Fatigue reliability updating based on inspection and monitoring results

Fatigue Reliability Updating Based on Inspection and Monitoring Results

Définition de la fiabilité à la fatigue basée sur les résultats d'inspection et de surveillance

Neubeurteilung der Ermüdungssicherheit aufgrund von Inspektions- und Überwachungsresultaten

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SUMMARY

Probabilistic models for fatigue crack growth consider the uncertainty in loading, material properties initial flaw size, and model uncertainty in calculation of stress intensity factors. The reliability against a defined failure event can be computed. As in-service inspection or monitoring results become available, reliability can be updated. The analysis is particularly useful for maintenance planning.

RÉSUMÉ

Les modèles probabilistes de la propagation de fissure de fatigue tiennent compte de l'incertitude des charges, des propriétés du matériau, de la taille des défauts initiaux et des incertitudes dans les modèles du calcul des facteurs d'intensité de contraintes. La fiabilité peut être évaluée pour un certain mode de rupture défini. Si des résultats lors des inspections de service ou des surveillances sont disponibles, la fiabilité peut être mise à jour. Cette analyse est particulièrement utile pour un programme de maintenance.

ZUSAMMENFASSUNG

Wahrscheinlichkeitstheoretische Modelle für das Risswachstum berücksichtigen Unsicherheiten der Lastmodelle und der Materialeigenschaften sowie Ungewissheiten über die anfängliche Rissgrösse und die Berechnung der Spannungsintensitätsfaktoren. Die Ermüdungssicherheit kann in Abhängigkeit einer wohldefinierten Versagensart berechnet werden. Falls Resultate aus der Überwachung einer Brücke im Betriebszustand vorhanden sind, kann ihre Ermüdungssicherheit neu beurteilt werden. Die Untersuchung ist besonders im Hinblick auf die Planung der Unterhaltung der Brücke von Nutzen.

1. INTRODUCTION

About one third of all steel bridges in the US are fifty years old or more and many others are nearing that age [1]. Also in Europe are many steel bridges past or near their design life. As the number of bridges entering old age grows the need for inspection and maintenance becomes of increasing importance. At the same time the resources which can be allocated to the proper maintenance of bridges is shrinking. There is thus a great demand for methods which helps in allocating these resources in a manner which gives the highest overall utility. The paper reports on an application of probabilistic methods for such maintenance planning. The methods account explicitly for uncertainties in material properties, loading, initial flaw sizes, inspection methods and analysis models. Successful applications of the methodology are emerging in the offshore industry.

2. FATIGUE ANALYSIS

There are two common approaches to fatigue analysis of steel structures: the *S–N* analysis mostly applied in design, and the fracture mechanics analysis mostly applied for structures in service. The *S–N* approach relates the life time to the distribution of stress ranges at a fatigue critical point through the *S–N* curve and the use of Miner's rule. This method has been used extensively in bridge fatigue studies, see e.g. [2,3]. Because *S–N* analyses do not relate to a measurable indicator of damage, it is difficult to incorporate inspection observations into the fatigue analysis. On the other hand, monitoring information about the loads and load effects can be incorporated. The use of a linear elastic fracture mechanics (LEFM) model for fatigue crack growth allows information on the presence or size of observed cracks to be incorporated into descriptions of both failure and inspection events. The LEFM approach to fatigue analysis of steel bridges has been applied by a number of researchers, see e.g. [4,5].

The LEFM approach to fatigue crack growth relates the range in the stress intensity factor ΔK at the crack tip to the rate of crack growth da/dN by the equation suggested by Paris and Erdogan, [6]

$$\frac{da}{dN} = C(\Delta K)^m, \quad \Delta K > \Delta K_{thr}, \quad a(N{=}0){=}a_0 \tag{1}$$

where C and m can be considered as material constants, ΔK_{thr} is a threshold value (in the following $\Delta K_{thr}=0$), N is the number of stress cycles, and a_0 is the initial crack size. A crack initiation period is easily included in the analysis by changing the initial condition $a(0)=0$ to $a(N_0)=0$. A separate stochastic model for N_0 can then be formulated. Alternatively a_0 can be considered as an equivalent initial crack size as is commonly done for analysis of aircraft structures. To achieve a good correspondence with experimentally derived *S–N* curves an initial crack depth of 0.1-0.2 mm must generally be assumed, [7].

