Zeitschrift: IABSE reports = Rapports AIPC = IVBH Berichte
Band: 60 (1990)

Artikel: Composite beam finite element method considering shear-lag effect
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DOI: https://doi.org/10.5169/seals-46502

Composite Beam Finite Element Method Considering Shear-Lag Effect

Méthode des éléments finis de la poutre mixte considérant l'effet de traînage de cisaillement

Finite Elemente für Verbundträger unter Berücksichtigung der Schubverschiebung

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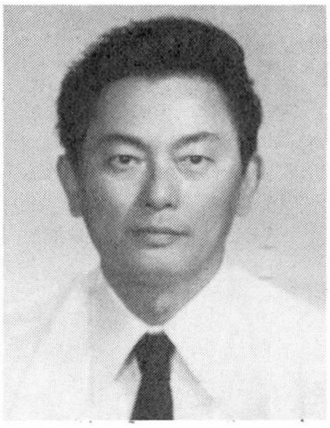

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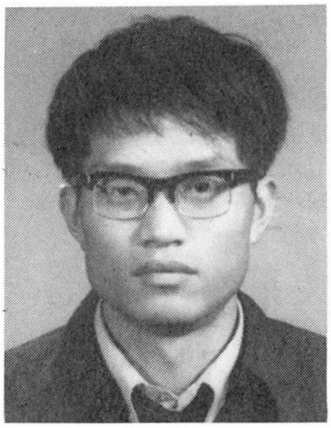

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SUMMARY
In the design of cable-stayed bridges with composite deck, the shear-lag effect in a reinforced concrete slab should be carefully considered. In order to analyze correctly the shear-lag effect in the composite beam, a special finite element of T-type girder is developed on the basis of Reissner's theory. The stress distribution in the deck slab of Huangpu River cable-stayed bridge in Shanghai, China, are calculated, and the results are verified by a conventional finite element analysis program, as well as compared with that from British Standards Code.

RÉSUMÉ
Dans le projet de pont à haubans avec tablier mixte, l'effet de traînage de cisaillement dans la dalle en béton armé doit être attentivement prise en considération. En vue d'analyser correctement cet effet dans la poutre mixte, un élément fini spécial a été développé sur la base de la théorie de Reissner. La distribution des contraintes a été calculée dans le tablier du pont à haubans sur la rivière Huangpu à Shanghai en Chine. Le résultat a été vérifié par le programme conventionel de la méthode des éléments finis puis comparé avec celui des normes britanniques.

ZUSAMMENFASSUNG
Im Entwurf der Schrägseilbrücken mit Verbundträgern sollte die Schubverschiebungswirkung (Shear-lag Effect) in der Stahlbetonplatte vorsichtig berücksichtigt werden. Um die Schubverschiebungswirkung richtig zu analisieren, wird in diesem Bericht ein besonderes finites Elementes für Verbundträger auf der Grundlage der Reissner-Theorie entwickelt. Die Spannungsverteilung in der Platte des Verbundträgers der Schrägseilbrücke über den Fluss Huangpu in Shanghai, China, wurde berechnet. Die Ergebnisse wurden durch ein übliches Finite-Element-Programm bestätigt und auch mit den englischen Normen verglichen.

1. INTRODUCTION

Since the Annacis Bridge in Vancouver, Canada with a record-breaking main span length of 465 m was completed in 1984, the composite design with a very simple, economical and easy-to-erect girder/deck framing system shows an outstanding advantage in some competition of long-span cable-stayed bridge. In the design of Huangpu River cable-stayed bridge in Shanghai with the main span of 423 m, the composite alternative beat an all-concrete design due to its lower weight, fewer cables, less massive towers, fewer foundation piles and shorter construction period required.

About half a century ago von Karman first studied the shear-lag effect in T-type girder, and established a basic theory of calculating the effective width of slab for design purpose. In after years many authors have made contributions towards this problem in simple-supported or continuous beam bridges, in which the Reissner's theory for analyzing the shear-lag in box girder by the principle of minimum potential energy is more effective and noticeable. But very few attention has been paid to the composite deck of cable-stayed bridges.

In order to analyze correctly the shear-lag effect in the composite beam, a special finite element of composite T-girder is developed in this paper on the basis of Reissner's theory. As an in-plane composite beam element, a new displacement parameter u_r reflecting the shear-lag effect of the flange slab, namely the relative longitudinal displacement between the root and end of the cantilever flange, is added in each end of the element. The stiffness matrix and loading matrix of this special beam element can be derived by the variational principle.

