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## Numerical Simulation of Fatigue Crack Growth

Simulation numérique de la propagation de fissures de fatigue

Numerische Simulation des Ermüdungsrisswachstums

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### SUMMARY

This paper compares fatigue test data on full-scale structural-steel details under both constant and variable amplitude loading with results obtained through numerical simulation. Objectives include modelling fatigue crack growth in structural details commonly used in steel constructions, such as coverplates and web attachments. Random variability of parameters governing the fatigue crack propagation process is accommodated.

### RÉSUMÉ

Cet article compare les résultats d'essais de fatigue sur des détails de structures en acier de dimension réelle sollicités sous charge d'amplitude constante et variable, avec les résultats obtenus à l'aide d'une simulation numérique. L'un des objectifs consiste en la modélisation de la propagation de fissures de fatigue dans des détails couramment utilisés en construction métallique, tels que des semelles de renfort et des liaisons à l'âme. La variation aléatoire des paramètres déterminants pour le comportement des fissures de fatigue est également prise en compte.

### ZUSAMMENFASSUNG

Der vorliegende Beitrag vergleicht Resultate aus Ermüdungsversuchen an Bauteilen, die sowohl unter konstanter wie auch variabler Spannungsamplitude durchgeführt wurden, mit Resultaten aus entsprechenden numerischen Simulationen. Eines der angestrebten Ziele ist es, das Wachstum von Ermüdungsrissen, ausgehend von gebräuchlichen Stahlbau-Konstruktionsdetails, wie beispielsweise Lamellen oder auf Stege aufgeschweisste Laschen, modellieren zu können. Die Streuung der für den Rissverlauf massgebenden Parameter wird berücksichtigt.



## 1. INTRODUCTION

Although in reality fatigue cracking is generally caused by random variable amplitude loading, to date there are very few available results of long life ( $N > 10^7$  cycles), variable amplitude loading fatigue tests carried out on full scale structural elements [1]. This is due to the fact that full scale, random loading fatigue testing, in the long endurance range, requires long testing periods, with very high costs. While experimental research is absolutely necessary and fundamental, the time and money needed for this type of approach induces to take into consideration the possibility of studying numerical procedures that, calibrated on few test results, may be able to supply useful indications and improve our understanding of the problem under investigation. Though a series of numerical models of fatigue crack growth are already available in the literature [2-10], they deal with cases of ideal growth and are not aimed at the study of details typically adopted in steel constructions. Furthermore, for one way or another, none of the models to the author's knowledge, can be regarded as completely satisfactory for the simulation of fatigue crack growth in structural steel details. In fact, the existing numerical models can be subdivided into three main groups:

- 1) deterministic models, like [2], that allow the simulation of the detail behavior during all phases of crack growth (fig. 1.1);
- 2) probabilistic approaches, capable of interpreting the low frequency random aspects of crack growth, by assuming the coefficients in the crack propagation law as random variables (fig. 1.2);
- 3) probabilistic approaches, capable of interpreting, during the intermediate, linear, phase of crack growth even the high frequency aleatory components, by means of the superposition of the Paris's Law with a random noise (fig. 1.3).

