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Calibration of Load Model for Fatigue Calculation

Calibration d'un modèle de charge pour le calcul à la fatigue

Kalibrierung eines Lastmodelles für die Ermüdungsberechnung

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SUMMARY

This paper explains how it is possible to calibrate a load model for fatigue calculation of road bridges without knowledge of stresses. The accuracy of this simple and efficient method is shown, and initial results are given.

RÉSUMÉ

Cet article expose une méthode qui permet de calibrer un modèle de charge pour le calcul à la fatigue des ponts-routes, sans avoir besoin de connaître les contraintes. La précision de cette méthode simple et efficace est démontrée et les premiers résultats sont donnés.

ZUSAMMENFASSUNG

Der Bericht erklärt, wie ohne Kenntnis der Spannungen die Ermüdungsberechnung einer Strassenbrücke anhand eines Lastmodelles durchgeführt werden kann. Die Genauigkeit dieser einfachen und leistungsfähigen Methode sowie erste Resultate werden vorgestellt.



1. INTRODUCTION

A traffic moving forwards on a bridge induces variable stresses that may produce fatigue damage. A bridge designer needs a load model that allows a stress calculation and a fatigue damage estimation. The question of calibration is to know if the damage produced by a load model is the same as the damage produced by real traffics. This paper considers the following hypothesis :

1° Counting method : the stress spectrum produced by the traffic is converted in a stress-range histogram ($\Delta\sigma_i, n_i$) using the Rain-flow counting method, that does not take in account the mean value [1].

2° Miner Rule : the damage calculation following this method is applicable on all shapes of stress-range histogram and S-N curve [2] :

$$D = \sum \frac{n_i}{N_i}, \text{ where,}$$

n_i , is the number of cycles in the stress range histogram ;

N_i , is the number of cycles corresponding to $\Delta\sigma_i$ in the SN curve. That is the number of cycles of a stress range $\Delta\sigma_i$ producing failure.

3° Equivalent stresses range : the total damage produced by a stress-range histogram is replaced by a couple of values ($\Delta\sigma_e, n_e$) that produces the same fatigue damage :

$\Delta\sigma_e$, is the equivalent stress range ;
 n_e , is the equivalent number of cycles.

4° Traffic load : using devices measuring axle load of moving vehicles, a high number of recorded traffics are available [3][4][5]. The method developed here under considers two of the more aggressive known traffics.

This paper shows how it is possible to consider equivalent load effects instead of equivalent stress ranges, that allows a calibration comparing equivalent load effects produced by a given traffic and by the model. Such a calibration is then independent of the sizes of the bridges structures.

2. EQUIVALENT LOAD EFFECT

2.1 Damage calculation

For bridge details submitted to fatigue damage some codes define S-N curves [6] [7] [8]. The shape of the existing S-N curves differs following the kind of detail and the codes.

The Eurocode 3 defines mainly an S-N curve with two slopes (Figure 1, curve 3) :

$$\begin{aligned} N \cdot \Delta\sigma^3 &= 5 \cdot 10^6 \cdot \Delta\sigma_D^3 & \text{if } \Delta\sigma \geq \Delta\sigma_D \\ N \cdot \Delta\sigma^5 &= 5 \cdot 10^6 \cdot \Delta\sigma_D^5 & \text{if } \Delta\sigma_D \geq \Delta\sigma \geq \Delta\sigma_L \\ N &= \infty & \text{if } \Delta\sigma_L > \Delta\sigma \end{aligned}$$

Where :

- $\Delta\sigma_D$, corresponding to $N = 5 \cdot 10^6$, defines the fatigue strength of the considered detail. The part of the curve below $\Delta\sigma_D$ is only applicable by a damage calculation under variable cycles, that is what occurs in bridges ;

- $\Delta\sigma_L$, corresponds to $N_L = 10^8$:

$$\Delta\sigma_L = \sqrt[5]{\frac{5 \cdot 10^6}{10^8}} \cdot \Delta\sigma_D = 0,55 \cdot \Delta\sigma_D \quad (2)$$

