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General Buckling of Fiber Reinforced Composite Plates

Voilement général de plaques composites renforcées de fibres

Beulen von faserverstärkten Verbundplatten

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SUMMARY

The paper presents a general formulation for the buckling of rectangular, orthotropic, laminated composite plates under linearly varying uniaxial compressive force using the method of differential quadrature. The results are reported for various combinations of simple and clamped boundary conditions.

RÉSUMÉ

Cet article présente l'exposé général du voilement des plaques stratifiées composites, de forme rectangulaire et orthotrope, soumises à une force de compression variant linéairement et agissant dans une seule direction, par utilisation de la méthode de la quadrature différentielle. Les résultats sont donnés pour de nombreuses combinaisons des conditions aux limites des plaques, sur appuis simples et encastrés.

ZUSAMMENFASSUNG

Der Beitrag stellt eine allgemeine Formulierung vor für das Beulen von rechteckigen orthotropen geschichteten Verbundsplatten unter linear ändernden Normalkräften. Es wird die Methode der quadratischen Differenzen verwendet. Die Resultate für verschiedenen Kombinationen von einfachen und eingespannte Rändern liegen vor.

1. INTRODUCTION

The effectiveness of fiber reinforced composites in performance improvement of weight-critical aerospace structures is well established. Its use in long span bridges, buildings and offshore structures is only now becoming popular. This emphasises the need for understanding the structural behaviour of various components made of orthotropic composite laminates to thereby develop efficient and reliable design methods. The present study is an effort in this direction.

In composite structures characterized by lightweight, thin walled members, the linear buckling load is one of the most important design consideration. The paper considers the buckling of rectangular, laminated composite plates subjected to uniaxial compression that varies linearly from one loaded edge to the other, a typical problem of local plate instability that might arise in a box beam subjected to nonuniform bending. To satisfy equilibrium under the load gradient, shear force must act along the unloaded edges, a suitable expression for which can easily be written. Libove et al [7] were the first to report the results on the buckling of simply supported isotropic plates under compression gradient using the Rayleigh-Ritz method. At present, the problem is generalized and results are reported for orthotropic laminated composite plates with various combination of simple and clamped boundary conditions.

The analysis of the present problem using classical energy methods is not straightforward since these methods require a priori selection of displacement functions satisfying the boundary conditions; an uneasy task, especially in cases of mixed boundaries. Whitney [8] has reported few cases of orthotropic plate buckling under uniform compression or pure shear with mixed boundaries. A common approach to problems of this class involved the use of beam vibration functions as displacement functions in the Rayleigh- Ritz [1] or extended Galerkin methods [9]. Thus, a priori selection of displacement functions and subsequent application of variational calculus often require a sound knowledge of the principles of mechanics. Computational methods such as finite elements or finite differences are less attractive due to excessive cost, storage requirements and data preparation. This has motivated the search for an effective approximate method for solving plate buckling problems in a direct manner without recourse to variational principles. The paper proposes the use of method of the differential quadrature (DQ) as introduced by Bellman and Casti [2] and further elaborated by Civan and Sliepcevich [4] for solving directly the partial differential equation governing the problem with prescribed boundary conditions. Bert and his coworkers have illustrated various applications of the DQ method in structural mechanics [5].

2. ANALYSIS

The differential equation of equilibrium of a rectangular anisotropic plate of length, a, and width, b, under uniaxial compression, N_x, and shear force , N_{xy},

$$d_{11}\, w_{,xxxx} + 2\,(d_{12} + 2\, d_{33})\, w_{,xxyy} + d_{22}\, w_{,yyyy} = N_x\, w_{,xx} + 2 N_{xy}\, w_{,xy} \tag{1}$$

combined with the boundary conditions defines the plate buckling problem of present interest. The subscripts preceded by a comma denote differentiation with respect to the corresponding coordinates. Here, d_{ij} , $i,j =$ 1..3 are the bending stiffness of the laminated plate [6], w is out of plane deflection and $\beta = a/b$ is the plate aspect ratio. In the case of orthotropic approximations, $d_{13} = d_{23} = 0$. It is assumed that the plate is subjected to linearly varying compressive forces per unit length N_{min} at x = 0 and N_{max} at x = a (Fig. 1) such that compressive stress at any section x can be written as

$$N_x = 2 N_{av}\,(R_1 X + R_2) \quad where \quad N_{av} = N_{max}\frac{(1+r)}{2} \tag{2}$$

