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## Analysis of a Plate Girder Bridge with a Reinforced Concrete Slab

Analyse d'un pont à poutres à âme pleine avec dalles en béton armé

Analyse einer Vollwandträgerbrücke mit Stahlbetonplatten

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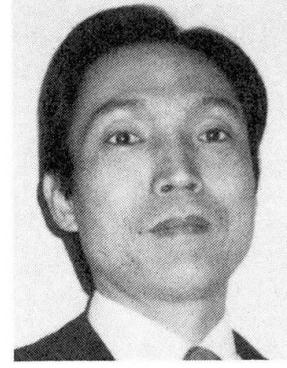
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### SUMMARY

A simple numerical method for evaluating the overall structural behavior and local stresses of a composite T-girder bridge is presented. The method is based on a finite element technique which can connect plate elements with a beam element. Reinforced concrete floor slabs are modeled by thin plate elements and main girders by beam elements. The effects of shear connectors are included by considering the difference of a horizontal displacement between a floor slab and steel girders. Validity and efficiency of the present method are shown by comparison with experimental results.

### RÉSUMÉ

Dans ce document, une méthode numérique simple est présentée pour évaluer le comportement d'ensemble et les contraintes locales d'un pont mixte à poutre en T. La méthode est basée sur la technique des éléments finis qui peut intégrer les éléments de plaque dans un élément de poutre. Les dalles du tablier en béton armé sont considérées comme des éléments de plaque minces et les poutres principales comme des éléments de poutre. Les effets des goujons de cisaillement sont calculés en tenant compte du déplacement horizontal différentiel entre la dalle du tablier et les poutres en acier. La validité et l'efficacité de la présente méthode sont vérifiées par comparaison des résultats expérimentaux.

### ZUSAMMENFASSUNG

Vorgestellt wird eine einfache numerische Methode zur Auswertung des strukturellen Verhaltens im Gesamtsystem wie auch von örtlichen Beanspruchungen einer Brücke mit Verbund-Doppel-T-Trägern. Die Methode basiert auf einer finiten Elementetechnik, die zur Verbindung von Plattenelementen mit Trägern angewandt wird. Stahlbeton-Fahrbahnplatten werden durch dünne Plattenelemente dargestellt und Hauptträger durch Trägerelemente. Die Wirkungen von Schubdübeln werden durch Berücksichtigung der Differenz des horizontalen Versatzes zwischen Fahrbahnplatte und Stahlträgern einkalkuliert. Gültigkeit und Effizienz der vorgestellten Methode lassen sich mittels Vergleich mit experimentellen Ergebnissen aufzeigen.



## 1. INTRODUCTION

In recent years, engineers often face to the serious decision making about some deteriorated or damaged bridges: to rehabilitate or replace it under the limited budget. In this occasion, the estimation of the expected remaining life span of the bridges is necessary. In order to estimate the remaining life of the bridges by a computer simulation, we have to compute the stresses and the deflections of the bridges as accurately as possible.

In girder bridges, concrete slabs and steel girders are completely or incompletely connected by shear connectors or slab anchors. Although many studies have been reported in reference to the analytical method for the behavior of composite girder bridges having complete or incomplete interaction[1-3], few methods are able to evaluate the 3-dimensional behavior by considering the effect of shear connectors accurately. Hence, it is important to study the method for analyzing the structural behavior including the effect of the stiffness of the cross beams or the cross frames and the effect of shear connectors.

This paper presents a simple numerical method for evaluating structural behavior as a whole system and local stresses of composite girder bridge. The method is based on a finite element technique which can connect the plate elements with a beam-column element[4]. Reinforced concrete (RC) floor slabs are modeled by thinplate elements and main girders are by beam elements. Thus we can reduce a number of freedom remarkably without losing the computational accuracy. The effects of shear connectors are included by considering the difference of a horizontal displacement between a floor slab and steel girders. The validity of this numerical method is studied through comparisons with the experimental results.

## 2. SUMMARY OF NUMERICAL METHOD

### 2.1 Theoretical Model

The theoretical model used for the analysis of a composite I-girder bridge which composed of RC floor slabs, main girders and cross beams or cross frames is shown in Fig.1. Floor slabs are modeled by triangular thin plate elements having six degrees of freedom for one node, derived by the consideration for the in-plane flexural stiffness. Main girders and cross beams are modeled by thin-walled beam-column elements of which details are described in Ref.[5].

