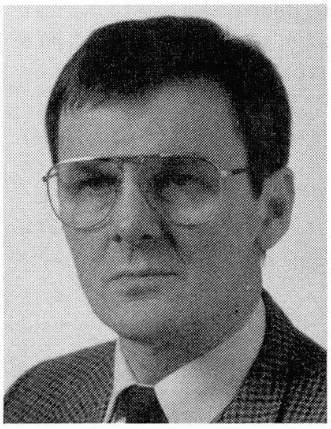
Systematic Fracture Mechanics Study of Shear Failure in Beams under Distributed Load

Rupture par cisaillement de poutres sous charge répartie

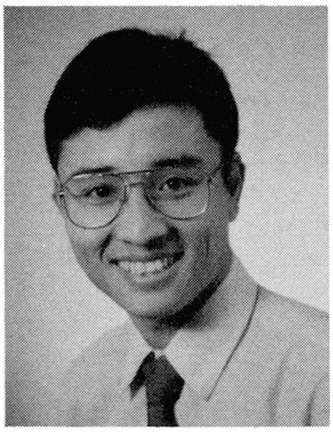
Bruchmechanische Studie des Schubbruches von Bauteilen unter Gleichlast

Johan BLAAUWENDRAAD
Professor
Delft Univ. of Technol.
Delft, The Netherlands

Q. B. WANG
Doctoral student
Delft Univ. of Technol.
Delft, The Netherlands



Johan Blaauwendraad, born 1940, obtained his civil engineering degree in 1962 at Delft University. He joined TNO for research, moved to Rijkswaterstaat, obtained his doctor's degree in 1973. Is now professor of Civil Engineering in Delft since 1979.



Qing Biao Wang, born 1962, received his civil engineering degree in 1988 at Delft University. He has been carrying out research investigating the shear strength of reinforced concrete beams using the finite element method.

SUMMARY

Shear failure is studied computationally in a systematic way for beams under distributed load with only longitudinal reinforcement. The shear capacity relies mainly on tensile tractions in the governing crack, aggregate interlock and dowel-action being neglectable. Scale effects are simulated correctly. Parameters are slenderness, brittleness, reinforcement ratio and the influence of a simultaneously acting normal compressive force.

RÉSUMÉ

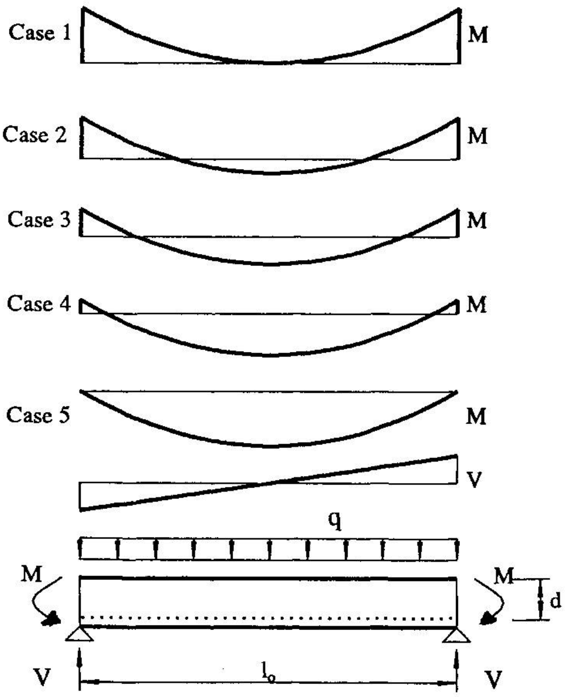
A l'aide de l'ordinateur, la rupture par cisaillement est étudiée systématiquement pour des poutres sous charge répartie ne comprenant qu'une armature longitudinale. La résistance au cisaillement dépend essentiellement des contraintes de traction dans le système de fissuration principal, l'adhérence de la pâte de ciment aux aggrégats et l'effet de goujon des barres d'armature ne jouant qu'un rôle négligeable. Les effets d'échelle sont correctement pris en compte, tandis que les paramètres étudiés sont l'élancement, la fragilité, le pourcentage d'armature et l'action simultanée d'une force de compression axiale.