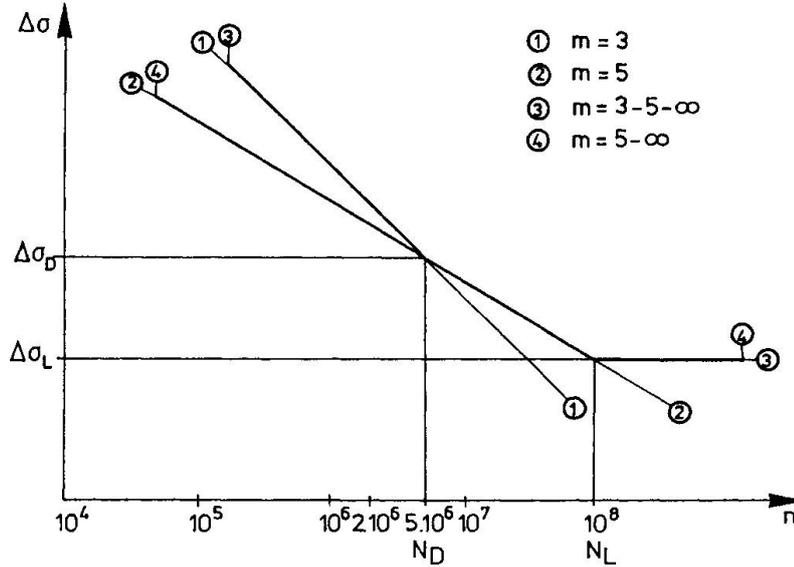


Figure 1 : SN curves

The damage produced by an histogram $(\Delta\sigma_i, n_i)$ is expressed by :

$$D_{EC} = \sum \frac{n_i}{N_i} = \sum_{\Delta\sigma_i = \Delta\sigma_L}^{\Delta\sigma_D} \frac{n_i \cdot \Delta\sigma_i^5}{5 \cdot 10^6 \cdot \Delta\sigma_D^5} + \sum_{\Delta\sigma_i = \Delta\sigma_D}^{\infty} \frac{n_i \cdot \Delta\sigma_i^3}{5 \cdot 10^6 \cdot \Delta\sigma_D^3} \quad (3)$$

The equivalent values must satisfy the relation :

$$D_{EC} = \frac{n_e \cdot \Delta\sigma_e^m}{5 \cdot 10^6 \cdot \Delta\sigma_D^m}, \text{ where } m = 3 \text{ or } 5 \quad (4)$$

For some kind of details (shear), an other S-N curve is given (Figure 1, curve 4) :

$$\begin{aligned} N \cdot \Delta\tau_L &= 5 \cdot 10^6 \cdot \Delta\tau_D^5 & \text{if } \Delta\tau \geq \Delta\tau_L \\ N &= \infty & \text{if } \Delta\tau_L > \Delta\tau \end{aligned} \quad (5)$$

Where $\Delta\tau_L$ corresponds to $N_L = 10^8$

The damage is expressed by,

$$D'_{EC} = \sum_{\Delta\sigma_i = \Delta\sigma_L}^{\infty} \frac{n_i \cdot \Delta\sigma_i^5}{5 \cdot 10^6 \cdot \Delta\sigma_D^5} = \frac{n_e \cdot \Delta\sigma_e^5}{5 \cdot 10^6 \cdot \Delta\sigma_D^5} \quad (6)$$

Other codes [8] consider S-N curves with one slope, and without cut-off (Figure 1, curves 1 and 2) :

$$N \cdot \Delta\sigma^m = 5 \cdot 10^6 \cdot \Delta\sigma_D^m, \text{ where } m = 3 \text{ or } 5 \quad (7)$$

The damage is expressed by



The equivalent values satisfy the relation :

$$n_e \cdot \Delta\sigma_e^m = \sum n_i \cdot \Delta\sigma_i^m \quad (9)$$

In this case, the values $\Delta\sigma_e$, n_e are independent of the fatigue classification of the detail ($\Delta\sigma_D$).