The local cartesian coordinate  $(x, y, z)$  for the plate element and beam-element is set up as shown Fig.1. Geometrical relations between the displacements of the middle plane  $N_p$  of the plate element and those of the neutral axis  $N_b$  of the beam element are shown in Fig.2. From this figure, the displacement of the point  $i_b$  can be expressed by the displacement of point  $i_p$ , according the compatibility of displacement field of a beam. That is,

$$\left. \begin{array}{l} u_{ib} = u_{ip} + \Delta u_{ip} - h\theta_{zip} \\ v_{ib} = v_{ip} \\ \theta_{zib} = \theta_{zip} \\ \theta'_{xib} = (\theta_{xip+1} - \theta_{xip})/l \end{array} \right\} \quad (1)$$

where  $u_{ib}$ ,  $v_{ib}$ ,  $\theta_{xib}$ ,  $\theta_{zib}$  and  $u_{ip}$ ,  $v_{ip}$ ,  $\theta_{xip}$ ,  $\theta_{zip}$  are two displacement components in directions of the  $x$ ,  $y$  axes and the two rotation components about the  $x$ ,  $z$  axes with respect to the nodal point  $i_b$  and  $i_p$  respectively, a prime denotes a derivative with respect to  $x$ ,  $l$  is a length of one plate element and  $h$  is the distance between the middle plane  $N_p$  and the neutral axis  $N_b$ . The displacement  $\Delta u_{ip}$  in Eq.(1) is the difference of a horizontal displacement between the floor slab and steel girder and caused by the horizontal shear  $X_{ip}$  which acts on shear connectors. The horizontal shear  $X_{ip}$  is related to the displacement  $\Delta u_{ip}$  by the slip modulus  $k$  of the shear connector as

$$X_{ip} = k\Delta u_{ip} \quad (2)$$

By substituting of Eq.(2) into Eq.(1), the relationship between the displacement vector  $\{\delta\}_{ib}$  of the nodal point  $i_b$  and the displacement vector  $\{\delta\}_{ip}$  of the nodal point  $i_p$  is expressed by

$$\{\delta\}_{ib} = [G_m]_{ip} \{\delta\}_{ip} \quad (3)$$

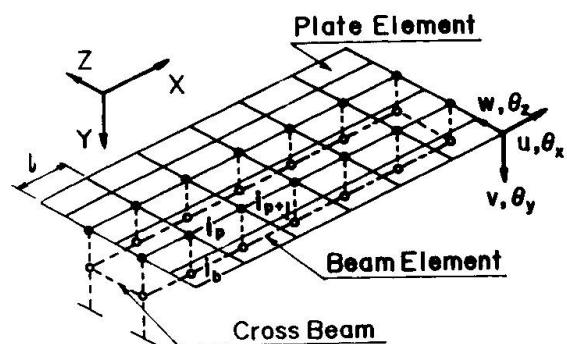


Fig.1 Theoretical model for analysis

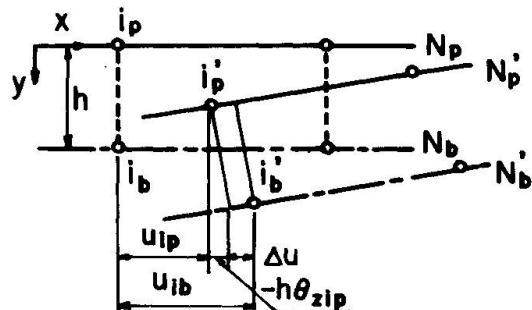


Fig.2 Geometrical relation between plate element and beam element

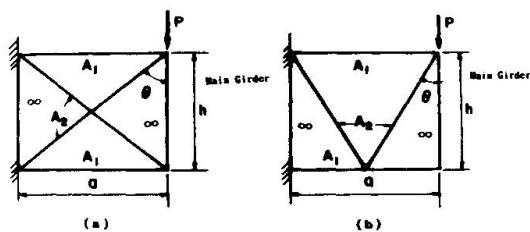


Fig.3 Modeling of cross frames

Table 1 Mechanical Properties of Materials

	E (kN/mm <sup>2</sup> )	G (kN/mm <sup>2</sup> )	$\nu$	k (kN/cm/cm)
STEEL GIRDERS	201.1	77.3	0.3	
REINFORCEMENT	206.0	79.3	0.3	283.5
CONCRETE	28.1	11.7	0.2	

