Using φ as abbreviation Index for the effects of creep, shrinkage and relaxation, the compatibility condition in the fibre of the bonded prestressing tendon is expressed by

$$\epsilon_{p,\varphi} = \epsilon_{cp,\varphi} = \lambda(\phi_t \cdot \epsilon_{cp,g+P} + \epsilon_s) = \lambda(\phi_t \cdot \epsilon_{cp,o} + \epsilon_s) \qquad (9)$$

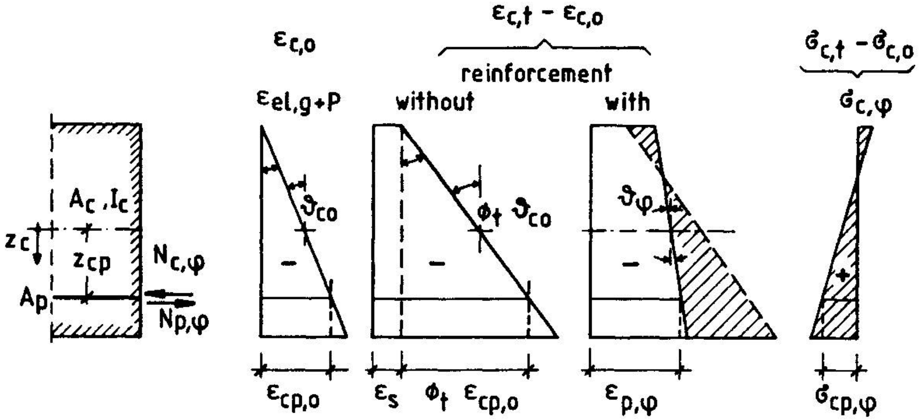
One can obtain with eq. (3) and the equilibrium for the state of Eigenstresses $N_{p,\varphi} = -N_{c,\varphi}$ the value of the redistribution-parameter λ by

$$1/\lambda = 1 + n \cdot \frac{A_p}{A_c} \left(1 + \frac{A_c}{I_c} \cdot z_{cp}^2\right)(1 + \chi \cdot \phi_t) \qquad (10)$$

with the maximum value $\lambda = 1$ for concrete without any reinforcement or by neglecting the restraining effects of the bonded steel. Then the so-called loss of prestress (see fig. 5 with all definitions, given in the EUROCODE 2 [11]) is calculated by

$$\Delta\sigma_{p,c+s+r} = \sigma_{p,\varphi} = \frac{N_{p,\varphi}}{A_p} = \lambda \cdot [n \cdot \phi_t \cdot \sigma_{cp,o} + E_p \cdot \epsilon_s + \Delta\sigma_{p,r}], \qquad (11)$$

where in addition the steel relaxation is introduced with $\Delta\sigma_{p,r}$. This derivation (see [3,6]) is demonstrated in fig. 4 and illustrated for a typical example, where the value of $\lambda = 0.55$ reduces the loss of prestress from 210 to 110 N/mm^2.



$$\varepsilon_{p,\varphi} \equiv \varepsilon_{cp,\varphi} = \lambda \, (\phi_t \cdot \varepsilon_{cp,o} + \varepsilon_s) \wedge \text{Compatib.}$$

$$\frac{\sigma_{p,\varphi} - \Delta\sigma_{p,r}}{E_p} = \frac{\sigma_{cp,o}}{E_c} \phi_t + \varepsilon_s + \frac{\sigma_{cp,\varphi}}{E_c} (1 + \chi \; \phi_t)$$

$$\frac{\sigma_{p,\varphi}}{E_p} \left[1 - \frac{E_p}{E_c} \; \frac{\sigma_{cp,\varphi}}{\sigma_{p,\varphi}} (1 + \chi \; \phi_t) \right] = \phi_t \cdot \varepsilon_{cp,o} + \varepsilon_s + \frac{\Delta\sigma_{p,r}}{E_p}$$

$$\text{Mit } [\;] = \frac{1}{\lambda} \text{ und } \frac{\sigma_{cp,\varphi}}{\sigma_{p,\varphi}} = - \frac{N_{p,\varphi}}{N_{p,\varphi}} \; \frac{A_p}{A_c} \left(1 + \frac{A_c \cdot z_{cp}^2}{I_c} \right) \wedge \text{Equil.}$$

$$\sigma_{p,\varphi} = \Delta\sigma_{p,c+s+r} = \frac{N_{p,\varphi}}{A_p} = \frac{n \cdot \phi_t \cdot \sigma_{cp,o} + E_p \varepsilon_s + \Delta\sigma_{p,r}}{1 + n \frac{A_p}{A_c} \left(1 + \frac{A_c \; z_{cp}^2}{I_c}\right) (1 + \chi \cdot \phi_t)}$$

Redistribution of stress in prestressed concrete member
(without $\Delta \sigma_{p,r}$)

$n = 6; \quad \phi_\infty = 2{,}5; \quad \sigma_{cp,o} = -10 \text{ N/mm}^2; \quad \varepsilon_{s\infty} = -30 \cdot 10^{-5}$

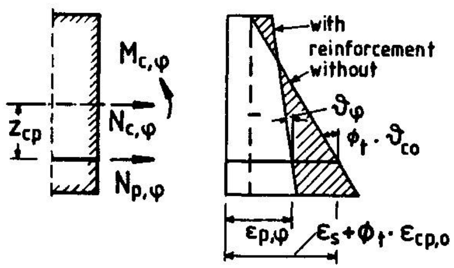
$$\sigma_{p,\varphi_\infty} = \frac{-6 \cdot 2{,}5 \cdot 10 - 2{,}0 \cdot 30}{1 + 0{,}1 \, (1+2) \cdot 3} = - \frac{210}{1{,}9} = -110 \text{ N/mm}^2$$

denominator: $n \cdot \frac{A_p}{A_c} = 0{,}1; \quad \frac{A_c}{I_c} z_{cp}^2 \approx 2; \quad 1 + 0{,}8 \cdot \phi_\infty = 3$

Deformations:

$$\varepsilon_{p,\varphi} = \frac{N_{p,\varphi}}{E_p A_p} = \frac{\sigma_{p,\varphi}}{E_p} = \lambda (\phi_t \cdot \varepsilon_{cp,o} + \varepsilon_s)$$

$$0 < \lambda < 1 \quad \text{here } \lambda = \frac{1}{1 + 0{,}9} = 0{,}55$$



$$\varepsilon_{p,\varphi} = \frac{N_{p,\varphi}}{E_p \cdot A_p}$$

$$M_{c,\varphi} = -N_{p,\varphi} \cdot z_{cp}$$

$$N_{c,\varphi} = -N_{p,\varphi}$$

Rotation: $\quad \vartheta_\varphi = \phi_t \cdot \vartheta_{c,o} + \frac{-N_{p,\varphi} \cdot z_{cp}}{E_c \, I_c} \cdot (1 + \chi \cdot \phi_t)$

Deflection: $\quad f_\varphi = \phi_t \cdot f_{o,g+p} + f_{N_{p,\varphi}} \cdot (1 + 0{,}8 \cdot \phi_t)$

Fig. 4: Change of stresses and deformations in prestressed concrete members due to creep and shrinkage and relaxation (for abbreviation Index φ = c+s+r)

4.2.3.5.5. Loss of prestress

(9) Time dependent losses should be calculated from:

$$\Delta\sigma_{p,c+s+r} = \frac{n\cdot\phi(t,t_0)\cdot(\sigma_{cg}+\sigma_{cp})+\epsilon_s(t,t_0)\cdot E_s+\Delta\sigma_{pr}}{1+n\cdot\frac{A_p}{A_c}\cdot(1+\frac{A_c}{I_c}\cdot z_{cp}^2)\cdot[1+0,8\cdot\phi(t,t_0)]}$$

where :

$\Delta\sigma_{p,c+s+r}$	is the variation of stress in the tendons due to creep, shrinkage and relaxation at location x, at time t.
$\epsilon_s(t,t_0)$	is the estimated shrinkage strain, derived from the values in Table 3.4 for final shrinkage (see also 2.5.5 and Appendix 1).
E_s	is the modulus of elasticity for the prestressing steel, taken from 3.3.4.4.
E_{cm}	is the modulus of elasticity for the concrete (Table 3.2).
n	is E_s/E_{cm}.
$\Delta\sigma_{pr}$	is the variation of stress in the tendon at section x due to relaxation.
$\phi(t,t_0)$	is a creep coefficient, as defined in 2.5.5, equation (2.9) (see also Appendix 1).
σ_{cg}	is the stress in the concrete adjacent to the tendons, due to self-weight and any other permanent actions.
σ_{cp}	is the initial stress in the concrete adjacent to the tendons, due to prestress (see 4.2.3.5.3 P(3)).
A_p	is the area of all the prestressing tendons at the level being considered.
A_c	is the area of the concrete section.
I_c	is the second moment of area of the concete section.
z_{cp}	is the distance between the centre of gravity of the concrete section and the tendons.

Fig. 5: Excerpt of the Final Draft of EUROCODE 2

In structural concrete members we have two kinds of reinforcement: the prestressing steel (active reinforcement) and normal unprestressed steel (passive reinforcement) in some different layers.

For a precise calculation of the change of stresses in these reinforcement layers one can summarize the active and passive reinforcement $A_p + A_s = A_r$ in a resulting steel fibre r, but then the moment of inertia of all steel layers I_r has to be taken into account. With the compatibility condition for strain and for rotation the change of stresses in the different fibres of steel and in the whole concrete section are resulting out of the combination of longitudinal forces $N_{r,\varphi}$ or $N_{c,\varphi}$ and bending moments $M_{r,\varphi}$ or $M_{c,\varphi}$, related to the steel or concrete section. Naturally this leads to more complicated formulas as can be seen in the detailed investigations in the publication [6] and [9], but usually one can neglect the effects of $M_{r,\varphi}$ with the assumption $I_r \approx 0$. For uncracked sections under permanent loads and with $E_s \approx E_p$ the redistribution of steel stresses is then given in the same form as in eq.(11) with

$$\sigma_{r,\varphi} = \frac{N_{r,\varphi}}{A_r} = \lambda_r \cdot [n \cdot \phi_t \cdot \sigma_{cr,o} + E_p \cdot \epsilon_s + \Delta\sigma_{p,r}] \tag{12}$$

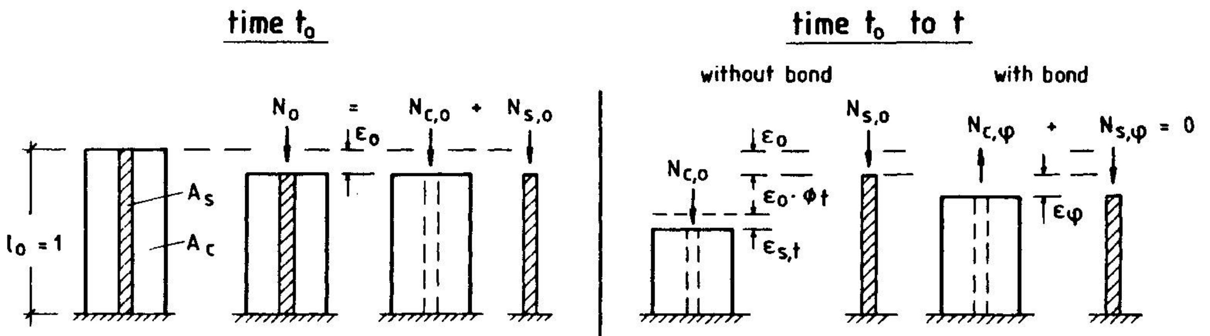
with the redistribution parameter in the resulting steel fibre r

$$1/\lambda_r = 1 + n \frac{A_s + A_p}{A_c} (1 + \frac{A_c}{I_c} \cdot z_{cr}^2)(1 + \chi \cdot \phi_t). \tag{13}$$

Because the redistribution parameter λ_r contains the sum $A_s + A_p$ the forces according to the state of Eigenstresses must be calculated with

$$N_{p,\varphi} = \sigma_{r,\varphi} \cdot A_p;\ N_{s,\varphi} = \sigma_{r,\varphi} \cdot A_s;\ M_{c,\varphi} = -N_{r,\varphi} \cdot z_{cr} = N_{c,\varphi} \cdot z_{cr}. \tag{14}$$

The change of internal forces and stresses in reinforced concrete columns due to creep and shrinkage can be computed by using the same procedure, as explained in fig. 6 for a concentrical load and a symmetrical reinforcement A_s. Because of the substantial amount of additional stress in reinforcement, which can add up to the two or threefold value of the initial stress, a careful design of stirrups is essential.



$N_o = N_{s,o} + N_{c,o}$

$\varepsilon_o = \varepsilon_{s,o} = \varepsilon_{c,o}$

$$\varepsilon_o = \frac{N_{s,o}}{E_s A_s} = \frac{N_{c,o}}{E_c A_c}$$

$$N_{s,o} = n \frac{A_s}{A_{i,o}} N_o \; ; \quad N_{c,o} = \frac{A_c}{A_{i,o}} N_o$$

with $A_{i,o} = A_c + n \cdot A_s$ and $n = \frac{E_s}{E_c}$

$0 = N_{s,\varphi} + N_{c,\varphi}$ Equilibrium

$\varepsilon_\varphi = \varepsilon_{s,\varphi} = \varepsilon_{c,\varphi}$ Compatibility

$$\varepsilon_\varphi = \frac{N_{s,\varphi}}{E_s A_s} = \frac{N_{c,o}}{E_c A_c} \Phi_t + \varepsilon_{s,t} + \frac{N_{c,\varphi}}{E_c A_c} (1 + \chi \cdot \Phi_t)$$

$$N_{s,\varphi} = - N_{c,\varphi} = \frac{n \frac{A_s}{A_c} \cdot \Phi_t \cdot N_{c,o} + E_s A_s \cdot \varepsilon_{s,t}}{1 + n \frac{A_s}{A_c} (1 + \chi \cdot \Phi_t)}$$

Fig. 6: Change of internal forces and stresses in reinforced concrete columns

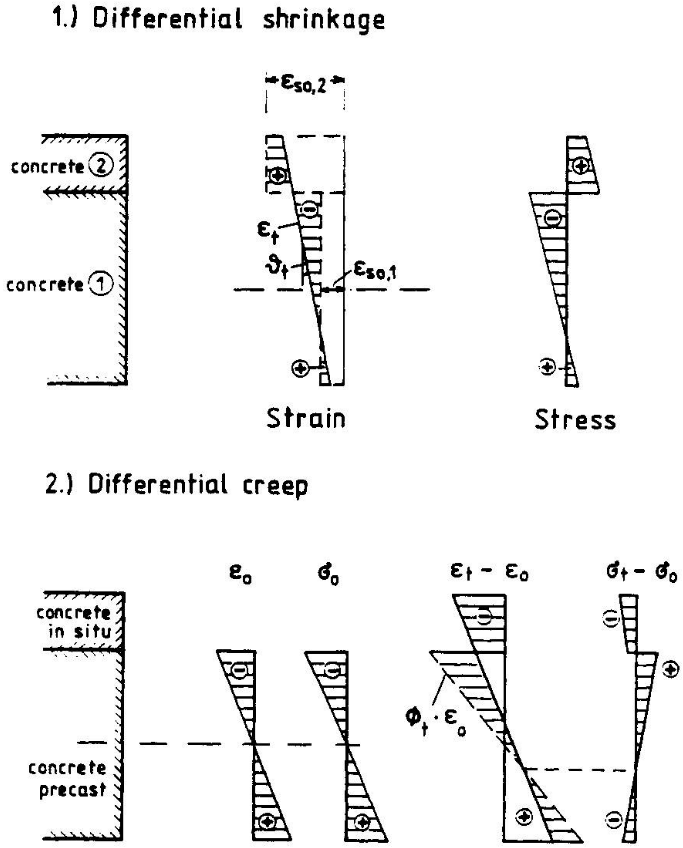


Fig. 7:

Redistributions of stresses in composite sections due to differential shrinkage and creep

In fig. 7 it is explained that the redistribution in composite sections due to differential shrinkage and creep can be calculated following the same principles.

5. Deflections of structural concrete

It is well known that deflections of structural concrete members can be calculated by double integration of the local rotations with $\delta \triangleq f = \iint \vartheta \; dx \; dx$.

The entire rotation of concrete sections including time dependent effects is given in analogy to eq. (3) by

$$\vartheta_t = \frac{M_o}{E_c I_c} (1+\phi_t) + \frac{M_t - M_o}{E_c I_c} (1+\chi\phi_t) = \vartheta_o (1+\phi_t) + \frac{M_{c,\varphi}}{E_c I_c} (1+\chi\phi_t). \qquad (15)$$

This expression contains the elastic rotation at t_o and the total creep rotation, which is known, if the redistribution moment $M_{c,\varphi}$ of the concrete cross section due to creep and shrinkage according to eq. (14) is put in. Then the creep rotation is

$$\vartheta_\varphi = \vartheta_o \cdot \phi_t + \frac{-N_{r,\varphi} \cdot z_{cr}}{E_c I_c} (1+\chi\phi_t). \qquad (16)$$

The steel force due to creep and shrinkage $N_{r,\varphi}$ in this formula is known from eq. (12); compare also fig. 4.

The initial rotation ϑ_o and the initial concrete stress in the steel fibre $\sigma_{cr,o}$ in eq.(12) are dependent on the kind of bond between steel and concrete when permanent loads are implemented. This means for pre-tensioned beams or reinforced beams without prestressing

$$\vartheta_o = \frac{M_o}{E_c I_i}$$

$$\sigma_{cr,o} = \frac{N_o}{A_i} + \frac{M_o}{I_i} \cdot z_{cr}$$

and for post-tensioned beams

$$\vartheta_o \simeq \frac{M_o}{E_c I_c}$$

$$\sigma_{cr,o} \simeq \frac{N_o}{A_c} + \frac{M_o}{I_c} \cdot z_{cr}$$

under the premise that the section properties taking the area of ducts and reinforcing steel into consideration are nearly equal to the concrete section properties A_c and I_c (exact value see [6]). Here N_o and M_o contain all permanent loads including the internal forces due to prestressing as an artificial load.

Thus the rotation of post-tensioned cross sections (compare fig.4) may be written after dividing into the different actions as

$$\vartheta_\varphi = \phi_t \cdot \frac{M_o}{E_c I_c} - \lambda_r \cdot n \cdot \phi_t \cdot \frac{M_o}{E_c I_c} \cdot \frac{A_r z_{cr}^2}{I_c} \cdot (1+\chi\varphi_t)$$

$$- \lambda_r \left[n \cdot \phi_t \cdot \frac{N_o}{A_c} + E_p \epsilon_s + \Delta\sigma_{pr}\right] \frac{A_r z_{cr}}{E_c I_c} \cdot (1+\chi\varphi_t) \qquad (17)$$

with the redistribution parameter λ_r from eq. (13).

Starting with this equation the time dependent rotation ϑ_φ is subdivided into the share ϑ_φ^M (due to the initial bending moment M_o) and the share ϑ_φ^N (due to the centric longitudinal force N_o and shrinkage ϵ_s), which is only caused by an eccentrical reinforcement. This is useful, because the share due to M_o as bending moment of an indeterminate system causes no additional redistribution of the reaction forces and internal forces, which remain "Eigenstresses". Otherwise, the share due to N_o and ϵ_s can produce some correction in the reaction forces in an indeterminate system if the excentricity of the longitudinal reinforcement is taken into account (see [6,8]). But usually in a reinforced structure these refinements are totally neglected.

The share of the bending moment leads from eq. (17) to

$$\vartheta_\varphi^M = \phi_t \cdot \vartheta_o \cdot \left[1-\lambda_r \cdot n \cdot \frac{A_r z_{cr}^2}{I_c} \cdot (1+\chi\phi_t)\right]$$

$$= \phi_t \cdot \vartheta_o \cdot \lambda_r \left[1+n \cdot \frac{A_r}{A_c} \cdot (1+\chi\phi_t)\right]. \qquad (18)$$

According to [6] and in analogy to [9,10] the deflection of post-tensioned structural concrete members in state I with one straight steel layer may be evaluated in relation to the inital elastic deflection $f_o = f_c$

$$f_\varphi^M = f_o \cdot \phi_t \cdot \lambda \left[1 + n \frac{A_r}{A_c} (1+\chi \cdot \phi_t)\right] = f_c \cdot \phi_t \cdot c \qquad (19)$$

with the reducing creep deformation coefficient $c < 1$ in the form

$$c = \frac{1 + n \cdot \frac{A_r}{A_c} (1 + \chi\phi_t)}{1 + n \cdot \frac{A_r}{A_c} \cdot \left(1 + \frac{A_c z_{cr}^2}{I_c}\right) \cdot (1 + \chi\phi_t)}, \text{ where } A_r = A_s + A_p. \qquad (20)$$

The reference deflections f_c are given in fig. 8c. This formula may also be used for post-tensioned curved tendons, if z_{cr} is taken from the middle of the span, as the variation of z_{cr} is nearly balanced by integration along the beam length. If $\chi = 0.8$ and $\phi_t = 2.5$ and $z_{cr} = 0.4 \cdot h$ are taken as regular values for long-term deflections, the coefficient c is only dependent on the factor $n \cdot A_r/A_c$ which is shown in fig.8a. The corresponding coefficient c for reinforced concrete (without prestressing) according to [9,10] is entered in fig. 8a as a broken line.

Deflections of state II for reinforced concrete may also be calculated with this coefficient c if the section properties of state II are taken into account. The result is also plotted in fig. 8a as a broken line. State II for prestressed members is much more complicated as the imposed longitudinal prestressing force has a strong influence on the shape of the compression zone. But pure state II is not a realistic premise for prestressed members under permanent loads, because as a rule state II is normally limited to short sections along the beam and the tension stiffening effect reduces local rotations even in these sections.

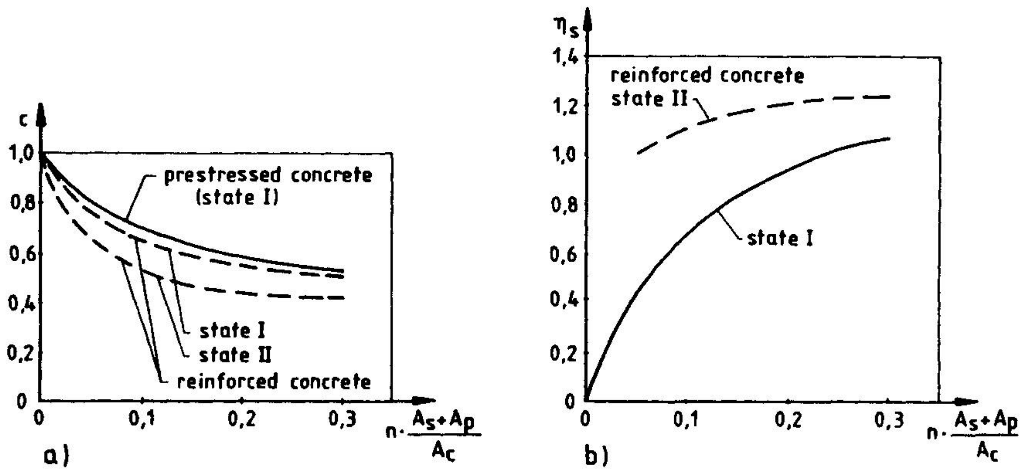
The evaluation of deflections - caused by eccentric reinforcement- due to longitudinal forces including prestressing force and shrinkage can be done in the same way solving eq. (17) in analogy to eq.(18). One has not to distinct between prestressed and reinforced members and describes the local rotations in state I with

$$\vartheta_{\varphi}^{N} = \frac{\phi_t \cdot \frac{N_o}{A_c E_c} + \epsilon_s + \frac{\Delta\sigma_{p,r}}{E_p}}{h} \left[\frac{h}{z_{cr}} \cdot \lambda_r \cdot n \frac{A_r}{A_c} \cdot \frac{A_c z_{cr}^2}{I_c} (1 + \chi\phi_t)\right] \quad (21)$$

The deflections can be directly calculated in analogy to eq. (19)

$$f_{\varphi}^{N} = - \alpha_s \cdot l^2 \cdot \frac{\phi_t \cdot \frac{N_o}{A_c E_c} + \epsilon_s + \frac{\Delta\sigma_{p,r}}{E_p}}{h} \cdot \eta_s \quad (22)$$

with α_s according to fig. 8c as coefficient for curved tendons and restraint conditions.



a: Creep deformation coefficient c acc. eq.(20)
b: Shrinkage deformation coefficient η_s acc. eq.(23)

	$f_c = \alpha \frac{max. M \cdot l^2}{E_c I_c}$	M x	M x	M x	M x
α	x	$\frac{1}{3}$	$\frac{1}{12}$	$\frac{1}{20,12}$	$\frac{1}{24}$
		$\frac{1}{4}$	$\frac{1}{9,6}$	$\frac{1}{23,08}$	$\frac{1}{16}$
		$\frac{1}{5}$	$\frac{1}{9,84}$	$\frac{1}{27,95}$	$\frac{1}{16,39}$
α_s	formula (22)	$\frac{1}{2}$	$\frac{1}{8}$	$\frac{1}{32}$	0
		$\frac{1}{3}$	$\frac{1}{9}$	$\frac{1}{14}$	$\frac{1}{18}$

c: Coefficient α and α_s

Fig. 8: Coefficients for a rational calculation of deformations in structural concrete members

The shrinkage deformation coefficient η_s for shrinkage ϵ_s and including longitudinal force N_o is given with the dimensionless value

$$\eta_s = \frac{h}{z_{cr}} \cdot \frac{n \cdot \frac{A_r}{A_c} \cdot \frac{A_c z_{cr}^2}{I_c} (1 + \chi\phi_t)}{1 + n \cdot \frac{A_r}{A_c} \cdot (1 + \frac{A_c z_{cr}^2}{I_c}) \cdot (1 + \chi\phi_t)} \qquad (23)$$

and can be taken from fig. 8b for mean values $\phi = 2.5$; $\chi = 0.8$ and $z_{cr} = 0.4 \cdot h$ in dependence of the parameter nA_r/A_c.

In consequence there are two ways to calculate the deflections in structural concrete: First the exact method analyzing the integral

$$\delta(t) - \delta_o = \delta_\varphi \hat{=} f_\varphi = \int\int (\vartheta_\varphi^M + \vartheta_\varphi^N)\, dx\, dx$$

with the local rotations according to eqs. (18) and (21) and second the simplified but less accurate solutions using eqs. (19) and (22) under reading off the required coefficients given in fig. 8.

Finally, it should be noted that the values of the coefficients c and η_s for the long-term deflections are strongly reduced if a compression reinforcement with the ratio A_s'/A_s is taken into account (compare [9,10]), while this is insignificant for the elastic or initial deformation. Due to the restraining effect of the existing bonded reinforcement the creep deflection caused by bending is only a fraction of the ϕ-fold initial one and the differences between state I and II are reduced with time, otherwise the deformation due to axial forces and shrinkage is only produced by the eccentricity of the resulting reinforcement in the structural member.

6. Summarizing commentary

With the described methods one can assess the influence of time-dependent material properties, especially the effects of limit values of the final creep and shrinkage coefficients, considering the known uncertainties in the design process.

For the whole range of the structural concrete, one could give the following recommendation: Only as much prestressing as necessary in order to achieve a favourable behaviour in the service state, but not less reinforcement as reasonable in order to assure the durability and the reliability concerning crack width control.

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Creep Effects on Structural Concrete

Effets différés dans les structures en béton

Auswirkungen des zeitabhängigen Materialverhaltens auf Betonverbundtragwerke

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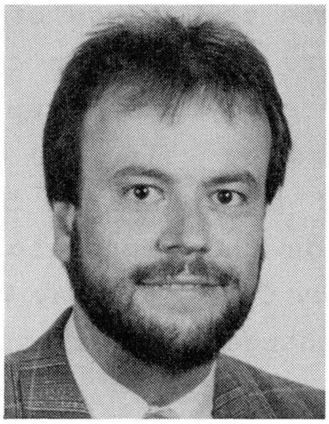

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SUMMARY

Starting with an analytical stress-strain-time relation of concrete, the numerical analysis of time-dependent effects on structural concrete using an equivalent-stiffness method and an iterative method are explained. If several abrupt loadings have to be superimposed these methods can be used with the help of an incremental constitutive law.

RÉSUMÉ

La relation algébrique contenant contrainte, déformation et temps intervient dans l'analyse numérique des effets différés du béton, et ceci en tant qu'élément de base; l'utilisation de la rigidité équivalente est expliquée, conjointement à la méthode itérative utilisée. Une relation constitutive est introduite afin de tenir compte de la superposition de plusieurs sauts de contrainte.

ZUSAMMENFASSUNG

Ausgehend von der algebraischen Spannungs-Dehnungs-Zeit Beziehung für Beton wird die numerische Berechnung der Auswirkungen des zeitabhängigen Betonverhaltens sowohl für die Methode der äquivalenten Steifigkeiten als auch für die iterative Methode erläutert. Anschliessend wird eine bei Anwendung dieser Methoden für die Superposition mehrerer Spannungssprünge vorteilhafte inkrementelle konstitutive Beziehung vorgestellt.

1. INTRODUCTION

The analysis of structural concrete at serviceability limit state regarding deflections, stress distribution and cracking due to creep and shrinkage is essential for a good performance. Nowadays two groups of methods are used for the numerical analysis of time-dependent effects on structural concrete. The first of them is the step-by-step method which is most often used for computer programs and allows to calculate the change of stress in short time steps. According to the nature of creep in concrete the change of stress has to be stored for every time step and every cross section or point of the structure to calculate the creep strain in further time steps. To avoid this storage of huge numbers of data the second method based on the algebraic stress-strain-time relation which was introduced by Trost [1] may be used. This relation allows the calculation of changes of stress as a result of creep and shrinkage in only one time step.

2. QUASI-ELASTIC METHODS OF CALCULATION OF TIME-DEPENDENT EFFECTS

Creep problems are generally solved by the incremental step-by-step analysis of structural concrete as a sequence of elasticity problems. Then the stress-strain relation within a time step is described by a linear function using the rectangle or the trapezoidal rule for time integration. These linear functions may be used to formulate an incremental elastic modulus according to the type of time integration (see [2]). The incremental method with short time steps based on the history integral is associated with the disadvantage that every preceding value of all stress components for each finite element must be stored. This may be reduced using differential-type formulations for the storage of stress history. But then computing time is still very long because only creep functions composed of e-functions can be used and short time steps are conditional.

The key idea to overcome these disadvantages and to obtain an efficient algorithm was to formulate an incremental constitutive law for long time steps in analogy to the stress-strain-time relation formulated by Trost [1]. Starting with

$$\varepsilon(t) = \frac{\sigma_c(t_0)}{E_c}[1 + \varphi(t,t_0)] + \frac{\sigma_c(t) - \sigma_c(t_0)}{E_c}[1 + \chi(t,t_0)\ \varphi(t,t_0)],$$

subtracting the elastic strain at loading age and replacing the change of strain $\varepsilon(t) - \varepsilon(t_0)$ by $\Delta\varepsilon(t,t_0)$ respectively the change of stress by $\Delta\sigma(t,t_0)$ results in

$$\Delta\varepsilon(t,t_0) = \frac{\sigma_c(t_0)}{E_c}\varphi(t,t_0) + \frac{\Delta\sigma_c(t,t_0)}{E_c}[1 + \chi(t,t_0)\ \varphi(t,t_0)] \tag{1}$$

This equation describes the physically measurable change of strain in the time interval from t_0 to t as a sum of three fictitious strains: First the unrestrained creep strain caused by the abrupt stress change at t_0 which is furthermore called primary creep. Second the elastic strain and third the unrestrained creep strain caused by the steady change of stress in the time interval from t_0 to t. The third part as well as the steady stress change itself is caused by primary creep and is therefore called secondary creep.

Two different ways of time dependent analysis of structural concrete may be adopted simply by transforming eq. (1). Furthermore the first of them is called equivalent-stiffness method. It requires the following conversion of eq. (1):

$$\Delta\sigma_c(t,t_0) = \frac{E_c}{1 + \chi(t,t_0)\ \varphi(t,t_0)}\left[\Delta\varepsilon(t,t_0) - \frac{\sigma_c(t_0)}{E_c}\varphi(t,t_0)\right]$$

This equation may be written as

$$\Delta\sigma_c(t,t_0) = E_{AAEM}\ \Delta\varepsilon(t,t_0) - E_{AAEM}\frac{\sigma_c(t_0)}{E_c}\varphi(t,t_0) \tag{2}$$

with the age adjusted effective modulus (AAEM, see [2])

$$E_{AAEM} = \frac{E_c}{1 + \chi(t,t_0)\ \varphi(t,t_0)} \tag{3}$$

The first part of eq. (2) corresponds entirely with Hooke's law and the second part contains the primary creep strain due to $\sigma_b(t_0)$ which can be treated like an imposed strain due to change of temperature. Thus the change of stress and strain in the time interval from t_0 to t may be calculated by the equivalent-stiffness method by considering primary creep as an imposed load and by modification of either the elastic modulus or more generally of the stiffness matrix of the structure in a way that secondary creep is enclosed. Then it is possible to calculate the change of stress and strain with conventional methods based on elastic theory.

The second method is called iterative method and may be explained with equation

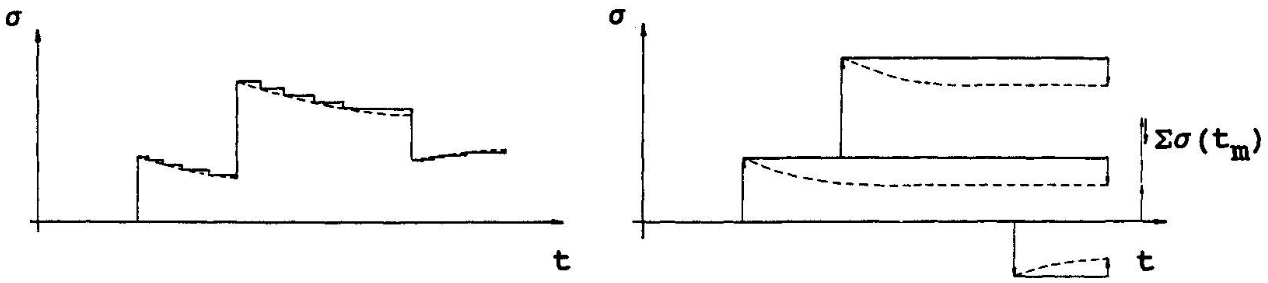
$$\Delta\sigma_c(t,t_0) = E_c\left[\Delta\varepsilon(t,t_0) - \frac{\sigma_c(t_0)}{E_c}\varphi(t,t_0) - \frac{\Delta\sigma_c(t,t_0)}{E_c}\chi(t,t_0)\varphi(t,t_0)\right] \qquad (4)$$

which can easily be obtained from eq. (1). The first part of eq. (4) is identical with Hooke's law, the second part contains primary creep and the third part secondary creep.