There is an infinity of $\Delta\sigma_e$, n_e values satisfying equation (9) ; some of them are remarkable :

1°) The usual definition considers $n_{en} = \sum n_i$

$$\text{and then : } \Delta\sigma_{en} = \sqrt[m]{\frac{\sum n_i \cdot \Delta\sigma_i^m}{\sum n_i}} \quad (10)$$

2°) We propose to consider $\Delta\sigma_e$ as the centre of gravity of the fatigue damage distribution obtained with an S-N curve with one slope :

$$\Delta\sigma_{em} = \frac{\sum n_i \cdot \Delta\sigma_i^{m+1}}{\sum n_i \cdot \Delta\sigma_i^m} \quad \text{then : } n_{em} = \frac{\sum n_i \cdot \Delta\sigma_i^m}{\Delta\sigma_e^m} \quad (11)$$

As it is shown below this second definition is less sensible to the slope of the S-N curve and to the cutting of the low stress values.

The fatigue damage distribution considering the load effect histogram (ΔS_i , n_i) instead of the stress histogram is exactly the same if the behaviour of the structure submitted to fatigue is linear : $\sigma_i = C \cdot S_i$.

The equivalent load effects is given by an equation similar to equation (11) where σ is replaced by S. Finally, we propose as definition of the equivalent load effect :

$$\Delta S_{e3} = \frac{\sum n_i \cdot \Delta S_i^4}{\sum n_i \cdot \Delta S_i^3} \quad \text{with : } n_{e3} = \frac{\sum n_i \cdot \Delta S_i^3}{\Delta S_e^3} \quad (14)$$

If this equation is exact for an S-N curve with one slope corresponding to $m = 3$, it will be an approximation if other S-N curves must be considered. But the advantage of this definition is that it is independent of :

- the sizes of the bridge detail ($\Delta\sigma_i$) ;
- the fatigue resistance of the bridge detail ($\Delta\sigma_D$) ;
- the shape of the S-N curve (m).

It allows a very simple method for calibrating a load model [9]. But before using it, it is necessary to check the differences for other S-N curves shapes and for a lot of levels of the stress-range regarding the S-N curves. On the other hand, the differences are influenced by the shape of the stress-range histogram. The estimation of these influences is treated below.

3. ACCURACY OF THE EQUIVALENT LOAD EFFECT.

3.1. Load effect histograms.

Considering the bending moment at the midspan in an isostatic beam ($L = 10$ m.) given by a simulation program [3] [10] [11], the moments produced under a traffic flow are calculated, and applying the Rain flow counting method, a moment range histogram is established (ΔM , n). The Fig. 2 shows the results obtained by two different traffics [5][12] :

- a) Auxerre traffic, recorded in France in 1986, that have a high number of high ranges,
- b) Rheden traffic, recorded in the Netherlands in 1978 that have a high number of low ranges.