A one-dimensional description of crack size is employed in Eq.(1), with a being the length of a through-crack or the depth of a surface crack. ΔK is expressed in LEFM as

$$\Delta K = Y(a)\sqrt{\pi a}\,S \tag{2}$$

where $Y(a)$ is the geometry function depending on the overall geometry of the joint including the presence and geometry of the weld, and S is the range of a far-field reference stress. For a surface crack the stress intensity factor is often written in the form suggested in [8]

$$K = (\sigma_t + H\sigma_b)\sqrt{\pi\frac{a}{Q}}\,F\left(\frac{a}{t},\frac{a}{c},\frac{c}{b},\theta\right) \tag{3}$$

where σ_t and σ_b are remote tension and bending stresses, t is the wall thickness, c is the half crack length, b is the half-width of the cracked plate, θ is a parametric angle for the ellipse, H is a function depending on a/t, a/c and θ, and Q is the shape factor for an elliptical crack. A more direct treatment of crack growth with a two dimensional description of the crack is introduced through two coupled differential equations of the form as Eq.(1) describing the growth at the deepest point and at the surface, respectively. The shape of the crack is assumed to be semi-elliptical initially and to remain such. In the case of bridge girder details a number of stress intensity factors have been compiled in [9] for relevant AASHTO fatigue sensitive details.

For a stiffened bridge girder the growth of a crack can take place in four stages:

- from an initial defect to the penetration of the girder web
- along the stiffener-to-web weld to the tension flange
- through the flange until it is penetrated
- as a through-crack towards the ends of the flange until the total remaining intact cross section fails due to yielding.

In the first stage the crack is described as a surface crack and a two-dimensional crack description is employed. In the second stage with a through-crack, a one-dimensional description is used, in the third stage a two-dimensional description is again used, while a one-dimensional description is used in the fourth stage. The main part of the life time is spent in stage 1, but the other stages are of importance in connection with the possibility of crack detection before failure. In an experimental study, [7], on large scale plate girders, stage 1 amounted to 92% of the life time.

By combining Eqs.(1) and (2), the number of cycles N_1 to reach a crack size a in the first stage is

$$N_1 = \frac{1}{C\overline{S^m}} \int_{a_0}^{a} \frac{dx}{Y(x)^m (\sqrt{\pi x})^m} \tag{5}$$

where $\overline{S^m}$ is the average value of the mth power of the stress ranges. The number of cycles before the crack has extended through the thickness of the web is determined with a equal to the thickness. The number of cycles N_2–N_4 in stages 2-4 are determined similarly, when a suitable initial condition is applied for each stage. The failure criterion becomes

$$M = N_1 + N_2 + N_3 + N_4 - \nu T \leq 0 \tag{6}$$

where ν is the frequency of load cycles, and T is the considered time period. M is called the safety margin.

3. RELIABILITY ANALYSIS

Many of the parameters entering the analysis can not be assessed with certainty, and in fact large uncertainties are present for some parameters. It is of importance to account for these uncertainties, and probabilistic methods provide tools for this. Each parameter is described as a random variable of a certain distribution type and with a mean value and coefficient of variation. The probability that the failure criterion in Eq.(6) is exceeded can then be computed, e.g by first- or second-order reliability methods (FORM and SORM), [10]. These methods are particularly useful as they are directly based on an available deterministic description of crack growth, they are fast, and besides a reliability measure they as a by-product provide importance factors for each source of uncertainty and sensitivity factors for each input parameter. It is thus directly clear which of the uncertainty sources are of highest importance, and without a re-analysis it is possible to give the change in reliability from a change in a deterministic design parameter or a statistical input parameter. FORM and SORM methods are easily extended to compute the probability and sensitivity factors for a parallel system $\{M_1 \leq 0 \cap M_2 \leq 0 \cap \cdots \cap M_k \leq 0\}$. The reliability is generally expressed in terms of the reliability index β, which is defined as

$$\beta = -\Phi^{-1}(P_F) = -\Phi^{-1}(P(M \leq 0)) \tag{7}$$

where the failure probability P_F is the probability for the event defined in Eq.(6), and $\Phi(\,)$ is the standard normal distribution function.

