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Local Buckling of Profiled Sandwich Plates

Voilement local de panneaux sandwich profilés

Lokales Beulen profilierter Sandwichplatten

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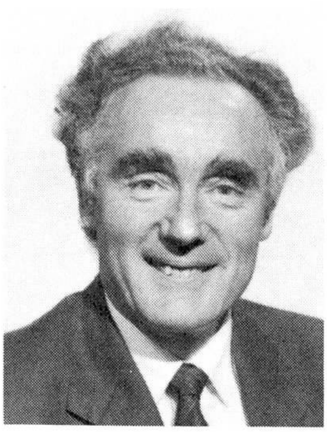

Professor Davies was awarded the degree of D.Sc. by the University of Manchester in 1979 for his research into the stability of steel structures. He is an active member of national and international standards committees and consults widely for the steel industry.

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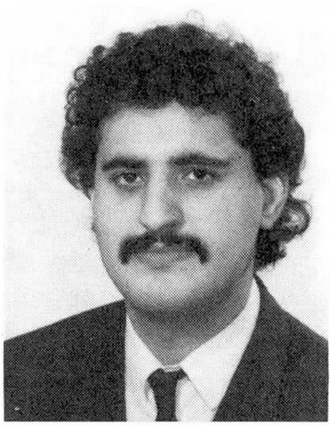


Dr Hakmi gained his Ph.D. in 1988 at the University of Salford for his research into the local buckling of sandwich elements. He is currently working on a sponsored project concerned with fire resistant panels for offshore structures.

SUMMARY

Sandwich panels used in building construction typically consist of two metal faces and a foamed plastic core. One or both faces may have a trapezoidal or similar profile for either structural or aesthetic reasons. When the panel is subject to static loading the profiled face may be compressed and therefore liable to failure by local buckling. This is directly analogous to one of the fundamental problems of thin-walled metal construction with the added factor that the face element in compression is supported by the core material. In this paper, the local buckling of a compressed plate element supported by a relatively weak isotropic medium is considered. A practical solution is presented.

RÉSUMÉ

Les panneaux sandwichs utilisés dans la construction des bâtiments se composent en général de deux faces métalliques et d'un noyau en mousse. L'une ou les deux faces peuvent avoir un profil trapézoïdal ou un profil semblable pour des raisons structurelles ou esthétiques. Lorsque le panneau est soumis à une charge statique, la face profilée peut être comprimée et par conséquent sujette à une rupture par voilement local. Ceci est analogue à l'un des problèmes fondamentaux de la construction métallique en éléments à paroi mince avec cette particularité que la face en compression est supportée par le matériau du noyau. Cet exposé tient compte du voilement local de la plaque métallique comprimée et supportée par un milieu isotrope relativement peu consistant; il propose une solution utilisable dans la pratique.

ZUSAMMENFASSUNG

Sandwichplatten in bautechnischer Anwendung bestehen in der Regel aus Metalldeckschichten und einem Kern aus Hartschaum. Eine oder beide Deckschichten können aus Gründen der Asthetik oder der Tragfähigkeit profiliert sein. Wenn das Paneel einer Biegebeanspruchung ausgesetzt ist, wird die prolifierte Deckschicht im Druckspannungsbereich örtliche Ausbeulerscheinungen zeigen. Hier zeigt sich die Analogie zu einem fundamentalen Problem dünnwandiger Bauteile unter Druckbeanspruchungen mit der Besonderheit, dass die Deckschicht durch den Kern abgestützt wird. Im vorliegenden Bericht wird das örtliche Beulen der druckbeanspruchten Deckschicht in der Interaktion mit einem verhältnismässig weichen und isotropen Schaum analysiert, und es wird eine praxisgerechte Lösung vorgestellt.

1. INTRODUCTION

The problem addressed in this paper is typified by Fig. 1. A sandwich panel has a thin profiled steel face in compression. This face is separated from the corresponding face in tension, which may be either profiled or flat, by a relatively thick lightweight core. The core ensures composite behaviour of the two faces and, at the same time, stiffens the compressed face with respect to local buckling. In order to design such panels efficiently it is necessary to be able to predict the local buckling stress of the individual plate elements supported by the core and the post-buckling behaviour. This paper provides a solution for the linear buckling problem and extends this to an empirical effective width formulation. A subsequent paper will address the non-linear post-buckling behaviour more formally.