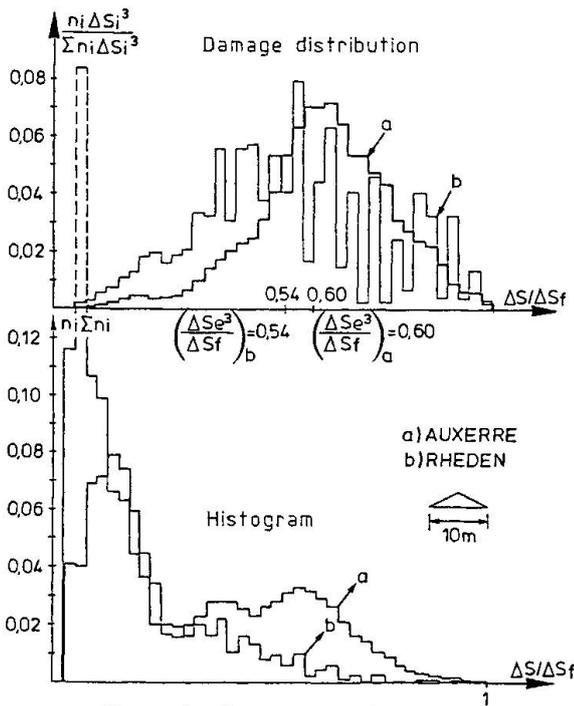


Figure 2 : Damage distribution
Load effect histograms

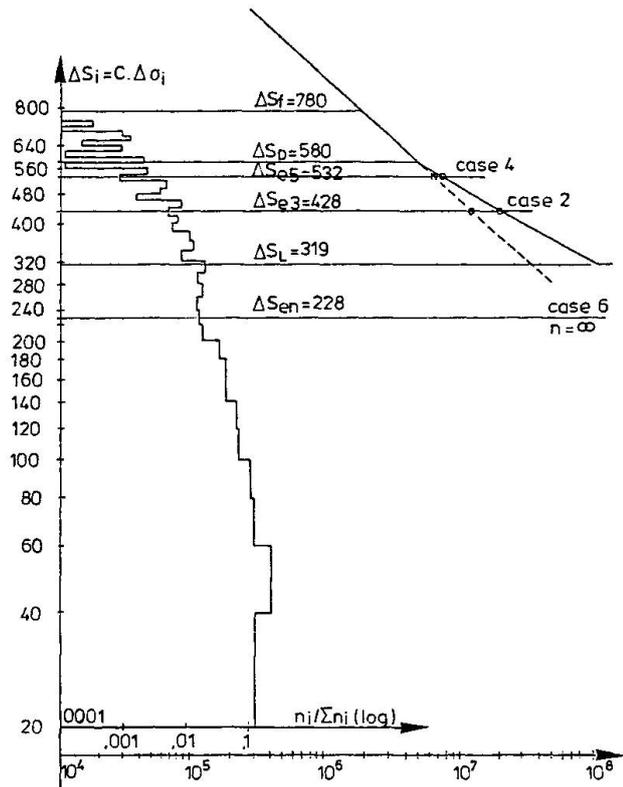


Figure 3 : Fatigue calculation-histogram b : $\Delta S_0/\Delta S_f=0.7$

These histograms may be considered as representative of all stress range histograms obtained in road bridges under traffic loads.

If the stress is proportional to the moment and if the S-N curve has one slope ($m = 3$), it is possible to calculate the fatigue damage distribution (Figure 2). In all cases, it appears that the high number of low cycles as well as the little number of high cycles produce low damage. The equivalent load effect defined following equation (14) is situated near the cycles that produces the most damage and is not influenced by the cutting of the high number of low cycles. These two considerations are not applicable by the usual definition of equivalent values (equation (10)).

3.2. Damage calculation.

The fatigue damage is calculated in different cases. In each case, different ratio $\Delta S_D/\Delta S_F$ (load effect corresponding to the fatigue limit/highest load effect) are considered. Following the sizes of the connection the histogram ΔS may be situated differently regarding the ΔS_D value : Figure 3 corresponds to one ratio.

Case 1 : Miner's calculation and Eurocode 3 main S-N curve.

The damage calculation considers the whole ΔS histogram and the S-N curve with 2 slopes of the Eurocode 3 (curve 3). The fatigue damage is expressed by an equation similar to equation (3) :



$$D_{EC} = \sum_{\Delta S_i = \Delta S_L}^{\Delta S_D} \left(\frac{n_i \cdot \Delta S_i^5}{5 \cdot 10^6 \cdot \Delta S_D^5} \right) + \sum_{\Delta S_i = \Delta S_D}^{\infty} \left(\frac{n_i \cdot \Delta S_i^3}{5 \cdot 10^6 \cdot \Delta S_D^3} \right) \quad (15)$$

Case 2 : Proposed equivalent load effect - Eurocode 3 - main S-N curve.
The equivalent load effect ΔS_{e3} is the gravity centre of the fatigue damage distribution calculated with an unlimited S-N curve with $m = 3$ (curve 1)

$$\Delta S_{e3} = \frac{\sum_{\Delta S_i=0}^{\infty} n_i \cdot \Delta S_i^4}{\sum_{\Delta S_i=0}^{\infty} n_i \cdot \Delta S_i^3} \quad \text{then} \quad n_{e3} = \frac{\sum_{\Delta S_i=0}^{\infty} n_i \cdot \Delta S_i^3}{\Delta S_e^3} \quad (14)$$