The first iteration may be done setting $\Delta\sigma_c(t,t_0) = 0$ on the right side of eq. (4). Thus changes of stress and strain caused by primary creep as external load in the time step from t_0 to t can approximately be calculated with conventional methods based on elastic theory. Then the third part of eq. (4) can be estimated with $\Delta\sigma_c(t,t_0)$ taken from the results of the first iteration step. The second iteration step follows with an improved external load containing now the sum of primary and secondary creep. Thus it is possible to improve the change of stress and strain and the load including secondary creep iteratively. Zienkiewicz [3] described this method already but he didn't take $\chi(t,t_0)$ into consideration. Therefore he had to limit the method to short time intervals with $\chi(t,t_0) \approx 1$ [4] what is not generally necessary.

3. SUPERIMPOSING OF SEVERAL ABRUPT LOADINGS

The superposition of several abrupt loads is easily possible if any step-by-step method is used. In connection with the algebraic stress-strain relation of Trost [1] or equations (2) or (4) several abrupt loads have to be treated separately according figure 1b and summed up for time t in question. If stress or strain of any other time is needed the whole calculation has to be carried out again.
These difficulties may be overcome by a combination of both methods using the incremental structure of the step-by-step method and the long time steps of Trost's method. Incremental structure means that the time axis is subdivided in intervals of any length and that the stress history is evaluated step after step from the beginning.



a) step-by-step method b) algebraic relation (Trost)
Fig.1 Superposition of several loads according to different methods

Limits of the time intervals should be set when abrupt loadings are implemented, when the cross section or the restraint conditions are modified and when values of stress or strain are needed. This means that the length of time intervals is optional - short or long.

4. INCREMENTAL METHOD WITH LONG TIME STEPS

An algebraic stress-strain-relation for the time interval from t_{m-1} to t_m which is needed for the incremental method was defined in [4]. It can be obtained from the integral equation (see [1]) by formulating the strain at t_m and subtracting the strain at t_{m-1} from it. Introducing the coefficient $\chi(t_m,t_{m-1})$ (exactly conform to the relaxation coefficient of Trost $\chi(t,t_0)$), the aging coefficient for the elastic modulus $k_E(t_m,t_{m-1})$ and the incremental aging coefficient for primary creep $\chi(t_m-t_{m-1},t_j)$ the remaining integrals can be transformed to algebraic expressions in analogy to Trost's algebraic relation.
The resulting constitutive law for uniaxial stress runs as follows

$$\Delta\varepsilon_c(t_m,t_{m-1}) = \Delta\varepsilon_{0,\varphi}(t_m,t_{m-1}) + \Delta\varepsilon_{0,S}(t_m,t_{m-1}) + \Delta\varepsilon_{0,T}(t_m,t_{m-1})$$
$$+ \frac{\Delta\sigma_{c\varphi}(t_m,t_{m-1})}{E_c} \cdot \left[\frac{E_c\ k_E(t_m,t_{m-1})}{E_c(t_{m-1})} + \rho(t_m,t_{m-1})\ \varphi(t_m,t_{m-1}) \right] \quad (5)$$

with the primary creep

$$\Delta\varepsilon_{0,\varphi}(t_m,t_{m-1}) = \sum_{j=1}^{m-1} \frac{\Delta\sigma_{cL}(t_j)}{E_c} \cdot [\varphi(t_m,t_j) - \varphi(t_{m-1},t_j)]$$
$$+ \sum_{j=1}^{m-2} \frac{\Delta\sigma_{c\varphi}(t_{j+1},t_j)}{E_c}\ \rho(t_m-t_{m-1},t_j)\ [\varphi(t_m,t_j) - \varphi(t_{m-1},t_j)] \quad (6)$$

and the unrestrained changes of shrinkage strain $\Delta\varepsilon_{0,S}(t_m,t_{m-1})$ and temperature strain $\Delta\varepsilon_{0,T}(t_m,t_{m-1})$ in the time interval from t_{m-1} to

t_m. Eq. (6) describes the unrestrained creep strain caused by changes of stress in the past, subdivided into abrupt changes $\Delta\sigma_{cL}(t_j)$ and steady changes $\Delta\sigma_{c\varphi}(t_{j+1},t_j)$. These steady changes took place during the time interval from t_j to t_{j+1} and do not exactly fit to the loading age t_j in eq. (6), but the incremental aging coefficient $\chi(t_m-t_{m-1},t_j)$ acts as a correction factor in this case. It's approximate value is 1,0 [4], differing from the well known average value of $\chi(t_m,t_{m-1})$ = 0,8 for the aging coefficient defined by Trost [1].

In consequence of the analogy between equations (5) and (1) the equivalent-stiffness and the iterative method may both be used in connection with the incremental constitutive law for the analysis of time-dependent effects in structural concrete. The first condition is that primary and secondary creep are strictly separated like in eq. (5) and treated properly as set forth in chapter 2.

5. CONCLUSIONS

Any algorithm or computer program based on elastic theory may be used for the analysis of time-dependent effects in structural concrete if quasi-elastic methods in connection with a suitable constitutive law are used. An incremental stress-strain-time relation like eq. (5) renders long time steps possible and reduces the necessity to store a lot of values in connection with short computing time. Some practical examples dealing with loss of prestress, stress redistribution in an inhomogenous cross section and bridges build in sections can be found in [4].

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Time-Dependent Behaviour of Prestressed Concrete Structures

Comportement différé des structures précontraintes

Zeitabhängiges Verhalten von Spannbetonkonstruktionen

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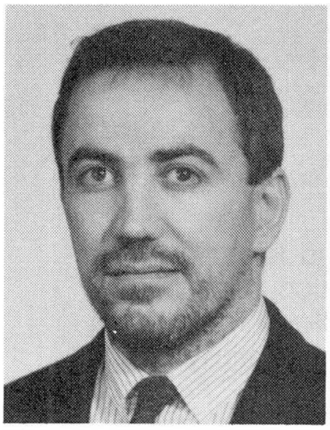

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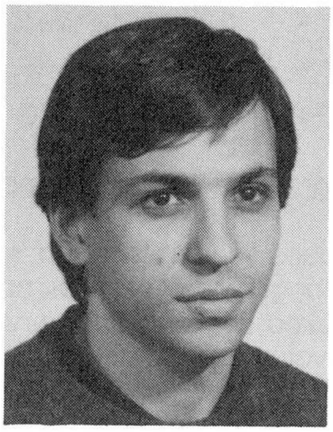

Alejandro Perez Caldentey, born in 1964, graduated as a Civil Engineer from the Polytechnical University of Madrid in 1989 and is now a Ph.D. candidate.

SUMMARY

This paper describes a model which permits the analysis of the time-dependent behaviour non-cracked prestressed concrete structures under service loads. The influence of the quality and reliability of the input data on the accuracy of the results obtained is stressed. Results obtained with the model are then compared to experimental data. Finally, from the comparison with experimental results it is concluded that such a model is a valuable computational instrument once its limitations are recognized.

RÉSUMÉ

Le modèle décrit permet l'analyse du comportement différé de structures en béton précontraint non-fissurées à l'état de service. L'accent est particulièrement mis sur la qualité des données introduites et sur la fiabilité des résultats obtenus, qui sont ensuite comparés aux données expérimentales. Un modèle comme celui présenté ne peut être utile qu'à condition de tenir compte de ses limites.

ZUSAMMENFASSUNG

Dieser Bericht beschreibt ein Modell für die Analyse des zeitabhängigen Verhaltens ungerissener Spannbetonkonstruktionen unter Gebrauchslast. Der Einfluss der Qualität und der Zuverlässigkeit der Daten auf die Präzision der Ergebnisse wird besonders hervorgehoben. Die Ergebnisse aus der Theorie und den Versuchen werden dann verglichen. Dieser Vergleich führt zu dem Ergebnis, dass dieses Modell, unter Berücksichtigung seiner Einschränkungen, gültig ist.

1. INTRODUCTION

When dealing with singular structures (cable-stayes bridges, cantilever construction, bridges built by phases) it is important to evaluate with good accuracy the evolution with time of strains and stresses. This sort of problems cannot usually be tackled in a simplified manner and therefore requires the use of more complete models which take into account all the phenomena involved.

This paper describes a model which permits the analysis of the behaviour with time of evolutive non-cracked prestressed concrete structures for service loads. The influence of the quality and reliability of the input data on the accuracy of the results obtained is stressed. Results obtained with the model are then compared to experimental data. Finally, from the comparison with experimental results it is concluded that such a model is a valuable computational instrument once its limitations are taken into account.

2. DESCRIPTION OF THE PROPOSED MODEL

2.1 Stress-Strain Diagram and Time-Dependent Behaviour of Concrete

For instantaneous loads, concrete is considered as an elastic and linear material characterised by its longitudinal modulus of elasticity. This assumption is justified by considering non-cracked sections and service loads. The model allows for different formulations of the evolution with time of the longitudinal modulus of elasticity.

Time dependent behaviour of concrete due to creep and shrinkage is handled according to the general approach proposed by CEB [1]. In order to avoid storing the whole stress history of the structure an alternate method, based on a Dirichlet series aproximation of the creep coefficient curves as proposed by Zienckiewicz and Watson [2] has also been implemented.

Several formulations for both creep coefficient and shrinkage strain are supported, including methods proposed by ACI[3] and CEB[4]. Experimental data may also be used through a Dirichlet series approximation.

2.2 Stress-Strain Diagram and Relaxation of Steel

Both prestressed and non-prestressed steels are considered as linear and elastic materials for both tension and compressive instantaneous loads. Again this assumption is valid while considering service loads only.

Relaxation of steel in prestressed concrete members occurs at variable length because of interaction with creep and shrinkage of concrete. The model uses a formulation explained in Fig. 1 and reference [5] which takes this into account. This method proposes an analytical formulation for relaxation at constant length at different initial stress levels and a procedure to represent the evolution of stress with time.

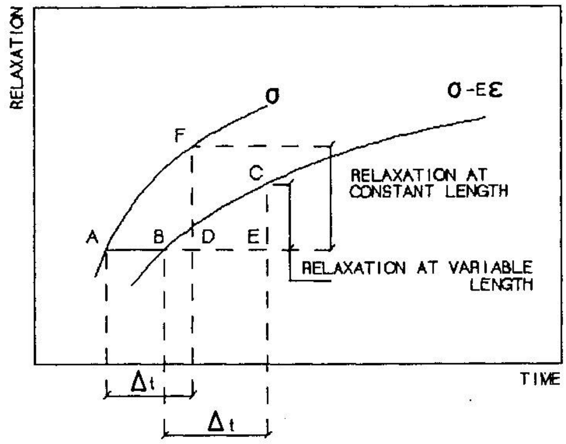

Fig. 1 Relaxation at variable length

2.3 Concrete-Steel Bond

Perfect bond between concrete and steel bars is assumed, both for prestressed and non-prestressed steel.

2.4 Analysis

The model allows changes in the geometry of both the structure (new nodes and bars) and of the different sections (composite sections), and can therefore simulate an evolutive construction process.

For each construction phase a different structure is considered. The structures are divided into a sufficient amount of sections. At each time interval incremental strains due to creep and shrinkage of concrete and relaxation of steel are determined for each section. These strains are then introduced on the structure and new bending moments and axial forces due to these time-dependent phenomena are obtained.

Strains, curvatures, bending moments and axial forces are referred to the same fiber throughout the analysis. This fiber does not necesarily coincide with the center of gravity of the sections which is itself subject to change with time. Using only one reference fiber makes stresses and strains which occur at different times additive without any need for transformation.

2.5 About the Data Required by the Proposed Model

As pointed out before the major drawback of a general model like the one proposed in this paper is the large amount of data required. Furthermore the results obtained will be no more precise than the data entered. It is therefore important to be aware of what data is necessary in order to measure the appropriate variables at the proper time if possible and, if not, to evaluate the possible errors due to this lack of precise data.

In reference to concrete, values are needed for:

- Evolution with time of the longitudinal modulus of elasticity. Analysis is specially sensible to this variable and care should be taken in determining its value.
- Creep coefficient.
- Shrinkage strain.

In reference to steel, data required includes:

- Modulus of elasticity of the different steel types used.
- Parameters defining the relaxation of prestessing steel.

3. COMPARISON WITH EXPERIMENTAL RESULTS

In this section reference is made to experimental results obtained at the E.T.S.I. Caminos, Canales y Puertos of the Polytechnical University of Madrid. These results are detailed in reference [6]. A more detailed comparison can be found in reference [7] including other experimental data dealing with a composite section and a two span continuous beam.

3.1 Available Experimental Data vs Required Data

As mentioned in paragraph 2.5, it is important to take into account the quality of the available data. The Corres-Rodríguez tests were very careful in this sense since they were designed with the purpose of testing an analytical model for determined prestressed concrete structures. All required data is therefore available.

Parallel to measuring strains on the beams themselves, a series of complementary tests were carried out. For concrete these included compression and tension strength, evolution of the modulus of elasticity as well as creep and shrinkage tests. Tension strength tests were also carried out for both prestressing and non-prestressing steels. Finally, relaxation of prestessed steel at constant length was measured for three different initial stresses.

3.2 Data for the Model

3.2.1 Evolution with time of the Modulus of Elasticity of Concrete

Because of the large amount of data available a particular model adjusted by means of the least squares method was used (see Equation (1)).

$$E_c = E_{c28}(2.5t/(t+42))^{0.10} \quad (1)$$

In this equation E_{c28} is the modulus of elasticity of concrete at 28 days and t is the age of concrete in days.

3.2.2 Creep Coefficient and Shrinkage Strain

Experimental results from creep and shrinkage tests were compared to analytical results given by ACI[3] and the former CEB[4] models. For the creep coefficient excellent agreement was found between experimental data and the model proposed by ACI while the 1978 CEB model was found to consistently overestimate creep.

For shrinkage strain neither ACI nor CEB models provided a close fit to experimental results although differences were not truly significant.

In view of these results the ACI models were used to represent both phenomena.

3.2.3 Relaxation of Steel

From the constant length relaxation tests enough information was provided in order to adjust by the least squares method the parameters required by the Elices-Sanchez-Gálvez model [5] mentioned in paragraph 2.2.

3.3 Discussion of Results

Figures 2 and 3 compare respectively the experimental and theoretical evolution of the curvature and deflection obtained at midspan of beams 1 and 2. As can be seen good agreement is obtained by application of the analytical model in both cases.

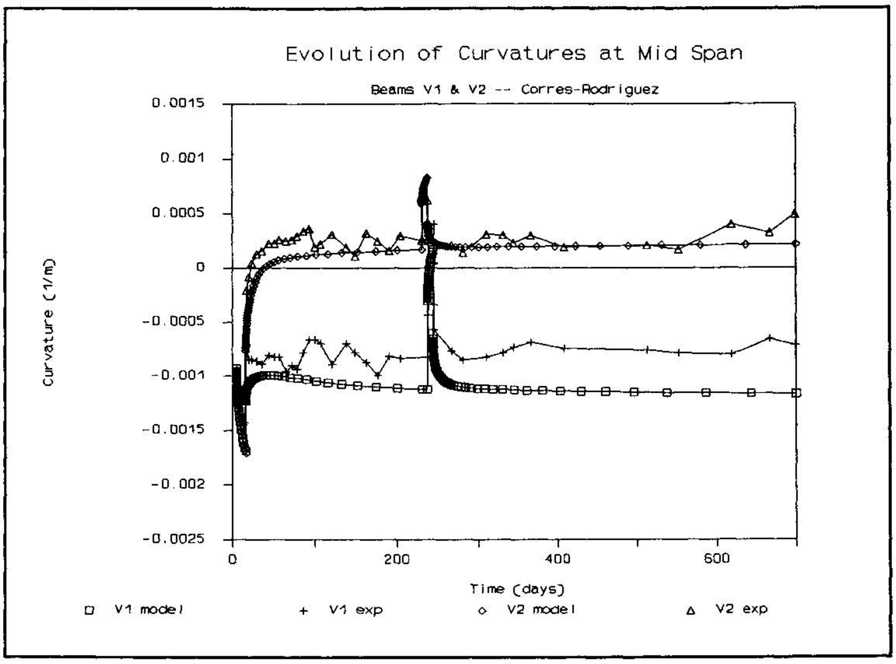

Fig. 2 Evolution of mid-span curvatures

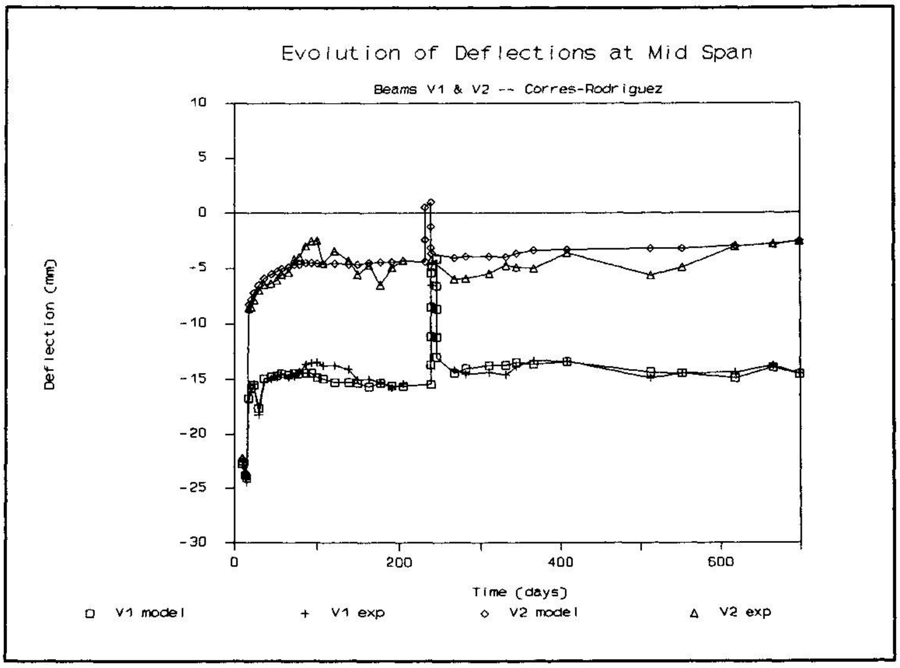

Fig. 3 Evolution of mid-span defelections

4. CONCLUSION

In this paper a flexible and general analytical model for the study of evolutive non-cracked prestressed concrete structures subject to service loads has been outlined. This model implements, as an alternative to Dirichlet series approximation of the creep coefficient, the general method proposed by CEB [1] and uses special procedure to represent relaxation at variable length.

Results obtained with the model have been compared with experimental tests obtaining good agreement.

As discussed above, this sort of model requires a large amount of data which in most cases cannot be accurately determined through analytical formulae and which greatly influence final results. It is therefore important to define the degree of precision required.

In this line it can be concluded that such a model is an ideal tool for the analysis in the service limit state of prestressed concrete structures if accurate data is available. In design, the use of such a model requires realistic estimates of creep coefficients, shrinkage strains and modulus of elasticity of concrete. In such conditions the accuracy provided by the model should be sufficient in most cases to point out the main aspects of the behaviour of the structures. Finally, in construction, experimental tests must be carried out to measure "in situ" the more important variables used by the model in order to obtain more accurate results.

5. ACKNOWLEDGEMENTS

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Deformations of T-shaped Beams under Sustained Loads

Déformations des poutres en T sous charge de longue durée

Durchbiegungen von Plattenbalken unter andauernder Belastung

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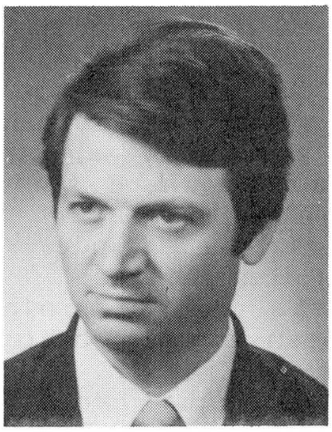

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SUMMARY

Experiments show that a considerable growth of strains occurs in long-term loaded reinforced concrete structures, as result of time-dependent changes of properties of concrete. Results of calculations were compared with both results from a finite element model and with experimental values.

RÉSUMÉ

Un modèle de calcul décrivant les poutres en T en béton armé sous charges de longue durée est présenté. Les résultats expérimentaux provenant d'un modèle spécialement développé ont été comparés à ceux obtenus analytiquement grâce à la méthode des éléments finis.

ZUSAMMENFASSUNG

In der Arbeit wird ein Berechnungsmodell für Stahlbetonplattenbalken unter dauernd wirkender Belastung formuliert. Die Messergebnisse werden mit Ergebnissen aus FEM-Berechnungen und aus Berechnungen anhand eines eigenen Modells verglichen.

1. INTRODUCTION

Experiments indicate that in the long term loaded reinforced concrete constructions, as a result of changes in time of properties of concrete, there occurs a considerable growth of strains in concrete.
The increase of the strains in concrete,leads to the deformation of the whole element and to the increase in deflection or additionally to the increase in the width of the opening of the cracks. The stresses also vary and therefore a strain in the materials may increase.
These phenomena must be predicted and taken into account during planning stage, and therefore it is necessary to express them analytically in order to make practical calculations possible.
In particular this situation applies to the flexed T-shape reinforced concrete beams, where the concrete closely cooperates with the steel during load transfer. Calculations of deformations for reinforced concrete beams under a long-term loading may be divided into two essentioal stages.
The first stage consists of determining the state of stresses and strains in freely-chosen cross-section of concrete beam. In this case the problem reduces itself to allocating the stresses and strains at any point of the cross section that,s consider at any point of time with regard to the most heterogeneous factors.
The second stage consists of observing the element as a whole. Here we may take into account rigidity, deflection, spacing as well as the width of the opening of the cracks. There we make use of considerations carried out at the first stage and therefore the assumptions made in the first stage will influence the obtained results obtained in an important way. It makes that correct solution of the problem of defining the changes of stresses and strains in any cross-section of concrete operating under long-term loading is the basic task having aa influence on all considerations.
In the present paper it has been decided to make an analysis of this problem on the basis of some new assumptions and existing solutions and to provide some means of formulation it comprehensively.
The proposed standardization of the method of presenting equilibrium equations in different phases and generalization of the method of presenting the dependence between creep stresses and strains allowed us to build the analytical model of problem and examine exactly the state of stresses and strains in reinforced concrete beam depending on the time duration of the load. To get a practical results of calculations there was made the computer program, which allowed to verify an influence of many different factors on the state of stresses and strains in the T-shape reinforced concrete beam.
The results of upper proposed traditional analytical calculations were compared with the result of calculations of the model built of structural concrete.

2. ASSUMPTIONS

The assumptions, which is necessary to make for the examination of the present problem, may be divided into 3 groups.
The first group is concerned with the method of making

allowances in the calculations for the rheological properties of concrete; shrinkage, creep or relaxation, under the influence of which changes in the strain of concrete are in time brought about. Customarily, making allowances for such influences has become generally accepted by means creep function, which in the general case may be shown as follows:

$$\varepsilon_c(t,\tau)= \frac{\sigma_c(\tau)}{E_c(\tau)} +F[\sigma_c(\tau)\delta(t,\tau)]+\int_\tau^t \frac{1}{E_c(\tau)}\frac{\partial\sigma_c(\tau)}{\partial\tau}d\tau +\int_\tau^t \frac{\partial F[\sigma_c(\tau)]}{\partial\tau} C(t,\tau)d\tau. \qquad (1)$$

In the presented paper thedependence (1) shown abovewas adopted as basic to the consideration. The second group of assumptions consists of assumptions concerning the stress-deformation dependence. The analysis of this problem carried out by Szechiński [1,2] allows us to state that until now the problem has been limited to examinations of cross-sections of concrete at different phases of operation, accepting the most varying shapes of the stress diagram. All these cases may be generalized so as to obtain the shape of the diagram as proposed by Szechiński [1,2] in the figure 1.

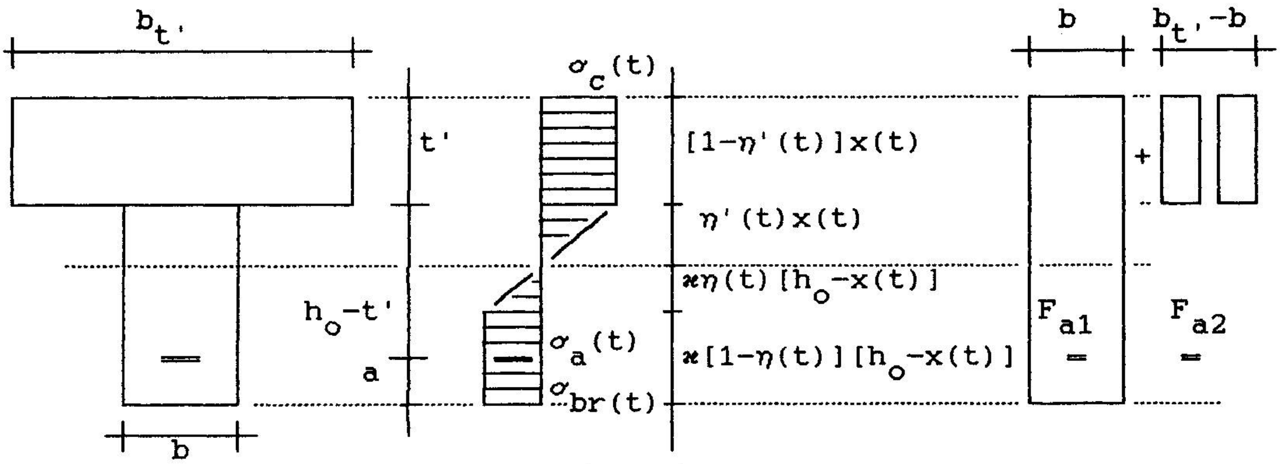


fig. 1
Assumptions about the concrete cross-section.

Acceptance of this generalization allows a completely universal notation of equilibrium equations,valid for every phase of operation of the concrete cross-section.
The third group of assumptions is concerned with deformations in the cross-section of the concrete beam. Customarily, it is accepted that the cross-section before and after deformation remains flat.

3 ANALYSIS OF THE CONCRETE CROSS-SECTION.

Accepting the assumptions given in previous section, fig.1, we get the following equilibrium equations.

$$\Sigma X = 0, \; -\frac{\sigma_c(t)}{2}\eta_o'(t)\, x(t)\, b + b\, \sigma_c(t)\, x(t) = \sigma_{ct}(t)\, [h_o - x(t)]b +$$

$$+ \varkappa\eta(t)\frac{h_o - x(t)}{} \sigma_{ct}(t)\, b + \sigma_a(t)\, F_{a1},$$

$$\sigma_c(t)\, t'\, (b_{t'} - b) = \sigma_a\, (t)\, F_{a1}. \qquad (2)$$

$$\Sigma M = M_1+M_2,$$

$$M_1= \sigma_c(t)bx[(h_o-x(t)/2] -\sigma_b(t)\eta'(t)x(t)\frac{b}{2}[h_o- x(t) +\eta'(t)\frac{x(t)}{3}] +$$

$$-\sigma_{ct}(t)[h_o-x(t)]^2 b/2 +\sigma_{ct}(t)\varkappa\eta(t)[h_o-x(t)b/2[h_o-x(t)-\varkappa\eta(t)x(t)/3],$$

$$M_2 = (b_t'-b)\ \sigma_c(t)t'(h_o - t'/2). \qquad (3)$$

Taking into consideration dependence (1) and geometrical dependence, that is assumption 3 from the previous section, the integral equation (4) was received.

$$\varepsilon_c(t,\tau)= \sigma_c(\tau)\delta(t,\tau)+\beta'\sigma_c^2(\tau)c(t,\tau)+\int_\tau^t\frac{1}{E(\tau)}\frac{\partial\sigma_c(\tau)}{\partial\tau}d\tau+\int_\tau^t c(t,\tau)\frac{\partial[\sigma_c(\tau)+\beta'\sigma_c(\tau)]}{\partial\tau}d\tau. \qquad (4)$$

The eq.(4) may be simplified to linear form assuming $\beta'= 0$. Furthermore eq. (4) was simplified to the algebraic form, from whihch the values of x(t) were calculated and further from equilibrium equations (2) values $\sigma(t)$, $\sigma_{ct}(t)$, $\sigma_a(t)$, $\varepsilon_c(t)$, $\varepsilon_a(t)$.

4. CALCULATING THE DEFLECTIONS.

From the geometrical dependences, for intermediate loads we may receive

$$\frac{M}{B_o} = \frac{\varepsilon_{co}+\frac{\varepsilon_{co}(1-\xi_o)}{\xi_o}}{h_o} = \frac{\varepsilon_{co}}{\xi_o h_o} = \frac{\sigma_{co}}{E_{co}\xi_o h_o} = \frac{\sigma_{ao}}{E_a (1-\xi_o)h_o}, \qquad (5)$$

It means that

$$B_o= B(t=0)= \frac{M\ E_a(1-\xi o)h_o}{\sigma_{ao}} \quad \text{or} \quad B(t) = \frac{M\ E_a[1-\xi(t)]h_o}{\sigma_a(t)}, \qquad (6)$$

From the equilibrium equations and after solving eq.(4) we may receive σ (t) that means we may get the cross-section stiffnes B(t).Furthermore taking into consideration the fact that stiffness is changing on the beam length thre was assumed according to [3] following stiffness function

$$B_b(t,\zeta) = f[B_{Ii}(t),m_{cr}] + h\ [B_{Ii}(t),B_{IIi}(t),m_{cr}], \qquad (7)$$

where

- m_{cr}= M/Mcr – relation between the moment on beam and the cracking moment,
- B_{Ii}, B_{IIi} - stiffness of beam cross-sections in different phases of work,
- B_b– stiffness function on the beam length.

Using the curvature equation and dependence (7) allows to calculate displacements of the beam due to the time flow.

5. FINITE ELEMENT MODEL OF THE BEAM.

Using the FEM program system [4], the computer model of the beam shown in fig. 2, was built, as it is shown in fig.3a. The three-dimensional solid elements, as shown in fig 3b, were used to describe concrete and reinforcement. Concrete and reinforcement were represented by different values of the modulus of elasticity, weight and the coefficient ν.

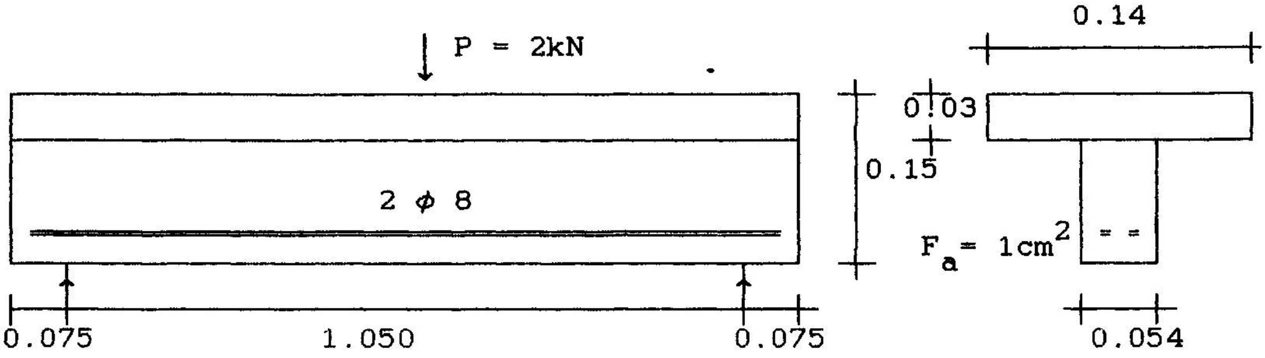
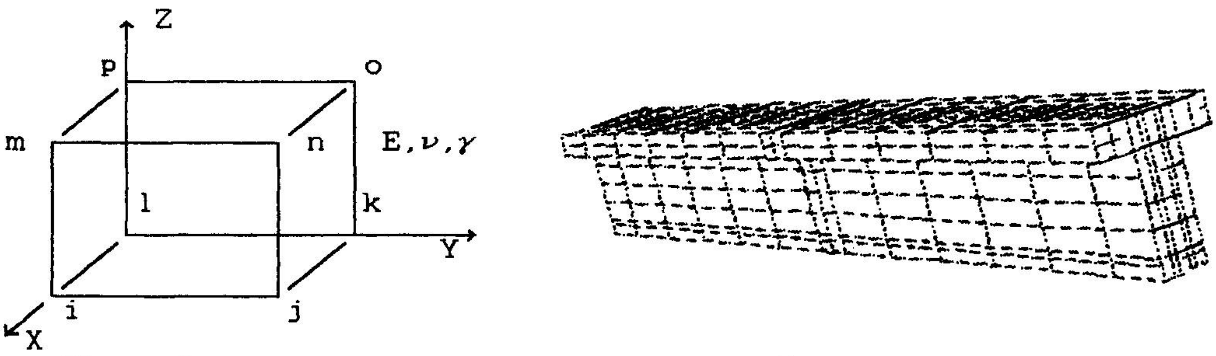


fig.2

Construction of the sample beam



a. Three-dimensional isotropic brick element

b. Nodal mesh on the beam

fig.3 a,b,

Finite Element Method model of the beam

This model, because of program limitations, was examined only under the intermedaite load (t=0). The results compared with results of traditional calculations and experimental results are shown in fig. 4,5.

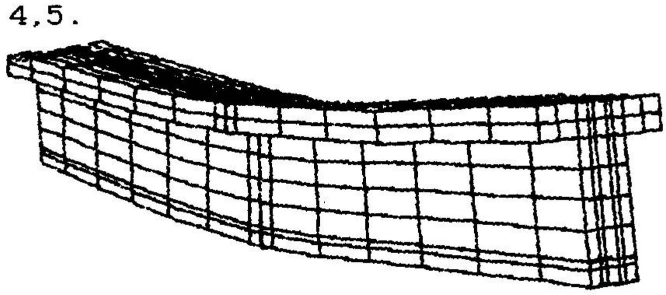
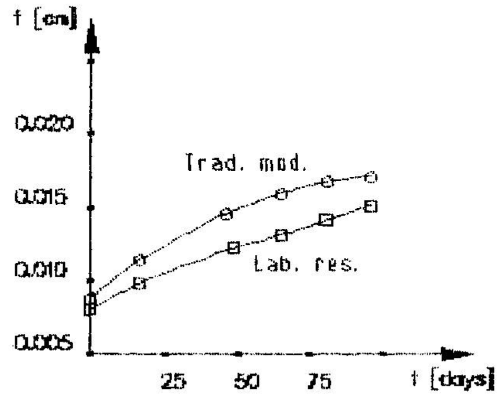


fig.4

Displacements of the beam. FEM model.

Time (days)	Maximum displacements		
	Trad. mod.	FEM mod.	Lab. res.
0	0.00824	0.00817	0.0078
16	0.01180		0.0095
32	0.01410		0.0110
48	0.01540		0.0124
64	0.01581		0.0128
80	0.01638		0.0134
96	0.01700		0.0144



Changes of displacements in time. Traditional model. Laboratory research.

fig.5

Comparison of the results

6.CONCLUSIONS.