4. INSPECTION RESULTS, EVENT MARGINS AND RELIABILITY UPDATING

The influence of in-service inspection results is introduced in the reliability analysis. Let an inspection be performed at time T_1 and a crack size A_1 be measured.

$$a(T_1) = A_1 \tag{8}$$

A_1 is generally random due to measurement error and/or due to uncertainties in the interpretation of a measured signal as a crack length. Measurements of the type in Eq.(8) can be envisaged for several times. For the jth measurement an event margin H_j can be defined similarly to Eq.(5) as, [11]

$$H_j = \int_{a_0}^{A_j} \frac{da}{Y(a)^m (\sqrt{\pi a})^m} - C\,\overline{S^m}\,\nu\, T_j = 0 \tag{9}$$

This event margin is zero due to Eq.(8).

A second type of inspection result is that no crack is detected. For an inspection at a time T_i this implies

$$a(T_i) \le A_d \tag{10}$$

expressing that the crack size is smaller than the smallest detectable crack size A_d. A_d is generally random since a detectable crack is only detected with a certain probability depending on the crack size and on the inspection method. The distribution for A_d is provided through the probability of detection curve (pod curve) for which experimental results exist for various inspection methods. Figure 1 shows experimental data and a pod curve for magnetic particle inspection (MPI).

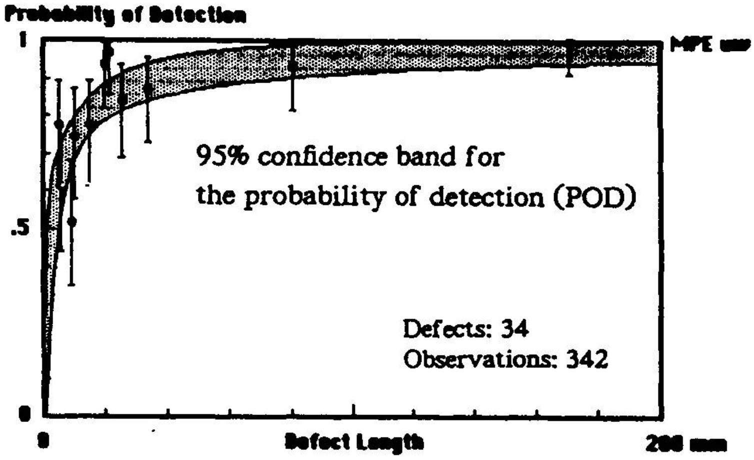

Fig. 1 Inspection reliability for MPI

Information of the type in Eq.(10) can also be envisaged for several times. For the ith measurement of the form in Eq.(10), an event margin H_i can be defined as, [11]

$$H_i = \int_{a_0}^{A_d} \frac{da}{Y(a)^m (\sqrt{\pi a})^m} - C\,\overline{S^m}\,\nu\, T_i \tag{11}$$

This event margin is negative when a crack is detected and positive when no crack is detected.

With one performed inspection where no crack is detected, the updated failure probability is

$$P(M{\le}0 \,|\, H_i{\ge}0) = \frac{P(M{\le}0 \cap H_i{\ge}0)}{P(H_i{\ge}0)} \tag{12}$$

Evaluation of the reliability of a parallel system (numerator) and a component (denominator) are thus required, and a FORM or SORM analysis can be directly applied.

With one inspection result of the type in Eq.(8) the updated failure probability is

$$P(M{\le}0 \,|\, H_j{=}0) = \frac{\frac{\partial}{\partial x} P(M{\le}0 \cap H_j{\le}x)}{\frac{\partial}{\partial x} P(H_j{\le}x)} \tag{13}$$

where the derivatives are computed at x=0. An evaluation of the sensitivity factor for a parallel system (numerator) and a component (denominator) are thus required, and a FORM or SORM analysis can be directly applied. The analysis is easily generalized to simultaneous consideration of several inspection results.

The interest is now on updating after repair and it is assumed that a repair takes place at time T_{rep} when a crack size a_{rep} is observed. An event margin H_{rep} is defined as

$$H_{rep} = \int_{a_0}^{a_{rep}} \frac{da}{Y(a)^m (\sqrt{\pi a})^m} - C\, \overline{S^m} \nu T_{rep} = 0 \tag{14}$$

The crack size present after repair and a possible inspection is a random variable a_{new} and the material properties after repair are m and C_{new}. The safety margin after repair is M_{new}

$$M_{new} = \int_{a_{new}}^{a_c} \frac{da}{Y(a)^m (\sqrt{\pi a})^m} - C_{new} \overline{S^m} \nu (T - T_{rep}) \tag{15}$$

where a_c is the critical size. The updated failure probability is

$$P(M_{new} \leq 0 \,|\, H_{rep} = 0) = \frac{\frac{\partial}{\partial x} P(M_{new} \leq 0 \cap H_{rep} \leq x)}{\frac{\partial}{\partial x} P(H_{rep} \leq x)} \tag{16}$$

where the derivatives are computed at x=0.