Fig. 1.1

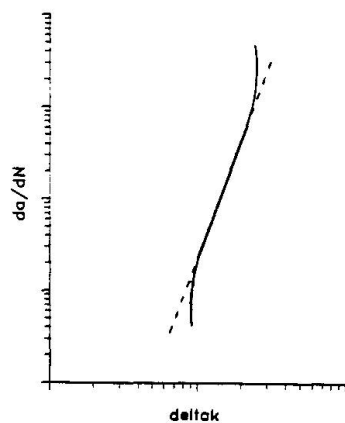


Fig. 1.2

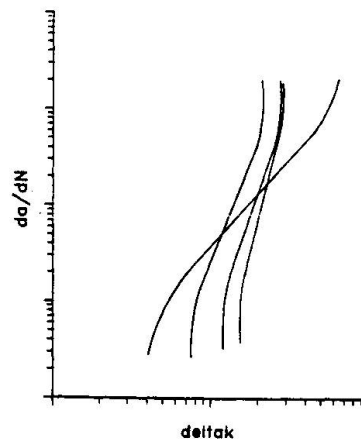


Fig. 1.3

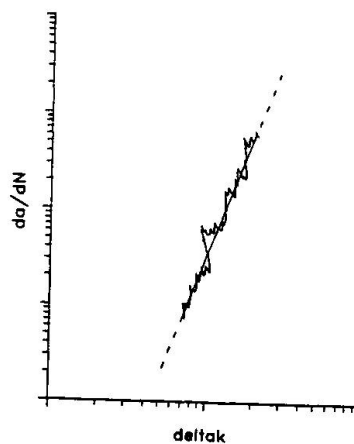
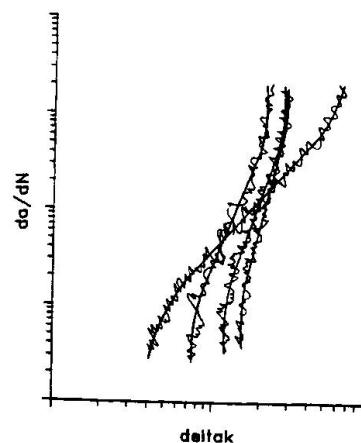


Fig. 1.4



Recently, Castiglioni and Rossi [11] proposed a numerical model that encompasses the characteristics of all the three types discussed above (fig. 1.4), i.e. capable of simulating both types of random non-homogeneity (at high and low frequency), even in the two extreme phases of crack growth, i.e. in proximity of the threshold value  $\Delta K_{th}$  and of the critical one  $\Delta K_c$  of the stress intensity factor range.

In fact, a propagation law is assumed of the type:

$$da/dN = f(\Delta K) Z(a) \tag{1}$$

in which  $Z(a)$  is a random function, as proposed in [9], and  $f(\Delta K)$  is the deterministic function proposed by Newman [12]:

$$f(\Delta K) = C (1 - R)^m \Delta K^n (\Delta K - \Delta K_{th})^p [(1 - R)K_c - \Delta K]^{-q} \tag{2}$$

where  $C$ ,  $m$ ,  $n$ ,  $p$  and  $q$  are material dependent parameters, considered to be random variables. It is immediately recognized that (2) as a whole includes the most commonly used propagation laws, as Paris's ( $m=p=q=0$ ), Forman's ( $m=p=q=1$ ) and Walker's ( $p=q=0, m=n[m_w-1]$ ).

The model presented in [11] takes into account:

- 1) various crack configurations and loading conditions (fig. 2)
- 2) retardation effects due to overloading according to Willenborg's model [13]
- 3) crack closure according to Newman's model [14]
- 4) stress concentrations due to geometric effects, and relative stress intensity factor's correction factors
- 5) stress corrosion