The results of structure analysis presented above show advantages and disadvantages of the used methods. Traditional method based on differential equations allows to find displacements at every point of beam length and at every time, but it doesn'tgive possibility to describe influence of cracks. There is no problem in modeling cracks in FEM model but the lack of time elements precluded to make analysis of time effects. Results of calculations in both methods were similar and were comparable with the results of the labaratory research.

During the shown process of the structural analysis it is possible to take into account many new factors and observe a structure response. Moreover it is possible to analyse the influence of a range of features of structure, e.g. material, shape or ways of reinforcing.

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Long-Term Strains of Compression Elements in Tall Buildings

Contraintes à long terme apparaissant dans les éléments comprimés des bâtiments élevés

Langzeitverformungen gedrückter Elemente in Hochhäusern

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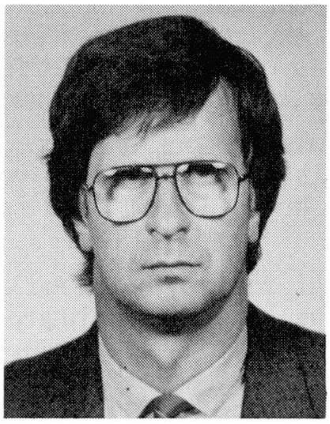

Stefan Gramblička, born 1948, received his M.Sc. and Ph.D. from the Slovak TU, Bratislava. His experience includes design, teaching and research of reinforced concrete and composite steel-concrete structures. He is the author of a number of original research works on effects of creep and shrinkage of concrete.

SUMMARY

This paper presents a theoretical solution for the determination of strains due to creep and shrinkage of concrete, in compression elements of tall buildings. The analysis of the creep and shrinkage effects takes into account the variability in percentages of cross-section reinforcement as well as the actual variation of the increasing normal forces. On the site of the Press Centre in Bratislava, selected compression elements were measured over a two year period. At the same time, measurements were made of unloaded specimens. The author presents an analysis of the measured strain values.

RÉSUMÉ

La solution théorique concernant la détermination des contraintes dues au fluage et au retrait est présentée, dans le cas des éléments comprimés de bâtiments élevés. Cette analyse tient compte des différents pourcentages d'armature dans les sections comprimées, ainsi que de l'augmentation de l'effort normal dans le temps. Pendant plus de deux ans, des mesures ont été effectuées sur des éléments comprimés sélectionnés du centre de presse de Bratislava. D'autres tests étaient menés conjointement sur des éléments non-chargés. Une analyse des résultats obtenus est présentée.

ZUSAMMENFASSUNG

Der Beitrag behandelt die theoretische Ermittlung von Betonverformungen infolge Kriechens und Schwindens in gedrückten Elementen von Hochhäusern. Bei der Analyse der Kriech- und Schwindauswirkungen werden verschiedene Bewehrungsgrade der gedrückten Elemente sowie verschiedene Annahmen über die zeitliche Belastungserhöhung untersucht. Beim Bau des Pressezentrums in Bratislava wurden im Zeitraum von 2 Jahren Langzeitverformungen ausgewählter gedrückter Elemente gemessen. Zum Vergleich wurden im gleichen Zeitraum Verformungen an unbelasteten Elementen ermittelt. Der Autor behandelt die Analyse der gemessenen Verformungen.

1. INTRODUCTION

Within the last years a large number of tall buildings exceeding 30 stories have been built. Such buildings of great height are very sensitive to cumulative differential length changes of their vertical elements. One of the influences affecting these changes in reinforced concrete structures are long-term strains due to volume changes, namely to creep and shrinkage of concrete which depend on a considerable number of influances. The overall contraction of the vertical load-carrying elements is the sum of a number of partial changes. The determination of elastic strains due to load does not present any difficulties. Therefore, in the paper we shall concentrate on the analysis and possibilities of determination of long-term strains which, under certain conditions, may exceed elastic strains several times. The extent of the creep and shrinkage of concrete is influenced by a variety of factors such as environmental effects, age of concrete during the exposure of the member to the load, concrete grade, reinforcement percentage, etc.

Prof. Bruggeling [1] presents the basic introduction of time-dependent effects. We would like to support his note that it is very important to understand when and why time-dependent effects are of importance for the behaviour and on the durability of a structure.

Traditionally, the effect of creep, shrinkage and temperature is considered in horizontal structures, such as long span bridges. These effects are usually neglected in multistory concrete buildings since, in the past, such structures seldom exceeded 20 stories. A number of recent ultra high rise buildings built without consideration of creep, shrinkage, and temperature effect in the vertical elements have developed partition distress, as well as structural overstress in horizontal elements. It is necessary for the structural engineer to consider the various differential movements and to develop acceptable structural, as well as architectural details, for the satisfactory performance of the building. We shall try to solve the specific problem of deformations which will complete the part 4.1 of the paper [2] .

2. ANALYSIS OF CONCRETE CREEP

In view of the calculation of effects of concrete creep on volume changes, the vertical elements in tall buildings can be characterized by several main factors:
- variability of reinforcement of the cross-sections of vertical elements,
- curve of the increase of load over time, depending on the progress in the construction of the building,
- stress due primarily to compressive forces.

Besides considering the effects of volume changes on length changes (shortening of the vertical elements in tall buildings), it is also necessary to account for these changes with respect to the distribution of the stresses over the cross-section of the member. The higher percentage of reinforcement reduces the increment of compressive stress acting on the reinforcement and increases the increment of tensile stress acting on concrete, adversely affecting the load-carrying capacity of the materials used.

This effect in combination with varying curves of possible load increases demonstrates Fig.1. In view of the extent of strain due to concrete creep it may be noted that increased percentage of reinforcement reduces the extent of deformation due to the creep of concrete (Fig.2).

The real course of load increasing over time can, in essence, be assumed for each element of the tall building already at the design stage. In general, this course can be modelled as shown in Fig.3. In the calculation of the effects of concrete creep, this course must be replaced by an appropriate function. The simplest solution is the one assuming a constant load increase, immediately from the onset of loading. However, such calculation is very unrealistic and can only provide very distorted date.

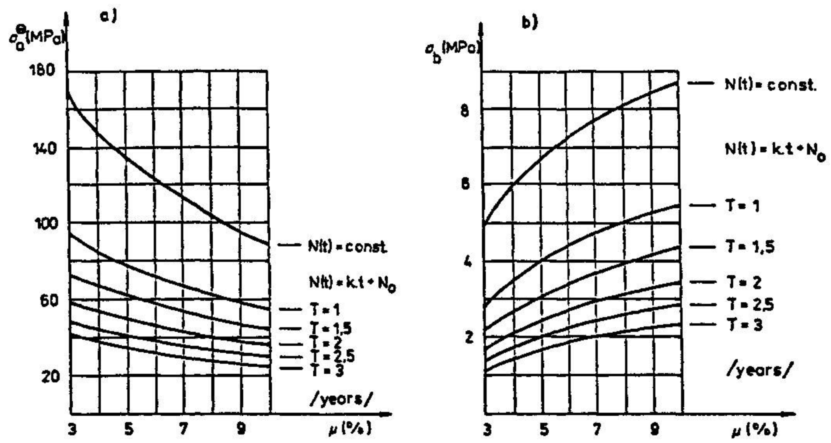

Fig. 1 Stress in the reinforcement (a) and stress in the concrete (b) due to concrete creep at varying percentages of cross-section reinforcement, accounting for the differences in the duration of building construction time

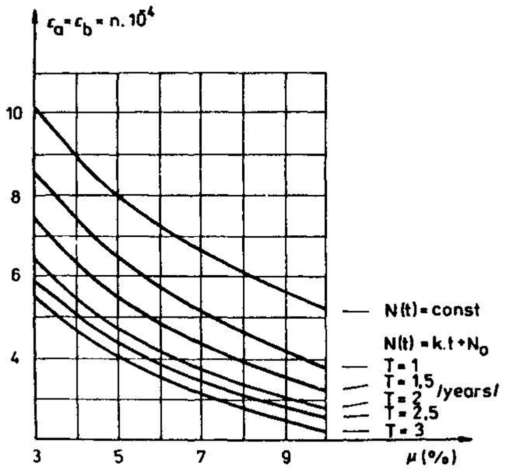

Fig. 2 Dependence of the extent of deformations due to creep on the percentage of cross-section reinforcement, accounting for the differences in the duration of building construction time

The most suitable is the possibility of substituting a bilinear relationships for the course of load increase on the member where, in the first part, the course of loading is a linear function of time and, in the second part, it is constant. This is a relatively most accurate expression of the real course of the load. The decisive factor is the time of construction T which it is relatively easy to determine in practice. The following conclusions can be drawn from the solutions of concrete creep effect at varying substitutions for the course of loading (Fig.1):

- the longer is the construction time T , the lower are the increments of compressive stress from creep on the reinforcement, and the increments of tensile stress from creep on the concrete. A similar dependence also applies to the strain generated by the creep of concrete.

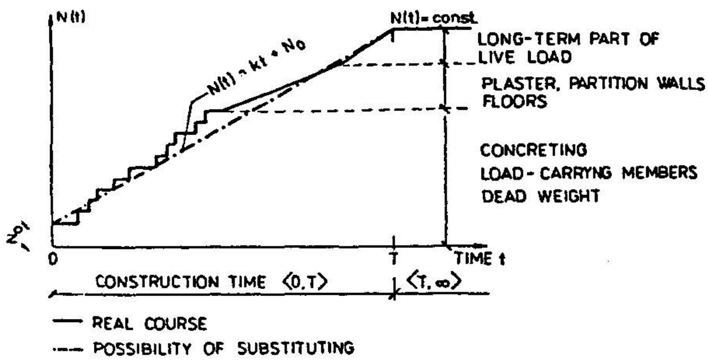

Fig. 3 The curve of normal force increase in the column of tall building

3. ANALYSIS OF SHRINKAGE EFFECTS

In relation to the effects of shrinkage, a higher percentage of reinforcement results in reducing the increment of compressive stress from shrinkage acting on the reinforcement and in increasing the increment of tensile stress acting on the concrete (Fig.4). Because the shrinkage is independent on the stress, it is also independent on the course of load increase. With respect to the strain, a higher percentage of reinforcement reduces the extent of strain from the shrinkage.

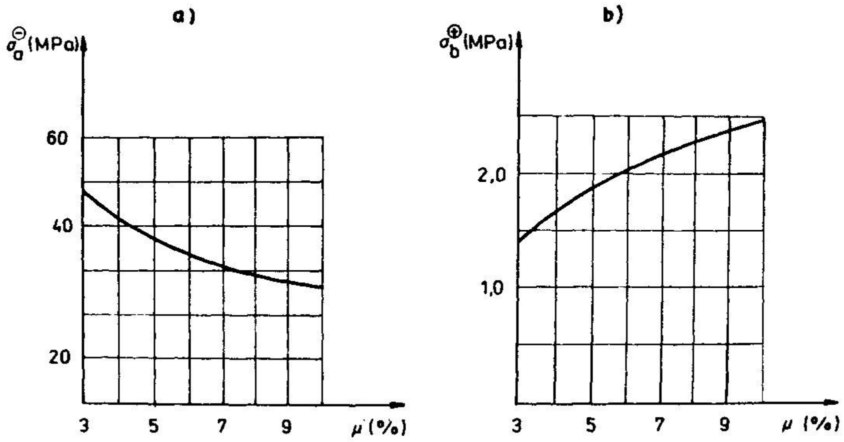

Fig. 4 Dependence of the stress in the reinforcement (a) and in the concrete (b) due to concrete shrinkage on the percentage of cross-section reinforcement

4. TALL BUILDING AND MEASUREMENT POINTS

The measurements of long-term strains of compression elements in the Press Centre tall building, Bratislava, may be listed, with regard to the investigated results, in the group of measurements whose conclusions offer a picture about the course of creep and shrinkage of concrete in a specific structure type. Due to this fact our attention was given to two structurally and statically important compression elements of the structure, i. e. load-carrying columns with a high percentage of reinforcement and gable walls with a low percentage of reinforcement.

The tall building has a height of 104 m. The measurements were conducted on the following floors (Fig.5): 4th floor, the measurement points being S1 to S4 columns and Š1 and Š2 walls, 11th floor, the measurement points being S5 and S6 columns and Š3 wall. S1 to S4 column dimensions were 1 400 x 700 mm (with welded I - section) and reinforcement 16 No 25 mm bars, having an overall reinforcement of 7,44 %. S5 and S6 column dimensions were 1 200 x 600 mm, the overall reinforcement 8,32 %. Wall thickness was 500 mm and the wall was reinforced with steel angles and reinforcing bars, the overall reinforcement being 1,15%. The selected structure points allowed only for overall strain measurements. These strains include a number of partial strains, i. e. strains due to tempe-

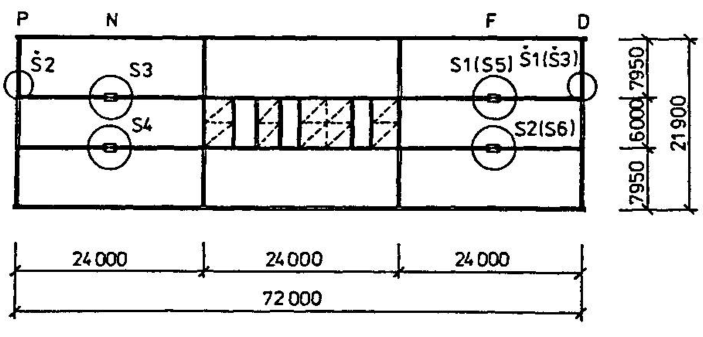

Fig. 5 Scheme of plan layout of tall building of Press Centre

rature changes, shrinkage namely due to changes in humidity, elastic strains due to load on a measured element, and strains due to creep of concrete.

5. THEORETICAL VALUES OF DEFORMATION

The growth of load of particular elements of the measured points in time was calculated according to the actual construction work sequence. Elastic strains were calculated from the assumption of central load of cross-sections of the measurement points. Modulus of elasticity of concrete in particular time was determined on the basic of laboratory results. Theoretical calculations of strains due to creep of concrete was conducted according to the e. g. Dishinger's theory of ageing. Differential equation of the 1st order with the right side will have the following form:

$$\frac{dN_{b\varphi}}{d\varphi} + \alpha N_{b\varphi} = (1-\alpha)\frac{dN(t)}{d\varphi} \qquad \text{where: } \alpha = \frac{A_a}{A_i} = \frac{A_a}{A_a + A_b/n} \tag{1}$$

The solution assumes the variability of the modulus of elasticity of concrete over time [3] :

$$E_{bN} = \frac{E_{b0}}{1+\varphi_t \psi_N} \qquad n_0 = \frac{E_a}{E_{b0}} \qquad n_N = \frac{E_a}{E_{bN}} \qquad \frac{n_N}{n_0} = 1 + \psi_N \varphi_t \tag{2}$$

where: E_{b0} - initial modulus of elasticity of concrete

E_{bN} - modulus of elasticity of concrete in time at axial force load

ψ_N - subsidiary creep coefficient

The differential equation (1) is solved in two intervals. In $\langle 0, T \rangle$ interval on the assumption that $N(t) = k \cdot t + N_0$ (Fig.3). After substitution

$$\frac{dN_{b\varphi}}{d\varphi} + \alpha N_{b\varphi} = (1-\alpha)\frac{0{,}625\,k}{\varphi_\infty - \varphi} \qquad I = \int_0^{\varphi} \frac{e^{\alpha\varphi}}{\varphi_\infty - \varphi}\, d\varphi \tag{3}$$

$$N_{b\varphi} = (1-\alpha)\, e^{-\alpha\varphi} N_0 + e^{-\alpha\varphi}(1-\alpha)\, 0{,}625\, k\, I \tag{4}$$

Subsidiary creep coefficient ψ_N^1 valid for interval $\langle 0, T \rangle$ is

$$\psi_N^1 = \frac{k\,t + N_0}{\varphi\alpha\, e^{-\alpha\varphi}(N_0 + 0{,}625\,k\,I)} - \frac{1}{\alpha\varphi} \tag{5}$$

k - coefficient of erection race, N_0 - initial value of normal force,
φ - creep coefficient in time t ,
ψ_N^2 shall be valid for the interval $\langle T, \infty \rangle$.

$$\psi_N^2 = \left[\frac{N}{(1-\alpha)e^{-\alpha\varphi_\infty}(N_0 + 0{,}625\,k\,I)} - 1\right]\frac{A_b}{A_a\varphi_\infty \cdot n_N^1} - \frac{1}{\varphi_\infty} \tag{6}$$

N - maximum value of normal force, φ_∞ - creep coefficient in time = ∞. The following is valid for the resultant value of the elasticity modulus ratios

$$n_{N\infty} = n_N^1 (1 + \psi_N^2 \varphi_\infty) \tag{7}$$

The calculation of strains due to creep is

$$\sigma_{b\infty} = \frac{1}{n_{N\infty}} \frac{N}{A_{i\infty}} \qquad \sigma_{a\infty} = \frac{N}{A_{i\infty}} \qquad A_{i\infty} = A_a + A_b / n_{N\infty} \tag{8}$$

$$\sigma_b^{creep} = \sigma_{b\infty} - \sigma_{b0} \qquad \sigma_a^{creep} = \sigma_{a\infty} - \sigma_{a0} \tag{9}$$

The calculation of strains due to shrinkage (restrained shrinkage) at uniaxial state of stress was conducted according to the following relation

$$\varepsilon_{t\tau_1} = \frac{\varepsilon_{st\tau_1}}{\mu\, n_{\tau_1}\, \varphi_{t\tau_1}} \left(1 - e^{-\alpha \tau_1 \varphi_{t\tau_1}}\right) \tag{10}$$

$\varepsilon_{st\tau_1}$ - relative deformation of concrete at free shrinkage at time t which started to appear at time τ_1 , n_{τ_1} - modular ration of reinforcement for the age of concrete at time τ_1 , $\varphi_{t\tau_1}$ - creep coefficient at time t for the load which started to act at time τ_1.

6. TEST AND MEASUREMENT RESULTS

Concrete compressive strenght and elasticity modulus tests in time were conducted to obtain material characteristics of concrete. Cubes and prisms were fabricated from identical concrete mixture and treated under conditions identical to those of the tall building. Temperature and humidity were measured by thermohydrographs located in the shed at the control non-loaded cubes and at the hole in the wall.

The measured deformations at the non-loaded plain concrete reference blocks include the effects of free shrinkage of concrete and temperature changes. Deformations due to temperature changes were calculated from the measured temperature values. Fig.6 shows plots of mean strains measured at two basic measurement points. A relatively good agreement can be stated when comparing the calculated values of deformations due to creep and shrinkage of concrete and elastic strains. The chosen method of calculation of the theoretical strain values due to creep and shrinkage of concrete for the given type of elements of the structure has been found as suitable and recommentable for use.

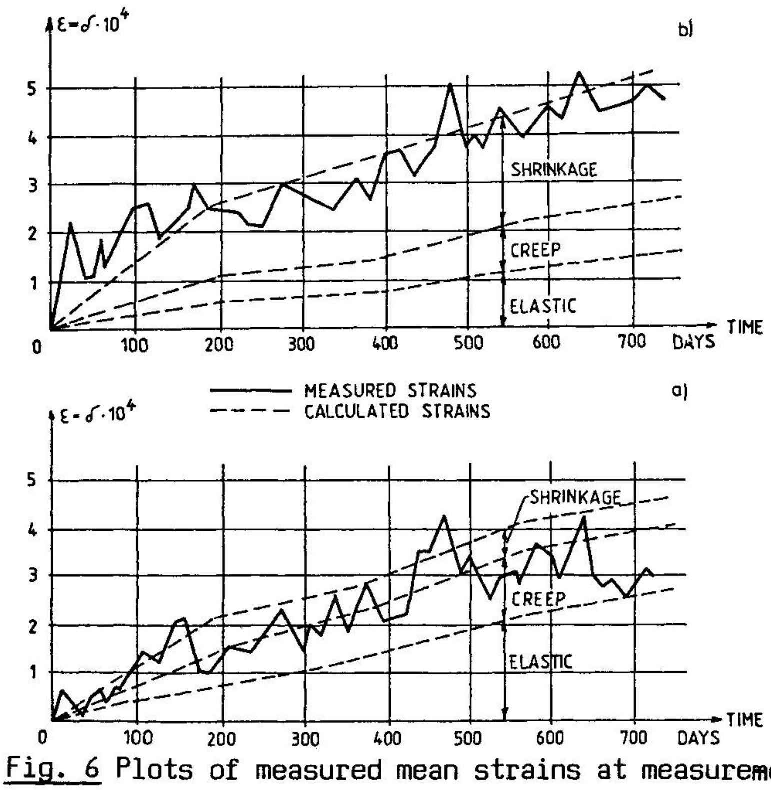

Fig. 6 Plots of measured mean strains at measurement points a/S1,S2,S3,S4 b/Š1,Š2

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Imposed Deformation and Cracking

Fissuration par déformation imposée

Rissbildung durch Zwang

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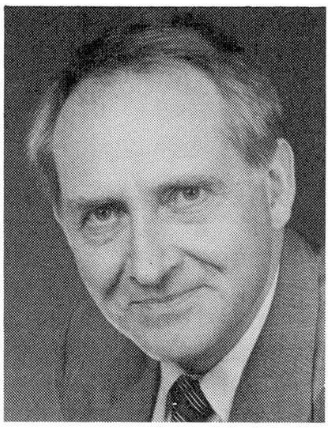

H.W. Reinhardt, born 1939, obtained his civil engineering degree at Stuttgart University. He performed research on concrete and concrete structures at Delft and Darmstadt. Since April 1990, he has been Professor for materials at Stuttgart University and the managing director of the Otto Graf Institute.

SUMMARY

Attention is focused on imposed deformation which may lead to cracking of concrete. Some typical situations are: stresses due to nonlinear temperature and shrinkage distribution, imposed deformation, and deformation with internal and external restraint. Some practical cases illustrate the relevance of these effects.

RÉSUMÉ

L'auteur traite les déformations imposées pouvant causer la fissuration du béton. On distingue des situations typiques: contraintes par distribution non-linéaire de la température et par retrait, déformation imposée, et déformation avec empêchement intérieur. Quelques cas pratiques illustrent l'importance de ces actions.

ZUSAMMENFASSUNG

Zentrischer Zwang mit möglicher Rissbildung wird behandelt. Dabei werden typische Fälle unterschieden: Spannungen infolge nichtlinearer Verteilung von Temperatur und Schwinden, aufgezwungene Verschiebung, innerer und äusserer Zwang. Praktische Beispiele verdeutlichen die Bedeutung dieser Belastungen.

1. MOTIVE AND SCOPE

There is an increasing number of structures with great demands on serviceability, especially on gas-tightness and liquid-tightness. We can think about structures for environmental protection such as catch basins, waste disposal sites, interim storage sites, treatment plants for contaminated water, about structures in chemical plants of refineries, but also about basements, pipes and ducts located in contaminated ground water. The common feature of these concrete structures is that they serve at least two purposes. First, they are structures which carry dead and life load, and second, they ought to be impermeable against gasses and fluids during a certain time. A main concern is cracking, i.e. occurence of cracks and crack width.

Besides direct actions which are usually taken into account in designing there are indirect actions originating in imposed and restraint deformations. Causes can be due to shrinkage and swelling of concrete, thermal movements, chemical reactions, and differential settlement. This contribution deals with eigenstresses and actions due to imposed and restraint deformation. It will be shown that the boundary conditions have an essential influence on these actions. Practical examples will illustrate the findings.

2. TYPE OF ACTIONS

2.1 Non-linear temperature distribution

Heating and cooling of concrete elements due to air temperature variation and thermal radiation cause non-linear temperature distribution.

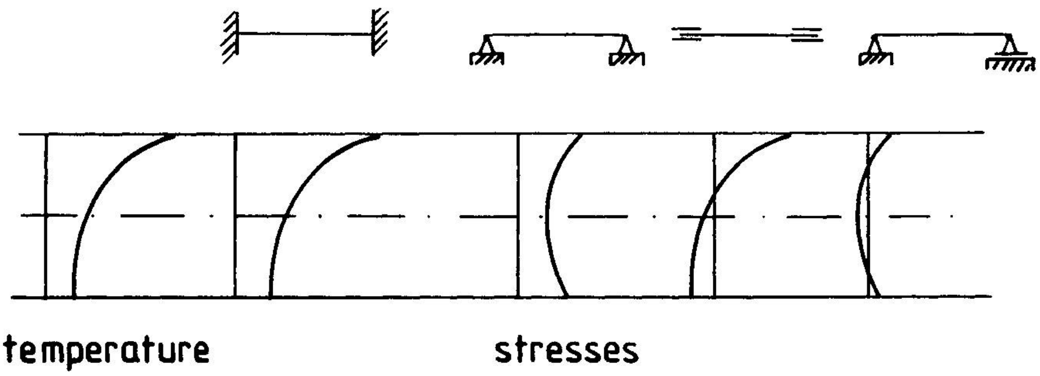

Fig. 1 Stresses due to non-linear temperature distribution with various boundary conditions.

Fig. 1 shows a typical example of a concrete slab on a foundation during nightly cooling. The surface temperature is lowest. Depending upon the boundary conditions the stress distribution is different. Even if the supports allow rotation and translation there are eigenstresses in the homogeneous cross-section, in this case tensile stresses at the surface.

In a composite cross-section with layers of material with different coefficients of thermal expansion there will be eigenstresses even if the temperature is raised uniformly. At the boundaries of the layers shear stresses develop.

Similarly to non-linear temperature distribution, drying and wetting of concrete cause eigenstresses due to shrinkage. However, the process is much slower. The governing parameter for temperature is the thermal diffusivity which is about $1*10^{-6}m^2\ s^{-1}$, and, for shrinkage, it is the diffusion coefficient which is about $2*10^{-10}\ m^2\ s^{-1}$. This means that shrinkage is about 5000 times slower than temperature movement.

2.2 Imposed deformation

If various concrete elements which are connected to each other undergo different thermal and shrinkage history there may mutually impose deformation. For instance, if an external column is heated up while the internal columns stay at the same temperature a deformation will be imposed on the slab resting on these columns. Those deformations are most relevant which cause tensile forces in an element or excentricity in a compressive part.

Common imposed deformations are due to differential settlement of hyperstatic structures.

The reaction of a structural member to an imposed deformation δ can be illustrated by Fig. 2. If a displacement δ_0 is imposed the member will crack

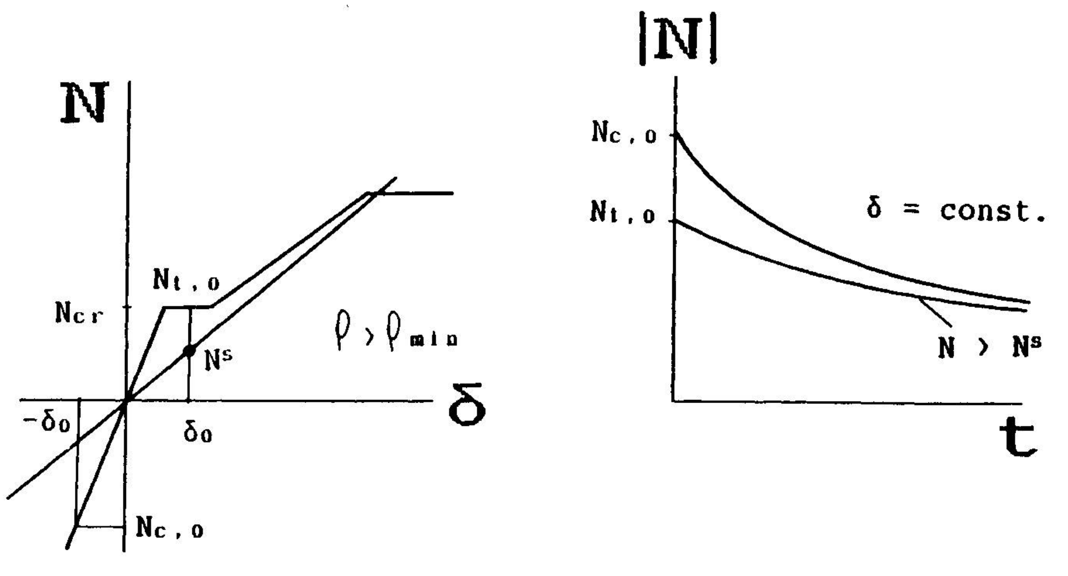

Fig. 2 a) Normal force vs. imposed deformation
b) Normal force vs. time with constant deformation

at N_{cr} (tension) or will react with $N_{c,o}$ (compression). The forces will decrease due to relaxation of the concrete. In case of tension the force cannot become lower than N^s. If the reinforcement ratio is smaller than $\delta_{min} = f_t/f_{sy}$ the steel will yield after cracking remaining at a constant force $N_y = A_s\ f_{sy}$. This is shown by Fig. 3

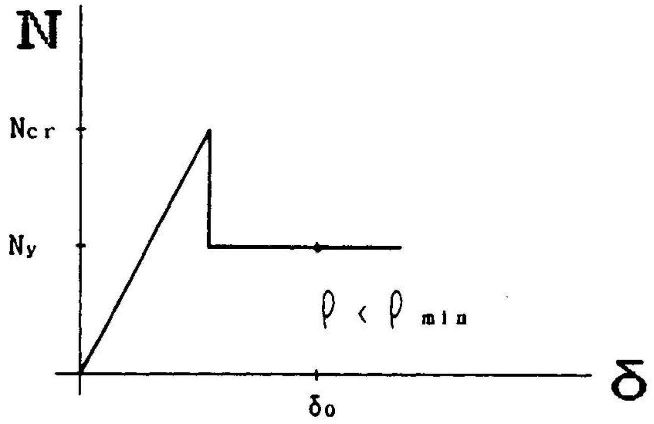

Fig. 3 Normal force vs. imposed deformation for $\delta < \delta min$.

To judge the effect of imposed deformations realistically the correct stiffness of the member has to be chosen. An analysis with initial elastic stiffness may lead to a great overestimation of reaction forces. Crack width and crack spacing can be obtained in the same way as for imposed loads [1].

2.3 Restraint deformation

If deformation is caused by temperature, humidity, chemical reactions, i.e effects which cause length changes of the concrete, and if this deformation cannot occur freely stresses will develop in the cross-section. There are two different cases: the movement will be restraint externally by supports or internally by reinforcement. The two cases lead to quite different stresses and crack widths. This will be elaborated in more detail in the following chapter.

3. UNIFORM SHRINKAGE IN A REINFORCED MEMBER

3.1 Internal restraint

It is assumed that the concrete shrinks and that this movement is restraint by embedded bars. Since the concrete member is simply supported there will be no external forces. Steel strain and concrete strains are equal, $\epsilon_s = \epsilon_c$. The forces are equal with opposite sign, $N_c = -N_s$. The concrete strain consists of the initial shrinkage strain ϵ_0 and the elastic strain due to composite action:

$$\epsilon_c = \frac{N_c}{A_c\ E_c} + \epsilon_0 \qquad (1)$$

The steel strain is given by

$$\epsilon_s = \frac{N_s}{E_s\ A_s} \qquad (2)$$

With $\varrho = A_s/A_c$, $n = E_s/E_c$, $\epsilon_s = \epsilon_c$ it yields

$$\epsilon_s = \frac{E_c\ A_c}{E_s A_s + E_c A_c} = \frac{1}{1 + n\ \varrho}\ \epsilon_0 \qquad (3)$$

The elastic stresses are

$$\sigma_s = \frac{E_s\ \epsilon_0}{1 + n\varrho} \quad \text{and} \quad \sigma_c = -\ \varrho\ \sigma_s \qquad (4)$$

As soon as the tensile strength of concrete is reached cracks will develop. This is true for $\sigma_c = f_{ct}$ and the appropriate strain ϵ_0^{CR} becomes

$$\epsilon_0^{CR} = \frac{1 + n\varrho}{\varrho}\ \frac{f_{ct}}{E_s} = \frac{1 + n\varrho}{n\varrho}\ \frac{f_{ct}}{E_c} \qquad (5)$$

In the vicinity of a crack, bond stresses between steel and concrete are activated. Assuming a constant bond stress τ_0 (for a more accurate treatment see [1]) the stress is linearly distributed between two cracks with the maximum $\sigma_{c,max} = f_{ct}$. The elastic steel deformation is then

$$\Delta l_s = \frac{1}{2}\ \frac{1 + n\varrho}{\varrho}\ \frac{f_{ct}}{E_s}\ l_{CR} \qquad (6)$$

and the mean steel strain

$$\bar{\epsilon}_s = \frac{\Delta l_s}{l_{CR}} = \frac{1}{2}\ \frac{1 + n\varrho}{\varrho}\ \frac{f_{ct}}{E_s} \qquad (7)$$

with l_{CR} = crack spacing.

The mean strain of concrete is at this moment

$$\bar{\epsilon}_c = \frac{1 + n\rho}{\rho} \frac{f_{ct}}{E_s} - \frac{1}{2} \frac{1 + n\rho}{\rho} \frac{f_{ct}}{E_s}$$

$$= \frac{1}{2} \frac{1 + n\rho}{\rho} \frac{f_{ct}}{E_s} \qquad (8)$$

Mean steel and concrete strain are the same, the crack width is zero.

With increasing ϵ_0, concrete will slip on the steel since τ_0 = const. Concrete strain increases according to

$$\epsilon_c = \epsilon_0 - \frac{1}{2} \frac{1 + n\rho}{\rho} \frac{f_{ct}}{E_s} \qquad (9)$$

while steel strain remains the same. The mean crack width becomes

$$\bar{w} = l_{CR} (\bar{\epsilon}_c - \bar{\epsilon}_s) = l_{CR} \left(\epsilon_0 - \frac{1 + n\rho}{\rho} \frac{f_{ct}}{E_s} \right) \qquad (10)$$

Crack spacing is given by

$$l_{CR} = \frac{1}{2} \frac{f_{ct}}{\tau_0} \frac{d_s}{\rho} \qquad (11)$$

with d_s = bar diameter. Three stages can be distinguished if concrete shrinkage increases continuously (see Fig. 4). These are:

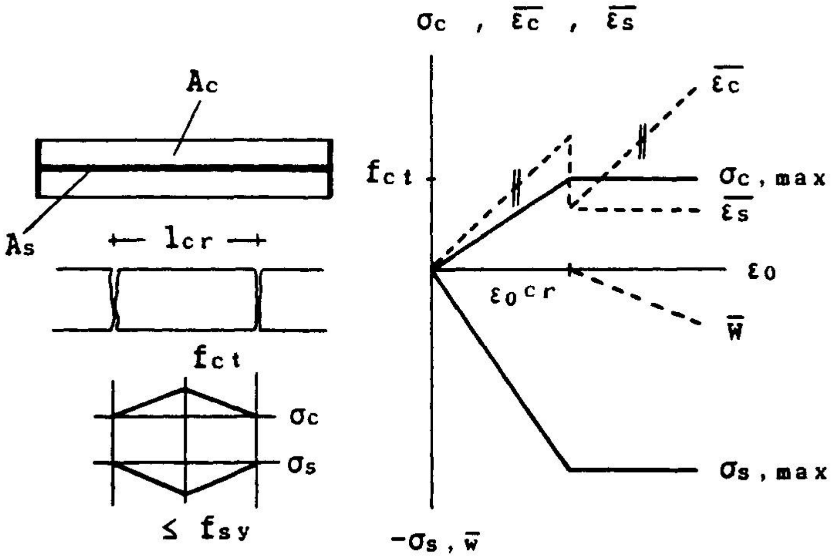

Fig. 4 Stress, strain, and crack width vs. initial strain ϵ_0

1) No cracks, shortening of the composite member.
2) At cracking, the member length increases again; crack width is zero.
3) The length of the bar remains constant and the crack width increases.