4.1 Example 1 - Cover plate

A 32 mm welded cover plate terminus (AASHTO category E, [12]) on a plate girder has been analysed. This cover plate is similar to the cover plates which were observed to develop cracks after only 12 years of service on the Yellow Mill Pond Bridge in Connecticut [9]. Table 1 shows the distributions for each of the random variables used in the reliability analysis and the subsequent updating. Failure was defined as the development of a through crack longer than 220 mm. Inspection times were chosen as the times when the reliability index fell below the value 2.0, i.e. when the failure probability in the period from the latest inspection became larger than 2.3%.

Quantity	Distribution
Effective stress $(\overline{S^m})^{1/m}$	Normal, μ=9.6 MPa, COV=20%
Material constant C	Lognormal, μ=$1.3\cdot 10^{-8}$ MPa $\sqrt{m}$, COV=7%
Material constant m	Normal, μ=3.2, COV=2%
Correlation coefficient ln$C$ and m	ρ=–0.97
Initial crack size a_0	Lognormal, μ=0.05 mm, COV=11%
Number of trucks per day	Normal, μ=5700, COV=10%
	Fixed values
Final crack size	110 mm
Crack aspect ratio	0.25
Thickness of cover plate	32 mm
Thickness of flange	32 mm
Width of flange	420 mm
Thickness of web	19.3 mm
Weld size	12.7 mm

Table 1: Yellow Mill Pond Bridge Data

Figure 2 shows the reliability index as a function of service time with no inspection (curve marked limit state). The figure also shows curves obtained by updating following inspections after 24, 33, and 40 years of service. It has been assumed that none of these inspections reveal a crack. The effect of updating is

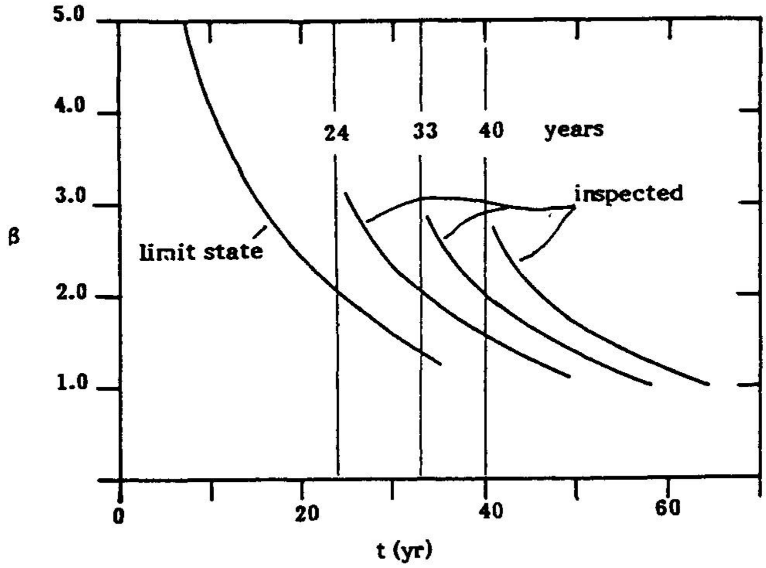

Fig. 2 Updated reliabilities for the Yellow Mill Pond Bridge.

important, but not as large as found in previous studies for offshore structures, see e.g. [11]. The main reason for this is the different behavior of the geometry function for a cover plate and a tubular joint with a high degree of local bending stresses.

The reliability of the inspection method has been expressed by an exponential pod curve

$$p(c) = F_{C_d}(c) = 1 - \exp(-c/8.9), \quad c \text{ in } mm \tag{17}$$

Such a quality is probably too optimistic for bridge inspection. With this inspection quality the probability of detecting a crack at the first inspection after 24 years is 30%. Because the crack growth rate is much higher for larger cracks, a decrease in the inspection quality (i.e. an increase in the smallest detectable crack size) causes the time between inspections to decrease and the necessary amount of inspection to increase.