ZUSAMMENFASSUNG

Schubversagen wird auf numerische Weise systematisch für Träger ausschliesslich verteilter Belastung mit Biegebewehrung untersucht. Die Schubkapazität hängt vornehmlich ab von Zugspannungen in dem bestimmenden Riss. «Aggregate interlock» und «dowel» action können nicht nennenswert beitragen. Massstabeffekte werden gut simuliert. Die untersuchten Parameter sind die Schlankheit, Sprödigkeit, Bewehrungsmass und der Einfluss einer gleichzeitig aktiven Druckkraft.

1. PROBLEM STATEMENT

This study originates from the construction of tunnels of rectangular cross-section crossing rivers and canals.
Rijkswaterstaat (Dutch State Public Works) commissioned the research organisation TNO to execute a number of tests on beams under distributed loading. In these tests 2 parameters have been varied [1]. One is the ratio of the positive mid-span moment and the end-of-span negative moment. So this parameter controls the amount of 'clamping-in'. The other parameter is the ratio of length-of-span and depth. This parameter controls the slenderness of the beams.
In figure 1 five different ratios are shown for the mid-span moment and the end-of-span moment (for a fixed value of the slenderness parameter), together with the crack pattern which occurs. The fat-drawn crack is due to shear failure. In all 5 cases the maximum shear force occurs at the beam end, but the shear failure does not always occur at that place. In stead the failure crack sometimes moves inwards the beam-span. This phenomenon is rather confusing for a designer, who is used to analyzing his diagrams for moments and shear forces and after that considers his structure on a section-by-section basis. In fact he is accustomed to checking the sections in which maximum moments and shear forces occur and to specify reinforcement demand at those positions. This approach appears to be meaningless in the case of a beam under distributed loading, at least for the prediction of the shear capacity.
The aim of the present study is to complete the experimental findings by computational investigations. It is hoped to better understand the available experimental results, but also to find new results for load cases and structures which have not been included in the experiments. In fact, the paper refers strongly to Hillerborgs publication [2].



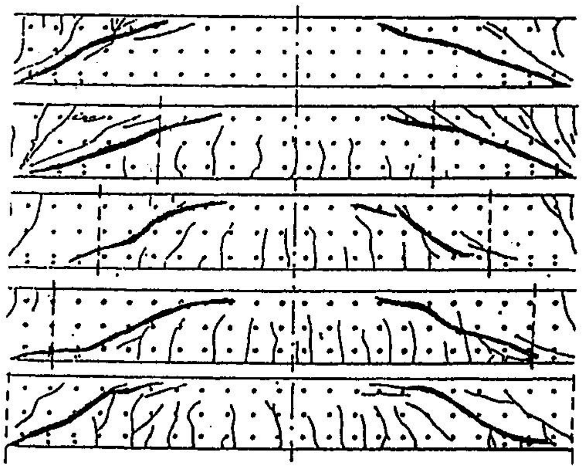


Fig.1
Five different moment lines (5 different moment parameters) and the respective crack patterns of the beams. All beams have the slenderness parameter $l_0/d = 4.5$ and d = 150 mm

2. ANALYSIS METHOD

The study has been started from the view-point of the predictor-corrector method, as proposed by Rots [3]. The program DIANA is used for all analyses.
In a discrete crack three different force transfer mechanisms can be taken into account:
- tensile traction (model Hillerborg)
- aggregate interlock (model Walraven)
- dowel action (results Fenwich)