It should be noted that $\varrho \geq f_{ct}/f_{sy}$ because otherwise the steel would yield.

3.2 External (and internal) restraint

If the composite member is fixed at the ends and shrinkage occurs then

$$\sigma_c = -\epsilon_0 E_c \quad \text{and} \quad \sigma_s = 0 \tag{12}$$

Cracks develop at $\sigma_c = f_{ct}$. Then, concrete gets shorter and forces the steel to become shorter. Since the ends are fixed a tensile force will develop in the steel. Steel stress and strain are

$$\sigma_{s,max} = -\frac{1 + n\varrho}{\varrho} f_{ct} \quad \text{and} \quad \bar{\epsilon}_s = -\frac{1}{2} \frac{1 + n\varrho}{\varrho} \frac{f_{ct}}{E_s} \tag{13}$$

The elastic elongation $\bar{\epsilon}_s = \dfrac{\sigma_s^{CR}}{E_s}$ has to cancel the shortening which makes

$$\sigma_s^{CR} = \frac{1}{2} \frac{1 + n\varrho}{\varrho} f_{ct} \tag{14}$$

which is half as much as in the simply supported case. However, the sign of the stress changes along the bar. The crack width can be calculated from the initial strain minus elastic elongation of concrete:

$$\bar{w} = l_{CR} \left(\epsilon_0 - \frac{1 + n\varrho}{2} \frac{f_{ct}}{E_c} \right) \tag{15}$$

Comparing eqs. (10) and (15) tells that crack width is larger in case of external restraint than it is at internal restraint. Crack spacing is the same. The minimum reinforcement ratio at external restraint is half of the one at internal restraint. The schematics of this situation are given by Fig. 5.

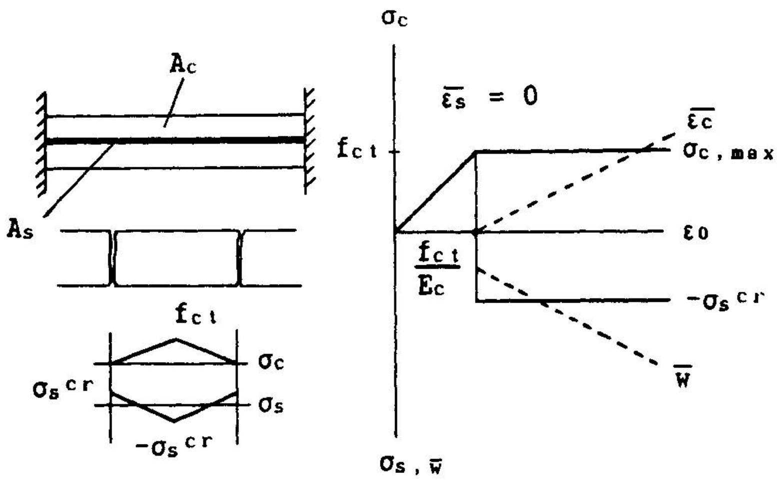

Fig. 5 Stress, strain and crack width vs. initial strain ϵ_0

3.3 General remark

The derivations made in the two preceeding chapters are intentionally based on crude material modelling in order to make the main point clear, the importance of the boundary condition. The results can be improved by assuming time dependent properties of concrete [2], realistic bond behaviour, and scatter of material properties.

4. PRACTICAL CASES

4.1 Car-park

The roof of an underground parking facility consists of a reinforced concrete slab supported by beams. On the roof, there is a water drainage but no isolation. Fig. 6 shows the situation after a year of service. Cracks have

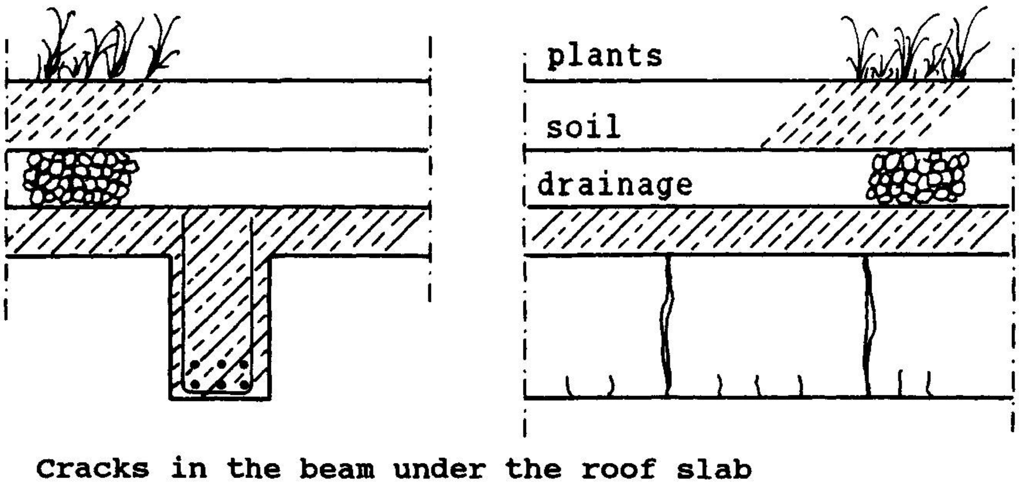

Fig. 6 Cracks in the beam under the roof slab

developed in the beam due to differential shrinkage. The slab does not shrink, it may even swell whereas the beam does shrink due to heating and ventilation of the parking deck. The crack width is the largest at places with no longitudinal reinforcement (w = 0.3 to 0.5 mm) and smallest near the longitudinal bars (w = 0.1 to 0.2 mm).

4.2 Baking furnace

Anodes for the electrolytic production of aluminium are manufactured in a baking furnace at about 1050°C. Although the interior of the furnace is strongly insulated the concrete structure is warmed up to about 200°C at the inner face. Temperature distribution gives rise to curvature of the walls (see Fig. 7). The horizontal displacement was large enough to touch the columns of the superstructure. The walls cracked and the columns were loaded by an almost constant force, i.e. imposed deformation on the walls and imposed force on the columns.

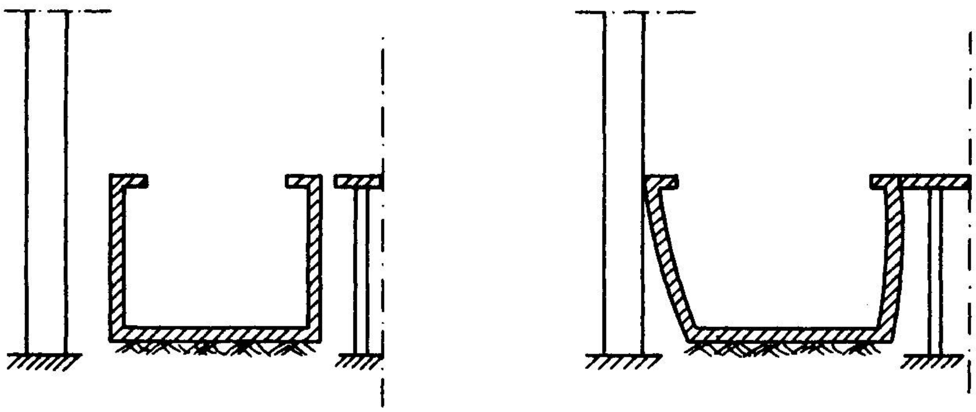

Fig. 7 Baking furnace with superstructure

4.3 Sedimentation-basin

The sedimentation basin of a process-water treatment plant cracked due to temperature differences between the hot water (38°C) and the cold air and structure. Fig. 8 shows the basin with vertical cracks at the

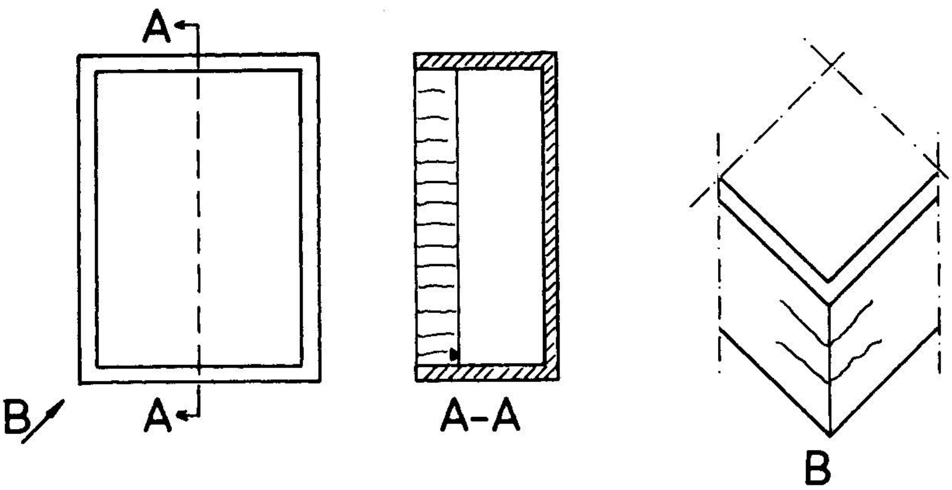

Fig. 8 Sedimentation-basin with cracks indicated

crown of the walls and horizontal cracks at the edges. The basin has been designed and constructed very carefully without any cracks due to early thermal movement. However, during operation cracks appeared the cause of which are rather obvious. There are thermal stresses in wall direction because the lower part of the structure warms up while the top part does not. In the middle of the basin the wall can bend and rotate freely whereas the edges restrain the curvature. Here the inner edge expands due to heating and the outer edge is stressed in such a way that cracks occur. It will be interesting to observe the cracks and to see whether they propagate into the concrete due to inelastic cyclic loading [3].

5. CONCLUSION AND OUTLOOK

Imposed deformations are usually treated as secondary effects which may be neglected in design of concrete structures. However, these effects should receive due attention in all cases where tightness against gasses and fluids play an important role.

It has been shown how cracks develop and that crack width is greatly influenced by boundary conditions. Models should be developed which enable the structural engineer to judge the behaviour of a structure under restraint deformations, similarly to what has been developed for imposed loads.

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Control of Crack Width in Deep Reinforced Concrete Beams

Contrôle de la fissuration des grandes poutres en béton armé

Beschränkung der Rissbreiten in hohen Stahlbetonträgern

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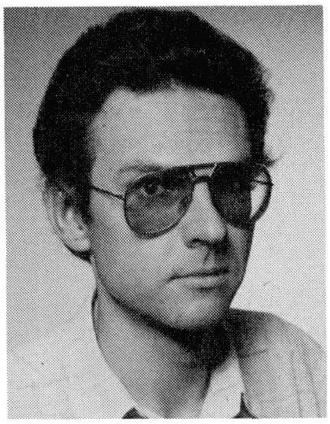

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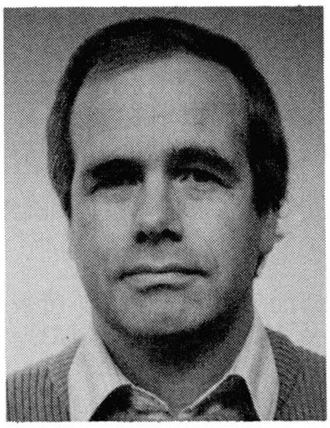

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SUMMARY

Most calculation methods for the control of crack widths in concrete structures are based on the behaviour of centrically reinforced concrete bars, subjected to tension. In deep beams, the validity of these methods is therefore mainly limited to the regions directly surrounding the main reinforcing bars. In the areas at some distance from the main reinforcement, however, crack control has to be carried out with the same attention. In this paper it is described how crack control in deep beams can be carried out on the basis of a rational model.

RÉSUMÉ

Plusieurs méthodes de calcul concernant le contrôle de la largeur des fissures sont basées sur le comportement de tirants armés axialement et tendus. Pour les murs porteurs, la validité de cette théorie est cependant limitée aux zones situées directement au voisinage des barres d'armature principale. On, pour les régions se trouvant à une certaine distance, le contrôle de la fissuration doit malgré tout être considéré avec le même soin. Ce problème est résolu de façon cohérente par un modèle rationnel adapté.

ZUSAMMENFASSUNG

Die meisten Rechenmodelle zur Beschränkung der Rissbreiten basieren auf dem Verhalten von zentrisch bewehrten Stahlbetonstäben, die auf Zug beansprucht werden. In grösseren Bauteilen ist die Gültigkeit dieser Rechenmodelle deshalb auf die Umgebung der Hauptbewehrung beschränkt. In den von der Hauptbewehrung weiter entfernten Bereichen ist eine ausreichende Untersuchung zur Vermeidung von klaffenden Rissen jedoch genau so wichtig. In diesem Aufsatz wird beschrieben wie man, aufgrund einer rationalen Modellierung, die Rissbreiten in hohen Stahlbetonträgern beschränken kann.

1. INTRODUCTION

The cracking behaviour of reinforced concrete structures has been investigated for many years. Most research was restricted to describing crack width and crack spacing in a semi-empirical manner. In recent years considerable progress has been gained with regard to the development of rational models for crack width control: Crack widths and spacings can be calculated as a function of the concrete tensile strength, bond strength, reinforcing ratio, bar size and load level. However, since laboratory experiments provide the basis for the tuning between theory and practice, most results are restricted to relatively small concrete specimens. With regard to members loaded in bending, the experiments have shown that crack widths are controlled in an effective area around the main reinforcement. However, outside this region the cracks 'collect' if too little web reinforcement is applied. Leonhardt [1] already pointed out this phenomenon in the early sixties and defined the wide cracks in the web as 'Sammelrisse'.
Since members in practice are generally considerably larger than test-beams in laboraties, this is a problem of particular importance. It is necessary to know the amount of horizontal web reinforcement that is required to control the cracking outside the 'effective area' of the main reinforcement.

2. CRACKING BEHAVIOUR OF TENSILE MEMBERS AND BEAMS

2.1 Tensile members

The first relations to predict the crack spacing and the crack width in tensile members were based on a relatively simple calculation model [2]. The basic principles of the model can be summarized as follows: At the instant of cracking, the concrete tensile force must be carried by the reinforcing steel. At a certain distance away from a crack, the so-called transfer length l_t [2], the bond stress is zero. The whole concrete section is assumed to be in uniform tension so that a new crack can occur. At increasing elongation new cracks are formed until the crack pattern is 'fully developed', e.g. all the crack spacings vary between l_t and $2l_t$. In this situation there are no parts where the concrete stress reaches the tensile strength. Thus, the mean crack spacing l_m in a fully developed crack pattern is $1.5l_t$ [2]. After the introduction of a lower-bound value, the following formula is obtained:

$$l_m = k_1 + k_2 k_3 \frac{d_s}{\rho} \qquad \text{[mm]} \qquad (1)$$

The mean crack width follows from the mean tensile strain ϵ_{sm}:

$$\epsilon_{sm} = \epsilon_s [1 - k_5 k_6 (\frac{\sigma_{s,cr}}{\sigma_s})^2] \qquad \text{[-]} \qquad (2)$$

where $\sigma_{s,cr}$ and σ_s are the steel stresses in a crack at the cracking load and the service load, respectively. The variation in crack widths is accounted for by the coefficient k_4:

$$w_k = k_4 w_m \qquad \text{[mm]} \qquad (3)$$

The coefficients k_1 to k_6 can be tuned so as to obtain close agreement with experimental results.

2.2 Beams

Formulae (1) to (3) can also be used to predict the crack pattern at the level of the main reinforcement of beams. In the formula (1) the mean crack spacing was found to depend on, among other factors, the reinforcing ratio ρ. In the case of a tensile member this ratio is given by $\rho=A_s/A_c$. For beams an 'effective' concrete area around the main reinforcement is defined. In most recent approaches, this area is based on the beam height h and the effective beam depth d [3,4]:

$$\rho_{eff} = \frac{A_s}{\alpha b(h-d)} \qquad [-] \qquad (4)$$

It is observed that the cracks initiated by the main reinforcement 'collect' outside the 'effective concrete area'. Thus, in the web of beams fewer cracks with larger widths occur, see figure 1 [5]. Several researchers have investigated the development of crack widths and spacings over the entire height of deep beams, e.g. [6,7]. However, the amount of experimental data is rather limited. Therefore, it was decided to perform experiments on deep reinforced concrete beams.

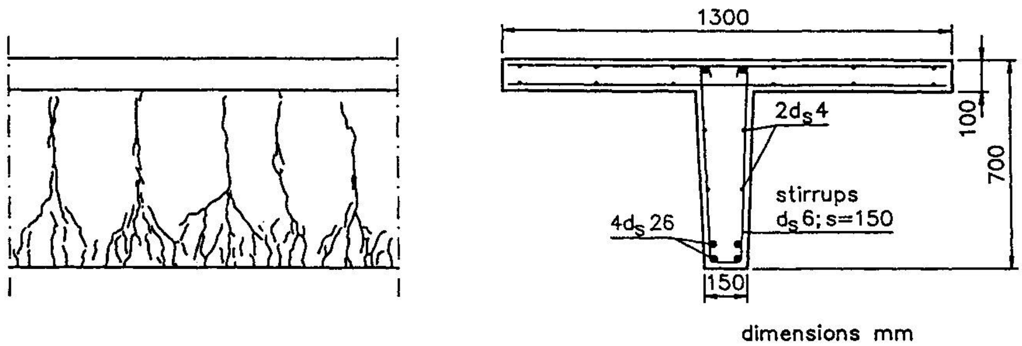

Fig. 1 Crack pattern and cross-section of a deep T-beam [5].

3. EXPERIMENTS

3.1 Test set-up

In the tests 15 beams were loaded in four-point bending, see figure 2. The beams were 5.5m long and 0.8m in height. Twelve beams had a T-shaped cross-section, whereas three beams were rectangular. For the main reinforcement either 4 bars d_s20mm or 3 bars d_s16mm were used. The diameter of the web rebars was 10, 12 or 16mm. The vertical bar spacing was 100, 150 or 200mm. One concrete mix was used. The average 28-day 150mm cube compressive and tensile splitting strength were 50.0 and 3.0MPa, respectively.

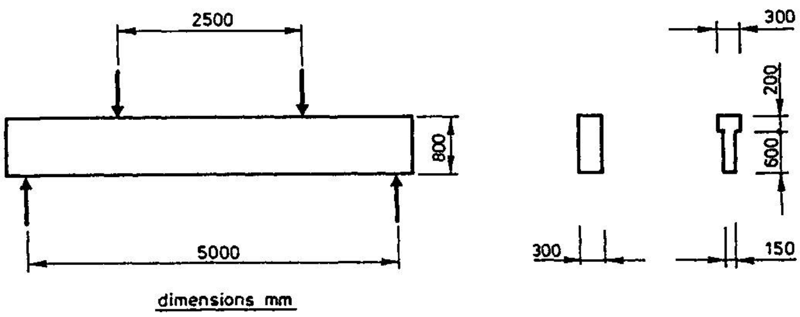

Fig. 2 Cross-sections and side-view of the beams tested

3.2 Measurements

The crack width measurements were restricted to the middle 2.3m of the uniform bending moment zone. Measurements were taken on both sides of the beams by means of a microscope. Nine horizontal lines were drawn, covering about the lower 450mm of the beams. Cracks were measured at each position where the cracks intersected these lines.

3.3 Experimental results

Figure 3 presents the influence of the web rebar spacing on the mean crack spacing. For comparison, the results of a beam without web rebars are also given. The dominant cracks are accompanied by minor cracks. Therefore, the sum of the widths of the dominant cracks is less than the measured beam elongation. This was also observed in [8]. It was found that :

$$w_m = 0.75 l_m \epsilon_{sm} \qquad [mm] \qquad (5)$$

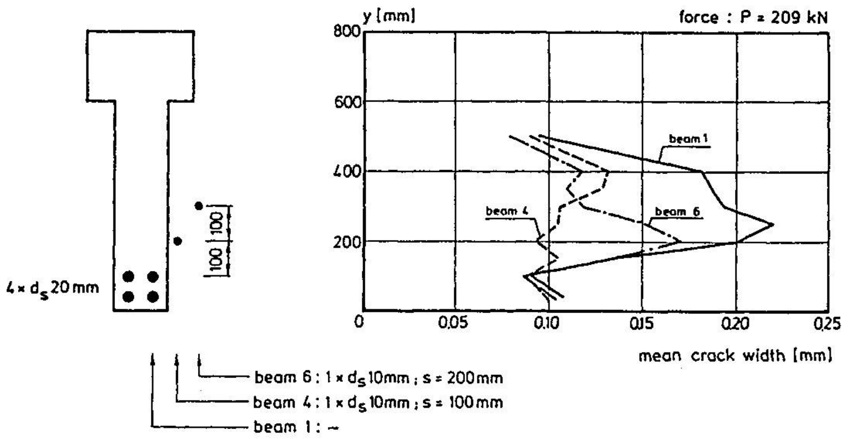

Fig. 3 The influence of web reinforcement on the mean crack width [9].

4. THEORETICAL MODEL

4.1 No web reinforcement

In the case no web reinforcement is applied, the crack pattern is similar to the one observed in a concrete wall cast on a hardened slab [10], see figure 4a. The mean crack spacing halfway down the web is approx. $l_m = h - h_x - (h-d)$. When comparing this value with the average crack spacing at the main reinforcement according to the Eurocode II a family of curves is obtained for various values of h and d_s (fig. 4b). The figure shows that the ratio $l_m$web / $l_m$main reinf.= 4, which is reported by several authors, applies for the region $0.6 < \rho < 1.0\%$.

4.2 Web reinforcement

If sufficient web reinforcement is applied, cracks can be forced to extend into the web. Figure 5a presents the corresponding relation between steel stress in the web rebars, the bar diameter and the bar spacing, whereas the transfer length is shown in figure 5b. It was assumed that f_{ct}=2.5MPa and that the distance from the side face of the member to the centre of the web rebars is b_1=50mm. In the case the actual parameters differ from these values, the results from the design curves (indicated by the superscript 'd') must be corrected by the following formulae:

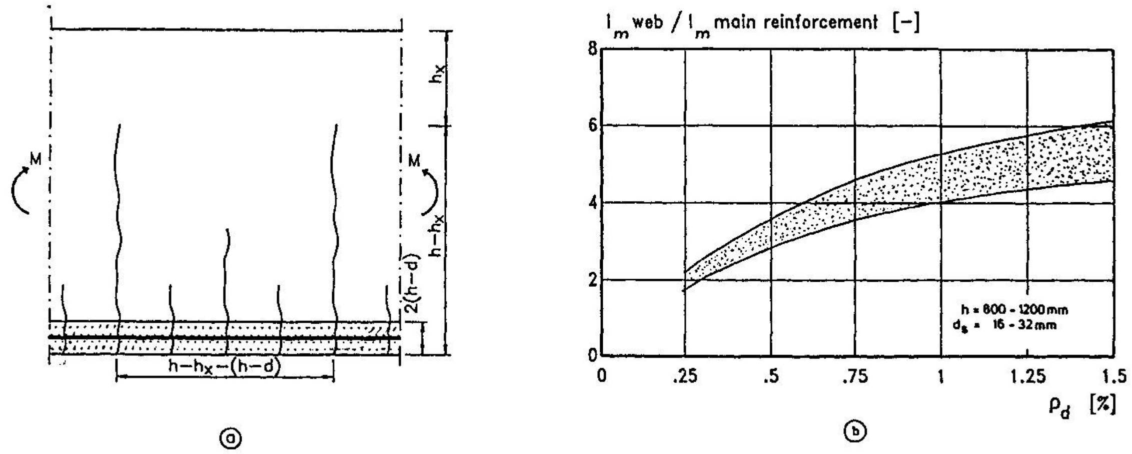

Fig. 4 Crack pattern in a deep beam without web reinforcement (a) and the ratio between the mean crack spacing in the web and at the bottom (b)

$$\sigma_{s,cr} = \sigma^d_{s,cr}(0.015b_1+0.125)\frac{f_{ct}}{2.5} \qquad [MPa] \qquad (6)$$

$$l_t = l^d_t(0.015b_1+0.125) \qquad [mm] \qquad (7)$$

The crack pattern is fully developed if the average surface strain exceeds $\sigma_{s,cr}/E_s$. The average crack spacing is then $l_m=l_t$. Since one aims at the use of a rather limited amount of web reinforcement, the web crack pattern is mostly not fully developed.

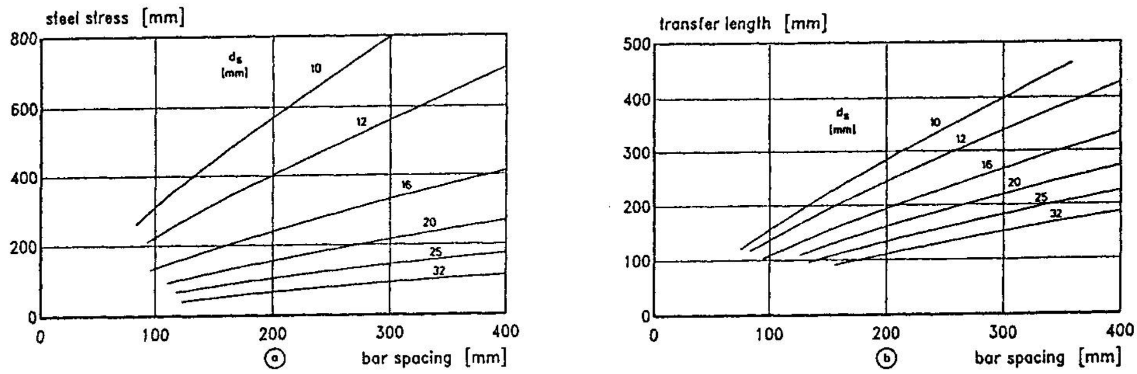

Fig. 5 The steel stress initiating cracking in the web (a) and the transfer length (b).

5. PRACTICAL DESIGN RULES

The previous sections provide the calculation method of web crack widths. Since the crack pattern is assumed not to be fully developed, long-term or varying loading is assumed to cause an increase of the transfer length by 40%. The characteristic long-term crack width is:

$$w_k = 1.3 l_t \epsilon_{s,cr} \qquad [mm] \qquad (8)$$

Formula (8) is presented in figure 6a in the case f_{ct}=2.5MPa and b_1=50mm. In the case the parameters differ from the assumed values, the result from the design curve is corrected as follows:

$$w_k = w_k^d(0.015b_1+0.125)^2 \frac{f_{ct}}{2.5} \qquad \text{[mm]} \qquad (9)$$

The part of the web h_w where web reinforcement is required can be calculated according to present design rules [12].

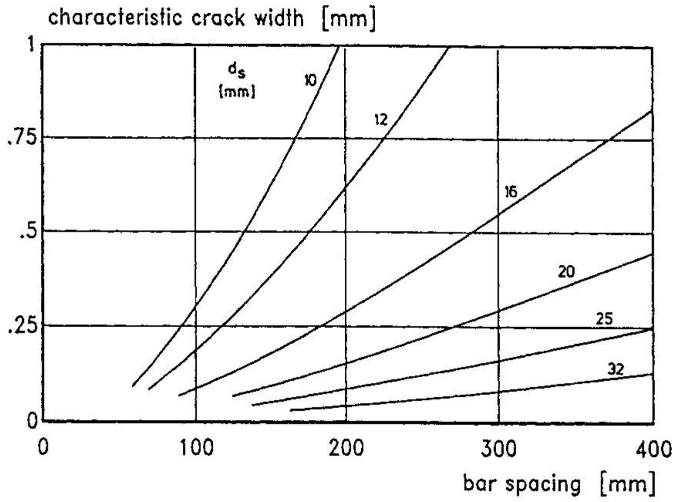

Fig. 6 Web crack width in a not fully developed crack pattern.

ACKNOWLEDGEMENT

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Influence of Temperature on the Cracking in Reinforced Concrete

Influence de la température sur la fissuration du béton armé

Temperatureinfluss auf das Rissverhalten von Stahlbeton

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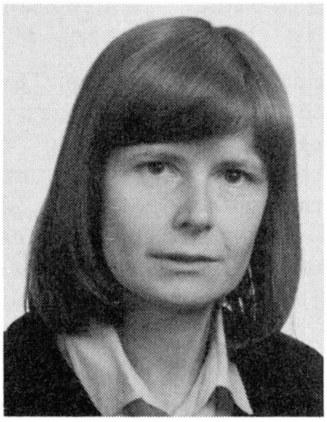

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SUMMARY

Crack spacings and crack widths in a reinforced concrete structure can be calculated by means of methods which are based on the τ-s-relation of a rebar in concrete. The τ-s-relation, presented in this paper, holds both for normal and cryogenic temperatures. From calculations it follows that the mean crack spacing at −165°C is almost the same as the value at +20°C, the mean crack width and the mean steel stress at −165°C, on the other hand are substantially greater than the corresponding values at +20°C.

RÉSUMÉ

L'espacement et l'ouverture des fissures peuvent être calculés à l'aide de méthodes basées sur la relation τ-s d'une barre d'armature dans le béton. La relation τ-s, présentée dans la présente communication, est valable pour les températures normales ainsi que pour des températures très basses. Des calculs démontrent qu'à −165°C l'espacement moyen des fissures est à peu près le même qu'à +20°C. Cependant l'ouverture moyenne des fissures ainsi que les contraintes moyennes dans l'armature sont plus élévées à −165°C qu'à +20°C.

ZUSAMMENFASSUNG

Rissabstände und Rissbreiten in einer Stahlbetonkonstruktion können mittels Methoden, die auf dem τ-s-Verhältnis eines Bewehrungsstabes im Beton beruhen, berechnet werden. Das in dieser Abhandlung beschriebene τ-s-Verhältnis gilt für normale und extrem tiefe Temperaturen. Aus Berechnungen ergibt sich, dass der mittlere Rissabstand bei −165°C ungefähr derselbe ist wie bei +20°C; die mittlere Rissbreite und die mittlere Stahlspannung sind im Gegenteil bei −165°C bedeutend grösser als die entsprechenden Werte bei +20°C.

1. INTRODUCTION

Because of the low tensile strength of concrete, both at normal and cryogenic temperatures, cracks are likely to occur in concrete constructions.

Crack control, which means the limitation of crack widths, plays a significant part in all types of structural concrete. Controlling crack widths can be applied for reasons of appearance, corrosion protection or tightness with regard to gas or liquid permeability. The cracking situation of the structural concrete has also an important influence on the extent of deformation. Deformation in cracked state can be a multiple of that in uncracked state [1,2]. The previous mentioned performance requirements are imposed by the supporting function of the structure in the serviceability limit state.

The following is an attempt to determine "quantitatively" crack spacings and crack widths in reinforced concrete members for temperatures ranging between +20°C and -165°C. The cracking behaviour of structural concrete is mainly dependent on the way a tensile force, exerted on a "reinforcement bar", is transferred to the "enveloping concrete" by way of "bond stresses". The bond between the reinforcement bar and the concrete is, consequently, one of the basic properties which make reinforced concrete possible. According to [1], concrete, reinforcing steel and bond behaviour can be called the subsystems of structural concrete.

Researchers [3,4,5,6] have found that the concrete properties, consequently also the bond stresses, are highly influenced by temperature. Especially the amount of chemically unbound water in the concrete plays an important part at low temperatures.

2. FROM τ-s RELATION TO CRACKING AT NORMAL TEMPERATURE

The bond between the reinforcement and the concrete may be described in an idealized way as a shear stress between the surface of the reinforcement bar and the surrounding concrete. The bonding mechanism may be expressed by the relation between the shear stress τ and the relative displacement s between the reinforcement bar and the concrete.

On the basis of the results of an extensive test programme of beam tests both at normal and cryogenic temperatures, executed at the Department of Civil Engineering of the K.U.Leuven, the τ-s-relation is mathematically approximated by the expression :

$$\tau = \tau_u \, (1 - \mu e^{-\lambda s}) \tag{1}$$

with μ = 0,78 and λ = 9,78. The ultimate bond strength τ_u is a function of the concrete cover on the rebar (c), the concrete quality (f_c,f_{ct}) and the temperature :

$$\frac{c}{\phi} \leq 3 \qquad \frac{\tau_u}{f_c} = \frac{\sqrt{K}}{2} \left[1 + (1 - K)\, 0,353 \, \frac{\frac{\phi}{2} + c}{\frac{\phi}{2}}\right] \tag{2a}$$

$$\frac{c}{\phi} > 3 \qquad \frac{\tau_u}{f_c} = \frac{\sqrt{K}}{2} \left[1 + (1 - K)\, 2,473\right] \tag{2b}$$

with $K = f_{ct}/f_c$. The temperature effect has been completely taken into account by way of the quantities f_c and f_{ct}. The expression (1) has the merit of describing the τ-s-course up to the bond fracture.

For a centrically loaded reinforced concrete tensile bar (Fig. 1) the transfer of the tensile force in the bar to the surrounding concrete is, for an elementary part dx, described by the following differential equation :

$$\frac{\phi\ E_s}{4(1 + \frac{E_s}{E_c}\omega)} \frac{d^2 s}{dx^2} = \tau_x \tag{3}$$

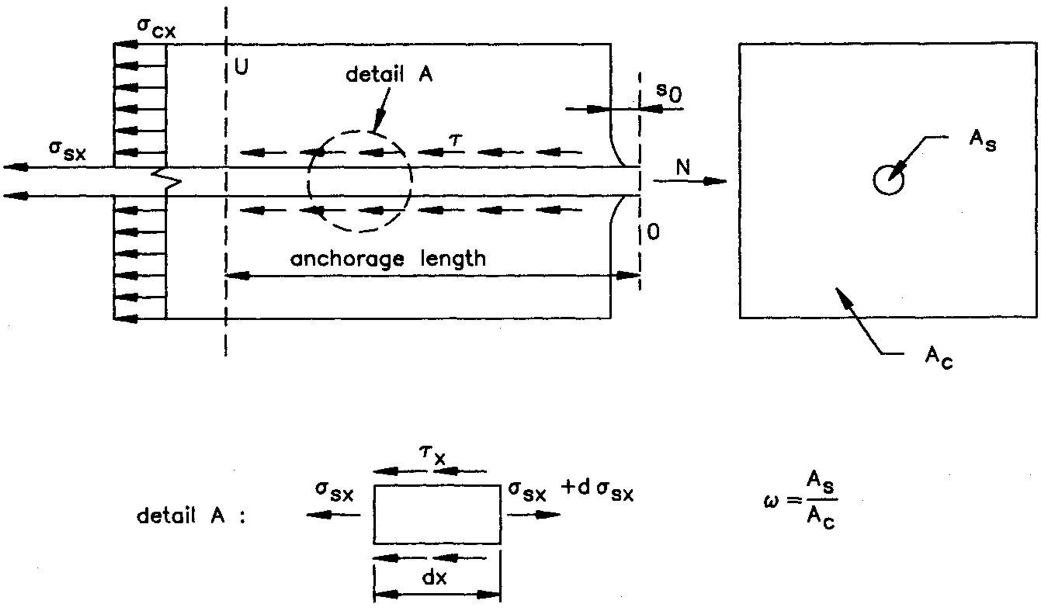


Fig. 1 Reinforced concrete tensile bar subjected to a force N.