The damage is calculated using the S-N curve with 2 slopes proposed by Eurocode 3 (curve 2) :

$$D_{e2} = \frac{n_{e3} \cdot \Delta S_{e3}^3}{5 \cdot 10^6 \cdot \Delta S_D^3} \quad \text{if} \quad \Delta S_{e3} > \Delta S_D$$

$$D_{e2} = \frac{n_{e3} \cdot \Delta S_{e3}^5}{5 \cdot 10^6 \cdot \Delta S_D^5} \quad \text{if} \quad \Delta S_D > \Delta S_{e3} > \Delta S_L$$

$$D_{e2} = 0 \quad \text{if} \quad \Delta S_L > \Delta S_{e3} \quad (16)$$

Case 3 : Proposed equivalent load effect - S-N curve with one slope ($m = 3$)
The equivalent load effect is the same as for case 2 : ΔS_{e3}
The damage is calculated by using the S-N curve unlimited with $m = 3$ (curve 1).

$$D_{e3} = \frac{n_{e3} \cdot \Delta S_{e3}^3}{5 \cdot 10^6 \cdot \Delta S_D^3} \quad \text{in all cases ;} \quad (17)$$

Case 4 : Modified proposed equivalent load effect - Eurocode 3 main S-N curve.
The equivalent load effect ΔS_{e5} is the center of gravity of the fatigue damage distribution calculated with an unlimited S-N curve with $m = 5$ (curve 2)

$$\Delta S_{e5} = \frac{\sum_{\Delta S_i=0}^{\infty} n_i \cdot \Delta S_i^6}{\sum_{\Delta S_i=0}^{\infty} n_i \cdot \Delta S_i^5} \quad \text{then} \quad n_{e5} = \frac{\sum_{\Delta S_i=0}^{\infty} n_i \cdot \Delta S_i^5}{\Delta S_e^5} \quad (18)$$

The damage is calculated using the S-N curve with 2 slopes proposed by Eurocode 3 (curve 3). The equations are similar to equation (16), where ΔS_{e3} is replaced by ΔS_{e5} and n_{e3} by n_{e5} .

Case 5 : Modified proposed equivalent load effect-S-N curve with one slope ($m = 3$).

The equivalent load effect is the same as for case 4 : ΔS_{e5} .

The damage is calculated using the S-N curve unlimited with $m = 3$.

$$D_{e5} = \frac{n_{e5} \cdot \Delta S_{e5}^3}{5 \cdot 10^6 \cdot \Delta S_D^3} \quad (19)$$

Case 6 : Usual equivalent load effect - Eurocode 3 - main S-N curve.

The equivalent number of cycles n_{en} is the total number of cycles :

$n_{en} = \sum n_i$ and the S-N curve is unlimited with $m = 3$:

The equation is similar to equation (10)

$$n_{en} = \sum n_i \quad \text{then} \quad \Delta S_{en} = \sqrt[3]{\frac{\sum_{\Delta S_i=0}^{\infty} n_i \cdot \Delta S_i^3}{\sum n_i}} \quad (20)$$

The damage calculation is made using the S-N curve with two slopes proposed by Eurocode 3. The equation are similar to equation (16), considering ΔS_{en} instead of ΔS_{e3} and n_{en} instead of n_{e3} .

Case 11 : Miner's calculation and Eurocode 3, 2d S-N curve.

The damage calculation considers the whole ΔS histogram and an S-N curve with $m = 5$ for $\Delta S > \Delta S_L$ (curve 4). The equation is similar to equation 6 :

$$D'_{EC} = \sum_{\Delta S_i = \Delta S_L}^{\infty} \frac{n_i \cdot \Delta S_i^5}{5 \cdot 10^6 \cdot \Delta S_D^5} \quad (21)$$

3.3. Conclusions.

The accuracy of equivalent load effects as proposed (case 2) is shown on Figures 4 to 6.