2. FORMULATION AND SOLUTION OF PROBLEM

The differential equations of displacement function reflecting shear-lag effect can be established by the variational principle for elastic structures. Let us consider a section of T-type composite beam as shown in Fig. 1.

2.1 Assumption for displacement functions

The displacement functions varying longitudinally in the composite beam are assumed as:

$w(x)$, vertical displacement of composite beam;

$u_{st}(x)$, axial displacement of steel girder;

$u_r(x)$, relative longitudinal displacement between the root and end of cantilever flange reflecting the shear-lag effect.

The displacement function varying transverly in the slab can be described as:

$$\tilde{u}_r(y)=1-(y/b)^i \quad (i=2,3,4) \tag{1}$$

in which, b is the width of slab on each side, and the coordinate system is shown in Fig. 1.

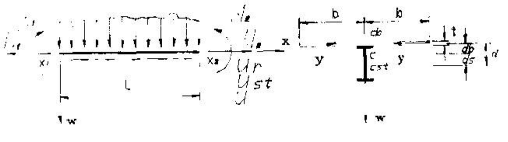

Fig. 1 Analysis model and Coordinate System

2.2 assumption for stress and strain

The cross sections of steel girder remain plane. As a result, the longitudinal displacement of slab at various points can be written as:

$$u_b(x,y) = w'(x)d + u_{st}(x) + u_r(x)\,\tilde{u}_r(y) \tag{2}$$

in which, d is a vertical distance between the middle surface of slab and the cross section centroid of steel girder.

The curvature of R.C. slab and of steel girder are identical in vertical bending.

The stress and strain in slab are regarded as a plane stress problem.

2.3 Differential equations of displacement functions

According to the assumptions mentioned above, the total potential energy of composite beam can be determined, and it may be written in the form as:

$$\pi = \pi(\, w'',\ u'_{st},\ u'_r,\ u_r,\ x\,) \tag{3}$$

By means of the calculus of variation, we obtain

$$\delta\pi = \int_{x_1}^{x_2} \{ \frac{\partial\pi}{\partial w''}\delta w'' + \frac{\partial\pi}{\partial u'_{st}}\delta u'_{st} + [\frac{\partial\pi}{\partial u_r} - \frac{d}{dx}(\frac{\partial\pi}{\partial u'_r})]\,\delta u_r \}dx + \frac{\partial\pi}{\partial u'_r}\delta u_r \Big|_{x_1}^{x_2} \tag{4}$$

The differential equations and boundary condition can then be established by making $\delta\pi=0$, and finally we obtain:

$$w'' + \frac{d_b}{I_c} F_b u'_r = -\frac{M_p}{E_{st}\, I_c} \tag{5}$$

$$u' + \frac{F_b}{A_c} u'_r = \frac{N_p}{E_{st}\, A_c} \tag{6}$$

$$u''_r - \tilde{k}^2 u_r = \frac{\tilde{k}\, M'_p\, d_b}{E_{st}\, I_c} - \frac{\tilde{k}\, N'_p}{E_{st}\, A_c} \tag{7}$$

$$[\, E_{st}\bar{F} u'_r + (\frac{N_p}{A_c} - \frac{M_p d_b}{I_c})\, F_b\,]\,\delta u_r \Big|_{x_1}^{x_2} = 0 \tag{8}$$

In which, M_p, N_p are the bending moment and axial force in composite beam;
$u=u_{st}+d_s w'$, is the axial displacement of composite beam;
A_c, I_c are the cross section area and moment of inertia of composite beam; E_{st} is the modulus of steel girder;
d_b is a vertical distance from the middle surface of slab to the cross section centroid of composite beam;
$F_b=t/n\int_b \tilde{u}_r\,dy$; $\tilde{k}=F_b/\bar{F}$; $\tilde{k}^2=\hat{F}'/(2+2\mu)$; $\bar{F}=\hat{F}-F_b^2(1/A_c+d_b^2/I_c)$,

where, d_s is a distance from the cross section centroid of steel girder to that of the composite beam; μ is Poisson's ratio;
$\hat{F}=t/n\int_b(\tilde{u}_r)^2\,dy$; $\hat{F}'=t/n\int_b(u'_r)^2\,dy$; $n=E_{st}/E_b$, E_b—the modulus of slab.