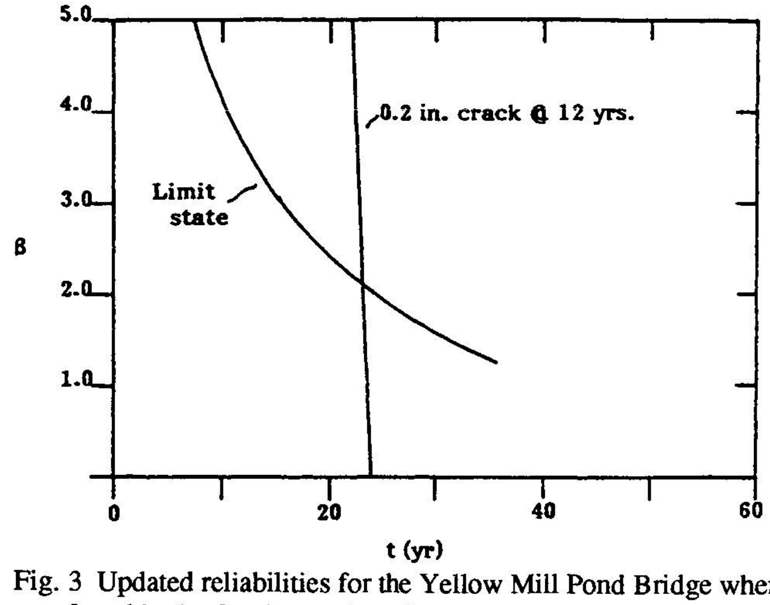

Fig. 3 Updated reliabilities for the Yellow Mill Pond Bridge when a 5 mm crack was found in the first inspection after 12 years of service.

Figure 3 shows results obtained when it is assumed that a crack of 5 mm is detected in the first inspection after 12 years of service. The reliability immediately after the inspection is elevated as the crack still has some distance to grow before failure. The drop in reliability with time is, however, very fast and a repair should be performed within a short period of time.

4.2 Example 2 - Rolled Beam

A W30×360 rolled section has been analysed at several levels of applied stress range and for a single inspection where no crack was detected. The failure criterion was the development of an edge crack of 64 mm in the flange. As in the case of the cover plate the inspection time was selected at the point where the reliability falls below the level β=2.0. Table 2 gives the applied input data, and Fig. 4 shows the reliability index for both the inspected and non-inspected detail. The inspection at 17 years lifts the reliability immediately after the inspection. The reliability level, however, soon approaches the level for the non-inspected situation and a second inspection is necessary after a few years.

Quantity	Distribution
Effective stress $(\overline{S^m})^{1/m}$	Normal, μ=68.9 MPa, COV=20%
Material constant C	Lognormal, μ=1.3·10^{-8} MPa $\sqrt{m}$, COV=7%
Material constant m	Normal, μ=3.2, COV=2%
Correlation coefficient ln$C$ and m	ρ=−0.97
Initial crack size a_0	Lognormal, μ=0.03 mm, COV=48%
Number of trucks per day	Normal, μ=500, COV=10%
	Fixed values
Final crack size	64 mm
Crack aspect ratio	0.67
Thickness of cover plate	43 mm
Thickness of flange	32 mm
Width of flange	423 mm
Thickness of web	24 mm

Table 2: Rolled Beam W36×360 Data

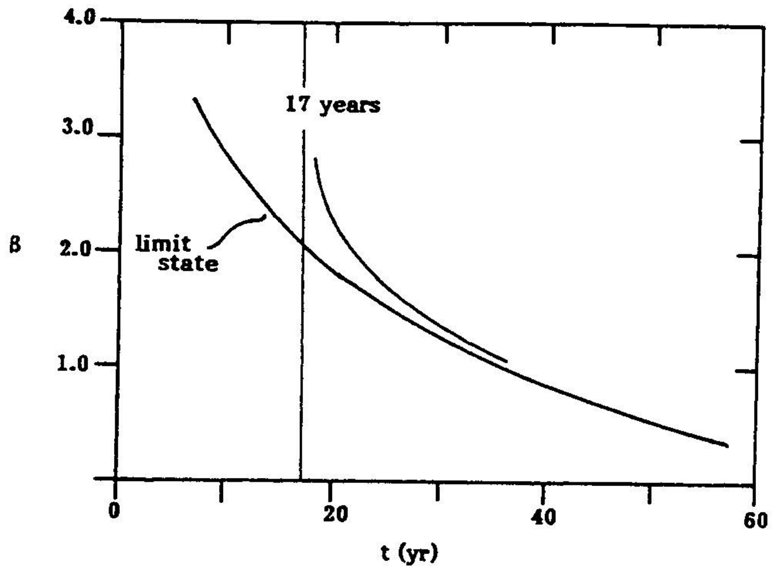

Fig. 4 Updated reliabilities for a W30×360 rolled section.