Fig. 4 shows that for the histogram obtained in Auxerre :

- 1°) The damage calculation given by the proposed equivalent values (case 2) is always a little higher than by the application of the Miner's rule (case 1) ; the difference reaches 19 % for $\Delta S_D = 0,60 \Delta S_F$.
- 2°) If the equivalent values are defined with $m = 5$ (case 5) instead of $m = 3$ (case 3), the damage is +/- 15 % lower. Comparing with the S-N curve of the Eurocode (case 1) the damage is +/- 15 % lower if $\Delta S_D < 0,60 \Delta S_F$ (case 5), and a little higher if $\Delta S_D > 0,60 \Delta S_F$ (case 4).
- 3°) The equivalent value defined for $n_{en} = \sum n_i$ (case 6) underestimates very much the damage for $\Delta S_D > 0,5 \Delta S_F$, and gives now damage for $\Delta S_D > 0,8 \Delta S_F$; this definition must be rejected.



Figure 4 : Fatigue calculation

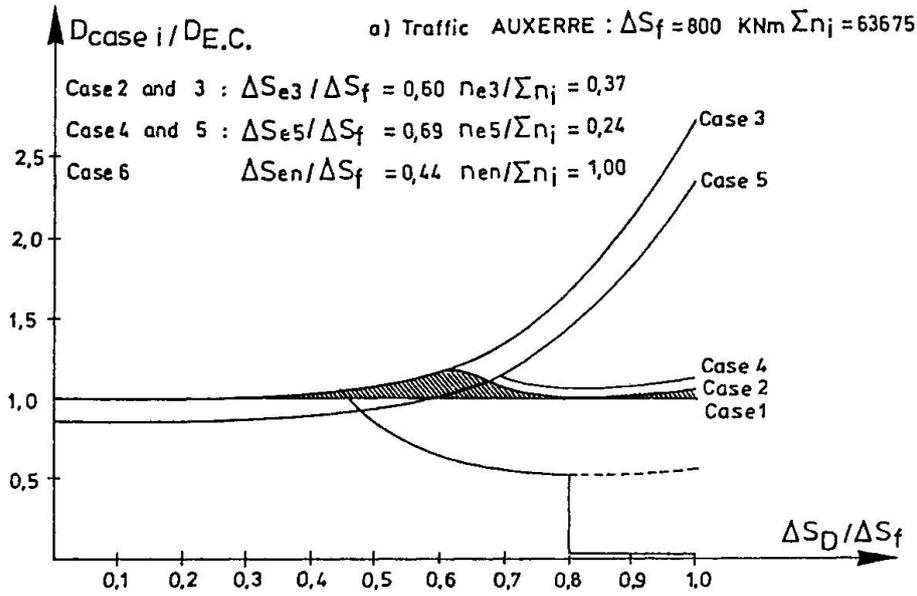


Figure 5 : Fatigue calculation

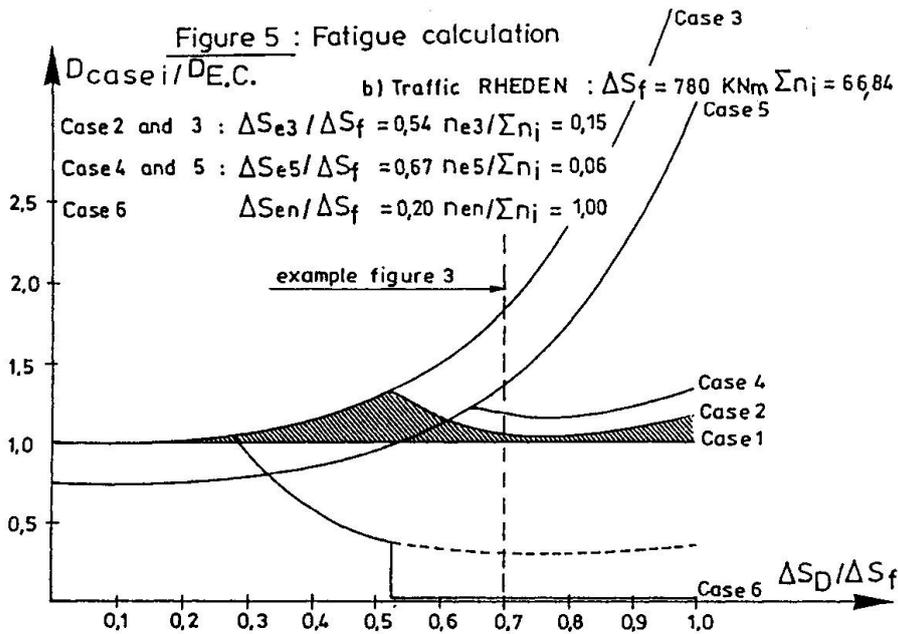
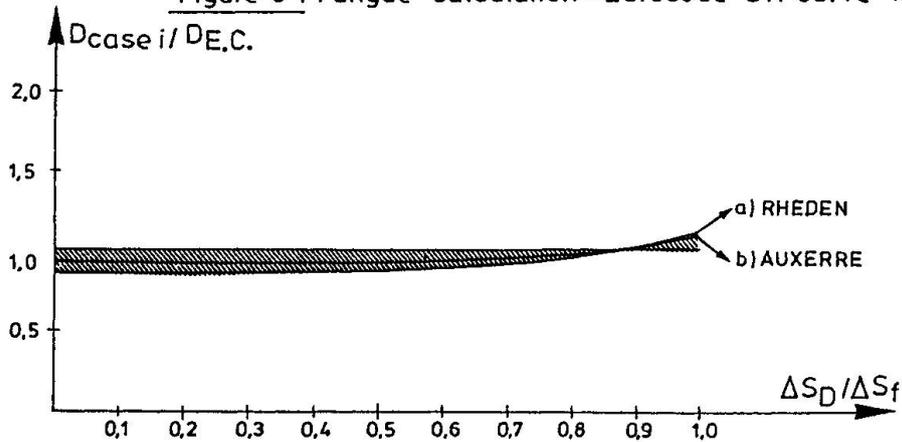


Figure 6 : Fatigue calculation - Eurocode SN curve with 1 slope (m=5)



The histogram obtained in Rheden, where exists a higher number of low values, gives larger differences. In this case, the low values represent a high percentage of the total number of cycles but a little percentage of the fatigue damage (Figure 2). The figure 5 shows :

1. The difference between the damage calculation given by the proposed equivalent values (case 2) and by the application of the Miner's rule (case 1) reaches 30 % for $\Delta S_D = 0,54 \Delta S_f$.