The length, needed for the transfer of the tensile force N, is called the anchorage length. At the end of the anchorage length (in U, see Fig. 1) the concrete and the steel strain are equal. If the force N is increased in such a way that the concrete tensile stress at U becomes equal to the concrete tensile strength, i.e. $N = N_r$, the length OU is equal to the anchorage length ℓ_T. After inserting (1) in (3) and numerically solving the differential equation (3), one obtains the anchorage length ℓ_T belonging to the force N_r :

$$N_r = A_s\ \sigma_{s,r} \tag{4}$$

with

$$\sigma_{s,r} = f_{ct}\ (\frac{1}{\omega} + \frac{E_s}{E_c}). \tag{5}$$

When subjecting a reinforced concrete tensile bar to a force N_r it is assumed that in the first instance all "first-order cracks" are formed. A first-order crack is by definition a crack at such a distance from the nearest crack that the transfer zones of both cracks do not influence each other (Fig. 2). This requires that the distance between the two cracks in question is greater than or equal to 2 ℓ_T. The crack width w_r, immediately after cracking is then equal to :

$$w_r = 2\ s_{r,0}.$$

After the completion of this first-order crack pattern, the so-called second-order cracks will be formed. The distance between two cracks is now smaller than 2 ℓ_T. With the second-order cracks the transfer zones overlap partly. After the completion of the second-order crack pattern the distance between two

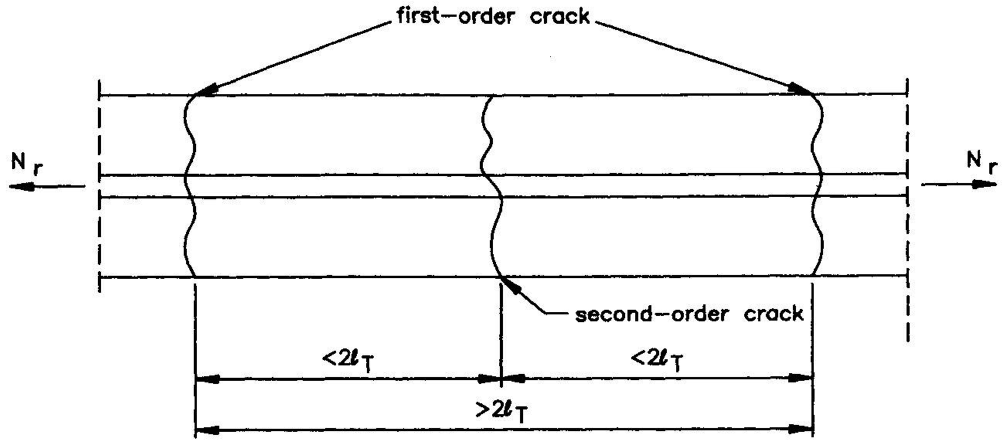


Fig. 2 First-order and second-order cracks.

cracks is at least ℓ_T. Only at this distance, taken from another crack, does the concrete stress attain the concrete tensile strength again. The crack width of second-order cracks varies, as a function of the crack spacing, between 75 % and 100 % from that of the first-order cracks.

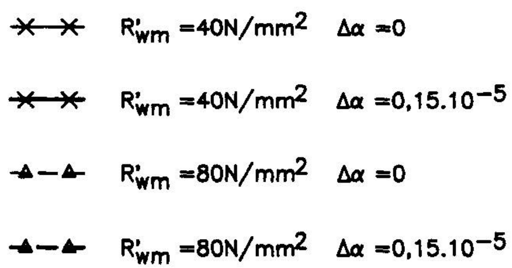
In reality the distribution of cracks is very irregular because it is determined by stochastic effects. The concrete tensile strength is, indeed, a quantity which is liable to a relatively great dispersion. Therefore strictly speaking, only minimum and maximum values can be given for the crack spacing (L_r) and crack width (w_r) respectively. A mean value for these quantities, obtained by making the calculations with the mean concrete tensile strength or another intermediate value is consequently to be interpreted with caution.

3. INFLUENCE OF TEMPERATURE ON THE CRACKING BEHAVIOUR OF STRUCTURAL CONCRETE

The above described theory may also be used for the determination of crack spacings and crack widths at low temperatures, provided that the coefficients of expansion of the concrete (α_c) and the steel (α_s) do not differ too much. This is indeed the case for concrete with a low moisture content. If, however, the concrete has a high moisture content, a correction has to be made on f_{ct}. The coefficient of expansion of the concrete is indeed at low temperatures a good deal smaller than the one of the reinforcement steel [5]. This has as a consequence that the concrete, whilst cooling off, is as it were prestressed. The tensile stresses which are formed in the steel when the temperature is going down from T_n (= normal temperature) to T_ℓ (= low temperature) may be calculated for a centrically reinforced concrete bar with :

$$\sigma_s = \int_{T_n}^{T_\ell} \frac{E_s(T)}{\left(1 + \frac{E_s(T)}{E_c(T)}\omega\right)} (\alpha_s(T) - \alpha_c(T))dT \qquad (6)$$

with $E_s(T)$: modulus of elasticity of steel at temperature T (N/mm²)
$E_c(T)$: modulus of elasticity of concrete at temperture T (N/mm²)
$\alpha_s(T)$: coefficient of expansion of steel at temperature T (1/°C)
$\alpha_c(T)$: coefficient of expansion of concrete at temperature T (1/°C).



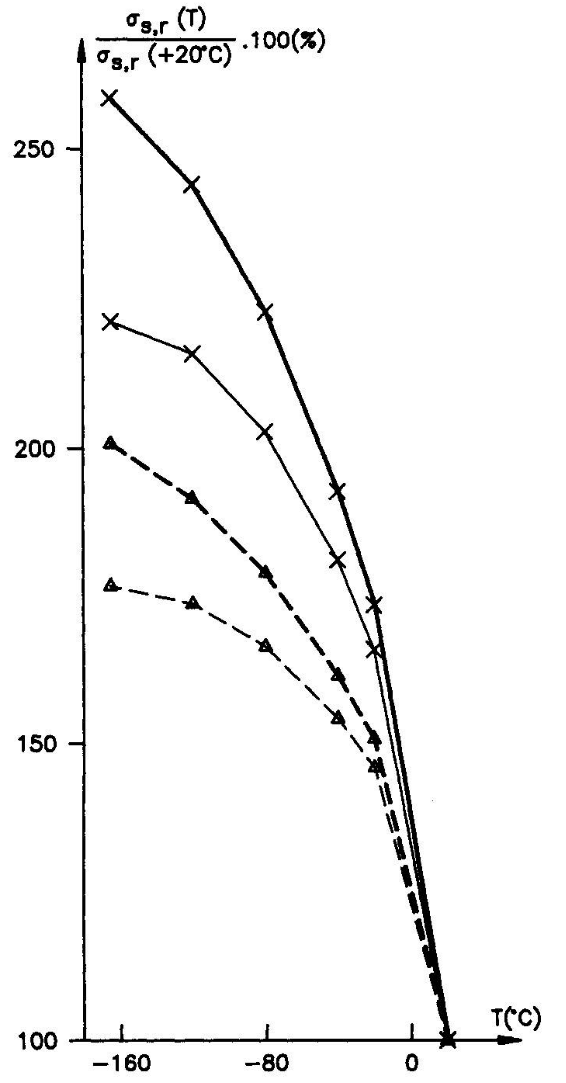


Fig. 3 Influence of temperature on the mean steel stress at the place of the crack ($\sigma_{s,r}$).

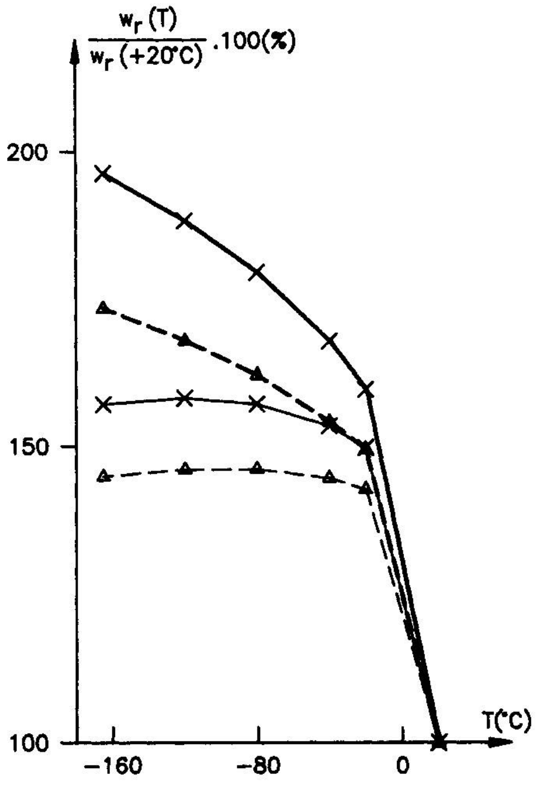


Fig. 4 Influence of temperature on the mean crack width (w_r) of first-order cracks.

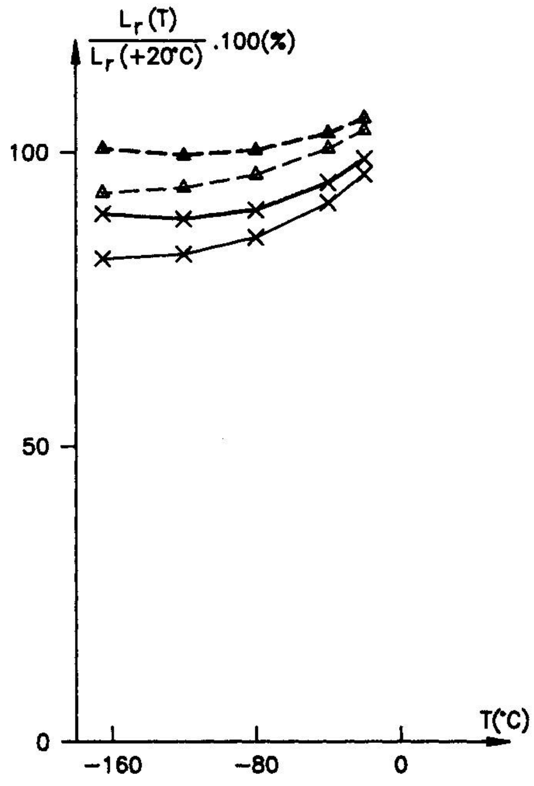


Fig. 5 Influence of temperature on the mean crack spacing (L_r) of first-order cracks.

The compressive stresses which are created in the concrete at the same time are then :

$$\sigma_c = -\sigma_s \omega . \qquad (7)$$

The internal prestress may be considered a virtual increase of the tensile strength of the concrete at that temperature and thus brings about a "postponement" of the crack formation at low temperatures. In the case of concrete with a high moisture content the uniaxial tensile strength f_{ct} has to be transformed into a virtual tensile strength $f_{ct}^*(T_\ell)$ for the calculation of $\sigma_{s,r}$ by using (6) and (7) :

$$f_{ct}^*(T_\ell) = f_{ct}(T_\ell) + \omega \int_{T_n}^{T_\ell} \frac{E_s(T)}{\left(1 + \frac{E_s(T)}{E_c(T)}\omega\right)} (\alpha_s(T) - \alpha_c(T))dT . \qquad (8)$$

The remaining characteristics of the concrete and the steel (f_c, f_{ct}, E_c and E_s) at low temperatures may be calculated on the basis of the corresponding values at room temperature [3,4].

Results of the qualitative course of the mean stress in the rebar at the place of the crack ($\sigma_{s,r}$), the mean crack width (w_r) and the mean crack spacing (L_r) as functions of temperature are shown in the Figs. 3, 4 and 5. At the calculation of the concrete tensile strength it is assumed that for one thing there is no difference between the coefficient of expansion of the steel and the concrete, consequently $\Delta\alpha = 0$, and for another that difference is constant over the whole temperature range and equal to $0{,}15.10^{-5}$ per °C [7].

From the Figs. 3, 4 and 5 it follows that the mean steel stress ($\sigma_{s,r}$) and the mean crack width (w_r) increase considerably, if temperature falls. This increase is so much the greater as the difference in the coefficient of expansion between steel and concrete increases and the mean compressive strength of concrete at room temperature decreases. The mean crack spacing (L_r) on the contrary varies relatively less (slight decrease) as a function of temperature.

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Cracking Analysis of Concrete Structures

Analyse de la fissuration des structures en béton

Rissbildungsanalyse von Betonbauten

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SUMMARY
A probabilistic approach based on an experimental model and on the variation characteristics of concrete strength and load, is presented. Traditional estimation of serviceability in terms of cracking is shown to yield significantly different reliabilities for concrete structures. The situation may be improved with the aid of suitable analysis; in any event, the traditional approach should be revised.

RÉSUMÉ
Une approche probabiliste basée sur un modèle expérimental est mise en évidence; elle tient compte de la variation des caractéristiques de résistance du béton, ainsi que de la charge. En effet, l'approche traditionnelle de l'état de service basée sur la fissuration est présentée, afin de montrer qu'elle peut mener à des sécurités différentes. Cette situation peut et doit être améliorée à l'aide d'une analyse appropriée.

ZUSAMMENFASSUNG
Eine probabilistische Methode, basierend auf einem experimentellen Modell und auf Variationscharakteristiken von Betonfestigkeit und Belastung, wird dargestellt. Die bisherige Erfassung des Gebrauchszustandes in Form von Rissbreitennachweisen liefert deutlich andere Aussagen für die Zuverlässigkeit von Betonkonstruktionen. Die Situation kann mit Hilfe einer geeigneten Analyse verbessert werden; auf jeden Fall sollten die traditionellen Verfahren überprüft werden.

1. INTRODUCTION

The cracking resistance of structures in bending defines their serviceability in terms of premature crack appearance and excessive deflections, depending on the cracking moment [1-4]. It is of prime importance in modern structures made of high-grade concrete [7]. In practice, this serviceability is assured by semi-probabilistic design methods, using deterministic values of loads, material strengths and criteria for the limit state. In reality, crack appearance in a structure complies with a particular probability. Probabilistic cracking analysis of some concrete structures has shown that the main influence on the probability of crack appearance is due to the lengthwise variation of concrete strength over the structure and to the variability of mean concrete strength of the members within their general population, as well as to the load variability [5-6].

In this paper the reliability of particular structures designed under the traditional approach are analysed on the basis of an experimentally obtained model and of the variation characteristics of tensile strength of concrete.

2. STRUCTURAL MODEL FOR ANALYSIS

Consider a simply-supported member of rectangular section under uniformly distributed load (Fig. 1).

Potential cracks in the member in bending may appear at ℓ_s - 40mm spacing (determined experimentally [4] in its middle part - ℓ_o (Fig. 2).

According to [5], the probability of crack appearance in the x-section is:

$$P_x = \frac{\Delta_i}{\sqrt{2\pi}} \sum_{i=-3}^{j} \exp(-i^2/2) \tag{1}$$

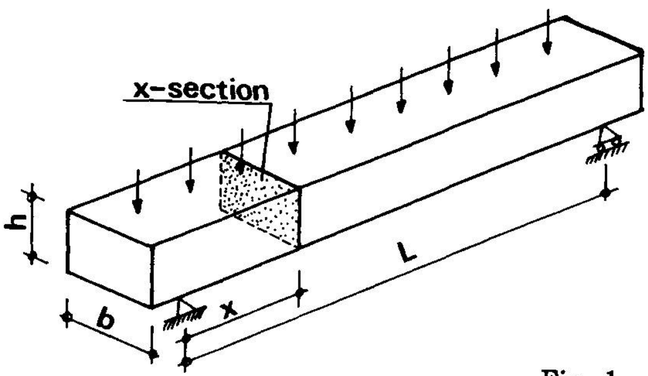


Fig. 1. Member for cracking analysis

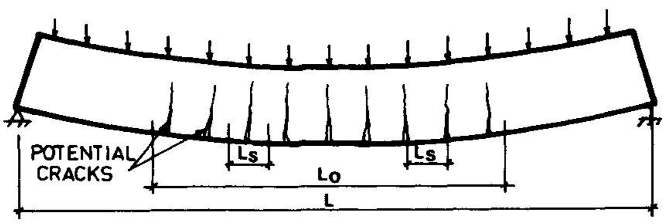


Fig. 2. Model of member with potential cracks.

In (1) Δ_i is the summation step, i is an independent parameter, j in the general case (including prestressed members) is given by:

$$j = [(\frac{M_x}{W} - f_p)/f_{cm} - 1]/C_{v1} \qquad (2)$$

and the Moment M_x in the x-section for a particular uniformly distributed load p is:

$$M_x = \frac{p}{2}(\ell x - x^2) \qquad (3)$$

f_p – residual prestress in most stressed fibre (for non-prestressed member f_p=0).

f_{cm} – mean concrete flexural-tensile strength in member.

C_{v1} – coefficient of lengthwise strength variation.

The probability of crack appearance in the considered member with particular mean concrete strength is:

$$P_{rm} = 1 - \prod_{x_1}^{x_2} (1 - P_x) \qquad (4)$$

According to [1], the mean flexural-tensile strength of concrete is:

$$f_{cmm} = f_{ctm} (.6 + .4h^{-.25}) \qquad (5)$$

and the mean tensile strength in the general population is:

$$f_{ctm} = .3 f_{ck}^{2/3} \qquad (6)$$

As may be inferred from [1], the overall variation coefficient of tensile strength C_v = 18.3%. The variation coefficient of the mean concrete strength in members is:

$$C_{vo} = \sqrt{C_v^2 - C_{v1}^2} \qquad (7)$$

3. PROBABILITY ESTIMATION OF CRACK APPEARANCE IN A MEMBER IN THE STRUCTURE POPULATION

The probability of crack appearance in a member (Fig. 2) with a given grade of concrete under a particular load is evaluated by:

$$P_r = \Sigma P_{rm} P_m \qquad (8)$$

where P_m –probability of occurrence of the specific mean concrete strength, namely:

$$P_m = \frac{\Delta_m}{\sqrt{2x}} \exp(-m^2/2) \qquad (9)$$

The mean concrete strength in a member is defined as: $f_{cm}=f_{cmm}(1 + m\, C_{vo})$.

The overall probability of crack appearance in a member within the general population is evaluated by:

$$P = \sum_n P_r P_q \qquad (10)$$

$$P_q = \frac{\Delta n}{\sqrt{2\pi}} \exp(-n^2/2) \quad \text{for} \quad q = q_m(1 + n\,C_q)\ , \qquad (11)$$

where: n – independent parameter, q_m – mean load, C_q – coefficient of load variation.

Fig. 3 shows the cracking probability for beams made of C-30 concrete under a characteristic load .0022 MN/m, versus C_q [5].

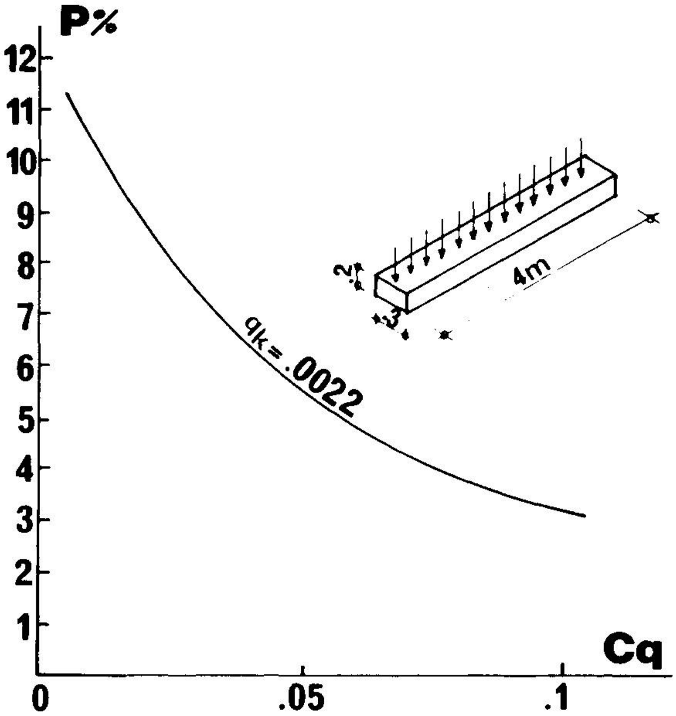
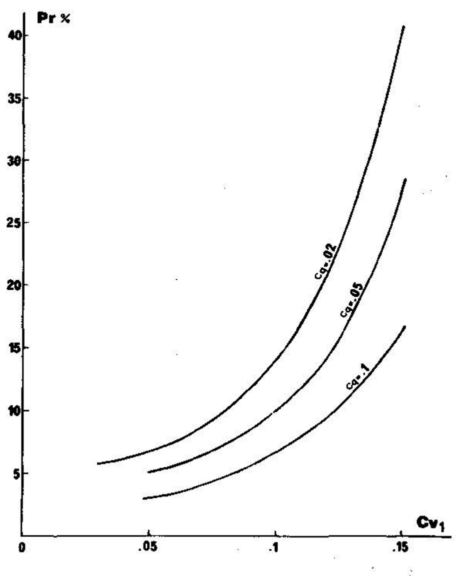


Fig. 3. Probabilities of crack appearance in beam made of C-30 concrete under 0.0022 MN/m characteristic load – versus coefficient of load variation.

4. PROBABILITY ANALYSIS OF MEMBERS WITH VARIABLE CONCRETE STRENGTH UNDER VARIABLE LOADS



The characteristic load for analysis is determined as the load causing crack appearance in the most stressed section of a member:

$$q_k = 8\ f_{cmk}\ W/\ell^2 \qquad (12)$$

Numerical analysis results for the members, as shown in Fig. 1 (h=.2m, b=.3m, grade C-50), with different lengthwise strength variations (C_{v1}) – are presented in Fig. 4.

Fig. 4. Probabilities of crack appearance in members vs. lengthwise strength variation (C_{v1}) under the same characteristic load with different variations (C_q).

It is felt that present codes favor design with a wide range of reliability in terms of cracking.

As shown in [6] for composite slabs, the probability of crack appearance in the spans of statically-indeterminate structures should be significantly lower than in simply-supported ones, and such structures may be more reliable. Their reliability depends on the variability of concrete strength in certain parts of the structure, and combinations of low strength make for very low probabilities.

Narrow variation of concrete strength may drastically reduce the probability of crack appearance in a structure. Analysis of cracking probability in members with narrower strength variability under the same mean load (Fig. 5) shows that the reliability of a code-designed structure may be significantly improved through improved homogeneity of the concrete in terms of tensile strength.

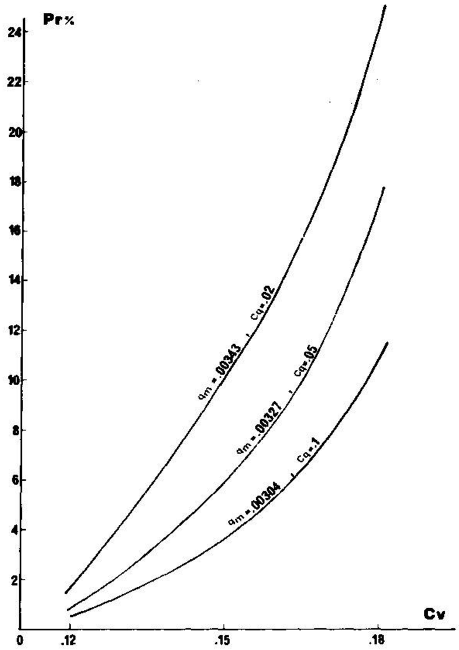


Fig. 5. Probabilities of crack appearance in members under the same mean loads appropriate to their variations - vs. real coefficient of tensile strength variation of concrete.

5. CONCLUSIONS

The analysis shows that traditional estimation of structure serviceability in terms of cracking, yields significantly different reliabilities for concrete structures in bending. Based on probabilistic criteria and statistical initial data, the above situation may be improved with the aid of suitable analysis. In any event the traditional estimation should be corrected by behaviour factors, taking into account the variabilities of concrete strength and load combinations.

The proposed design approach calls for supplementation of the codes by suitable statistical data and by probabilistic restrictions.

As the reliability of a real structure depends on the variability of concrete strength in practice, gradation and strength control of the concrete on the basis of its tensile strength are of extreme importance.

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Watertight Concrete Structures

Etanchéité des structures en béton

Wasserundurchlässige Betonkonstruktionen

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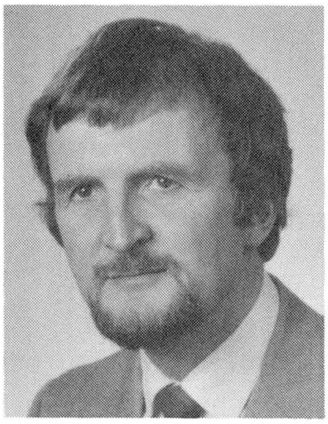

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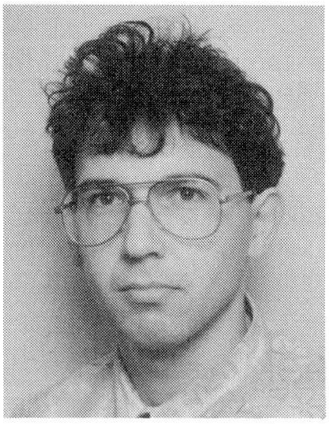

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SUMMARY

For various projects like basements or subway-tunnels, which are built and remain permanently in the groundwater, watertight concrete structures are of increasing importance. The paper presents general design criteria for such constructions. It deals, in particular, with loading due to restraint under construction and final conditions, waterproof designs and their consideration in different design codes.

RÉSUMÉ

L'étanchéité est d'importance grandissante pour des projets variés, qui tels que fondations et tunnels, restant en permanence sous le niveau de la nappe phréatique. Cet article souligne les critères généraux de dimensionnement concernant ces types de constructions: l'accent est mis sur les contraintes en phase constructive et finale, la conception de l'étanchéité elle-même ainsi que sa prise en compte dans diverses normes de dimensionnement.

ZUSAMMENFASSUNG

Im Rahmen verschiedener Bauaufgaben wie Gründungen und U-Bahn-Tunnel, die in das Grundwasser einbinden, gewinnen wasserdurchlässige Betonkonstruktionen immer mehr an Bedeutung. Im folgenden sollten daher Entwurfskriterien für derartige Bauwerke vorgestellt werden. Insbesondere wird auf die in den verschiedenen Bau- und Endzuständen auftretenden Zwangsbeanspruchungen, auf die Nachweise der Dichtigkeit sowie deren Behandlung in den Normen eingegangen.

1. INTRODUCTION

The production of watertight concrete structures without additional waterproofing provisions is being practised successfully since many years [2], [4]. Projects which need watertight structures may be classified in three groups:

- As protection of structures against underground water penetration in cases in which the ground water level is located above the foundation slab (underground garages, cellars, subway tunnels).
- As protection of underground water against contaminating substances from purification plants, from manure dumps or from catch basins of chemical plants.
- Water reservoirs require a certain grade of impermeability (leakage rate) because of operating conditions, although their contents do not endanger the underground water.

The determination of the impermeability can result from two procedures, which differ on their practical evaluation:

- Limitation of the moisture content at the air side regarding the impermeability criteria: - complete dry - dry to a great extent - capillarily soaked -
- Limitation of the amount of moisture penetrated through the cross section (leakage rate).

The permeability or leakage of a buildung can by caused by its materials (because of porosity, that is the concrete texture) or by the construction (because of cracks, improperly performed construction joints or expansion joints, leakages within the range of perforations for installations).

The production of a concrete with enough tight texture is primarily a concrete technological problem. The causes of leakage dependent on the construction can be obviated by means of a corresponding clear construction, that harmonizes optimally with the building function (through states of stress due to loads and restraints which are statically easy to survey and through a practicable construction).
The following report deals mainly with the possible proof and examination of usefulness.

2. STATES OF STRESS

2.1 Causes of the states of stress

The states of stress acting on reinforced concrete structural parts may be classified according to the instant at which they occur and according to their causes. According to the instant of occurrence one may distinguish between:

- states of stress during the construction
- states of stress after the end of the construction

According to the causes one may distinguish between:

- states of stress due to loads
- states of stress due to restraints

Stress resultants due to loads (for example owing to dead load or to traffic load) can be easily calculated in most cases. The realistic calculation of stress resultants due to restraints, however, cause considerable difficulties. The dissipation of the heat of hydration [1], shrinkage, temperature and settlements can be regarded as the main reasons for the appearance of restraint stresses.

2.2 Restraint stress resultants

2.2.1 Calculation

The restraint stresses of a thick structural member of concrete (Fig. 1) often have a non-linear development over the thickness of the structure. These non-linear restraint stresses can be divided into a constant part (shortening by restraint), a linear part (change of curvature by restraint) and a non-linear part (residual stresses). It is characteristic for the state of residual stresses, that these stresses do not create stress resultants.

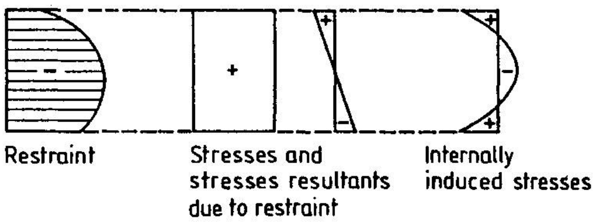


Fig. 1 Stress-strain of a thick concrete member due to non-linear restraint

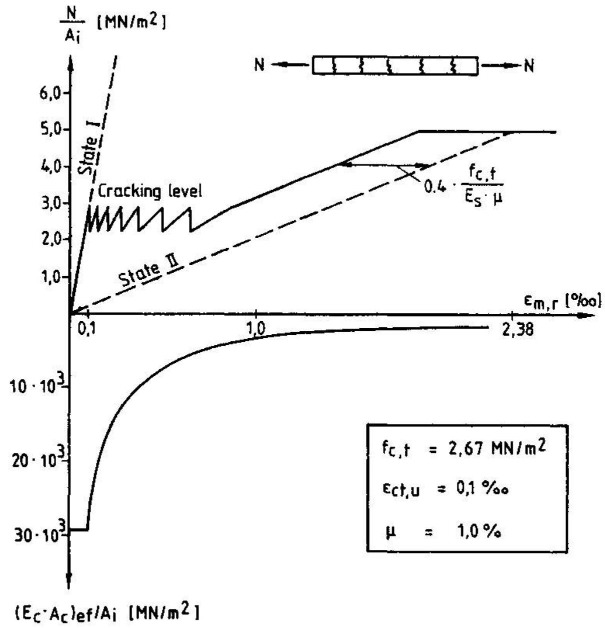
The following denotations are valid for the calculation of the restraint stress resultants:

$$N_{zw} = \epsilon_r \cdot (EA_c)_{ef} \cdot \delta \cdot c_s \qquad (1)$$

$$M_{zw} = \chi_r \cdot (EI_c)_{ef} \cdot \delta \cdot c_s \qquad (2)$$

$(EA_c)_{ef}$: effective longitudinal stiffness
$(EI_c)_{ef}$: effective bending stiffness
c_s : reduction of restraint by creep
δ : grade of impediment by the structural member

The effective stiffness used in equations (1) and (2) can be studied clearly by a centrically loaded member in Fig. 2.



Up to the load of the first crack ($\sigma_c = f_{c,t}$) the structural member remains in state I. After that a successive cracking starts. When the fully developed cracking pattern is reached, there is another ascent of the stress - strain - diagram, which is almost parallel to the plain state II, at a distance of $0{,}4 \cdot f_{c,t}/E_s \cdot \mu$ (tension-stiffening-effect)

f_{ct} : tensile strengh of concrete
ϵ_{ctu} : corresponding strain of concrete
E_s, E_c : modulus of elasticity of steel and of concrete
μ : percentage of reinforcement

Fig. 2 Stress-strain-relationship and effective stiffness of a concrete member under longitudinal force ($A_i = A_c \cdot (1 + n \cdot \mu)$; $n = E_s/E_c$)

At the final state the restraint effects (change of temperature, settlements) are of minor importance for the constructions, which are mostly box-shaped and bedded to the ground. During the construction however, the restraint effects, such as dissipation of the heat of hydration and shrinkage, are to be analysed more carefully. That is especially important for monolitically connected structures, which are cast however at different times. Regarding the dissipation of the heat of hydration, the calculation of the corresponding restraint stress resultants is very difficult because reliable informations about modulus of elasticity, reduction by creep and coefficient of thermal expansion for new (not matured) concrete are hardly available. An estimation of the restraint forces through shrinkage is possible in form of a difference value for shrinkage at structural components which are cast at different times (foundation slab/walls).

2.2.2 Evaluation of the limits of stress resultants for special structural components

- Regarding the load for first cracking

The upper limit of the restraint stress resultant at the instant of appearance of the first crack can bei determined through the sectional forces of the crack for state I.

$$N^I_{Crack} = A_c \cdot f_{ct,ef} \quad (3)$$

$$M^I_{Crack} = W_c \cdot f_{ct,ef} \quad (4)$$

- Regarding the interaction soil - building

For foundation slabs it is possible to get further limitations for the normal force due to restraint considering the effects of frictional forces. In this connection it has to be examined, whether slide of the foundation slab with respect to the soil results or whether a total bond exists. Fig. 3 shows the diagram of the forces at the contact surface with the soil and the forces at the foundation slab in the case of friction, bond and a combination of both effects.