3. COMPOSITE BEAM FINITE ELEMENT METHOD CONSIDERING SHEAR-LAG EFFECT

The composite beam finite element can be established by using the differential equations of displacement functions and the boundary condition derived above. It can be seen from equations (5) to (8) that three displacement functions of w, u, and u_r are fundamental for determining the displacements and internal forces of composite beam. If we take an extra parameter u_r in addition to the conventional three displacement of in-plane beam element u, v, θ, as the basic displacement parameters of a special composite beam element, namely(see Fig. 2)

$$\{\delta\}^e = [\ \theta_i\ ,\ u_i\ ,\ v_i\ ,\ u_{ri}\ ,\ \theta_j\ ,\ u_j\ ,\ v_j\ ,\ u_{rj}\ ]^T \tag{9}$$

which will determine solely the stress state of composite beam element.

The stiffness matrix of composite beam element $[k]^e_{8\times8}$ can be obtain by using the solutions of Eqs. (5) to (7) under boundary conditions related to determining elements of stiffness matrix. It should be pointed out that the elements k_{ij} (i=4,8;j=1,2,...8) in the matrix are related to the additional displacement u_{ri} and u_{rj}.

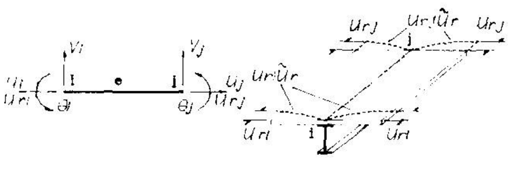

Fig. 2 Displacement Parameters

The elements of loading matrix $\{P\}^e$ can be calculated by using Eqs. (5) to (7), making influence lines of reactions M, R and R at beam ends, and loading on lines, which are carried out by computer program.

Finally, the equilibrium equation of composite beam element can be written as:

$$[k]^e \{\delta\}^e - \{P\}^e = 0\ , \tag{10}$$

in which, $$\{P\}^e = [\ M_i\ ,\ R_{ix}\ ,\ R_{iy}\ ,\ N_{ui}\ ,\ M_j\ ,\ R_{jx}\ ,\ R_{jy}\ ,\ N_{uj}\ ]^T \tag{11}$$

N_{ui} , N_{uj} are a generalized elastic resistance corresponding to the displacement $\tilde{u}_r$ and may be defined as work done by normal stress in slab at ends of element caused by M, R_x and R_y on the displacement $\tilde{u}_r$.

It can be proved that the equilibrium equation relative to N_{ui} or N_{uj} in Eq.(10) is the another form of Eq.(8). So that when an element is in equilibrium, it must satisfy the boundary condition from the variation of the total potential energy.

As a verification of the composite beam finite element method developed in this paper, a simple-supported composite beam subjected to a concentrated load or a fully uniform load is taken, and the results of effective width ratio in different cases are obtained. Compared with the BS 5400 code, under the uniform loading, the results are in good agreement with the BS code; under the concentrated loading, the results in range of width-span ratio $b/L < 0.5$ are close to that in the BS code. The best choice of i in equation (1) is 4.

4.APPLICATION TO HUANGPU RIVER BRIDGE

The Huangpu River Bridge is a T-type composite beam cable - stayed bridge with a main span of 423 m (Fig. 3) in construction.

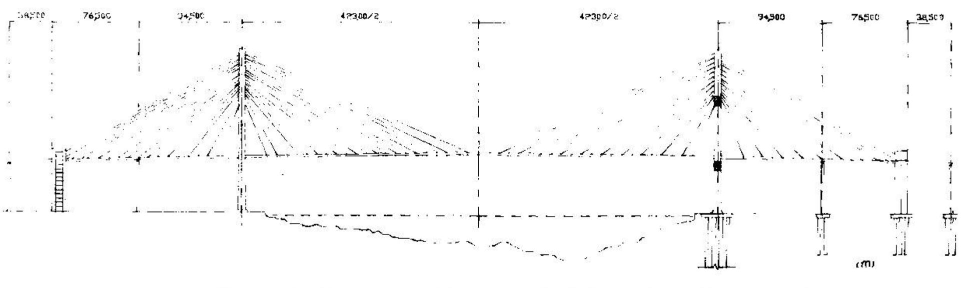

Fig. 3 Huangpu River Bridge in Shanghai

4.1 Effective width ratio of slab and internal forces in the composite beam under dead load

The conventional FEM is used for calculating three cases:
Case (a), (b) and (c) express that the effective width ratio (E.W.R.) of slab Φ=1.0, 0.05-0.25 and 0.1-0.5 are taken respectively in determining the bending stiffness of composite beam, and Φ =1.0 in axial stiffness for three cases. In which case(b) with Φ=0.05-0.25 is obtained according to the BS code when the composite beam is regarded as a continuous beam supported rigidly on the cables.