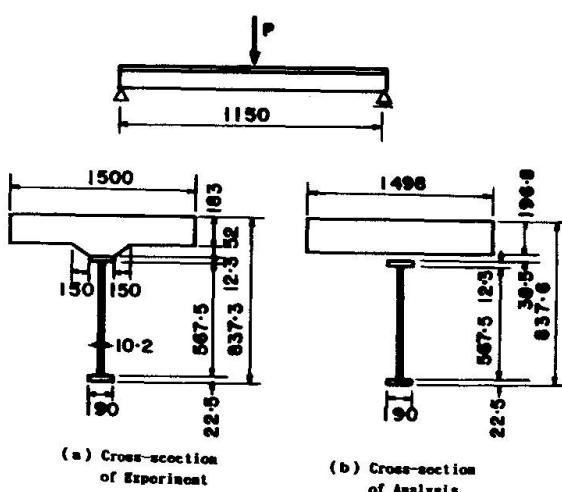


Fig.4 Composite beam model with one steel girder (unit in mm)

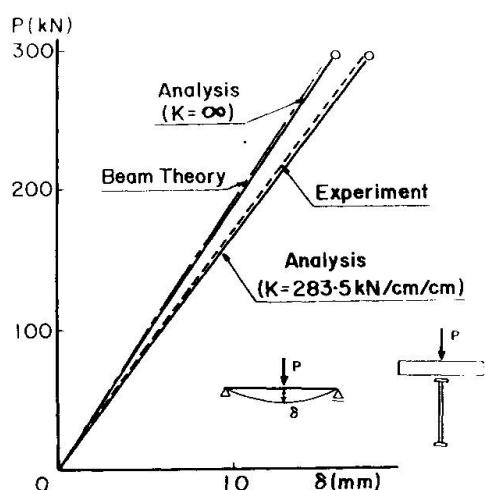


Fig.5 Load versus displacement diagrams at midspan

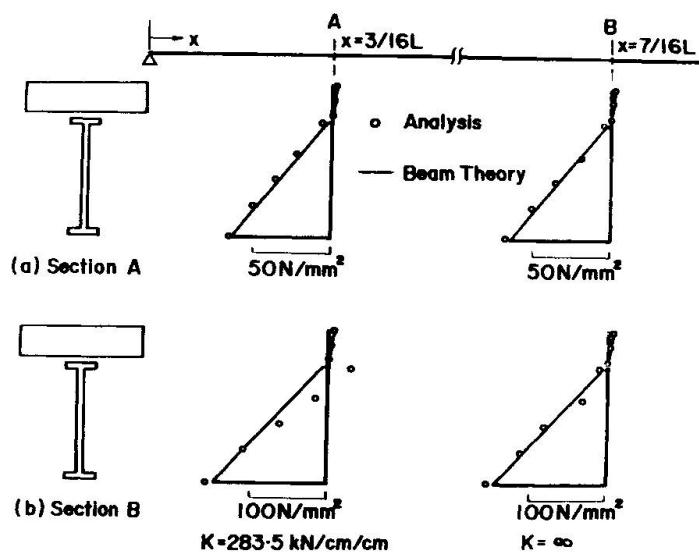


Fig.6 Distribution of normal stresses



in which the matrix  $[G_m]_{ip}$  is called herein the coupling matrix and represents the stiffness matrix which includes the effect of the slip modulus of shear connectors. The displacement vector  $\{\delta\}$  is partitioned into two vectors  $\{\delta_n\}$  and  $\{\delta_m\}$ . The vector  $\{\delta_n\}$  of the beam element consists of displacements of dependent degrees of freedom and the vector  $\{\delta_m\}$  of the plate element consists of displacements of independent degrees of freedom. Since the stiffness matrix  $[K]$  is similarly partitioned, the stiffness equation is partitioned in the following manner:

$$\left\{ \begin{array}{l} \bar{F}_n \\ F_m \end{array} \right\} = \left[ \begin{array}{cc} \bar{K}_{nn} & [0] \\ [0] & K_{mm} \end{array} \right] \left\{ \begin{array}{l} \delta_n \\ \delta_m \end{array} \right\} \quad (4)$$

$$\{\delta_m\} = [G_m]\{\delta_n\} \quad (5)$$

in which  $\bar{K}_{nn}$  and  $K_{mm}$  are the stiffness matrices, and  $\bar{F}_n$  and  $F_m$  are force vectors. A bar over the symbol is used to denote matrices that are replaced in the reduction process. The matrix  $[G_m]$  in Eq.(5) is obtained by assembling Eq.(3) for a whole bridge system. The total displacement of beam elements is represented by the corresponding displacement of plate elements at which beam elements are connected. The elimination of  $\{\delta_m\}$  of Eqs.(4) and (5) yields

$$\{F_n\} = [K_{nn}]\{\delta_n\} \quad (6)$$

where

$$\left. \begin{array}{l} [K_{nn}] = \bar{K}_{nn} + [G_m]^T [K_{mm}] [G_m] \\ \{F_n\} = \{\bar{F}_n\} + [G_m]\{F_m\} \end{array} \right\} \quad (7)$$

The initial partition of  $[K]$  and operations indicated by Eq.(7) are performed by appropriate modules of the program. The Eq.(6) are the governing equilibrium equations to be solved.