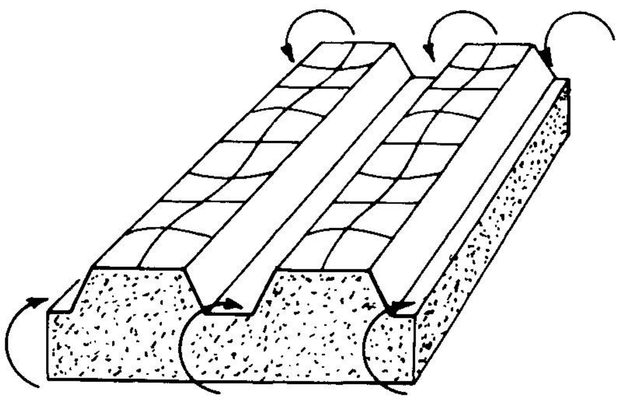


Fig. 1. Typical sandwich panel with a profiled face in compression

Typical core materials for practical sandwich panels are rigid plastic foams (polyurethane, polyisocyanurate, polystyrene) with densities in the range 30–60 kg/m^3. These materials are assumed to be isotropic and linear so that their behaviour can be described in terms of the elastic modulus, E_c, and the shear modulus G_c. While not strictly true, this assumption has been found useful for most practical purposes. Where it has proved unsafe, theoretical solutions obtained on this basis have been modified by an empirical factor.

The mathematical problem to be solved is shown in Fig. 2. A simply supported rectangular plate of length a and width b is subject to an applied stress p along the two transverse edges. The longitudinal edges of the plate are assumed to be simply supported. The tendency of the plate to buckle locally is resisted by the core. It is required that the critical value p_{cr} of the stress at which the buckles illustrated in Fig. 1 first form should be determined. The length of the plate in the x-direction is generally large compared with the width. The wavelength of the buckles is then an unknown in the solution.

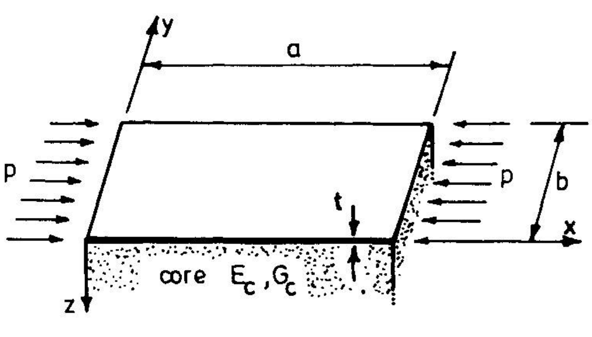


Fig. 2. Plate element in compression with core support

2. LINEAR BUCKLING ANALYSIS

The solution which follows is related to that given by Timoshenko [1] for a thin plate without core support. The buckled shape is represented by the double series

$$w = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} a_{mn} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \qquad \ldots\ldots (1)$$

which automatically satisfied the boundary condition of simply supported edges. The strain energy of bending is

$$U_B = \frac{\pi^4 abD}{8} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} a_{mn}^2 \left[\frac{m^2}{a^2} + \frac{n^2}{b^2}\right]^2 \qquad \ldots\ldots (2)$$

The work done by the applied axial forces during buckling is

$$V = p\ t \left[\frac{ab}{8} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} a_{mn}^2 \frac{m^2\pi^2}{a^2}\right] \qquad \ldots\ldots (3)$$

The strain energy in the core material is found as follows. It is assumed that the core is an infinite elastic half-space and that, within that space, the displacements decay exponentially so that

$$w = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} a_{mn} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b}\, e^{-kz} \qquad \ldots\ldots (4)$$

where the decay factor k is to be determined. The strain energy in the core is given by

$$U_c = \tfrac{1}{2} \int_o^a \int_o^b \int_o^{\infty} \left[E_c \left[\frac{\partial w}{\partial z}\right]^2 + G_c \left[\frac{\partial w}{\partial x}\right]^2 + G_c \left[\frac{\partial w}{\partial y}\right]^2\right] dx\, dy\, dz \qquad \ldots\ldots (5)$$