In the case of skidding friction the maximum normal force in the foundation slab results:

$$N_\mu = 1/2\, \mu\, g\, l_\mu \quad (5)$$

Assuming a total bond between foundation slab and soil the following value results:

$$N_B = \frac{\epsilon_m}{\dfrac{1}{E_{cf}\, t_f} + \dfrac{1}{(2\, E_s\, t_s + E_{cb}\, t_b)}} \quad (6)$$

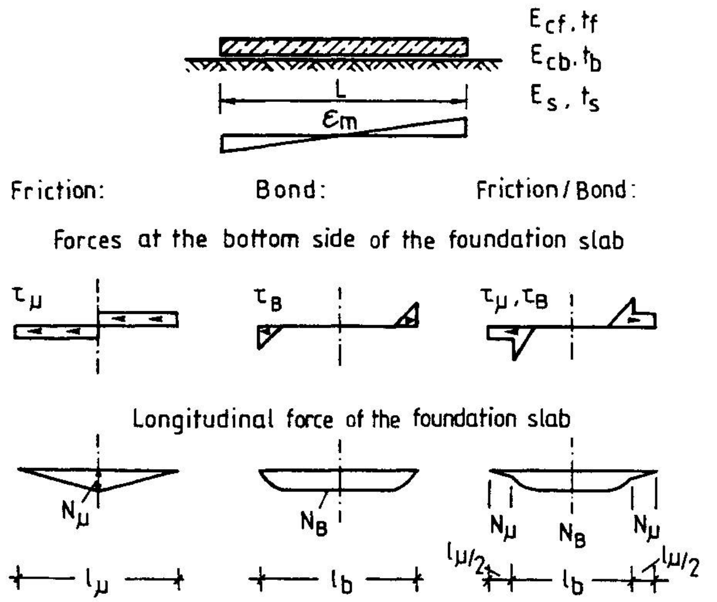


Fig. 3 Longitudinal forces due to restraint of a foundation slab

3. PROOF OF WATER TIGHTNESS

3.1 Method of proof

The proof of watertightness is made for the limit state of serviceability. In additon to the demonstration of an available sufficient tight concrete texture, the leakage caused by construction is limited further through the following criteria.

- sufficient depth of the compression zone: $x > x_{nec}$
- limitation of the width of separating cracks: $w < w_{adm}$
- limitation of tensile stresses in concrete: $\sigma_{ct} < f_{ct}/\nu$

3.2 Proof of the depth of the compressive zone

The procedure is based upon the requirement that the resulting depth of the compression zone in working conditions must be greater than the depth of penetration of water in the watertight concrete, which is produced according to technological points of view. The determination of the depth of the compression zone in working conditions for bending with normal forces leads to a cubical equation if linear elastic behaviour is assumed for concrete and for steel.

Determinant parameters:
- the percentage of reinforcement ratio μ_1/μ_2
- the relative edge distances $d_{1,2}/h$
- the E-moduli ratio $n=E_s/E_c$
- the relative eccentricity e/h

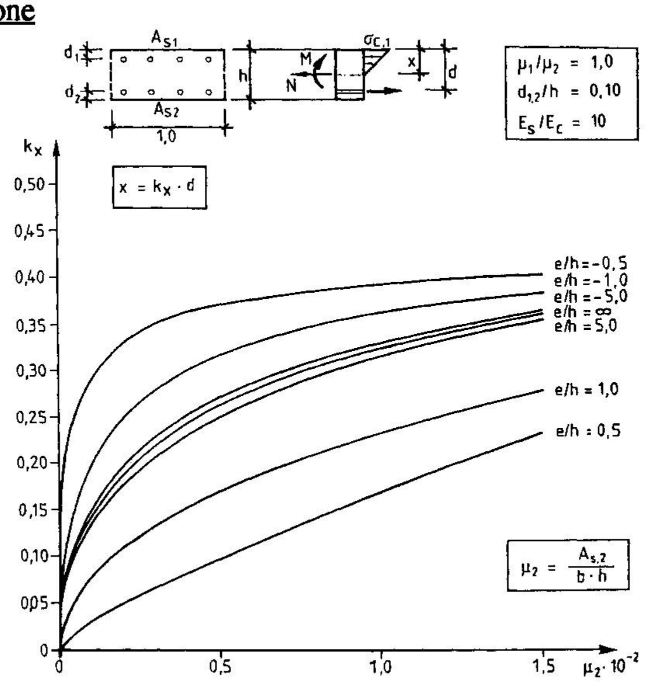


Fig. 4 Depth of compression zone under servie load

3.3 Proof of the width of crack

Separating cracks are critical with respect to the watertightness of a concrete structure. All national and international standards have renounced to fix an admissible width of crack w_{adm} in relation to watertightness (see paragraph 4). The determination of such a limit value needs the careful examination of the special function of the building and of environment conditions as for example the value of the water pressure acting on the examined structural members. In the references ([3], [4], [5], [7]) however, values for w_{adm} = 0,1 to 0,2 mm are given. There are many theories which calculate the theoretical width of crack at the limit state of serviceability. The principles of equation (7) based on fig. 5 are common to all theories. Based upon this fundamental equation it is possible to control cracking through concrete technological provisions and through the adequate choice of reinforcement (steel stresses, diameters of bars) with minimum steel areas according to the restraint stress resultants given in paragraph 2.

$$w_m = s_{rm} \cdot \epsilon_{sm} \quad (7)$$

w_m : average width of crack
s_{rm} : average distance between cracks
ϵ_{sm} : average strain of steel

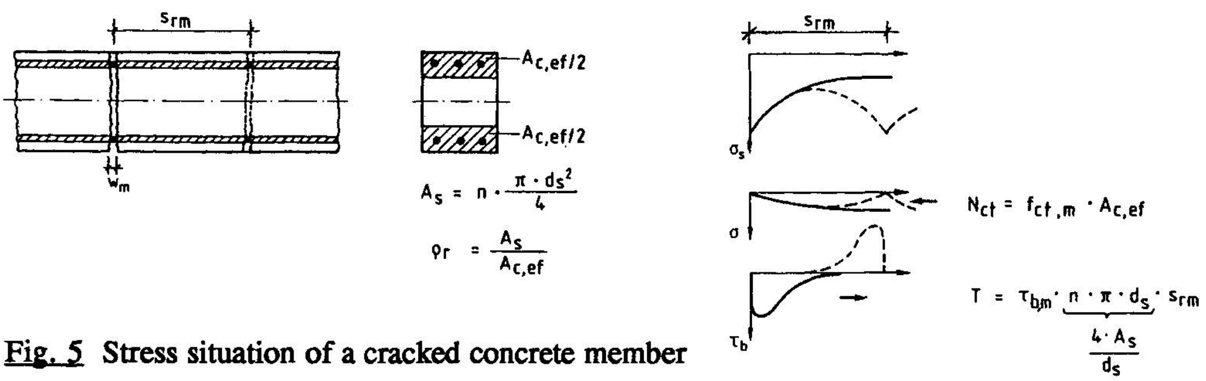


Fig. 5 Stress situation of a cracked concrete member

3.4 Limitation of the concrete tensile stresses in state I

On the one hand this criterium leads to relativ thick strctural members, which are sensible to states of stress due to restraint, and on the other hand the accurate knowledge of the tensile strength of concrete is necessary, although the values show the well known large statistical dispersion. For that reason the proof based on the latter criterium should be given up.

4. COMPARISON OF NATIONAL AND INTERNATIONAL STANDARDS

Whereas in many standards the production of a concrete with tight texture is mentioned, as Tab. 1 shows, nearly all standards which have been studied have renounced to define requirements for a numerical proof of the watertightness. The corresponding proof of limitation of crack width from most standards serves therefore only to insure the durability.

Code/Country/ Year	Water-tightness	Permeability of Concrete	Minimum reinforcement	Crackwidth-Control	Tensile stresses of Concrete
MC - 90/ - / 1990	No	Yes	Yes	Yes	No
EC - 2 / - / 1989	No	Yes	Yes	Yes	No
DIN 1045/D/ 1988	No	Yes	Yes	Yes	No
SIA 162/CH/ 1989	Yes	Yes	Yes	Yes	No
CP-110/GB/ 1972	No	Yes	Yes	Yes	Yes
B 4200/A/ 1979	No	Yes	No	No	No
ACI 318/USA/1983	No	Yes	Yes	Yes	No

Tab. 1 Comparison of standards

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Effects of Residual Strength of Cracked Concrete on Bond

Effets sur l'adhérence des contraintes résiduelles du béton éclaté

Einfluss der Restzugfestigkeit gerissenen Betons auf den Verbund

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SUMMARY

The local bond-slip law of an anchored ribbed bar after the complete cracking of the surrounding concrete is studied. The theoretical approach is based on the confining effects due both to the transverse reinforcement and the residual tensile strength of cracked concrete. Experimental confirmations and theoretical results are presented. In particular the confining effects produced by the residual strength of cracked concrete are investigated and discussed.

RÉSUMÉ

On étudie ici la loi localisée d'«adhérence-glissement» apparaissant dans le cas d'une barre nervurée ancrée après éclatement complet du béton d'enrobage. L'approche théorique se base sur des effets confinants dûs conjointement à l'armature transversale ainsi qu'aux contraintes de traction résiduelles du béton éclaté. Des confirmations expérimentales et des résultats théoriques sont présentés; les effets confinants provoqués par la contrainte résiduelle du béton éclaté sont examinés et discutés en détail.

ZUSAMMENFASSUNG

Das örtliche Verbundgesetz für die Verankerung eines gerippten Bewehrungsstabs in vollständig gerissenem Beton wird untersucht. Grundlage für die theoretische Untersuchung sind die Umschnürungseffekte infolge der Querbewehrung und der Restzugfestigkeit des gerissenen Betons. Theoretische Ergebnisse und experimentelle Bestätigungen werden vorgestellt. Insbesondere werden die Einflüsse einer Umschnürung infolge der Restfestigkeit des Betons untersucht und diskutiert.

1. INTRODUCTION

The importance of the tensile strength of concrete on bond was underlined in [1]. In the same paper this phenomenon was theoretically modelled and the bond strength was evaluated taking into consideration the tensile strength of solid concrete that surrounds the split core. Experimental tests [2] showed that the bond stress-slip relationship is influenced both by the amount of stirrups and by the thickness of the concrete cover. A theoretical interpretation and modelling of the phenomena involved around an anchored bar were proposed in [3,4], where the relevance of the confining action produced by the residual tensile strength of the split concrete was underlined. At the beginning, the splitting crack opens near the anchored bar (Fig.1a,b) and propagates both transversally and along the bar. When it is completely propagated throughout the cross-section, bond strength is still locally possible owing to the confining actions produced by the transverse reinforcement and the residual stress transmitted by the crack faces [5,6]. For light or no transverse reinforcement, bond stress decreases as slip increases, so that an unstable local behaviour occurs. When the split zone is limited as in Fig.1, bond stress redistribution along the bar can occur and a ductile global behaviour of the anchorage is still possible. The bigger the concrete cover and bar spacing become, the more relevant the confining contribution of cracked concrete is. This is because a small residual stress acting on a large split surface can produce a considerable confining action. Since the splitting crack opening is variable both across the transverse section and along the anchored bar, the local response of the cracked concrete is also variable. The cracked concrete confining contribution should be evaluated by means of the tensile stress-crack opening law. The well known specific fracture energy $\mathcal{G}_F$, which is the integral of this law, is not sufficient to express the cracked concrete confining capacity, at least in the present theory on bond.

2. ANALYTICAL FUNDAMENTALS

In anchorages with completely propagated splitting cracks, bond is still possible when an adequate transversal confining action is assured. This confinement can be produced both by the transverse reinforcement (secondary bars or stirrups) and by the residual strength of cracked concrete.

The modelling of the local bond behaviour in anchorages when splitting occurs is developed on the basis of the following assumptions:
1. The splitting crack is completely propagated along the bar spacing and cover in influence zone Δz of one transverse bar (Fig.1c).
2. Δz is small and has the same value of stirrup spacing, so that average crack opening w and bond stress τ can be assumed as the local values.
3. All the principal bars have the same diameter ϕ_p and all the transverse bars have the same diameter ϕ_{st}.

According to these assumptions, the following equations were proposed in [3,4]. For bond:

$$\tau = \tau_{m,o}\,(1 - \gamma_1\, w/\phi_p)\,(1-e^{-(\beta_1+\beta_2 w/\phi_p)(s/\phi_p-\gamma_2 w/\phi_p)}) \quad (1)$$

$$\tau = \tau_0(1/(1+K_1 w/\phi_p)) + \tau_1\sigma_n(1/(\ 1+K_2 w/\phi_p)) \quad (2)$$

where $\tau_{m,o}$ = maximum bond stress for w=0; γ_1, γ_2, β_1, β_2, K_1 and K_2= coefficients experimentally determined on the basis of the curves plotted in Fig.2a,b and obtained in [7]; s = principal bar slip and σ_n = radial stress produced by the principal bar. The limitation $\tau = \tau_{m,o}$ when $\tau > \tau_{m,o}$ was adopted. For stirrup stress (1st confining action) equation:

$$\sigma_{st} = E_s\sqrt{a_2(w/(\alpha\ \phi_{st}))^2 + a_1(w/(\alpha\ \phi_{st})) + a_0} \quad (3)$$

plotted in Fig.2c, was assumed according to [8] where E_s = Young's modulus for steel; a_0, a_1 and a_2 = coefficients of the ideal trilateral local bond stress-slip law of the transverse bars and α = factor characterizing the position of the splitting crack (Fig.4d). For tensile stress transmitted by the splitting crack faces (2nd confining action) equation:

$$\sigma_{rc} = f_{ct0}/(\kappa\ w/\phi_a + 1) \quad (4)$$

plotted in Fig.2d, was adopted according to [6] where f_{ct0} and κ = coefficients experimentally determined and ϕ_a = maximum aggregate size.

Eq.1 is based on the following similitude criterion: both crack opening w and slip s are proportional to bar diameter ϕ_p. In this way, for the same value of ratios s/ϕ_p and w/ϕ_p all the coefficients γ_1, γ_2, β_1 and β_2 should be independent of ϕ_p. Even coefficients τ_0, τ_1, K_1 and K_2 in Eq.2 should be independent of ϕ_p, having adopted ratio w/ϕ_p in the place of w. The first confining action produced by the stirrup legs increases with the splitting crack opening (Eq.3) and this phenomenon is governed by the progressive unsticking of the bar studied in [8]. For the second confining action due to the tensile strength of cracked concrete (Eq.4), a similitude criterion for the relationship between w and the maximum aggregate size ϕ_a was also proposed in [6], so that coefficient κ turned out to be be independent of ϕ_a.

For equilibrium, the global confining action in zone Δz, given by Eqs.3 and 4, is equal to the global radial force produced by the anchored bars, so that:

$$\sigma_n = \Omega\, \sigma_{st} + B\, \sigma_{rc} \quad (5)$$

where Ω = stirrup index of confinement, defined as the ratio between global cross section area A^*_{st} of the stirrup legs and area A^*_p of the principal bar in the split plane (Fig.4f), B = concrete index of confinement, defined as the ratio between the net area $(b-n_p\phi_p)\Delta z$ of concrete in the split plane and the afore mentioned area A^*_p.

From Eqs.2 and 5, bond stress τ as a function of σ_{st}, σ_{rc} and w can be obtained:

$$\tau = \tau_0(1/(1+ K_1\ w/\phi_p)) + \tau_1(\ \Omega\, \sigma_{st} + B\, \sigma_{rc}\ )\ (1/(1 + K_2\ w/\phi_p))$$

Owing to the nonlinear equations involved, the relationship of bond stress τ as a function of slip s is obtained for the principal bar by means of a numerical approach which is based on the following procedure. Attributing a value w to crack opening, Eqs.3 and 4 give σ_{st} and σ_{rc}. Then bond stress τ can be calculated by means of Eq.6 and finally slip s is obtained from Eq.1.

3. RESULTS

In Fig.3 curves τ-s obtained by the present theory fit the experimental results well. Curves 1-4 (Fig.3a) concern the cases examined in [2] with different transverse reinforcement diameters ϕ_{st}. Fig.3b, referring to a specific test studied carefully in [3] to check this theory, shows a very good agreement also for crack opening and stirrup stress.This agreement still emphasizes the importance of the confining contribution due to the residual tensile strength of split concrete.

Theoretical diagrams of Fig.4 show the role of some significant parameters. Curves τ-s, w-s, σ_{st}-s refer to the following governing parameter values:

- Eq.1: $\tau_{m,0}$ = 18 MPa $\quad\beta_1$ = 75 $\quad\beta_2$ = 0 $\quad\gamma_1$ = 42 $\quad\gamma_2$ = 0.8;
- Eq.2: τ_0 = 1.8 MPa $\quad\tau_1$ = 0.8 $\quad k_1$ = 115 $\quad k_2$ = 35;
- Eq.3: τ_{02} = 2.5 MPa $\quad\tau_{12}\phi_{st}$ = 500 MPa $\quad\tau_{12}/\tau_{11}$ = 0.3;
- Eq.4: f_{ct0} = 1.0 MPa $\quad\kappa$ = 250;
- geometrical and mechanical characteristics:

n_p=2 $\quad\phi_p$=20 mm $\quad\alpha$=2 $\quad E_s$=206000 MPa $\quad\Delta z$=100 mm $\quad\phi_a$=15 mm $\quad$b=200 mm

Different values of the geometrical or mechanical characteristics adopted for each curve are indicated in Tab.1. Figs.4a,b,c show the role of the transversal extension of the concrete split-area dependent on section width b. Three different amounts of confining reinforcement are adopted and expressed through stirrup index of confinement Ω. Figs.4g,h,i show the influence of fracture energy $\mathcal{G}_F$ obtained by integrating the σ_{rc}-w curve (Eq.4) from w=0 to $w=w_u$. High values of $\mathcal{G}_F$, correspondent to an appreciable residual strength of cracked concrete, and large values of width b both increase the value of bond stress τ.

Note that fracture energy $\mathcal{G}_F$ could be assumed as one of the governing parameters of the present bond stress-slip relationship, but some specifications and remarks are necessary. In reality, the bond stress-slip relationship obtained in [4] showed the importance of the parameters f_{ct0}, κ and ϕ_a, characterizing the σ_{rc}-w relationship. These results were independent of the ultimate crack opening w_u (correspondent to stress-free crack surface). In fact, the maximum value of the splitting crack opening involved was 0.2-0.3mm, which was remarkably less than values w_u=0.4-0.7 mm indicated by experiments [6]. Fracture energy $\mathcal{G}_F$ depends on the same governing parameters f_{cto}, κ and ϕ_a, but also on w_u. This ultimate crack opening seems to be only variable with maximum aggregate size ϕ_a, according to both the similitude criterion introduced in Eq.4 and to some experiments in progress, so that the ratio w_u/ϕ_a could be assumed as a constant for every type of concrete, as well as coefficient κ. In this way $\mathcal{G}_F$ and ϕ_a can become the only governing parameters involved in σ_{rc}-w relationship. Diagrams of Fig.4g,h,i refer to an aggregate size ϕ_a=15mm (w_u/ϕ_a=0.05) and three values of $\mathcal{G}_F$ (50,100,150 J/m^2) correspondent to low, medium and high residual strength.

4. CONCLUDING REMARKS

The analytical model here proposed for the local bond stress-slip relationship after concrete splitting gives results which have a good agreement with the experimental tests (Figs.3a,b). The theoretical curves considerably depend on the residual tensile strength of cracked concrete especially when light or no transverse reinforcement is present. This residual tensile strength of split concrete is here introduced by means of two governing parameters which are fracture energy $\mathcal{G}_F$ and maximum aggregate size ϕ_a. In the present theory the single parameter $\mathcal{G}_F$ is not sufficient to describe this confining action due to the split concrete.

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Diag.	ϕ_{st} [mm]	n_{st}	f_{ct0} [MPa]	G_F [J/m²]
4,a	0	0	1.00	150
4,b	6	1	1.00	150
4,c	8	2	1.00	150
4,g	0	0	1.00 0.66 0.33	150 100 50
4,h	6	1	1.00 0.66 0.33	150 100 50
4,i	8	2	1.00 0.66 0.33	150 100 50

Table 1

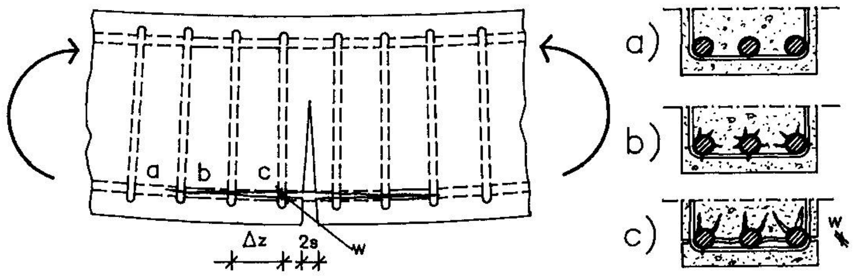


Fig. 1 – Splitting crack propagation in anchorages: a)no splitting crack; b)partially propagated splitting crack; c)completely propagated splitting crack.

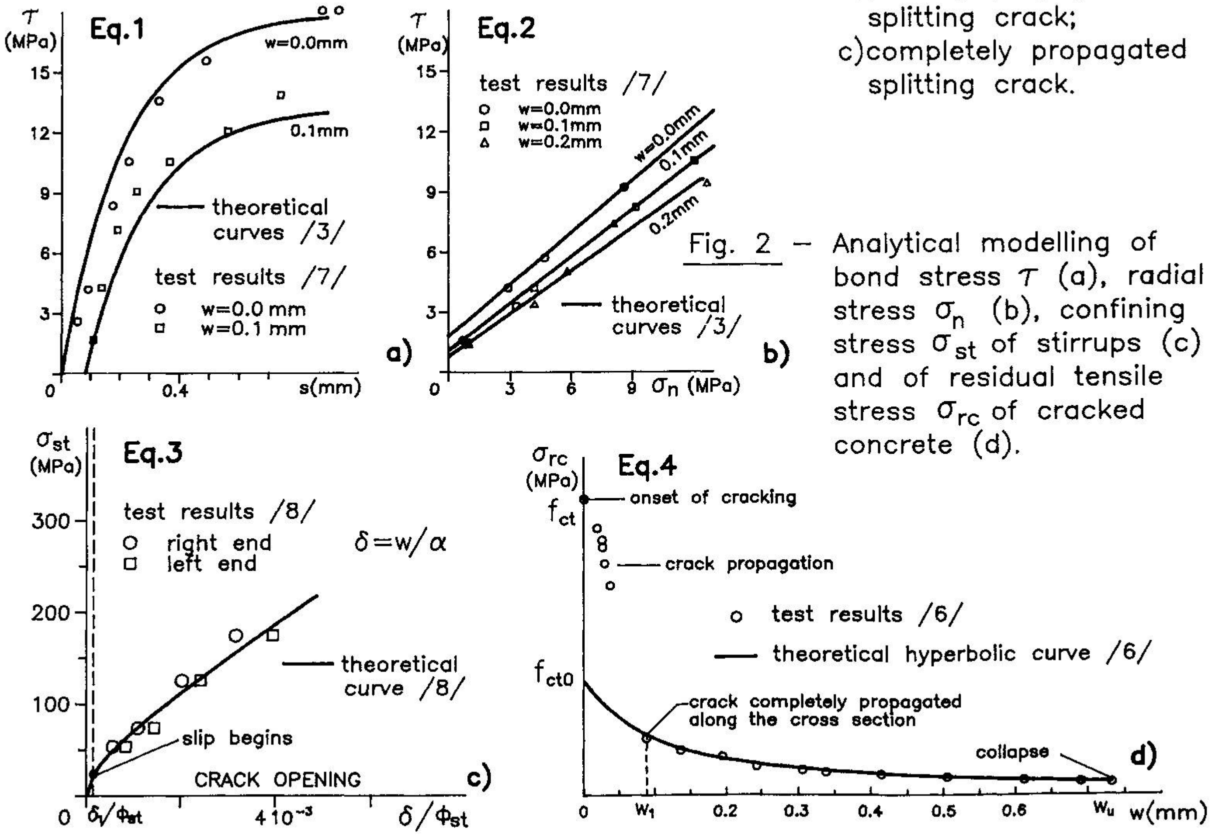
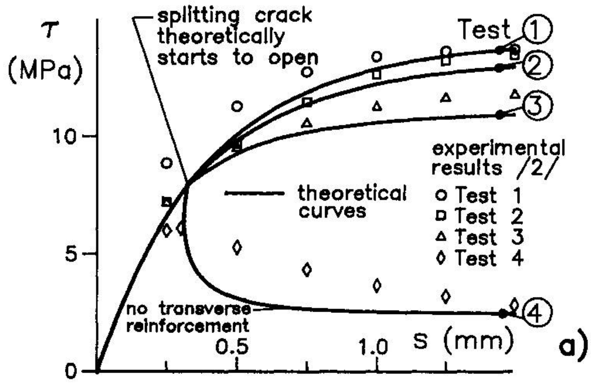


Fig. 2 – Analytical modelling of bond stress τ (a), radial stress σ_n (b), confining stress σ_{st} of stirrups (c) and of residual tensile stress σ_{rc} of cracked concrete (d).



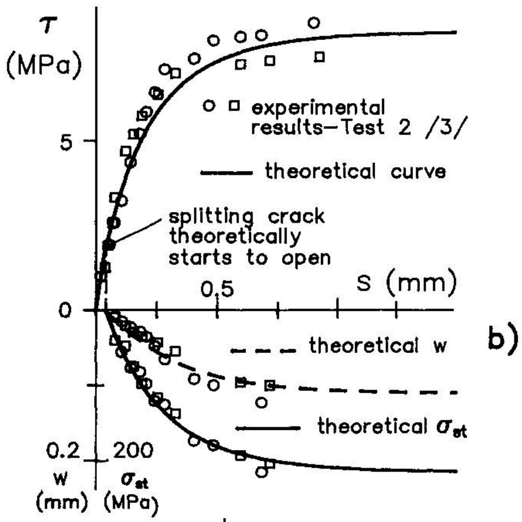


Fig. 3 – Comparison between theoretical and experimental results.

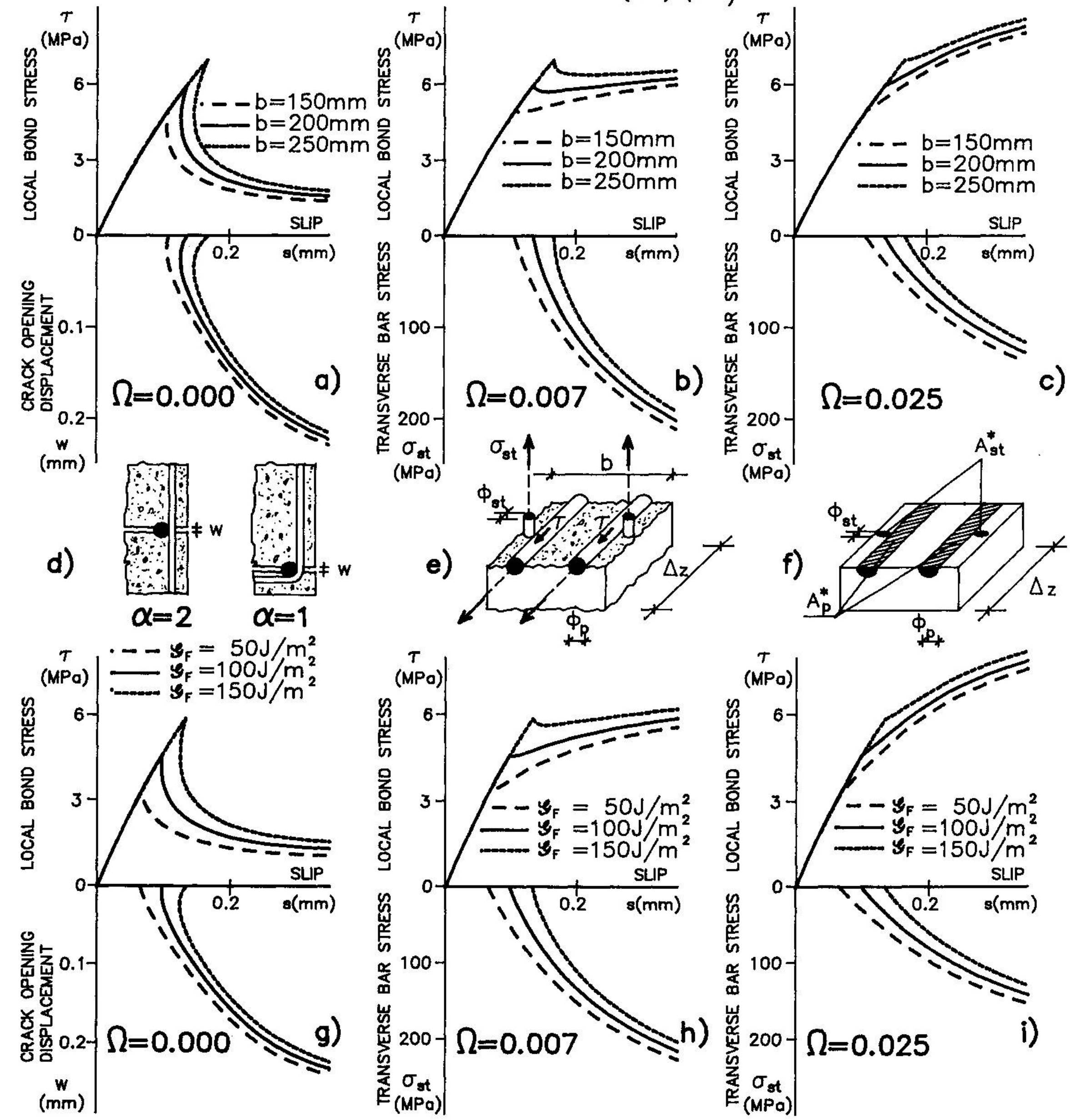


Fig. 4 – Influence of cross section width b and fracture energy $\mathcal{G}_F$ on bond after splitting.

Local Bond between Reinforcing Steel and Concrete

Adhérence localisée acier-béton

Lokaler Verbund zwischen Bewehrungsstahl und Beton

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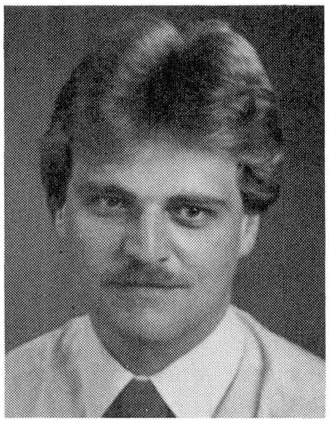

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SUMMARY

Experimental and analytical investigations on the local bond behaviour are presented. The permanent magnet – Hall sensor measuring system was developed for this purpose. This system allows local relative displacements in axial and radial directions between steel and concrete inside the test specimen to be determined. The test results yielded basic knowledge for a better understanding of bond behaviour. Furthermore, a material model was developed which is especially suitable for investigations with the aid of the Finite Element Method.

RÉSUMÉ

L'adhérence localisée acier-béton est présentée à la lumière des recherches expérimentales et analytiques. Grâce à l'emploi de la sonde Hall munie d'un aimant permanent, il a été possible de déterminer les déplacements relatifs locaux dans les directives axiales et radiales apparaissant entre béton et armature, et ceci à l'intérieur-même des échantillons testés. Les résultats du test ont permis d'établir des connaissances fondamentales pour une meilleure compréhension du mécanisme d'adhérence. Un modèle a d'ailleurs été mis au point qui permet d'effectuer des recherches assistées par éléments finis.

ZUSAMMENFASSUNG

In dem Beitrag werden experimentelle und analytische Untersuchungen zum lokalen Verbundverhalten vorgestellt. Mit dem hierfür entwickelten Permanentmagnet-Hallsonden-Messverfahren war es möglich, die lokalen Relativverschiebungen in axialer und radialer Richtung zwischen Stahl und Beton im Inneren der Versuchskörper zu bestimmen. Aus den Versuchsergebnissen wurden grundlegende Erkenntnisse zum besseren Verständnis des Verbundverhaltens gewonnen. Ferner konnte ein Materialmodell erstellt werden, das besonders für Untersuchungen mit der Finite Elemente Methode geeignet ist.

1. INTRODUCTION

Cracks form in reinforced concrete structures when the concrete tensile strength is exceeded. Besides the visible cracks on the surface, inner cracks form at the reinforcing bar ribs. The more the crack formation has progressed, the smaller are the bond forces between steel and concrete. As the crack development depends directly on the concrete strength, the bond stiffness is also influenced by this factor.

The analytical consideration of bond behavior of steel and concrete gains considerable importance. Therefore, the knowledge of realistic models of material behavior is necessary. With these models realistic deformation analyses and economical load-carrying capacity analyses can be executed.

2. GENERAL REMARKS ON THE EXPERIMENTAL INVESTIGATIONS

A detailed description of both experiments and results can be found in [1]. The tension specimens were made of Portland cement PZ35F and natural, unbroken aggregate with a maximum grain size of 16 mm and were centrically reinforced. The following parameters were varied:

- specimen length (short tension specimens without separating cracks of different lengths, long tension specimens with intended separating cracks of different lengths),
- concrete (cover and strength),
- reinforcements (related rib area and diameter),
- loading (number of repeated loads).

For the measurement of the local relative displacements between steel and concrete a small permanent magnet is fixed in the groove of an austenitic reinforcing steel. A Hall sensor is fastened to the concrete at a certain distance to the magnet (see Fig. 1). The magnetic induction influencing the Hall sensor changes with the distance between the measuring system elements. For the determination of the relative displacements a relation between distance and Hall voltage dependent on the magnetic induction is established by calibration before casting. The magnetic induction decreases with increasing distance to the permanent magnet. The relative displacements between steel and concrete at the edge of the specimen were determined using LVDTs.

The forces in the reinforcing steel were determined with the aid of strain gauges glued into two opposite grooves. The bond stress related to one relative displacement measuring point (permanent magnet at reinforcing steel) was analyzed applying the force difference of the strain gauges arranged on both sides of the measuring point.

3. RESULTS

Fig. 1 shows the bond stress - axial displacement relations of two experiments. It can be seen that these relations depend considerably on the concrete strength.

The measuring points arranged at a distance of 35 mm to the edge of the specimen mostly reach the maximum bond stress or even exceed it at the maximum external load of 100 kN. For the measuring points nearer to the center of the specimen (at a distance of 70 mm to the edge of the specimen) the maximum bond strength was reached only with those specimens with a concrete compressive strength of less than 35 N/mm^2 at the maximum external load of 100 kN.