2. The difference between $m = 5$ (case 5) and $m = 3$ (case 3) is also higher : 30 %.
3. The value $\Delta S_{en}/\Delta S_f$ is very sensitive to the number of low cycles. In this case $\Delta S_{en}/\Delta S_f = 0,20$ and the damage calculated by this method (case 6) equal zero for $\Delta S_D > 0,52 \Delta S_f$.

In an other hand, if we treat histograms with high number of high values, the differences between case 1 and case 2 decrease. That has been shown in previous works [9].

If the S-N curve to consider in the Eurocode has one slope ($m = 5$ for $\Delta\sigma > \Delta\sigma_L$), the difference between the proposed equivalent value (case 2), and a damage calculation, is always lower than 12 % (Figure 6).

After having considered different shapes of histograms, and different positions of these histograms regarding the S-N curves (ratio $\Delta S_D / \Delta S_f$ or $\Delta\sigma_D / \Delta\sigma_f$), it appears that a load effect defined independently from the bridge details is able to give a damage very close to a complete Miner calculation. The simple calculation gives a higher damage, but the difference is generally lower than 10 %. Only if the stress-range histogram has a very high number of low values (shape b, on Figure 2), the difference reaches 30 % for $\Delta\sigma_D/\Delta\sigma_f$ near of 0,54. On the other hand, as a difference of 30 % in damage means a difference of 9 % in the level of stresses, we may conclude that this method is applicable for the calibration of a simple load model, independently of the bridge behaviour to fatigue.