Substituting for w from equation (6) and observing the well-known rules for integrating products of sines and cosines gives

$$U_c = \frac{ab}{16} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} a_{mn}^2 \left[k\, E_c + \frac{\pi^2 G_c}{k} \left[\frac{m^2}{a^2} + \frac{n^2}{b^2}\right]\right] \qquad \ldots\ldots (6)$$

The decay factor k is determined from the condition $\frac{\partial U_c}{\partial k} = 0$

i.e. $$k^2 = \frac{\pi^2 G_c}{E_c} \left[\frac{m^2}{a^2} + \frac{n^2}{b^2}\right] \qquad \ldots\ldots (7)$$

and therefore $$U_c = \frac{ab\pi}{8} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} a_{mn}^2 \sqrt{E_c G_c} \left[\frac{m^2}{a^2} + \frac{n^2}{b^2}\right]^{\frac{1}{2}} \qquad \ldots\ldots (8)$$

The potential energy is $U_B + U_C - V$ and minimising this with respect to each of the coefficients a_{mn} in turn gives the following series of equations.

$$p_{cr}\left[a_{mn}\frac{m^2\pi^2}{a^2}\right] = \frac{a_{mn}}{t}\left[\pi^4 D\left[\frac{m^2}{a^2}+\frac{n^2}{b^2}\right]^2 + \pi\sqrt{E_c G_c}\left[\frac{m^2}{a^2}+\frac{n^2}{b^2}\right]^{\frac{1}{2}}\right] \qquad \ldots\ldots (9)$$

We now make the assumption that, in the x-direction, the plate buckles into a series of m half-waves. We therefore consider only the equations with a particular value of m containing coefficients a_{m1}, a_{m2}...etc. For the case of uniform compression, the plate also buckles into a single sine wave in the y-direction and it is only necessary to consider a single equation in the dominant coefficient a_{m1}. The more general derivation including the full series of terms is necessary if non-uniform distributions of applied stress are considered. Thus, the equation for p_{cr} is

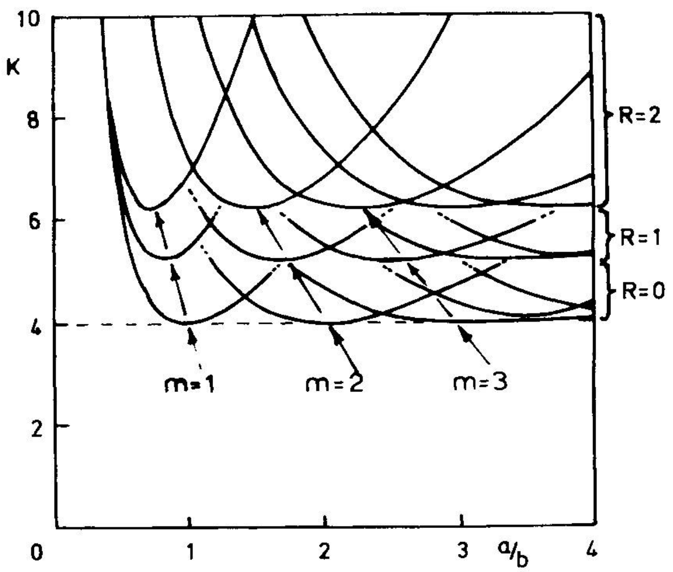
$$p_{cr} = \frac{a^2}{m^2 t}\left[\pi^2 D\left[\frac{m^2}{a^2}+\frac{1}{b^2}\right]^2 + \frac{1}{\pi}\sqrt{E_c G_c}\left[\frac{m^2}{a^2}+\frac{1}{b^2}\right]^{\frac{1}{2}}\right] \qquad \ldots\ldots (10)$$