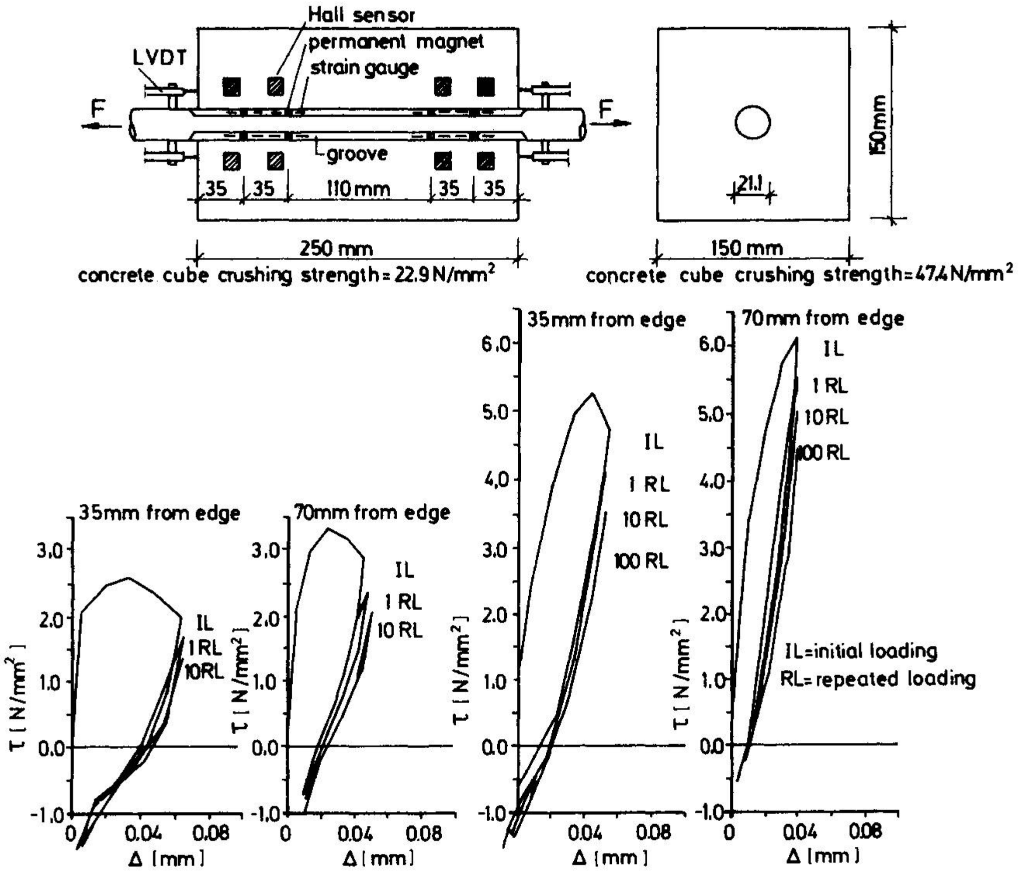

Fig.1 Bond stress - axial displacement relations dependent on concrete strength

An influence of the related rib area on the bond stress - axial displacement relations could not be found. Obviously a certain minimum size of rib area is necessary for the introduction of bond stresses into the concrete. This miminum size at the concrete ribs must be large enough to prevent them from deforming. A deformation of the concrete ribs was not observed with any of the concrete specimens which were split after the tests.

For the determination of the influence of the specimen length specimens which only differed in their lengths (175, 250, and 400 mm) were tested. At equal concrete strength and equal distance to the edge of the specimen smaller bond strengths resulted with increasing specimen length.

Smaller negative or no negative bond stresses occur with specimens with intended cracks contrary to short tension specimens with free edges. This can be traced back to the fact that at a certain unloading of the long specimens an almost constant course of steel stress results. When no external load affects the tension specimens with intended cracks, a nearly constant tensile stress remains in the steel owing to the cracks which do not close completely. Consequently, the concrete is in compression at this point. The intended cracks in the long specimens were arranged in a way that they corresponded to the average crack spacing. This is because the bond stress - axial displacement relations depend on the specimen lengths.

The bond stress - axial displacement relations of long specimens with average crack spacings are decisively dependent on the concrete cover. The larger the concrete cover, the larger the transfer of bond stress between steel and concrete (see Fig. 2).

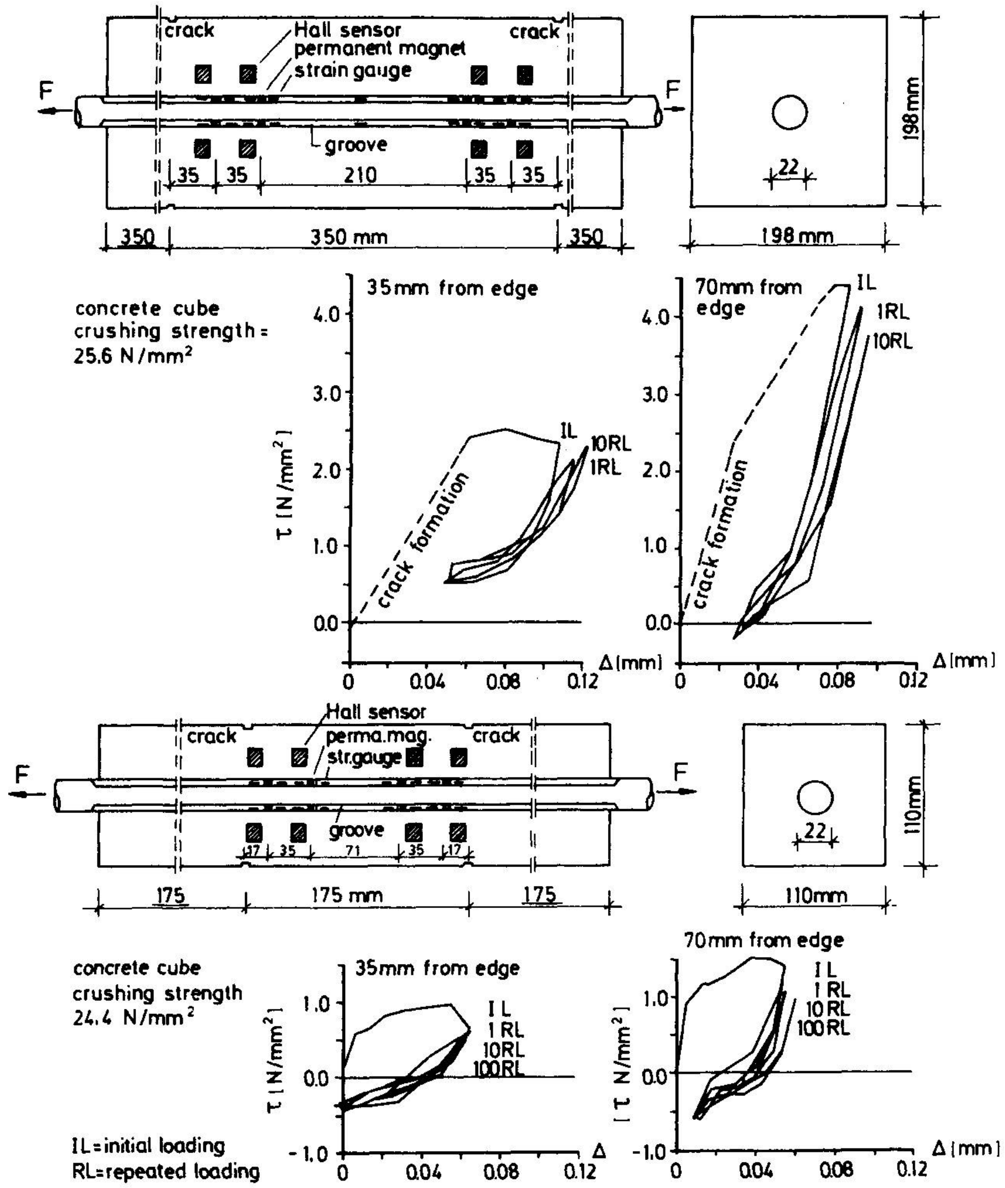

Fig.2 Bond stress - axial displacement relations dependent on concrete cover

For specimens with average crack spacings and a concrete cover of the triple size of differing reinforcing bar diameters the differences of bond stress - axial displacement relations at equal distance to the crack were small. This can be explained by the fact that the relations of concrete cross sectional area and steel surface over which the bond forces are introduced into the concrete have a similar amount.

The courses of relative displacements in radial direction which can be seen in Fig. 3 show that the relative displacements in radial direction at the edge of the specimen increase with increasing specimen length. The relative displacements in radial direction at the edge of a short specimen, however, are larger than those of a long specimen at equal distance to the specimen center. Particularly with the specimens with a length of 400 mm longitudinal cracks form during loading owing to which relatively large relative displacements in radial direction resulted. This explains the lower bond stiffness of edge areas compared to inner areas and the worse bond strength transfer of long specimens compared to short ones at equal distance of the measuring points to the specimen edge.

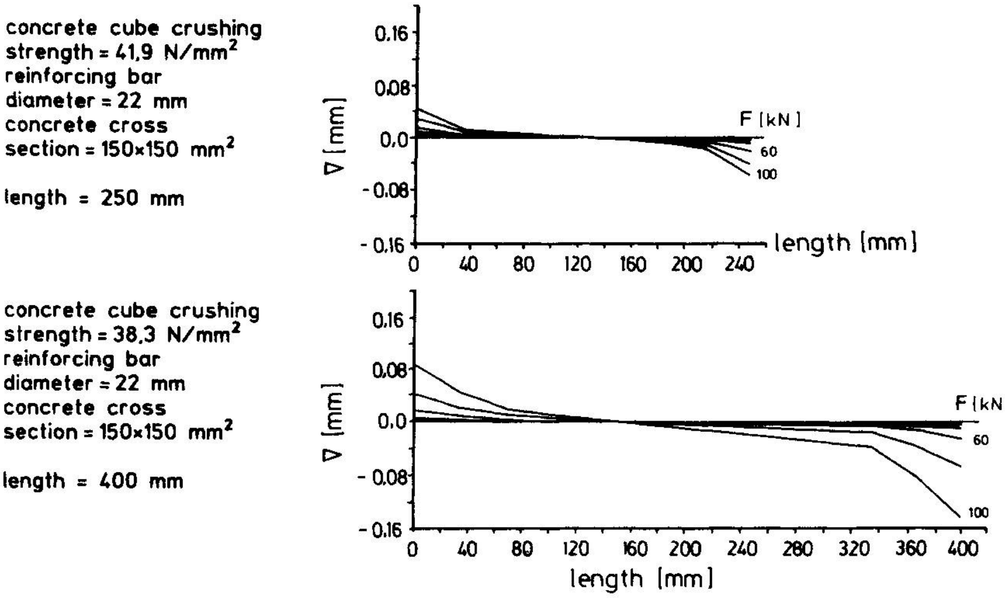

Fig.3 Relative displacements in radial direction dependent on specimen lengths

Fig. 4 shows the relation between relative displacements in axial and radial direction at initial loading. It can be seen that the relative displacements in radial direction are smaller than the relative displacements in axial direction. A distinct increase of relative displacements in radial direction is present only at relative displacements in axial direction of about 0.03 mm or more.

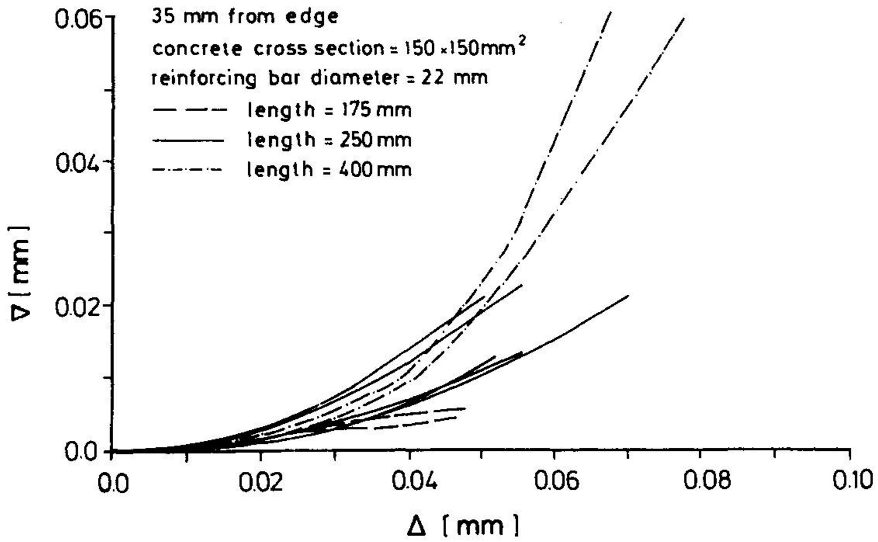

Fig.4 Relation between relative displacements in axial and radial direction for initial loading

4. MATERIAL MODEL

The material model shown in Fig. 5 was determined on the basis of the specimens with average crack spacing. It describes approximately the average bond stress - axial displacement relation between two cracks. The place dependence of bond was not considered as this requires a great deal of additional work in Finite Element analyses. More detailed information for special analyses can be taken from [1].

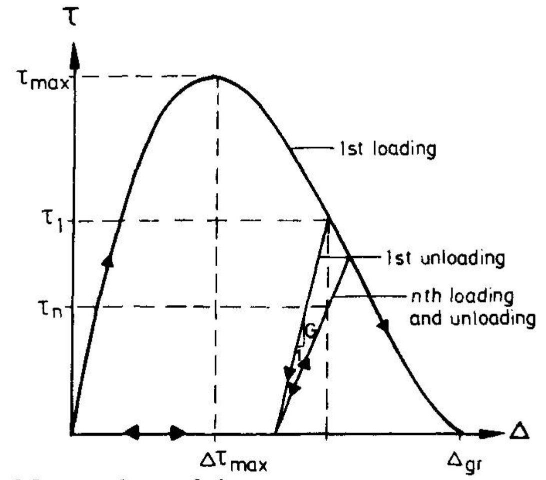

Fig.5 Material model

1st loading

$\tau_{max} = \frac{f_{cube}}{k}$ with

k	c
16	$2 \times d_b$
8	$3 \times d_b$
6	$4 \times d_b$

$\tau = \tau_{max}(4300\Delta^3 - 1000\Delta^2 + 58\Delta)$; Δ[mm]

$\Delta\tau_{max} = 0.04$mm

$\Delta_{gr} = 0.11$ mm

1st unloading

$\tau = G \cdot \Delta$; $G = 200$ N/mm³

nth loading and unloading

$\tau_n = 0.04\tau_1^2 + c \cdot \tau_1$; τ [N/mm²]

c = 0.65 10th loading and unloading

c = 0.50 100th loading and unloading

5. EXAMPLE FOR THE USE OF THE MATERIAL MODEL IN FINITE ELEMENT ANALYSES

A tension specimen was chosen to demonstrate the use of the material model (see Fig. 6). The calculation was made with the Finite Element Program ADINA which has been enlarged by the contact element [2]. Axisymmetric elements were used for the idealization of the reinforcement and the concrete. The nodes of a concrete element are connected to the nodes of a reinforcement element by a contact element which has no physical dimension in transverse direction. Fig. 6 shows that steel stresses, crack spacings, deformations etc. can be calculate if there are realistic material models.

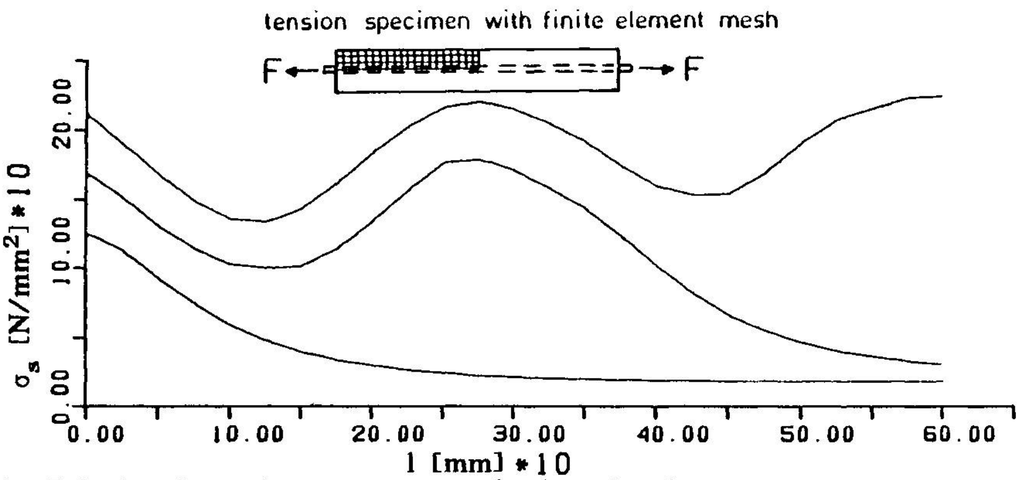

Fig. 6 Calculated steel stresses over the length of a tension specimen

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Tension Stiffening in Reinforced Concrete Elements

Contribution du béton tendu dans des éléments en béton armé fissurés

«Tension-Stiffening» in Stahlbeton-Strukturen

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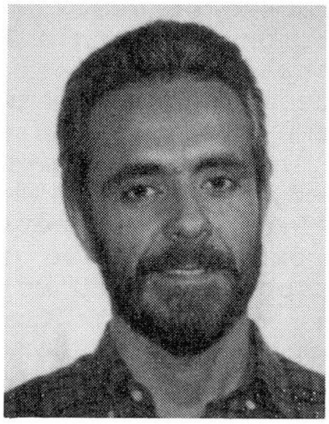

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SUMMARY

The methods to evaluate tension stiffening effects in reinforced concrete structures, are generally based on empirical formulations derived from experimental tests. Recently, an analytical study was presented for modelling the post-cracking behaviour of reinforced concrete bars and membrane elements in a general manner, without a priori hypotheses. The aim of the present paper is to develop an analytical model for reinforced concrete beams subject to bending and axial load, whose formulation is based solely on the classical hypothesis of cross sections remaining plane using suitable constitutive laws for materials and bond characteristics.

RÉSUMÉ

L'évaluation de la contribution du béton tendu dans des éléments en béton armé fissurés se base généralement sur des formules empiriques dérivées d'essais expérimentaux. Une étude analytique a été récemment présentée afin de modéliser le comportement après fissuration de tirants, ainsi que d'éléments-membrane dans la phase suivant la fissuration, mais sans introduire des hypothèses à priori. Le modèle analytique est donc développé pour des poutres en béton armé soumises à des charges axiales et flexionnelles; leur formulation repose uniquement sur l'hypothése classique de la conservation des sections planes, tout en adoptant certaines lois constitutives ainsi que des mécanismes d'adhérence appropriés.

ZUSAMMENFASSUNG

Die Beurteilung der «tension stiffening»-Auswirkungen in Stahlbeton-Strukturen stützt sich im allgemeinen auf empirische Formulierungen, die aus Versuchen abgeleitet wurden. Vor kurzem wurde eine analytische Untersuchung zur allgemeinen Bestimmung des Verhaltens von Zugstangen und Membranstrukturen nach dem Auftreten von Rissen bekannt, die sich nicht von vornherein auf irgendwelche Annahmen stützt. Das Ziel dieser Arbeit ist, ein analytisches Modell für Stahlbeton-Träger zu entwickeln, die Biege- und Normalkraftbeanspruchung unterworfen sind und dessen Formulierung auf der klassischen Hypothese des Ebenbleibens der Querschnitte und der Verwendung geeigneter Material- und Verbundgesetze basiert.

1. INTRODUCTION

Generally the studies to evaluate the tension stiffening effects in reinforced concrete cracked elements are mainly based on semiempirical formulations established to fit test data.[1÷9]
These studies, even if show a good agreement between the theoretical model and experimental results, don't fulfil the requirement of a rational approach to the phenomena [10].
Recently Gupta and al. [11÷12], referring to reinforced concrete bars and membrane elements, presented a theoretical model for the tension stiffening, without resorting to any empirical hypotesis.
The aim of the present paper is similar to the Gupta's one.
A mathematical model for r.c. beams subject to bending with and without axial load is developed, on the basis of the hypotheses generally agreed in studying the behaviour of r.c. cross sections in uncracked and cracked stages.
The basic assumptions concern a linear relationship between concrete compressive strains and steel tensile strains and a linear behaviour of concrete in tension between cracks; no further restrictive hypothesis, of empiric type, is introduced.
In the paper a linear stress-strain behaviour for concrete and steel was adopted, as well as a linear bond stress-slip relationship at the steel-concrete interface.
These crude approximations are used only to reduce the complexity that would be associated with more general relationships, but still keeping a general validity to the formulation.
The proposed model gives reasonably good results, allowing to improve the mathematical formulation by adopting non linear constitutive laws for materials and bond behaviour.
The paper is mainly intended to run an unified approach to describe the post-cracking behaviour of elements in bending in more general way, according to a comprehensive theoretical treatment of the complex phenomena.

2. THE ANALYTICAL MODEL

Fig. 1 shows a part of a reinforced concrete beam between two cracks. The total length of this part is the same as crack spacing 2a and the origin of the x axis is taken midway between the cracks.
The beam has a rectangular, singly reinforced cross section and is subjected to combined axial force and bending moment.

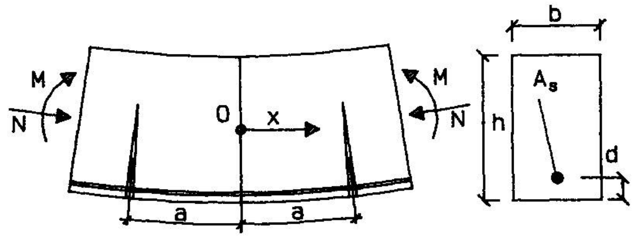

Fig. 1 A part of r.c. beam between cracks and cross section

Let us consider an infinitesimal element of length dx, at distance x from the origin. Fig. 2 shows the stress distributions on the opposite sides of this element. The force F_{bx} is the resultant of the steel-concrete interface stresses in the element and, under the hypothesis of linear bond relationship, is given as

$$F_{bx} = E_b \cdot s \cdot p \cdot dx \qquad (1)$$

where E_b = the slope of the linear bond stress-slip curve, generally called the slip modulus and having unit of FL^{-3}, $s = u_s - u_c$ = the slip between steel and concrete, defined by axial displacements u, p = the sum of the perimeters of the steel bars.
The governing parameters of the problem are the stress state and the slip. If the numerical values of these parameters on one side section of the element are fixed, the unknown quantities are the corresponding values on the other side or their variations : e.g. of the concrete stress in compression $d\sigma_{cc}$ and in tension $d\sigma_{ct}$, of the neutral axis depth dy and of the slip ds. So a set of four equations, relating these four variations of governing parameters, are necessary to solve the problem.

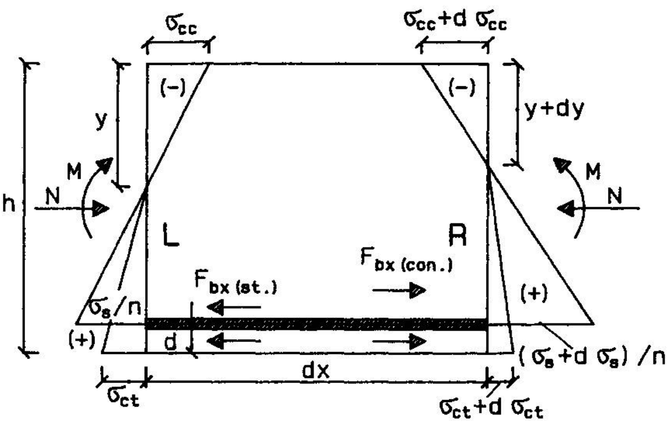

Fig. 2 Free body diagrams for cracked element

The equilibrium of axial forces leads to the first equation:

$$(\sigma_{ct}-\sigma_{cc})\frac{dy}{dx} - (h-y)\frac{d\sigma_{ct}}{dx} - y\frac{d\sigma_{cc}}{dx} - \left[\frac{2E_b p}{b}\right]s = 0 \qquad (2)$$

The equilibrium of moments of the same forces, referred to the steel in tension, leads to the second equation:

$$\left[-\sigma_{cc}(-h+\frac{2}{3}y+d) - \sigma_{ct}\left[\frac{2(h-y)}{3}-d\right]\right]\frac{dy}{dx} + (h-y)\left[\frac{h-y}{3}-d\right]\frac{d\sigma_{ct}}{dx} + y\left[h-\frac{y}{3}-d\right]\frac{d\sigma_{cc}}{dx} = 0 \qquad (3)$$

The third equation is related to the hypothesis of a linear relationship between concrete compressive strains and steel tensile ones.
From the geometric relation $\epsilon_s = -\epsilon_{cc}\frac{(h-y-d)}{y}$ it follows

$$\sigma_s = -\sigma_{cc}\frac{E_s}{E_c}\frac{(h-y-d)}{y} = -n\sigma_{cc}\frac{(h-y-d)}{y}$$ and by differentiating

$$\frac{d\sigma_s}{dx} = -n\frac{(h-y-d)}{y}\frac{d\sigma_{cc}}{dx} + n\sigma_{cc}\frac{(h-d)}{y^2}\frac{dy}{dx} \qquad (4)$$

The equilibrium for the steel axial forces gives:

$$\frac{d\sigma_s}{dx} = \frac{(E_b p)}{A_s}\, s \qquad (5)$$

By introducing eq. (5) in eqs. (2),(4), the final form of third equation can be worked out:

$$\left[\sigma_{ct} - \left[1+\frac{2nA_s}{b}\frac{(h-d)}{y^2}\right]\sigma_{cc}\right]\frac{dy}{dx} - (h-y)\frac{d\sigma_{ct}}{dx} + \left[\frac{2nA_s}{b}\frac{(h-y-d)}{y} - y\right]\frac{d\sigma_{cc}}{dx} = 0 \qquad (6)$$

The last equation refers to the slip between concrete and steel, both in tension. The basic differential equation is:

$$\frac{d^2 u_s}{dx^2} - \frac{d^2 \bar{u}_{ct}}{dx^2} = \frac{d^2 s}{dx^2} = \frac{1}{E_s}\frac{d\sigma_s}{dx} - \frac{1}{E_c}\frac{d\bar{\sigma}_{ct}}{dx} \tag{7}$$

where $d\bar{\sigma}_{ct}$ and $d^2\bar{u}_{ct}$ are referred to concrete at steel level.
From the relation between tensile stresses at different levels

$\bar{\sigma}_{ct} = \sigma_{ct}\dfrac{(h-y-d)}{(h-y)}$, by differentiating it follows:

$$\frac{d\bar{\sigma}_{ct}}{dx} = \left[\frac{h-y-d}{h-y}\right]\frac{d\sigma_{ct}}{dx} - \sigma_{ct}\frac{d}{(h-y)^2}\frac{dy}{dx} \tag{8}$$

By introducing the relationships (5),(8) in the slip equation (7) the last equation required to define the problem becomes:

$$-\left[\frac{\sigma_{ct}\, d}{E_c(h-y)^2}\right]\frac{dy}{dx} + \left[\frac{(h-y-d)}{E_c(h-y)}\right]\frac{d\sigma_{ct}}{dx} - \left[\frac{E_b\, p}{E_s\, As}\right] s + \frac{d^2 s}{dx^2} = 0 \tag{9}$$

3. BOUNDARY CONDITIONS AND NUMERICAL SOLUTIONS

At the onset of a new crack between the existing ones of Fig. 1 the section at x=0 (midway of the element) is still uncracked and the end section at x=a is fully cracked, as in the classical theory for r.c. elements in the II stage.
The boundary conditions at x=0 are quite known because $\sigma_{ct}=f_{ct}$, being f_{ct}=concrete strength in tension, s=0 for symmetry condition, σ_{cc}, σ_s and y have to fulfil equilibrium conditions for fixed values of axial load N and bending moment M.
In the section at x=a $\sigma_{ct}=0$, because of cracking, the slip value s(a) is unknown and is determined together with the distance "a" from the origin.
For solving equations (2),(3),(6),(9) a finite differences technique is used. By substituting the derivatives with the corresponding increment ratios, the set of four differential equations is transformed into multiple sets of four linear algebraic equations.
Each set allows to determine the values at section $x=x^{i+1}=x^i+\Delta x$ if the corrisponding ones are known at section $x=x^i$. So if we start from section x=0 (section L in Fig. 2), where all the boundary conditions are known, the stress state and slip in the opposite section (R) can be easily calculated and from these, in turn, the values in other sections, by proceeding in the same manner.
The distance "a" is unknown, so a value has to be guessed and an iterative numerical procedure is used which stops when in a section the fully cracked condition $\sigma_{ct}=0$ is fulfilled with a fixed degree of accuracy.
The sum of Δx gives half-length between the first cracks and the slip value in this section is the final one: s=s(a).
From the stress state it is possible to calculate u_{cc} and u_s displacements, the rotations and from these the mean cross section curvature in the element betweeen two cracks.
If one starts the procedure from a value of $M=M_{cr}$, being M_{cr}=the cracking moment, and continues until steel yields, a complete bending moment versus mean curvature diagram can be evaluate and, as a consequence, the bending deformability changes for increasing ap-

plied moments.

4. NUMERICAL RESULTS

To test the model a beam is examined having the following characteristics : h=500 mm, b=300 mm, d=30 mm, σ_{ct}=1.6 MPa, E_b=20,40,80 MN/mm^3, E_c=2000 MPa, E_s=20000 MPa. The beam is subjected to bending moment only, without axial load and the steel reinforcement consists of n. 3 bars, placed in tension zone; three different diameters were considered ϕ=10,14,20 mm, leading to the following:

Case	A	3 ϕ 10	A_s=235.6 mm²	p= 94.25 mm	A_s/A_c = 0.157%
	B	3 ϕ 14	A_s=461.8 mm²	p=131.95 mm	A_s/A_c = 0.308%
	C	3 ϕ 20	A_s=942.5 mm²	p=188.50 mm	A_s/A_c = 0.628%

The "a" values when E_b=40 MN/mm^3 and cracking moments are applied, result

A M_{cr}=21 MN·m a=269 mm
B M_{cr}=22 MN·m a=235 mm
C M_{cr}=24 MN·m a=193 mm

and well agree, within the crude approximations adopted, with existing formulae.
In Fig. 3 the moment-mean curvature relationships for the examined cases in uncracked and cracked stage are plotted. When tension stiffening effects are considered, according to the hypoteses of linear constitutive laws for materials and bond, the curves shift parallel from the fully cracked curves (II stage) at a distance depending on the geometrical percentage of steel reinforcement.

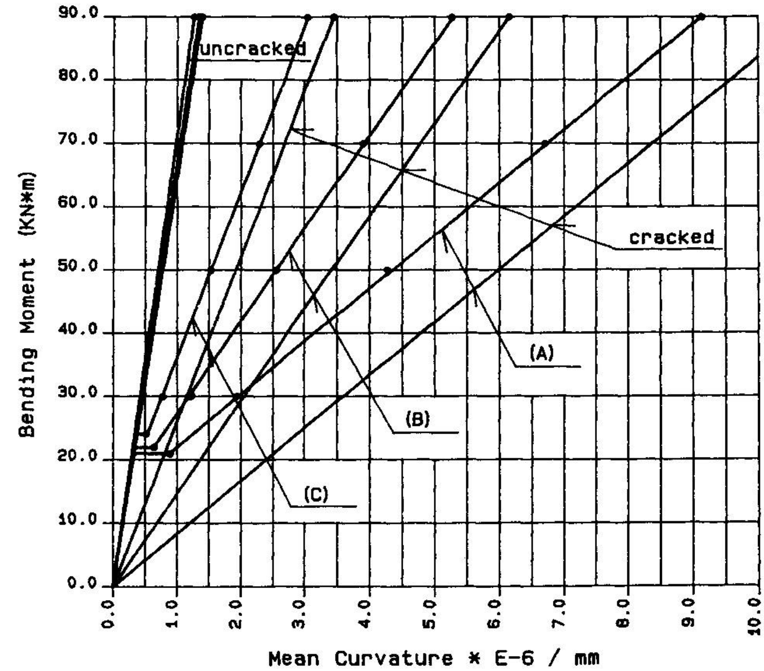

Fig. 3 moment-mean curvature relationships

The model shows that the bond parameter doesn't affect the slope of the curve but only the crack spacing, just as in the most common formulations. If E_b is halved or doubled the following values are obtained:

E_b=20 MN/mm^3	case A	a= 380 mm	E_b=80 MN/mm^3	case A	a= 190 mm
	B	a= 332 mm		B	a= 166 mm
	C	a= 272 mm		C	a= 136 mm

while the moment-curvature diagrams remain the same, whithin the numerical approximations.

5. CONCLUDING REMARKS

The proposed analytical model, which is based only on the classical hypothesis of cross section remaining plane, seems to be a good starting-point for a rational approach to tension stiffening phenomena. Owing to its quite general formulation the model may be improved by adopting more realistic non linear constitutive laws for materials and bond behaviour. From this point of wiew it's well suited for studying the tension stiffening effects even after steel yields.

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Influence of Tension Stiffening on Behaviour of Structures

Influence de la contribution du béton tendu sur le comportement des structures

Der Einfluss des Tension-Stiffening-Effekts auf das Verhalten von Tragwerken

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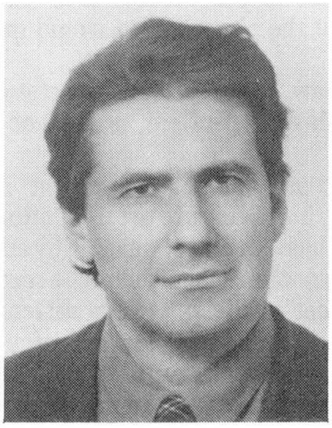


Aldo Cauvin, born 1939, obtained his Civil Engineering Doctorate from Politecnico in Milan and is Full Professor since 1986. Author of several papers on nonlinear analysis of reinforced concrete structures and on buckling. Active for several years in the CEB Commission 2 and in the «Council on Tall Buildings» Committee 22. Currently devoted mainly to the problem of practical applications of nonlinear analysis in design.

SUMMARY

The problem of a simplified simulation of the so-called «Tension Stiffening» effect in the nonlinear analysis of reinforced concrete structures is treated. Its importance on the behaviour of Structural Concrete is emphasized, with reference to different kinds of loads and limit states. The uncertainties in the evaluation of the parameters which influence cracking of concrete and tension stiffening are discussed, with reference to the necessity to perform a safe design using simplified methods of nonlinear analysis.

RÉSUMÉ

L'article concerne la simulation par une méthode simple de l'effet de la contribution du béton tendu dans l'analyse non-linéaire des structures en béton armé fissurées. Son importance est mise en évidence, et ceci par rapport aux différentes situations de charge et d'appui. Les incertitudes dans l'évaluation des paramètres influençant la fissuration du béton et la contribution du béton tendu sont prises en compte par rapport à la nécessité de réaliser un projet qui respecte les conditions de sécurité, tout en utilisant des méthodes simplifiées d'analyse non-linéaire.

ZUSAMMENFASSUNG

Es wird das Problem der vereinfachten Erfassung des sogenannten «Tension stiffening» Effekts bei der Analyse von Stahlbetontragwerken behandelt. Die Wichtigkeit dieses Effekts für die Reaktion des Konstruktionsbetons wird unter Bezugnahme auf Belastungen und Grenzzustände hervorgehoben. Die Unsicherheit bei der Einschätzung der Parameter, die die Rissbildung des Betons beeinflussen, und das «tension stiffening» werden im Zusammenhang mit der Notwendigkeit betrachtet, ein sicheres Bemessungskonzept zu erarbeiten, das vereinfachte Methoden der nichtlinearen Analyse verwendet.