Here, $R_1 = (1-r)/(1+r)$, $R_2 = r/(1+r)$ and $r = N_{min}/N_{max}$. N_{av} is the average compressive stress and r is the ratio of minimum to maximum compressive stresses. The conditions of equilibrium of mid-plane stresses

$$\frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = 0 \quad \text{and} \quad \frac{\partial N_y}{\partial y} + \frac{\partial N_{xy}}{\partial x} = 0 \tag{3}$$

provide the following expression for shear stress distribution [7] in the prebuckling state

$$N_{xy} = -\frac{N_{av}}{\beta} R_1\,(2Y-1) \tag{4}$$

X = x/a and Y = y/b are non-dimensional coordinates. The boundary conditions may be defined, for simple supports, as

$$w = 0 \quad at \ x = 0,a \ \text{and} \ y = 0,b$$
$$M_x = d_{11} w_{,xx} + d_{12} w_{,yy} = 0 \quad at \ x = 0, a \tag{5}$$
$$M_y = d_{12} w_{,xx} + d_{22} w_{,yy} = 0 \quad at \ y = 0,b$$

and, for clamped supports, as

$$w = 0 \quad at \ x = 0,a \ \text{and} \ y = 0,b$$
$$w_{,x} = 0 \quad at \ x = 0, a \ \text{and} \ w_{,y} = 0 \quad at \ y = 0, b \tag{6}$$

Mixed boundaries can be defined in a similar way.

3. METHOD OF DIFFERENTIAL QUADRATURE

In this method, partial space derivatives of a function are approximated by means of a polynomial expressed as the weighted linear sum of the function values at a preselected grid of discrete points. For example, the first partial derivative of a function, f(x), at the i th discrete point is approximated by

$$\frac{\partial f(x_i)}{\partial x} \cong \sum_{j=1}^{N} A_{ij} f(x_j) \qquad i = 1, 2..N \tag{7}$$

where, x_i are the set of discrete points in the x-direction and A_{ij} are the associated weighting coefficients which can be derived by assuming the function, f(x), of the following polynomial form :

$$f(x_i) = x_i^{k-1} \qquad k=1, 2..N \tag{8}$$

From eqn. (7) and (8), one can write

$$\sum_{j=1}^{N} A_{ij} f(x_j) = (k-1)\, x_i^{k-2} \qquad i,k = 1, 2..N \tag{9}$$

This represents N sets of N linear, simultaneous algebraic equations which have a unique solution for the weighting coefficients, A_{ij}. The weighting coeff. for higher order partial derivatives can be obtained by an alternative technique of utilizing individual quadratures [4] which, in essence, implies that, for example, the second order derivative can be written as

$$\left(\frac{\partial^2 f}{\partial x^2}\right)_i = \sum_{j=1}^{N} B_{ik} f(x_k) \tag{10}$$

where B_{ik} is obtained by simple matrix multiplication, [B] = [A] × [A]. The weighting coeff. for 3 rd and 4 th order derivatives, C_{ij} and D_{ij}, respectively, can be derived in a similar way, [C] = [A] × [B] and [D] = [B] × [B].

Using the nondimensional coordinates, X,Y and W, the differential equation of equilibrium, eqn.(1), under a known prebuckling stress state is approximated as follows

$$d_{11} \sum_{k=1}^{N} D_{ik} W_{kj} + 2(d_{12} + 2d_{33})\beta^2 \sum_{m=1}^{N} B_{jm} \sum_{k=1}^{N} B_{ik} W_{km} + d_{22}\beta^4 \sum_{k=1}^{N} D_{jk} W_{ik}$$
$$= 2N_{av} a^2 (R_1 X_i + R_2) \sum_{k=1}^{N} B_{ik} W_{kj} - 2N_{av} a^2 R_1 (2Y_j - 1) \sum_{m=1}^{N} A_{jm} \sum_{k=1}^{N} A_{ik} W_{km} \qquad i,j = 3..(N\text{-}2) \tag{11}$$

The boundary conditions for simple supports are defined as

$$W_{1j} = W_{Nj} = W_{i1} = W_{N1} = 0 \qquad i,j = 1..N$$
$$d_{11} \sum_{k=1}^{N} B_{ik} W_{kj} + d_{12}\beta^2 \sum_{k=1}^{N} B_{jk} W_{ik} \quad i = 2,(N\text{-}1) \ \text{and} \ j,k,m = 2..(N\text{-}1) \tag{12}$$
$$d_{12} \sum_{k=1}^{N} B_{ik} W_{kj} + d_{22}\beta^2 \sum_{k=1}^{N} B_{jk} W_{ik} \quad j = 2,(N\text{-}1) \ \text{and} \ i,k,m = 2..(N\text{-}1)$$

and for clamped supports

$$W_{1j} = W_{Nj} = W_{i1} = W_{N1} = 0 \qquad i,j = 1..N$$

$$\sum_{k=1}^{N} A_{ik} W_{kj} = 0 \qquad i = 2,(N\text{-}1) \ \text{and} \ j,k = 2..(N\text{-}1) \tag{13}$$

$$\sum_{k=1}^{N} A_{jk} W_{ik} = 0 \qquad j = 2,(N\text{-}1) \ \text{and} \ i,k = 2..(N\text{-}1)$$

The above equations lead to the generalized eigen-value problem which is solved for the minimum buckling load using NAG subroutine F02BJF based on the QZ algorithm. The mixed boundary conditions can easily be accommodated by combining equations (11) and (13). The method can be extended to include rotational boundary restraints as well.