For the tensile tractions normal to the crack faces a linear softening branch is adopted in the σ-w diagram for the stress σ and crack opening w. The tangent should be close to the value at tensile strength in the real curvilinear softening branch.
The aggregate interlock model of Walraven has been adopted, as reported in [4], which defines the tangential shear stress and the normal compression in a rough crack as nonlinear functions of the crack opening w, the crack-sliding Δ and the concrete compressive strength f_c.
For dowel-action a nonlinear spring has been programmed, whose stiffness is obtained from simulation of the dowel-experiments of Fenwich [5].
From a series of analyses it has been concluded that aggregate interlock and dowel-action do not contribute noticeable to the shear capacity. If the discrete crack is chosen properly (according to what is found in experiments), the crack faces are only loaded by tensile tractions (mode I). Only in the cover layer of concrete a crack-sliding Δ of some importance can occur, but not sufficient to mobilize an important contribution of aggregate interlock and dowel-action to the shear capacity. At peak load these contributions are negligible. They first start to grow after peak load has been reached, but at that time the contribution of the tensile tractions drops down rapidly. The conclusion is that the shear capacity must be fully provided by the tensile softening stresses.

3. PARAMETER STUDY FOR DISTRIBUTED LOAD

From preliminary calculations a number of lessons have been learned. To find a correct failure load the following applies:

- the smeared crack approach correctly simulates the beam stiffness, but is very time-consuming and does not converge properly for all analyses
- the analyses using one single discrete crack do not simulate the stiffness correctly, but predict the ultimate peak load well in short run time
- the crack tip does not reach the compressive zone for the cases studied, and the inclination of cracks is important.
- in a correct crack path no aggregate interlock can develop
- dowel action can only develop after passing of the peak load and can therefor be excluded
- scale effects are due to tensile softening in the crack and to the fact that bond does not follow scaling. Good performance of the model is found if a constant bond-interface is used equal to the beam width.

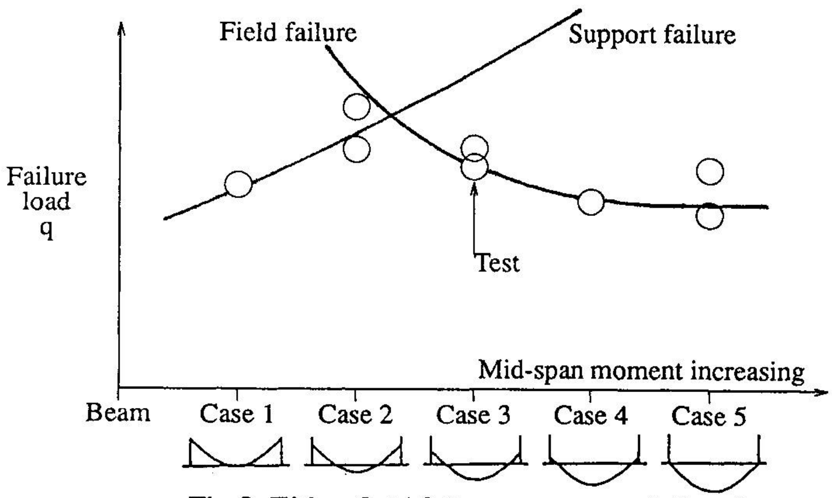


Fig.2. Either field failure or support failure is determining, depending on the moment parameter (the amount of clamping-in)

Starting from these findings a parameter study has been started for beams under distributed loading on basis of the predefined crack approach. When we closely inspect the crack patters in figure 1, it is noticed that two different failure mechanisms can occur, depending on the moment parameter (ratio of end-span moment and mid-span moment). In figure 2 is expressed the expectation that the two different failure types, field failure and support failure, can be studied separately and that for each moment parameter must be determined which one determines the lowest capacity and consequently failure. Test results support this expectation.
In this paper only the field failure is subject of examination. From here the length of the shear span is called *l (the distance between two counter-flexure points)*, the depth of the beam is d, the material property l_{ch} is a characteristic length defined by

$$l_{ch} = \frac{EG_f}{f^2_{ct}}$$

In this expression E is the elasticity modulus, G_f the fracture energy release and f_{ct} the concrete tensile strength. The reinforcement ratio for the longitudinal bottom reinforcement is ρ. The bond is determined by an elasto-plastic τ-Δ diagram. The stiffness in the diagram D and the maximum value of the bond stress τ_{max} are kept constant in all analyses. The width b of the beam is used for the total bond-interface. A normal force N can occur, which yields an additional constant compressive stress σ_N. The following parameters are studied:

(1) Slenderness

$$\frac{l}{d} = 6,9,12$$

(2) Brittleness number

$$\frac{d}{l_{ch}} = \frac{3}{4}, \frac{3}{2}, 3, 6$$

(3) Reinforcement ratio

$$\rho = \frac{1}{2}, 1, 2$$

(4) Axial compression

$$\frac{\sigma_n}{f_{ct}} = 0, 1, 2$$

The computations are all done using a constant reference depth d=1.0 m and width b=1.0 m.

A major decision is the choice of the shape and the position of the predefined crack. The inclination is chosen according to the elastic principle stresses in the centerline of the beam when no axial force is present. This appears to be in close agreement with experiments. A small vertical part is chosen near the longitudinal reinforcement. The length of this part is d/15.
From experiments the position of the cracks can be chosen very convincingly. There is no need to run many smeared crack analyses to produce the same results.
It is not necessary to draw the complete half beam into the computations. A beam part is chosen which roughly extends over a length d to the right and left from the beginning of the crack, and this part is loaded in accordance with the elastic cross-section stresses. A mesh of triangular 3-noded elements is applied in the surroundings of the crack path.
A series of results has been got. In figure 3 the influence of the slenderness l/d, the reinforcement ratio ρ and the brittleness number d/l_{ch} on the shear capacity q/q_{ref} is shown in one diagram. The reference load q_{ref} is the maximum load for the combination

$$\frac{l}{d} = 9, \qquad \frac{d}{l_{ch}} = 6 \qquad \text{and } \rho = 1\%$$

The influence of the reinforcement is moderate (about 5% between ρ = 1% and ρ = 2%). Doubling the slenderness from 6 to 12 increases the capacity by about a factor 3. Increase of the brittleness number by a factor 8 from 0.75 to 6 does drop down the shear capacity roughly by 30%. So the most important parameters are the slenderness and the brittleness number.
Figure 4 plots the shear strength against the slenderness l/d for ρ=1% and different d/l_{ch}. It appears that d/l_{ch} does not interfere much with the effect of l/d. Similar observation is obtained for ρ=2%.
Figure 5 has been made to investigate the influence of the reinforcement ratio ρ in a larger domain. The ratio ρ has been varied from 0 to 3 for a beam with

$\frac{l}{d} = 9$, $\frac{d}{l_{ch}} = 3$. Again q_{ref} refers to the combination $\frac{l}{d} = 9$, $\frac{d}{l_{ch}} = 3$ and, ρ = 1%.

Hereby ΣC is chosen to be equal to b for practical reason. For smaller ρ it is a reasonable assumption to reduce ΣC linearly with ρ and for larger ρ the limit value of b is used. In the same diagram is shown the curve according to Zutty and the rule of the Eurocode.
Figure 6 shows the influence of normal compressive stress. It is seen that the influence may modeled with a linear function.

Meanwhile continuing effort is made to conclude on an engineering model, such that designers can understand and use either the model or rules derived from it.

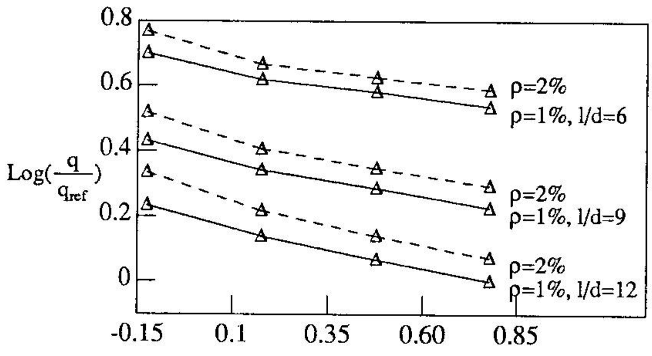


Fig.3. Influence of d/l_{ch}, ρ and l/d
(For q_{ref}: l/d = 12, d/l_{ch} = 6.0, ρ = 1%)