1.INTRODUCTION

As known concrete in tension between cracks has a stiffening effect on cracked reinforced concrete members.
As experience in nonlinear analysis of r.c. practical structures has demonstrated this phenomenon,known as "tension stiffening", may have a considerable influence on results both in terms of displacements and action effects.
On the other hand "tension stiffening" is not easy to simulate not only because it is in itself a complicated mechanism of interaction between the two materials,but also because it is necessarily based upon the cracking pattern which in turn depends on tensile strength of concrete (variable whose dispersion is very high),distribution and size of tensile reinforcement,and also the so called "size effect",according to the recent developments of fracture mechanics.
It is not true that disregarding or underevaluating tension stiffening will always lead to conservative results:certainly the opposite is true in some cases such as the evaluation of stresses due to thermal variations,where the disregarding of the phenomenon would inevitably produce grossly underevalued action effects.
In this discussion we intend to draw some relevant conclusion on the subject,basing on the accumulated experience, recent investigations in this specific field, and the objectives of this Colloquium[1][2] on which the author is partially in agreement.
As one of the basic objectives is to individuate a "trasparent" model for the design of structural concrete,it is necessary to choose,among the many approaches to the simulation of structural behaviour,the ones which best fit the need for both simplicity and adequate accuracy,meaning by adequate a degree of accuracy suitable for design.
Therefore the possible approaches to nonlinear analysis in the cracked stage will be first examined and classified.
Secondly,as an example, a "trasparent" model of tension stiffening simulation will be briefly described,which seems to meet the objectives of simplicity and sufficient accuracy for design purposes.
Thirdly the importance of an adequate simulation of tension stiffening within the framework of a simplified method of nonlinear analysis is emphasized and an example is given.
At last the uncertainties in the assumption of basic input data are considered and final conclusions are drawn.

2.NONLINEAR ANALYSIS WITH REFERENCE TO CRACKING-SMEARED AND DISCRETEMODELS

As well known,the static-dynamic behaviour of reinforced concrete structures is markedly nonlinear in nature even in the elastic stage and difficult to model with accuracy,due to the micro-anisotropic,quasi brittle nature of concrete and the composite nature of the material.Problems arise mainly from uncertainties in definition of concrete constitutive law, complexities in interaction phenomena between the two materials, extensive cracking in tensile areas.
To describe this behaviour two different basic approaches are possible(fig.1),using micro or macro-elements.
Only the first approach,coupled with a "discrete" representation of cracking, can(potentially at least) describe accurately the interaction between the two materials,as each element model a very small area of steel or concrete and the discontinuous nature of cracking is described.
This approach however can only be used in simulation of laboratory tests and for very simple structures or structural elements and is unsuited for design and analysis of real structures.
It is therefore necessary to formulate the stiffness of macro-elements of r.c. (and not concrete or steel only) which takes into account as well as possible the previously mentioned local phenomena.
In this formulation a "smeared" approach needs be adopted and therefore tension stiffening must be introduced in a suitable way.
Several approaches are possible[13].In the following paragraph a simple and "trasparent" method is briefly described.More details can be found in[7],[8] and [12].

3.AN EXAMPLE OF "TRASPARENT" MODEL FOR "TENSION STIFFENING"

The problem of "tension stiffening" is concerned with the behaviour of steel bars embedded in concrete and subject to tensile forces.In a given crack,all the tension is trasmitted by the bars;between one crack and the following part of the tension is transferred to concrete,thus reducing the stresses in the steel.

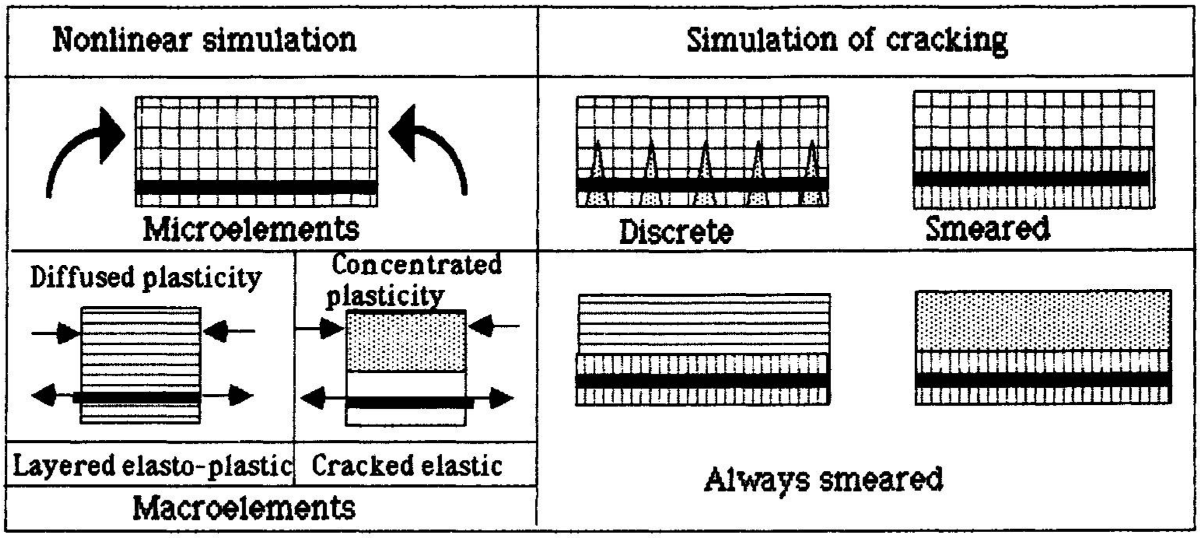


Fig. 1-Simulation of cracking

It is quite natural to think that things behave as if the tensile reacting part of the beam be constituted by the tensile reinforcement and an additional "virtual" reinforcement which represents the contribution of the concrete in tension between cracks. The method can be described as follows:
Each element belonging to the frame is divided in a given number of short macro-elements behavin elastically(as plastic behaviour is considered concentrated in "plastic hinges"). For each element the moment of inertia of the cracked section is introduced when, within the element, the adopted tensile strength of concrete in tension is exceeded. In computing the moment of inertia of the cracked section the influence of "tension stiffening" is simulated by introducing a "virtual" additional steel area AS^* which can resist a constant force $K_1 b(a-x)$, K_1 being the mean tensional stress between cracks which will be subsequently called "tension stiffening coefficient" (seefig.2)

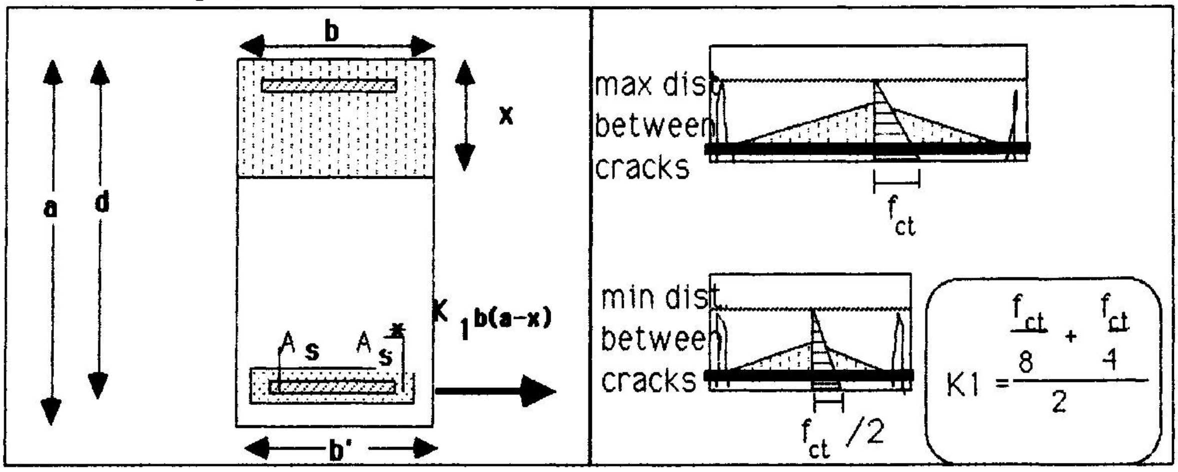


Fig. 2-"Trasparent" simulation of "Tension Stiffening"

A_s^* decreases as loads increase, thus simulating the decreasing influence of " tension stiffening" with increasing loads(which can be verified experimentally).
The tension stiffening coefficient can be computed in function of the tensile resistance of concrete, adopting the simplified assumption of uniform distribution of adherence stresses along tensile reinforcement between one crack and the following. If this assumption is made tensile stresses in concrete vary linearly along the element axis. If again the assumption is made that tensile stresses also vary linarly along the section depth, the mean value of these stresses is equal to $f_{ct}/4$ in the case two consecutive cracks form at the maximum possible distance(upperlimit situation) and equal to $f_{ct}/8$ in the lower limit situation(minimum possible distance between

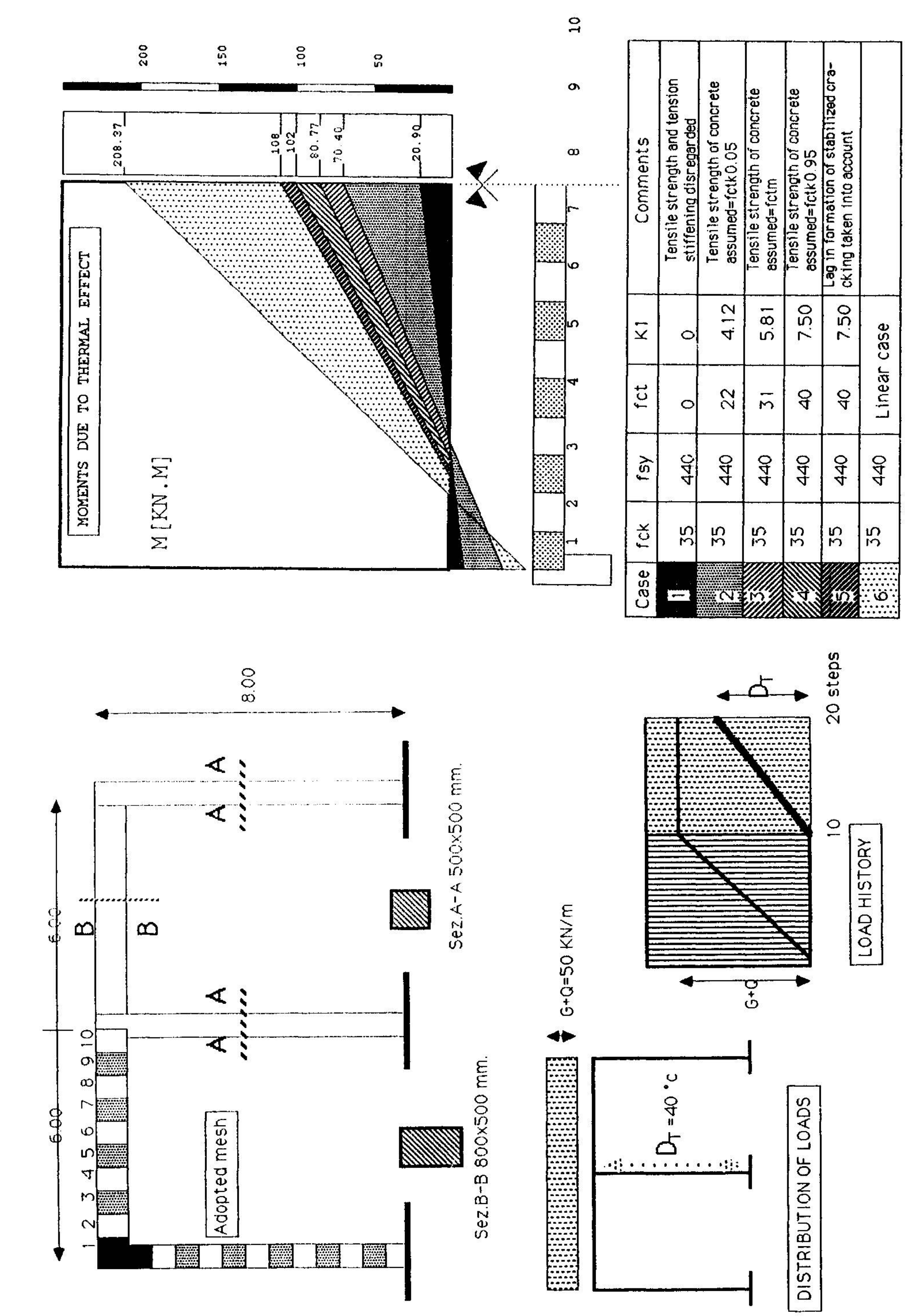
cracks). If again a mean value is assumed between these two extremes a value of the tension stiffening coefficient equal to $K1=3/16.f_{ct}$ is obtained.

4. IMPORTANCE OF "TENSION STIFFENING" ON STRUCTURAL BEHAVIOUR

One might wonder whether it is really important, to evaluate the structural behaviour of frames, to model accurately the influence of cracking and the related phenomenon of "tension stiffening", that is to evaluate the nonlinear behaviour in the elastic stage.
In fact experience tells us that these effects are essential to evaluate correctly, among others, the following phenomena:
-redistribution of moments due to cracking in beams, which can influence considerably the behaviour at Ultimate Limit State.
-elastic displacements at service load level
-Influence of cracking on second order effects in slender columns
-Thermal structural effects.
With reference to the latter phenomenon an example taken from [15] is briefly illustrated.
In the frame of fig.3 vertical distributed loads were applied to the beams first and then a thermal variation was applied to the column.
The frame was analyzed both linearly and non linearly and for different values of f_{ct} and corresponding values of K1 (tension stiffening coefficient). The 6 cases which were considered are summarized in the table of fig.4.
The lag in formation of stabilized cracking (see following paragraph) was simulated in case n.5.
The load history is represented in fig.3: vertical loads were applied first to the beams in 10 steps so that cracking might take place due to external loads. An uniform thermal field of +40°C was then applied to the central column (again in 10 steps), so that moments of the same order as those produced by external loads (and of the same sign in the central section) were induced in the beams.
In figure 4 the moments due to thermal effect only for the various cases are represented graphically
As may be seen from this diagram the extreme cases (linear analysis of uncracked structure and nonlinear analysis disregarding tensile stress of concrete and tension stiffening) lead, on opposite side, to completely unreliable results. Taking into account these factors according to different evaluation of tensile strength of concrete leads to considerably different results. This consideration leads to the problem of tensile strength evaluation.

5. UNCERTAINTIES IN THE SIMULATION OF CRACKED BEHAVIOUR AND TENSION STIFFENING
ASSUMPTION OF A SUITABLE VALUE FOR TENSILE STRENGTH

Every simulation of the cracking formation process must necessarily be based on an assumed value of concrete tensile strength. On the other hand it is well known that this parameter can only be determined with much uncertainty given the considerable dispersion of test results. Besides other factors influence crack formation and crack propagation [14] (size and quantity of reinforcement, cover, size effect, moment gradient)
Therefore we cannot, in evaluating structural behaviour in the cracked stage, reach the same degree of accuracy that we can obtain in the calculation of yielding moments in critical sections, which are in most cases mainly influenced by steel strength fsy, and to a lesser degree by concrete compressive strength fck.
As we cannot hope, even using sophisticated methods, to obtain very accurate results, the choice of the tensile strength fct must be based on safety probabilistic considerations, which are in turn influenced by the kind of results we need from structural analysis.
Also it is not convenient to adopt a very sophisticated procedure for tension stiffening simulation
If the purpose of analysis is to calculate deflections, the choice on the safe side is to overevaluate them, therefore to underevaluate fct and, as a consequence, tension stiffening. The most logical choice according to CEB Model Code philosophy [5][6] is to adopt a characteristic value with a 5% probability of not being exceeded (fctk.05)
If the purpose is to evaluate redistribution of moments due to cracking, it is not easy (and probably not even possible) to establish which value of fct would yield the safest distribution of action effects. In this case it is most reasonable to look for the "most probable" result and therefore choose a mean value of resistance (fctm)
If at last we need to evaluate thermal effects, the choice on the safe side is of course to overevaluate them and therefore to overevaluate fct (and tension stiffening).
A value of fct with a 95% probability of not being exceeded can be adopted (fct0.95).
This may however not be enough.
The method of simulating stiffness reduction due to cracking which was described in the previous paragraph (as well as most other methods of this kind) are based on the assumption that, as soon that, in a given element, the



Case	fck	fsy	fct	K1	Comments
1	35	440	0	0	Tensile strength and tension stiffening disregarded
2	35	440	22	4.12	Tensile strength of concrete assumed=fctk0.05
3	35	440	31	5.81	Tensile strength of concrete assumed=fctm
4	35	440	40	7.50	Tensile strength of concrete assumed=fctk0.95
5	35	440	40	7.50	Lag in formation of stabilized cracking taken into account
6	35	440	Linear case		

Fig. 3/4-Example: structural effect of thermal variation

tensile strength of concrete is reached in the most stressed fiber,fully opened "stabilized" cracks form in the element and its stiffness can be reduced accordingly.
However this assumption does not correspond to results of experimental tests In fact there is a "lag" between the reaching of the limit value of fct in a zone with constant momentand the formation of stabilized cracking in the same zone. An approximate method for simulating this lag is given in[15].
It may be concluded that a situation of fully open and stabilized cracks takes place gradually in the element and is fully developed for values of maximum tensile stress in concrete that can be much higher then the tensile strength fct.
This fact may be explained intuitively:in fact when fct is reached in the most stressed fiber a crack must begin;however an increase in stress in the adjacent fibers is required for the extension of this crack toward the neutral axis;this can happen only by furtherly increasing external actions also because tensile reinforcement is an obstacle to this propagation.

6.CONCLUSIONS:HOW "TENSION STIFFENING" SHOULD BE TREATED ACCORDING TO THE PHILOSOPHY OF THIS SYMPOSIUM

From the above considerations the following conclusions can be drawn:
-The influence of cracking and "tension stiffening" on structural behaviour must be taken into account in many practicallyimportantcases.
-Given its intrinsic complexity,the phenomenon must be simulated using simplified methods,which,to be accepted and correctly used by the engineer,must be sufficiently "trasparent",as the one which has been,as an example,presented.
-Given the uncertainties related to the prevision of the stress level at which a stabilized crack patterns takse places,the adopted input data and,in particular,the concrete tensile strength should be chosen so that results on the safe side be obtained.This means that values to be adopted must depend on the kind of load to be applied and the kind of situation to be studied.

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Fatigue Strength of Structural Concrete Girders

Résistance à la fatigue des poutres en béton

Ermüdungsfestigkeit von Betonträgern

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SUMMARY
This paper presents a general approach to fatigue design in structural concrete. A consistent approach can be used for both non-prestressed and prestressed concrete girders if the effective prestress force is treated as an external load. The paper reviews a series of studies on the fatigue performance of prestressing strand, when subjected to cyclic loading in air, in pretensioned girders, and in post-tensioned girders.

RÉSUMÉ
Une approche générale du calcul à la fatigue est présentée; c'est ainsi qu'une prise en compte cohérente du problème peut s'appliquer de façon égale aux poutres non-précontraintes et précontraintes, si la force de précompression est considérée comme une charge extérieure. Une série d'études est présentée au sujet des effets de fatigue sur les câbles de précontrainte soumis à des charges cycliques, et ceci pour les cas de poutres précontraintes et postcontraintes.

ZUSAMMENFASSUNG
In dieser Veröffentlichung wird ein allgemeines Bemessungsverfahren von Trägern aus Konstruktionsbeton gegen Ermüdungsversagen präsentiert. Spannbetonträger und nicht vorgespannte Träger können einheitlich behandelt werden, wenn die Vorspannkraft als äussere Kraft betrachtet wird. In der Veröffentlichung werden eine Reihe von Studien über das Ermüdungsverhalten von Spannstahllitzen zusammengefasst. Diese Studien umfassten Versuche an freien Litzen, an Spannbetonträgern mit sofortigem Verbund und an Trägern mit nachträglichem Verbund.

1. INTRODUCTION

With the trend towards ultimate load design, materials are better used and generally experience higher stresses at service load levels. Consequently fatigue has become a concern in the design of structural concrete.

In non-prestressed concrete bridges, fatigue is generally only a problem of the fatigue strength of the tensile reinforcement. The stress range in such reinforcement can be readily determined for service load conditions using a fully cracked, transformed section analysis. Based on extensive tests of deformed reinforcement, the AASHTO Bridge Specifications [1] present the following expression for the allowable stress range in deformed bars:

$$f_f = 145 - 0.33 f_{min} + 55(r/h)$$

where f_f = stress range in MPa;
f_{min} = minimum stress level in MPa;
r/h = ratio of base radius to height of rolled-on transverse deformations, equal to 0.3, when the actual value is unknown.

For prestressed concrete girders fatigue is generally not a problem if the girder remains uncracked. However, with the tendency to increase allowable tensile stresses in the precompressed concrete tensile zone and the possibility of larger prestress losses than anticipated in design, prestressed concrete has become more susceptible to cracking and subsequently to fatigue of the prestressing tendon. This paper presents a survey of several studies investigating the behavior of prestressing strand when subjected to fatigue loading in air, in pretensioned girders, and in post-tensioned girders.

2. PERFORMANCE REQUIREMENTS AND DESIGN APPROACH

To preclude fatigue failure, the stress range under service loads in the various components of the structure must be controlled and must be kept below a certain limit, depending on the number of load cycles the structure has to sustain during its lifetime.

In current U.S. practice tendon fatigue in prestressed concrete girders is addressed indirectly by assuming an uncracked section and then by limiting the nominal tensile stresses computed in the concrete tensile zone. This method does not account for the possibility of cracks in the girder due to modest overloads, fatigue of concrete in tension, or other unforeseen effects. A more logical approach would be to treat the prestressing force as an external load on the cross section, as suggested by Bruggeling [2], and to determine the tendon stress range from a cracked section analysis, ignoring any concrete tensile strength contribution. With this approach all levels of prestress including no prestress can be readily handled, without imposing any artificial limits on the allowable concrete tensile stresses.

3. STRAND-IN-AIR TESTS

Paulson, Frank, and Breen conducted a study of the fatigue characteristics of prestressing strand tested in air [8]. Previously reported data and additional strand-in-air test results were used to compile a database comprising over 700 prestressing strand samples from different manufacturers. Based on statistical analysis of this database a lower bound design equation for strand-in-air was developed (Figure 1).

The design equation is recommended for checking fatigue stress ranges in uncracked pretensioned concrete girders and in stay cables. The authors point out that stress concentrations in cracked girders and at anchorages or gripping systems may reduce the fatigue life significantly. These effects were not considered in the development of the fatigue model.

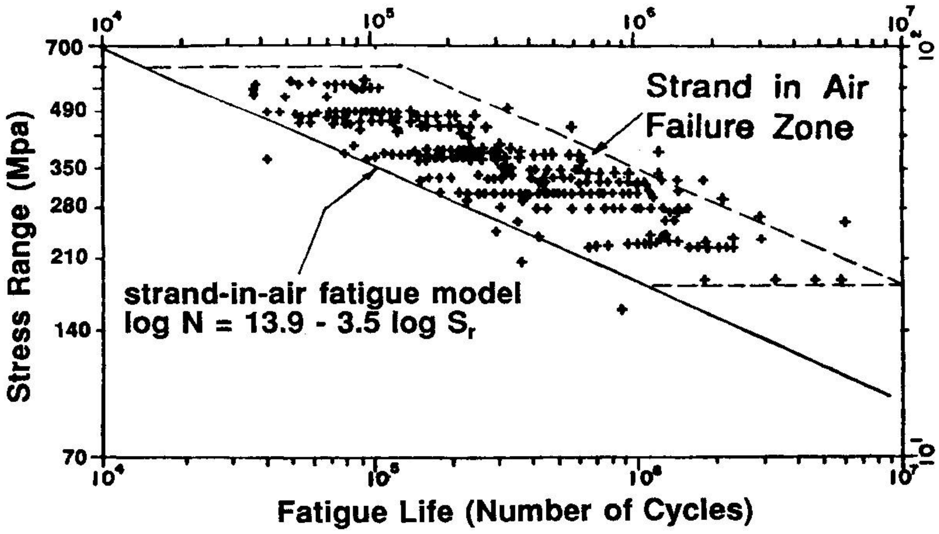

Figure 1 Strand in Air Fatigue Model (from [8])

4. TENDONS IN PRETENSIONED CONCRETE

Rabbat, Karr, Russel, and Bruce reported on a study of the fatigue strength of pretensioned concrete girders in 1978 [9]. This study was continued by Overman, Breen, and Frank and completed in 1984 [7]. The main variables included maximum nominal concrete tensile stress levels, tendon stress range, tendon layout, presence of passive reinforcement, degree of precracking, prestress losses, and presence of occasional overloads.

An important objective of the study was to evaluate current U.S. bridge specifications. In these specifications fatigue of prestressed concrete girders is addressed indirectly by limiting the nominal tensile stresses in the concrete tensile zone to $0.50\sqrt{f'_c}$ MPa, assuming an uncracked concrete section.It has been implicitly assumed that this limitation will ensure adequate fatigue performance of prestressed concrete girders. However, the results of the study indicate that this approach may be unconservative. Three out of eight girders with nominal concrete tensile stresses at approximately $0.50\sqrt{f'_c}$ MPa failed after less than three million load cycles, and one of these girders failed at less than two million load cycles. In general the maximum nominal concrete tensile stress level was not a good indicator of the fatigue performance.

The authors also point out the detrimental effect of modest overloads and excessive prestress losses. Both effects accelerate the formation of cracks in the concrete. At crack locations tendon stress concentrations occur which greatly aggravate the fatigue problem. In addition, prestress losses reduce the decompression moment of the section and subsequently cause an increased tendon stress range.

Presence of passive reinforcement was found to have a twofold beneficial effect on the fatigue performance of prestressed girders. It provides additional steel area in the tensile zone and thus reduces the tendon stress range, and it reduces prestress losses by controlling creep deformations.

The authors recommend to determine the tendon stress range from a cracked section analysis with a conservative estimate of the effective prestress level. The test results indicate that allowable fatigue stress ranges for strand-in-air are also adequate for pretensioned cracked concrete girders, provided a cracked section analysis is used (Figure 2).

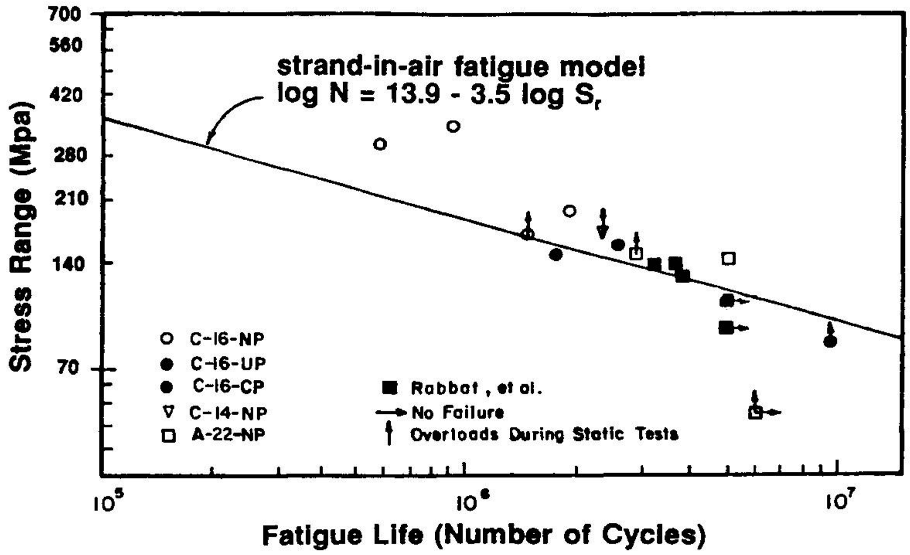

Figure 2 Fatigue of Strands in Pretensioned Girders (from [7])

5. TENDONS IN POST-TENSIONED CONCRETE

The fatigue life of prestressing strand as expected from strand-in-air and pretensioned concrete girder tests can be substantially lower in post-tensioned concrete girders with curved tendons. This observation was first reported by Magura and Hognestad in 1966 [4], but attempts to quantify it have only be made in recent years [3,5,6,10,11].

In post-tensioning tendons lateral pressure due to tendon curvature and friction stresses act on the prestressing steel, in addition to the fluctuating axial stresses. At the location of cracks stress concentrations occur, and debonding allows individual strands or wires to slip relative to each other and relative to the duct. The combined action of contact pressure, axial and friction stresses, and slip is well known in mechanical engineering as fretting. Fretting greatly accelerates the initiation of cracks in the prestressing steel and consequently reduces the fatigue performance of post-tensioning tendons.

Wollmann, Yates, Breen, and Kreger compiled data from previous post-tensioning tendon tests and conducted additional tests on girder specimens and reduced beam specimens [11]. The variables of the experimental study included tendon curvature, stress range, type of duct, and presence of strand coating. The reduced beam specimen was originally conceived by Oertle, et al., to alleviate the difficulties of determining the effective prestressing force and the tendon stress range in prestressed girders [6]. As shown in Figure 3 the reduced beam specimen allows accurate determination of tendon force and stress range from simple equilibrium conditions, provided the concrete is fully cracked.

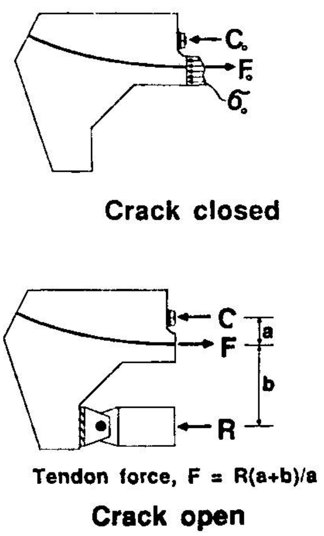

Figure 3 Reduced Beam Specimen (from [11])

The results of the study confirm that fretting fatigue can be a serious problem in cracked concrete sections at location of tendon curvature. With metal ducts rubbing between duct and strand greatly aggravates fretting. Figure 4 shows that strand-in-air test results are not adequate for the evaluation of allowable tendon stress ranges. Use of plastic duct improved the fatigue performance of single strand tendons by eliminating fretting fatigue between strand and duct. However, with multiple strands in more than one layer, fretting occurred between layers of strand, and use of plastic duct did not improve the fatigue performance significantly (Figure 5). Epoxy coating of the strands alleviated fretting and improved the fatigue performance.

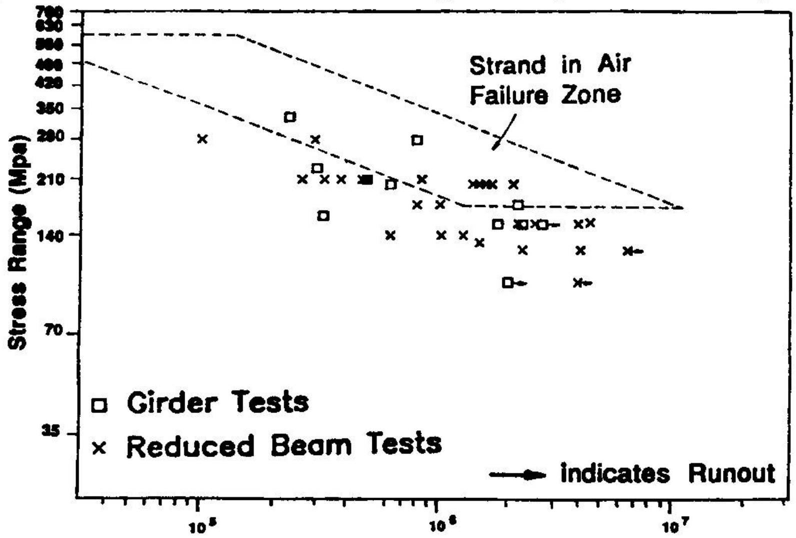

Figure 4 Fatigue of Strands in Post-Tensioning Tendons with Metal Duct (from [11])

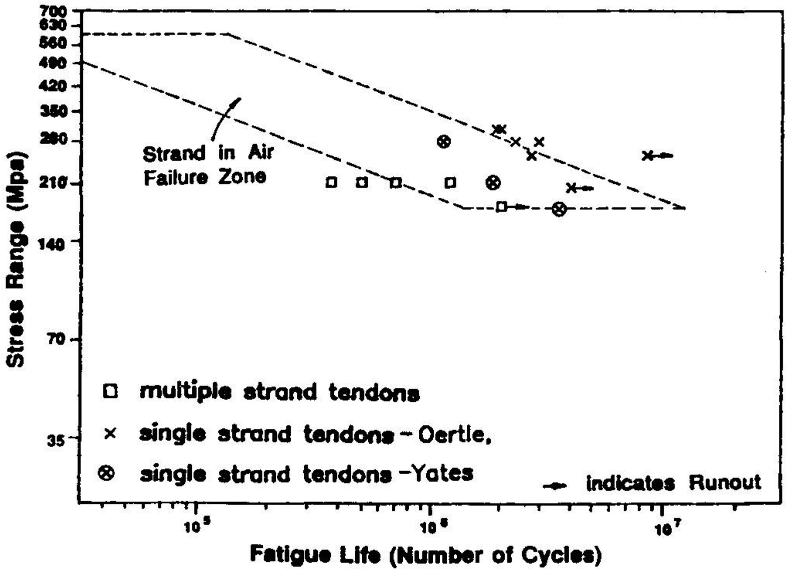

Figure 5 Fatigue of Strands in Post-Tensioning Tendons with Plastic Duct (from [11])

6. SUMMARY

A series of studies on the fatigue characteristics of prestressing strand when tested in air, in prestressing applications, and in post-tensioning applications is reviewed. Fatigue is not a problem in uncracked girders, but needs attention if the girder may become cracked. Occasional modest overloads and excessive prestress losses impair the fatigue performance of prestressed concrete girders due to accelerated crack formation in the concrete and increased tendon stress ranges. Passive reinforcement is effective in improving the fatigue performance.

A lower bound model for strand-in-air tests is also adequate for pretensioning tendons, provided the tendon stress range is determined from a cracked section analysis. In post-tensioned concrete applications fretting further impairs the fatigue performance, and lower allowable tendon stress ranges are necessary. Use of plastic duct does not significantly improve the fatigue performance of multiple strand tendons due to fretting between individual strands of the tendon.

A consistent procedure can be used for checking the fatigue strength of the structural concrete member if any long term prestressing force present is applied as a load, as suggested by Bruggeling. Subsequent analysis assumes a possible cracked section and no concrete tension contribution. Stress ranges in the reinforcing bars or prestressing tendons are compared to allowable stress ranges based on laboratory fatigue tests which must include realistic variables like duct material, cross reinforcement, and multiple strands where used.

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