4. APPLICATION

As the details that are the most sensitive to fatigue are the details influenced by local effects, it is necessary to analyse in a first step effects produced by one vehicle, or one axle.

The equivalent values obtained by the analysis of 20 European traffics are given in the table [5].

	ΔQ_e (kN)	n_e/n_L
One axle	80 - 130	0,80 - 2,40
Tandem axle	150 - 250	0,12 - 0,30
Tandem axle	210 - 260	0,06 - 0,27
Vehicule	330 - 440	0,20 - 0,66

n_L : number of lorries.



The worst case may be covered by a four axles vehicle of $4 \times 120 = 480$ kN ; with spacings of 1,2 - 4 to 6 and 1,2 m. To check this very simple load model, a set of influence lines representative of real influence lines are simulated with the Auxerre traffic.

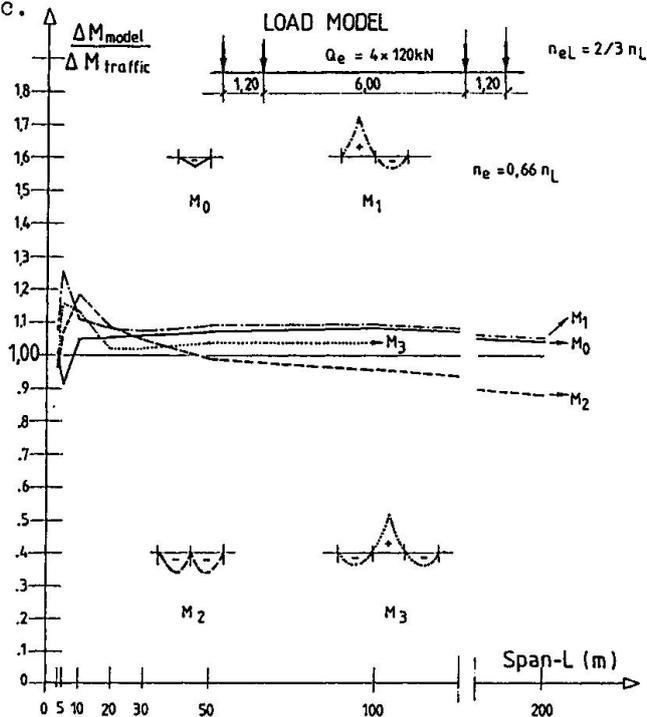


Figure 7

Figure 7 shows that the equivalent load effects obtained by the simulation are very close to the load effect produced by the simple vehicle. The equivalent load effects is calculated for one equivalent number of cycles obtained by 66 % of the number of vehicles. For short spans ($L < 10$ m), it seems necessary to examine if a set of vehicle with different geometry is not able to give a better accuracy.

More informations are given in the report for the Eurocode Commission (in preparation).

Comparing these results with previous work [10][11][12][13], it appears that the load of the vehicle of the load model is now higher, because the Auxerre traffic is more aggressive; but it is not necessary to consider a complementary distributed load acting simultaneously with the vehicle.

5. CONCLUSION

Figures 4 - 5 and 6 show clearly that the calibration based only on influence lines of load effects, excluding stresses, gives a very good accuracy : the difference reaches exceptionally 30 % in life time, that means 9 % in stress-ranges level. This difference is small regarding the differences between the measured traffics (see table). The definition of a load model in fatigue must consider different types of traffics following the number of lorries expected during a life time ; a difference in the load is perhaps not necessary. These problems are treated now by the Eurocode Commission and a definitive proposition will be made during this year.

The final aim is to have in a code two solutions for a fatigue verification of road bridge : a very simple load model that gives results on the safe side, and a sophisticated load model that has recourse to a set of vehicles and an computer calculation.



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