By using the special FEM considering shear-lag effect, the internal forces, the variation of effective width ratio along the span, and the normal stress distribution in slab in the middle of span are also given, which is named as case (d) as shown in Fig. 4 and Fig. 5.

It can be seen that under dead load a little difference of internal forces exists among four cases except the region near the tower. Compared with case (d), the biggest difference appears in case (a), and the least in case (c). In fact, the amount of axial force in deck is much larger than under live loads, and the bending moment is relatively small. So it may be accepted that the bending and axial stiffness are determined by Φ =1.0, however a better result of normal stress in slab from case (b) or (c) is more close to the reality.

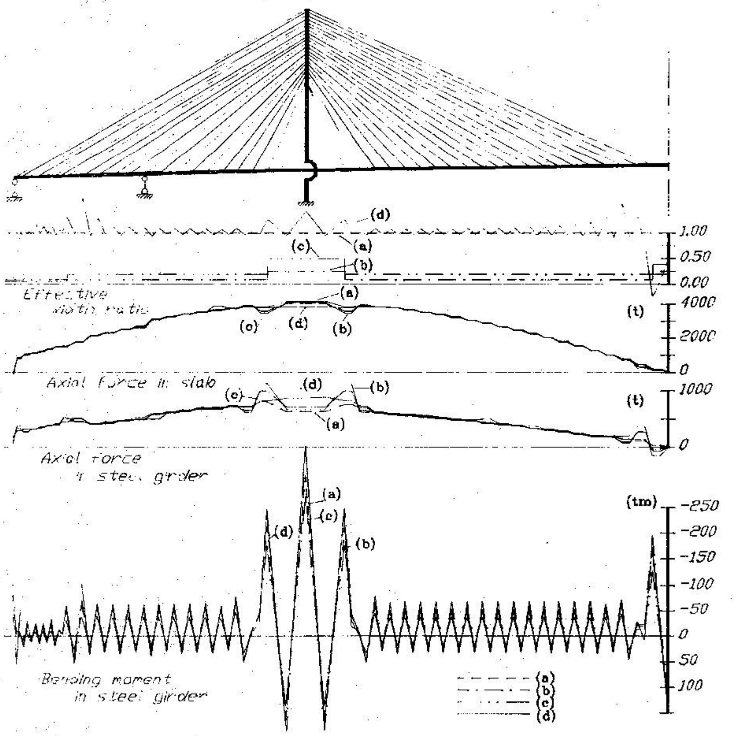

Fig. 4 E.W.R. Curves and Internal Forces

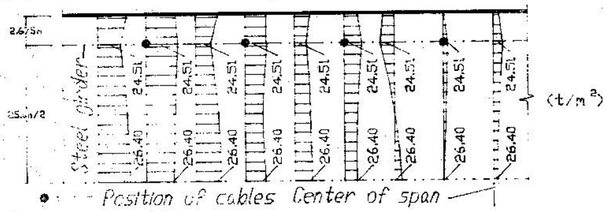

Fig. 5 Normal Stress in Slab

4.2 Effective width ratio of slab and internal forces in the composite beam under live loads

The truck loads are taken as live loads, and the sections 1,2,3 and 4 are chosen for calculating the internal forces. In Table 1 the bending moments of four sections obtained by FEM are listed with the effective width ratio Φ=0.4 to 1.0. The results obtained from special FEM considering shear-lag effect are also shown.