## 2.2 Modeling of Cross Frame

Cross frames are modeled by beam elements having the flexural rigidity equivalent with the actual frames. Let us consider a plane frame consisting of main girders and a cross frame as shown in Fig.3. The contribution of the flexural rigidity of the diagonal and the vertical members to the flexural rigidity of the plane frame can be neglected and those of the upper and the lower members are considered. The flexural rigidity  $I_{Qf}$  of the equivalent cross beam element was obtained by assuming the deflection of a cantilever beam with a concentrated at the free end equals the deflection of the plane frame at the free end.

$$I_{Qf} = \frac{I_t}{1.0 + CI_t/(A_2 a^2 \sin \theta)} \quad (8)$$

where

$$\left. \begin{array}{l} I_t = A_1 h^2/2 \\ C = 1.5 \text{ (Fig.3(a))}, 3.0 \text{ (Fig.3(b))} \end{array} \right\} \quad (9)$$

in which  $A_1$  and  $A_2$  are cross sectional areas of upper(= lower) and diagonal members respectively, and  $a$  and  $h$  are the length and the height of the plane frame respectively.

## 3. NUMERICAL RESULTS AND DISCUSSIONS

### 3.1 Analysis of Composite Beam Model with One Steel Girder

A simply supported composite beam model subjected to a lateral load at the center of the span as shown in Fig.4 is analyzed. The cross sectional shape of the model as shown in Fig.4(b) is determined according to the condition that the moment inertia and the neutral axis of the transformed composite section of the model equal those of the experiment by Maeda et al.[6]. The RC slab plate is divided into 8 elements along the length and 2 elements along the width. The steel beam is divided 8 beam elements. The material properties and the slip modulus of shear connectors are given in Table 1 as obtained in the experiment. Fig.5 shows the load  $P$  versus displacement diagrams at the midspan. The results of the present method correspond to the experimental results including the effect of the displacement due to the horizontal shear and the solution of the beam theory fairly well. The distribution of normal stresses on the cross section at  $x = 3/16L$  and  $x = 7/16L$  are shown in Fig.6.

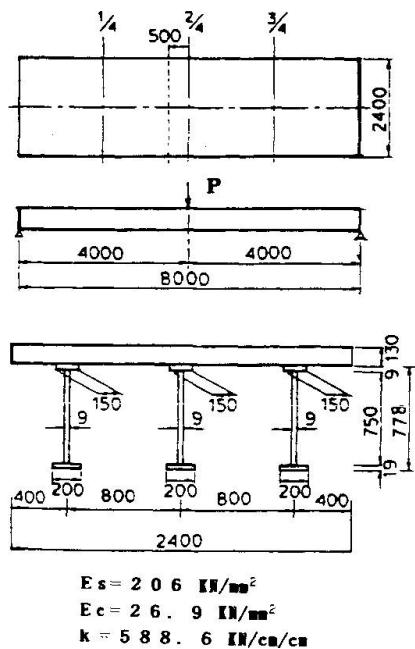


Fig.7 Composite beam model with three steel girders (unit in mm)