Introducing $\Phi = \frac{a}{mb}$, the above equation can be rewritten

$$p_{cr} = K\,\frac{\pi^2 E}{12\left[1-\nu^2\right]}\left[\frac{t}{b}\right]^2 \qquad \ldots\ldots. (11)$$

$$\text{where}\quad K = \left[\frac{1}{\Phi}+\Phi\right]^2 + R\Phi^2\left[\frac{1}{\Phi^2}+1\right]^{\frac{1}{2}} \qquad \ldots\ldots. (12)$$

$$\text{and}\quad R = \frac{12\left[1-\nu^2\right]}{\pi^3}\,\frac{\sqrt{E_c G_c}}{E}\left[\frac{b}{t}\right]^3 \qquad \ldots\ldots. (13)$$



R is a non-dimensional ratio that reflects the relative stiffness of the core and the plate. When the stiffness of the core is zero, R = 0 and equations (11) to (13) reduce to the well known equations for the buckling of a thin plate. The values of K for different values of m and R are shown in Fig. 3. For R = 0, the minimum value of K = 4 is obtained when the plate buckles into square waves. For increasing values of R, the critical wavelength reduces and K increases.

Fig. 3 Variation of the buckling parameter K

For a long plate, the critical value of the buckling stress occurs when the equation for p_{cr} is minimised with respect to the wavelength parameter Φ, i.e., $\partial K/\partial\Phi = 0$, which gives

$$2\left[\frac{1}{\Phi}+\Phi\right]\left[1-\frac{1}{\Phi^2}\right] + R\left[\frac{1}{\Phi}+2\Phi\right]\left[\frac{1}{\Phi^2}+1\right]^{-\frac{1}{2}} = 0 \qquad \ldots\ldots. (14)$$

Equation (14) does not have an explicit solution and is best solved by a numerical method. Newton iteration was chosen by the authors and a small number of cycles starting with $\Phi = 1$ was usually sufficient to obtain convergence. Equation (14) has the form $f(\Phi) = 0$ and the iteration rule is simply

$$\Phi_{i+1} = \Phi_i - \frac{f[\Phi_i]}{f^1[\Phi_i]} \qquad \ldots\ldots (15)$$

As the derivative of equation (21) also has an explicit form, the solution is simple to program. Having found the wavelength parameter Φ, the critical buckling stress p_{cr} follows dirctly from equations (11), (12) and (13).

3. APPROXIMATE BUCKLING STRESS FOR A UNIFORMLY COMPRESSED PLATE

The solution given above is somewhat complex for practical design purposes and an approximate solution has evident attractions. The following equation for the buckling constant K has been found to be sufficiently accurate for all practical purposes.

$$K = [16 + 11.8\,R + 0.055\,R^2]^{\frac{1}{2}} \qquad (0 \leqslant R < 200) \qquad \ldots\ldots (16)$$

The relationship between the accurate and approximate solutions is shown in Fig. 4.

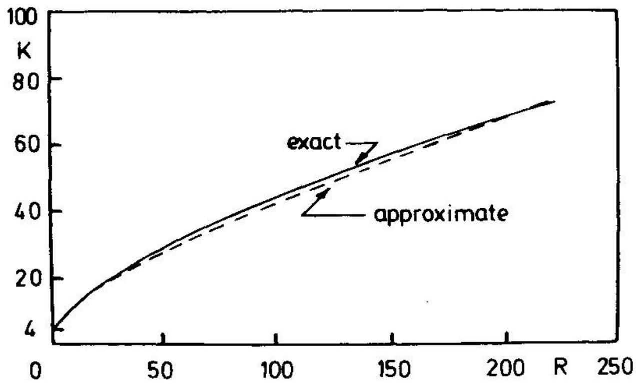


Fig. 4. Accurate and aproximate values of the buckling coefficient K

4. PRACTICAL SIGNIFICANCE OF INCREASED BUCKLING STRESS

In conventional light gauge steel applications, local buckling problems of the type shown in Fig. 1 are treated for design purposes by utilising the concept of effective width. A typical effective width formula, used in many national and international standards for cold-formed steel design is the Winter formula [2]. This has the form

$$\left.\begin{array}{lll} b_{eff} = \rho b & & \\ \text{where} \quad \rho = \frac{1}{\lambda}\left[1 - \frac{0.22}{\lambda}\right] & \text{for } \lambda > 0.673 \\ \rho = 1.0 & \text{for } \lambda \leqslant 0.673 \\ \lambda = 1.052 \left[\frac{b}{t}\right] \sqrt{\frac{f_c}{E\,K}} & \end{array}\right\} \qquad \ldots\ldots\ldots (17)$$