5. MAINTENANCE OPTIMIZATION

The examples have operated with a threshold value of β=2 from one inspection to the next. Such a limiting value could be specified in codes. Alternatively one can perform a formal cost optimization, minimizing the total cost of design and inspection and the expected cost of repair and failure. Such an optimization is illustrated in [13] for four different repair strategies. The optimization results in an optimal value for element thickness, inspection times and qualities and a limiting crack size for choice of repair method.

6. CONCLUSIONS

The method of reliability updating described here can be used to estimate reliabilities for fatigue sensitive details conditioned on the results of inspections which result in either no crack detection or detection and possibly also repair of a crack. Beacuse the LEFM based fatigue analysis relates physical quantities such as crack size and stress range, the method of updating estimated reliabilities using LEFM is straight forward.

The examples showed the limited effectiveness of evenly fairly high quality inspections of details for which a short fatigue life had been observed. Because the inspection quality used for bridges is such that only rather large cracks are detected, the effect on the estimated reliability of an inspection which detects no damage is limited to a fairly short time after the inspection. The gain in reliability is shorter than experienced for analysis of offshore jacket structures.

The example of the inspection resulting in a crack detection gave updated reliabilities which quickly fell off following the discovery of the crack. In such a case, the rate of decrease of the estimated reliability can be used to determine the available time for repair in order to maintain an acceptable level of safety.

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REFERENCES

1. GALAMBOS, C.F., Bridge Design, Maintenance and Management. Public Roads, Vol. 50, No. 4, 1987.
2. MOSES, F., Probabilistic Load Modelling for Bridge Fatigue Studies. Proceedings, IABSE Colloquium on Fatigue of Steel and Concrete Structures, Lausanne, Switzerland, 1982.
3. NYMAN, W. and MOSES, F., Calibration of Bridge Fatigue Design Model. Journal of Structural Engineering, ASCE, Vol. 111, No. 6, 1985.
4. FISHER, J., Fatigue and Fracture in Steel Bridges - Case Studies. John Wiley & Sons Inc., New York, 1984.
5. YAZDANI, N. and ALBRECHT, P., Risk Analysis of Fatigue Failure of Highway Steel Bridges, Journal of Structural Engineering, ASCE, Vol. 113, No.3, 1987.
6. PARIS, P. and ERDOGAN, F., A Critical Analysis of Crack Propagation Laws. Journal of Basic Engineering, ASME, Vol.85, 1963.
7. WESSEL, H.-J. and MOAN, T., Fracture Mechanics Analysis of Fatigue in Plate Girders. Proceedings, 13th IABSE Congress, Helsinki, 1988.
8. NEWMAN, J.C. and RAJU, I.S., An Empirical Stress Intensity Factor Equation for the Surface Crack. Engineering Fracture Mechanics, Vol. 15, No. 1-2, pp. 185-192, 1981.
9. ALBRECHT, P.and YAZDANI, N., Risk Analysis of Extending the Service Life of Steel Bridges. Maryland Dept. of Transp. Report No. FHWA/MD-84/01, 1986.
10. MADSEN, H.O., KRENK, S. and LIND, N.C., Methods of Structural Safety. Prentice-Hall Inc., Englewood Cliffs, NJ, 1986.
11. MADSEN, H.O., TALLIN, A.G., SKJONG, R. and KIRKEMO, F., Probabilistic Fatigue Crack Growth Analysis of Offshore Structures with Reliability Updating. Proceedings, Marine Structural Reliability Symposium, SNAME, Arlington, VA, 1987.
12. AASHTO, Standard Specifications for Highway Bridges. 12 th Ed., American Association of State Highway and Transportation Officials, Washington, D.C., 1977.
13. MADSEN, H.O. and SORENSEN, J.D., Probability-Based Optimization of Fatigue Design, Inspection and Maintenance. Proceedings, Integrity of Structures Symposium, Glasgow, 1990.