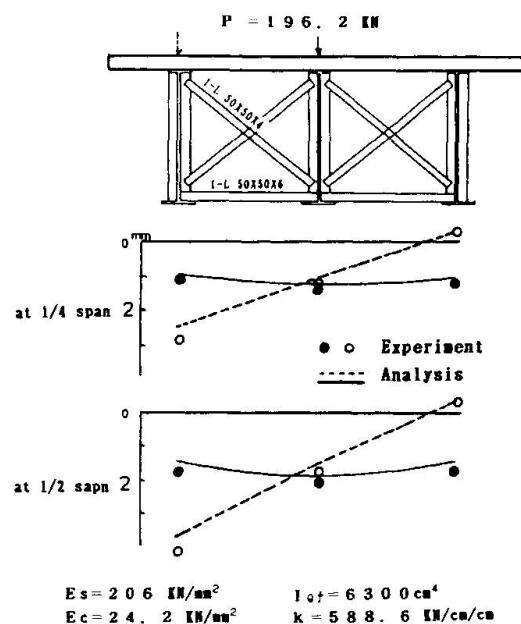


Fig.9 Deflection along the cross section at quarter span and midspan

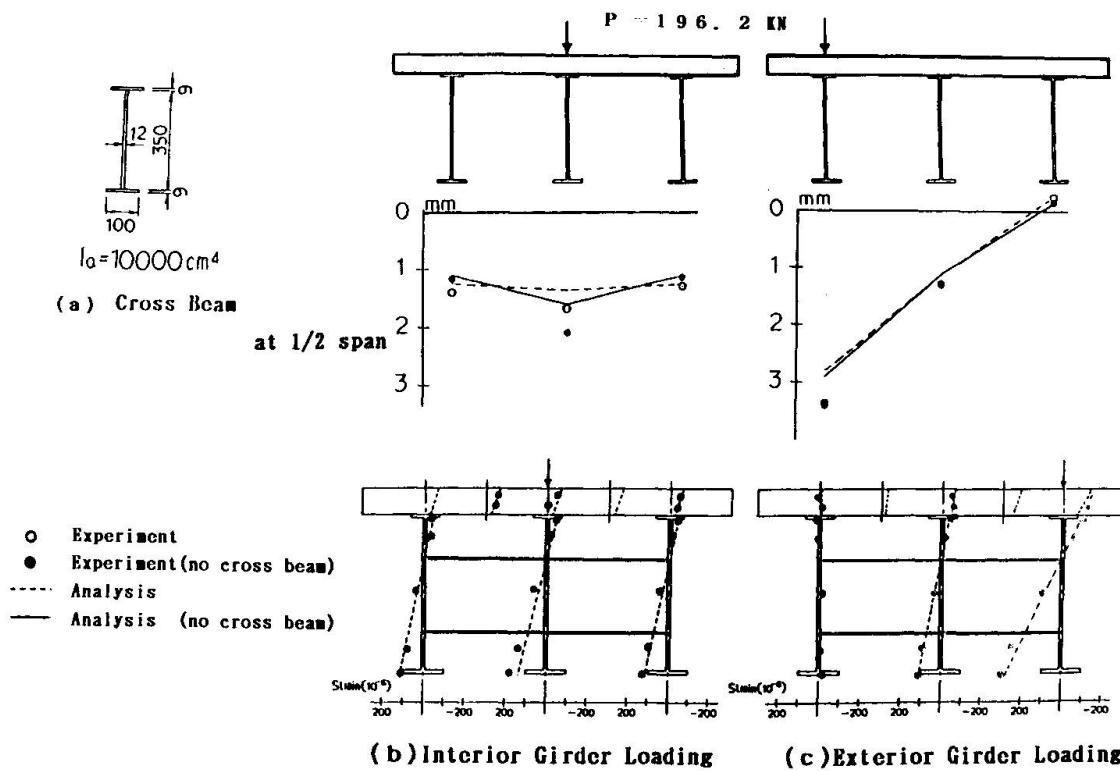


Fig.8 Deflection at central cross section and distribution of normal strains



The results of the complete composite analysis by the present method( $k = \infty$ ) show fairly good correspondence with the solution of the beam theory.

### 3.2 Analysis of Composite Beam Model with Three Main Girders

The composite beam models with three main girders and cross beams or cross frames subjected to a lateral load at the center of the span, as shown in Fig.7, are analyzed. The number of the discretization of the RC plate and the beam elements along the length is 8 and those of the plate element along the width is 6. The material properties and the slip modulus  $k$  of shear connectors are given as obtained in the experiment by Sato et al.[7]. Numerical computations are carried out in three cases, with and without the cross beams located at the center of the beam as shown in Fig.8 and with the cross frames, located at three points which divide the span into four equal parts as shown in Fig.9. The deflection at the midspan and the distribution of normal strains on the cross section at a distance 50 cm from the center of the span, subjected to a lateral load at the interior or the exterior girder of the midspan are shown in Fig.8. The deflections at the section at  $x = L/2$  and  $x = L/4$  for the model with cross frames are compared with the results of the experiment as shown in Fig.9. It is seen that the results of the present method have fairly good correspondence with the experimental results as a whole. The validity of the present method can be recognized.

## 4. CONCLUSIONS

A finite element method which can analyze the structural behavior as a whole system and local stresses of composite girder bridges is developed. The validity and efficiency of the present method are studied in numerical examples and the following facts are found. 1)The results of present method show fairly good correspondence with the experimental results. 2)It was shown that this method can analyze the structural behavior including the effect of the stiffness of the cross beam or the cross frame and the effect of shear connectors. 3)The results show that this is a promising method for evaluating the current conditions of the composite bridges and for evaluating the effect of repair or strengthening

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