It can be seen in the Table that from Φ =0.4 to 1.0, the bending moments change little and about Φ=0.5 the results approach the values by FEM considering shear-lag effect. It should be noticed that if we take a length of an influence line (same symbol) of a section concerned as the span of an equivalent continuous beam, we may obtain an effective width

Table 1 Moments and effective width ratios

section	1	2	3	4
E.W.R. Φ	moments (tm)			
1.0	-696.02	763.70	-246.72	668.62
0.6	-676.20	745.41	-244.03	651.83
0.5	-667.36	737.08	-242.50	642.66
0.4	-655.20	725.57	-240.60	630.05
FEM considering shear-lag effect				
moments	-671.62	739.87	-242.37	633.97
E.W.R. Φ	0.914	0.852	0.750	0.805

ratioϕ=0.5 from the BS code.

According to the position of live loads, under which the bending moment of the section 3 or 4 arrives its most unfavourable value, the variation of effective width ratio obtained from FEM considering shear-lag effect are shown in Fig. 6.

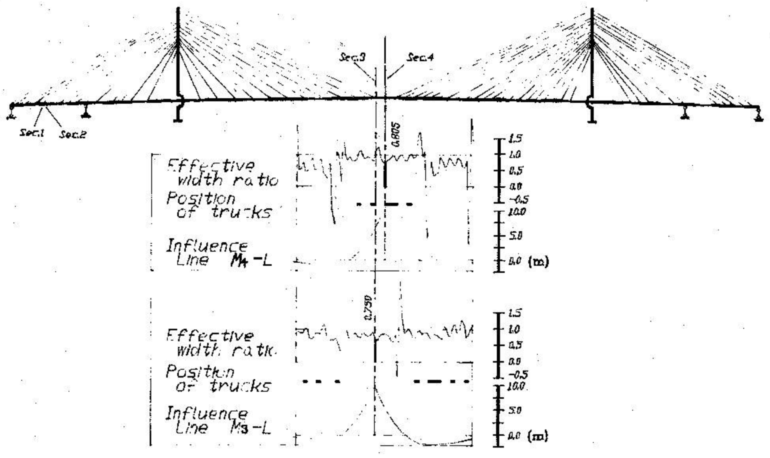

Fig. 6 E.W.R. Curves (Loading on M_3-L, M_4-L)

Because of the interaction caused by the wheel loads, vertical and horizontal componets of cables, the curves wave along the span. We are much interested in the normal stress in slab at the sections 3 and 4, where the effective width ratios show ϕ =0.805 and 0.75. So it makes clear that the normal stress distribution in slab is also uniform enough, just as shown in Fig. 7.

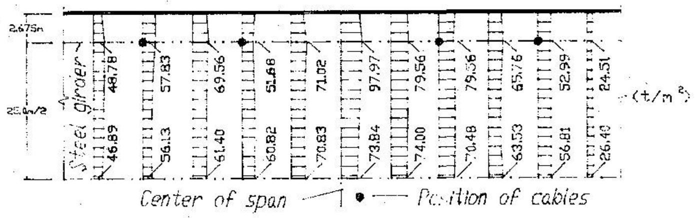

Fig. 7 Normal Stress in Slab (Loading on M_4-L)

5.CONCLUSIONS

In the composite deck of cable-stayed bridges, the effective width related to the normal stress in slab is very complicated. For dead or live loads, axial force or bending memont, the effective width should be different theoretically, and varies along the span.

A special FEM considering shear-lag in slab of a T-type composite beam is established on the basis of Reissner's theory. An additional displacement parameter reflecting the shear-lag is included in the beam element.

Under dead load, the normal stress in slab due to axial force can be calculated as distributed uniformly on the full width of slab, but the normal stress due to bending moment should be considered by using effective width corresponding to a rigidly-supported continuous beam.

In determining the internal forces under live loads, the different value of effective width is not sensitive, and the full width can be taken for making the inflnence lines,but care must be taken in determining the stress. An equiavalent continuous beam with a span equals to the length of influence line concerned can be used for calculating the effective width on the safe side.

REFERENCES

1. Shanghai Municipal Engineering Design Institute(SMEDI): Design materials of Huangpu River Bridge in Shanghai.
2. Zhang, Dr. S.D.: Theory of Bridge Design (in chinese) 1984.
3. Reissner, E.: Analysis of Shear-lag in Box Beams by the Principle of Minimum Potential Energy. Quarterly of Applied Mathematics, Vol. 4 Oct. 1946.
4. Moffatt, K.R. and Dowling P.J.: British Shear-lag Rules for Composite Girders. ASCE. No. ST7. July 1978.
5. BS 5400. Steel, Concrete and Composite Bridges. 1978-1983.