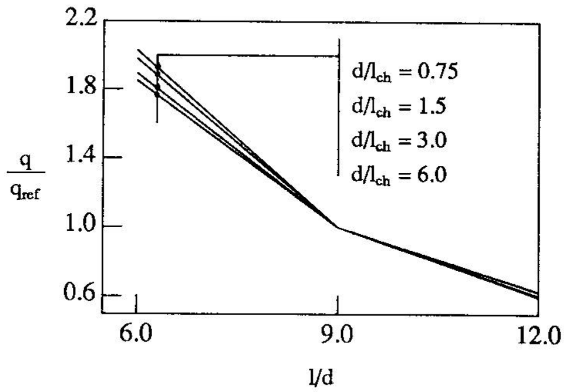


Fig.4. Influence of slenderness l/d
(ρ = 1%, for q_{ref}: l/d = 9)

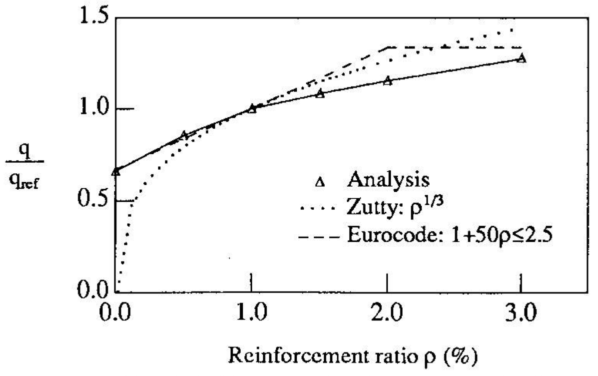


Fig.5. Influence of reinforcement ratio ρ
(l/d=9, d/l_{ch}=3.0, for q_{ref}: ρ = 1%)

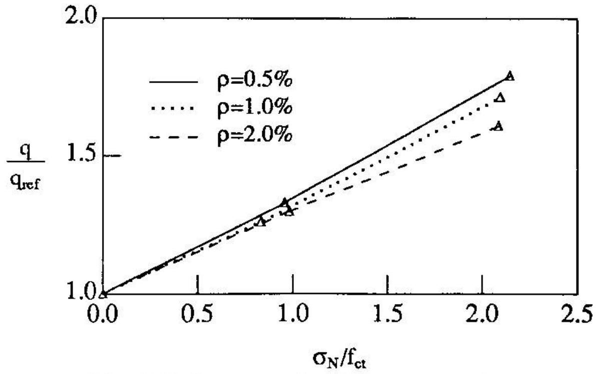


Fig.6. Influence of axial compressive stress
(l/d=9, d/l_{ch}=3.0, for q_{ref}: σ_N = 0)

4. CONCLUSIONS

1. Beams under distributed load and clamped-in conditions can fail in shear in two modes, here called support failure and field failure. The field failure can be modeled as a diagonal cracking failure. The field failures occur at positions which are supprizing to designers. Their convential section by section approach can not be applied in judging the shear capacity of beams.
2. The predictor-corrector method in which a smeared crack approach and a predefined discrete crack path are used sequentially, appears to be a necessary and successful way to predict the diagonal cracking shear failures.
3. The inclination of the predefined crack path must be chosen carefully. Shear failure occurs when the crack is so inclined that it cannot provide sufficient resistance.
4. The effect of dowel-action and aggregate interlock is negligible in the well-chosen crack path. Bond and tension-softening are important factors for shear capacity of diagonal cracking.
5. Size effect can be modelled correctly. The tension softening is most important. Bond has an additional effect.
6. The parameter study for field failure so far has revealed that the brittleness number and the slenderness have significant importance. Doubling the longitudinal reinforcement ratio does not have much effect. In the practical range the order of 10%
7. From continued research it is hoped to derive design rules or engineering models to be applied by designers.

5. ACKNOWLEDGEMENT

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6. REFERENCES

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