3.1 Computation

A computer program is developed which, for an assumed grid of points in the X and Y directions, computes weighting coefficients for partial derivatives, generates the bending stiffness matrix from specified lamination parameters and finally solves the eigenvalue problem defined by eqn. (11) with appropriate B.C. using the QZ algorithm. Zero edge displacement conditions are obviously applied at X,Y = 0,1 but zero moment or slope conditions, as the case may be, are applied at points very close to the plate boundaries which require grids of nonuniformly spaced points as shown in Figure 1.

4. RESULTS AND DISCUSSION

A numerical method for buckling analysis under general loading and boundary conditions is developed. The results are reported for the plate with four different combinations of simple and clamped boundary conditions (B.C.) under three loading conditions corresponding to r = 1, 0.5, -0.5. The edge conditions are denoted by the letter S for simple and C for clamped along the four edges in the following order, x = 0, x = a, y = 0 and y = b. The accuracy of the method is established by comparing results for isotropic, square plates under uniform, uniaxial compression, i.e., r = 1, with those obtained by the classical methods [3]. The comparison, as shown in Table 1, is found to be satisfactory. The average buckling stress, $K_{av} = N_x a^2 / \sqrt{d_{11}\, d_{22}}$, for square, symmetric angle-ply for the fiber angle θ = 0 to 90 in steps of 15 are presented in Figures 2 - 5 for B.C. SSSS, CCCC, SSCC and SSCS respectively. In general, the average buckling stress in the case of compression gradient is found to be less than that of uniform compression. Of course, the value of maximum stress for buckling under prescribed gradient is much higher than the uniform buckling stress. For simple supports and its combinations, the sharp optima associated with $\theta = 45$ starts diminishing when one of the stress becomes tensile, e.g. for r = -0.5. This might have interesting implications in the design - optimization of laminated plates.

The quadrature method proves to be a simple and efficient tool for handling complex combinations of simple and clamped boundaries. The basic concept of this method is the polynomial fit to the derivative of a function. The accuracy of the approximation increases as the number of grid points or the order of the polynomial increases. Beyond a certain extent numerical ill conditioning can result which introduces gross errors. At present, results are reported using grids of 8 and 9 points which appears to be within acceptable accuracy. The proposed analysis awaits further verification by classical methods which assure the converging eigenvalue problem.

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Table 1: Uniaxial Plate Buckling : Comparison

B.C.	No. of Points	Buckling Load Coeff. Kav	
		DQM	Classical [3]
SSSS	9	39.484	39.478
CCCC	8	101.510	99.389
SSCC	8	84.326	84.878
SSCS	8	57.060	56.750