Of particular note is the presence in the formulation of the buckling parameter K. it would appear that such an effective width formula could be extended to include plates stiffened by core material by replacing K by a more appropriate value which includes for the effect of the core in raising the buckling stress p_{cr}. In order to investigate this possibility, the authors conducted a series of tests on thin-walled steel beams in which the compression flange was stiffened by foam infilling [3]. Fig. 5 shows the results of these tests superimposed on a diagram showing the effective width concept extended as described above.

The simple combination of equations (16) and (17) resulted in values of the effective width which were unsafe when compared with the test results for large values of b/t. In plotting the curves on Fig. 5, the value of K given by equation (16) has therefore been reduced by replacing R by 0.6 R giving

$$K = \left[16 + 7R + 0.02R^2\right]^{\frac{1}{2}} \quad \ldots\ldots (18)$$

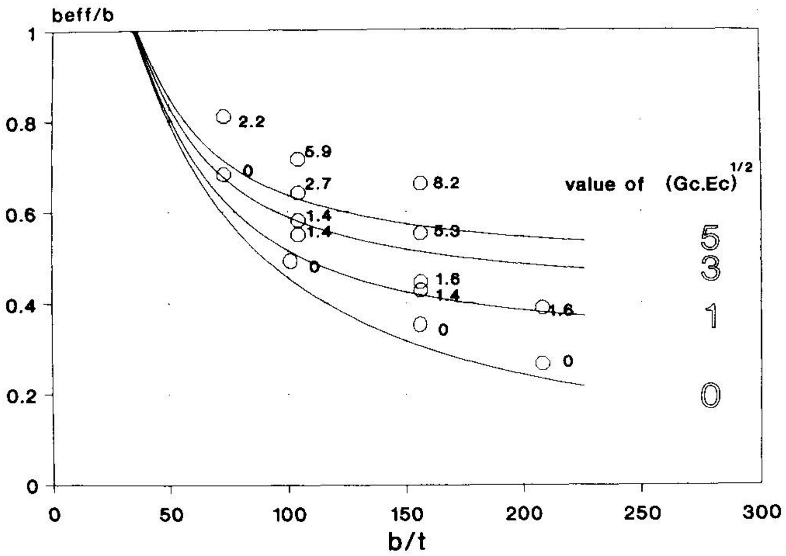


Fig. 5. Effective width of steel elements stiffened by foam

Similar empirical reductions have been found necessary for the related problem of wrinkling in flat or quasi-flat plates. Equations (17) and (18) are offered as a practical solution to the problem in question that is suitable for inclusion in codes of practice.

NOTATION

a = length of plate
a_{mn} = displacement coefficient
b = width of plate
b_{eff} = effective width of plate
C = elastic foundation constant
D = $Et^3/12(1-\nu^2)$
= flexural rigidity of plate
E = Young's modulus of plate
E_c = Young's modulus of core
G_c = shear modulus of core
f_c = maximum edge stress
k = decay factor in core
K = plate buckling coefficient
m,n = integers controlling terms in displacement function
p_o = maximum applied edge stress
R = non-dimensional ratio of core stiffness to plate stiffness
t = thickness of plate
U_B, U_C = strain energy in plate bending and core respectively
V = potential energy of edge load
w = displacement of plate in z direction
x,y,z = coordinates as defined in Fig. 2
λ = term in effective width equation
ν = Poisson's ratio of plate
Φ = a/mb = wavelength parameter
ρ = b_{eff}/b = effective width ratio

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