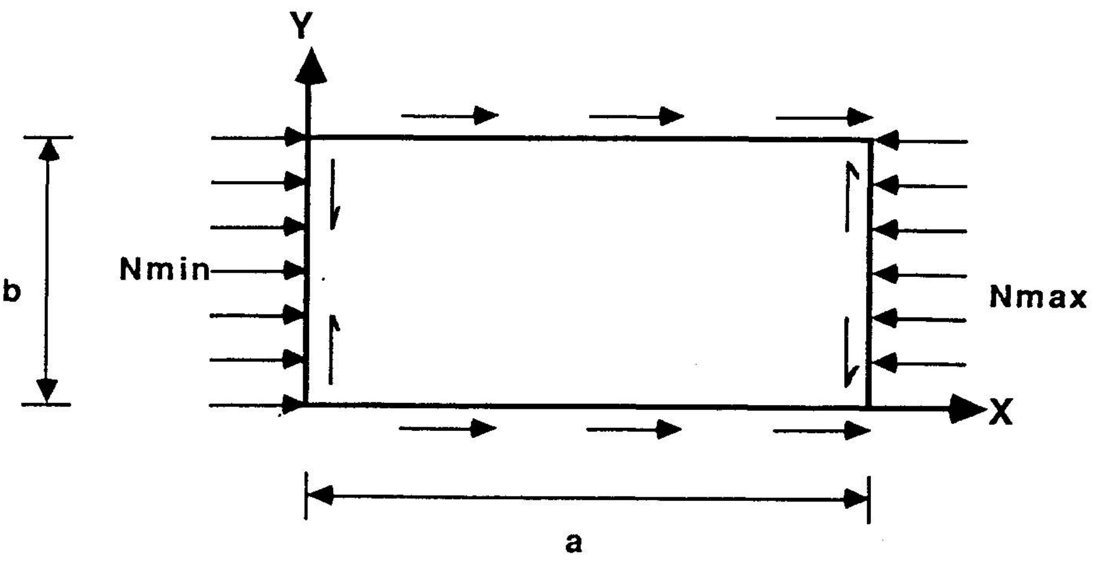

Figure 1 : Plate Buckling under Compression Gradient

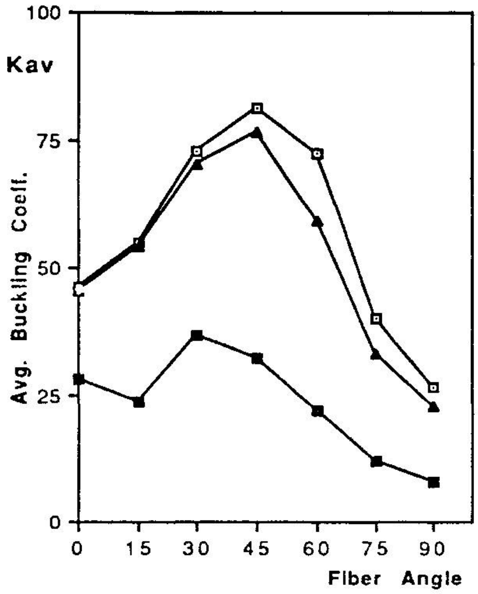

Figure 2 : SSSS Plates

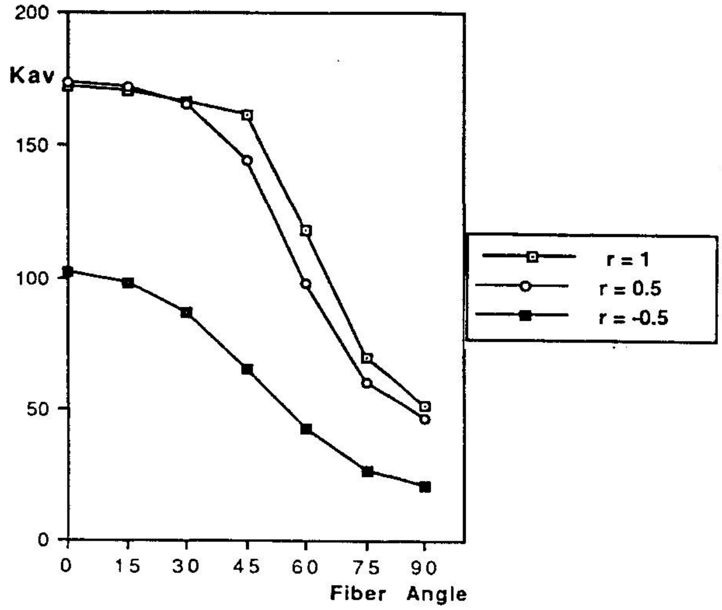

Figure 3 : CCCC Plates

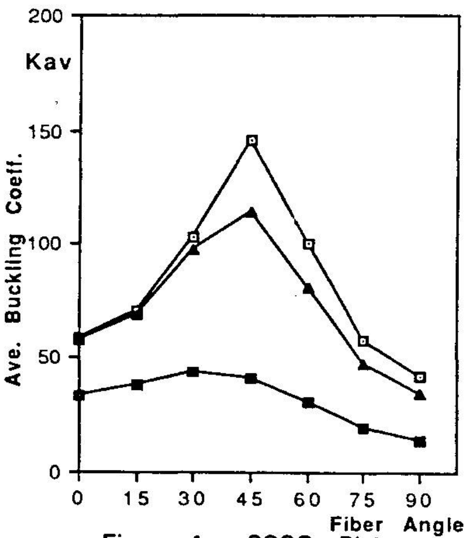

Figure 4 : SSCC Plates

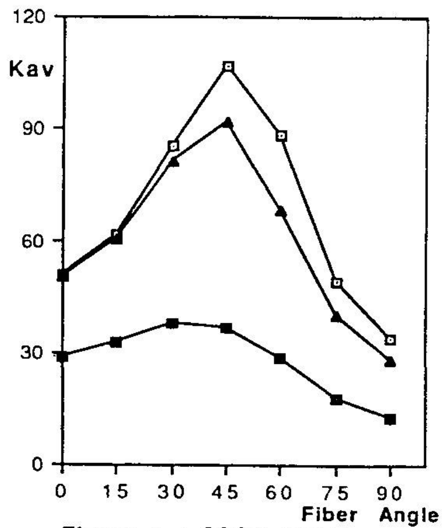

Figure 5 : SSCS Plates