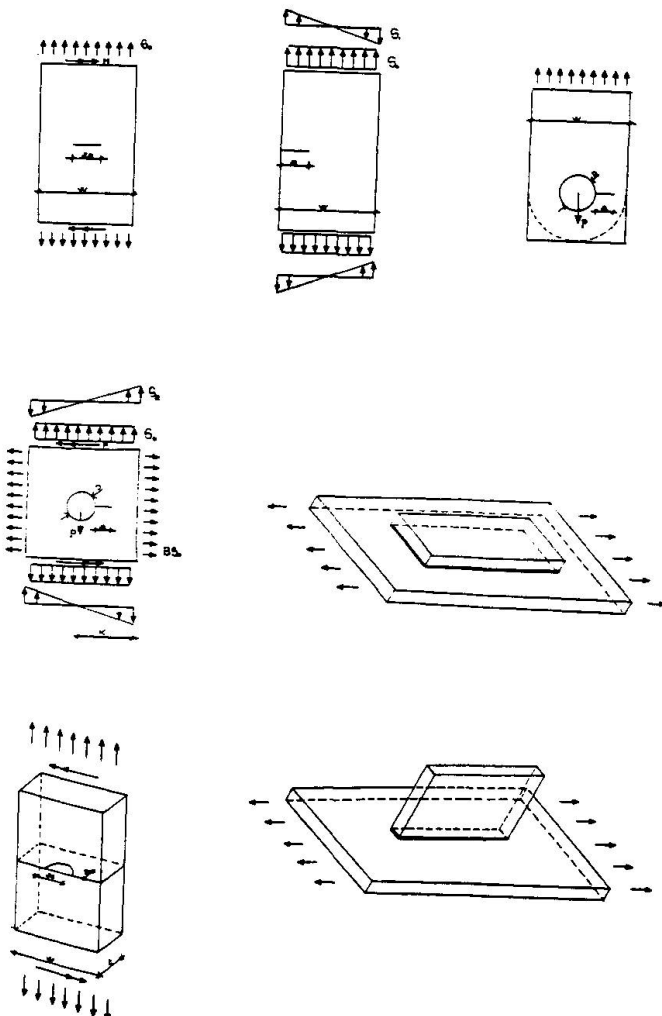


Fig. 2



In this paper, some results are presented obtained by means of model [11]. The behavior of typical structural steel details is simulated and the numerical results are compared with those of available experimental tests. In the case of fatigue under constant amplitude loading cycles, the results reported in the literature [15-17], relating to the weld toes at the ends of web attachments and of coverplates on beam flanges, are taken into consideration. Finally the influence of the loading history on the fatigue life of some structural details is investigated. To begin with, in order to determine the correspondence of the model with the physical reality, the case is considered of a plate with a through crack, for which the test results are widely presented in the literature. Attention is then focused on longitudinal web attachments and, in order to make a comparison, geometries similar to those adopted in the tests presented in [18] are considered.

## 2. ANALYSIS OF THE RESULTS

In order to correctly simulate fatigue tests on typical structural details, the model was previously calibrated [11] on the basis of available experimental results. In [11], at first the effect of the single parameters governing the model is investigated, with reference to the simple case of a plate with a through crack. It is concluded that the variability of the parameters of the propagation law (functions of the material), during crack growth, does not substantially influence the fatigue life of the structural detail. On the contrary, the superposition to the propagation law of a high frequency noise  $Z(a)$ , defined by a stationary stochastic process with a log-normal statistics has an influence on the fatigue life that is comparable with that of the initial defect size.

In this paper, the simulation of some experimental tests on full scale beams is presented. Furthermore, the influence of the loading history is investigated, with reference to the case of a plate with a through crack.

### 2.1 Constant Amplitude Loading

The test results reported in [15-17], concerning coverplated beams and web attachments are taken into consideration.

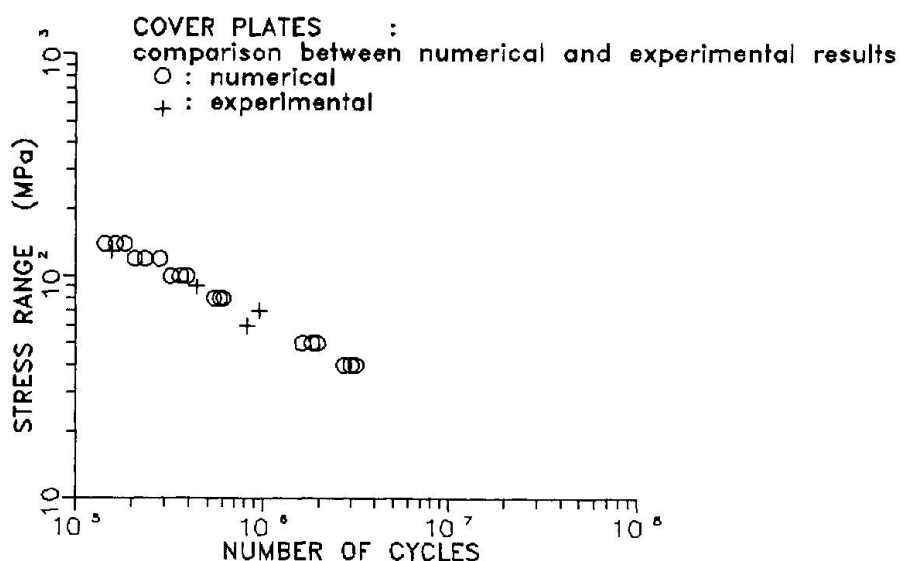


Fig. 3

Various numerical simulations were carried out at the same stress range level, considering different initial crack depths chosen from a sample having a normal distribution, 0.1 mm mean value and 40% standard deviation. The crack shape correction factor was computed, as a function of the crack size ( $a$ ), with reference to the relationships between crack depth ( $a$ ) and crack width ( $c$ ) proposed, for both coverplates and web attachments by Fisher [19].

The parameters of the crack propagation law were calibrated on the test data presented in [18] for a plate with a central through crack, while the material parameters were obtained from [16,17].

Fig. 3 shows a comparison between numerical and experimental results in the case of coverplates on beam flanges.

It can be noticed that the numerical results interpret fairly well the trend of the experimental data, at all the stress range levels considered.

Fig. 4 shows the comparison between experimental test data and numerical simulation results in the case of web attachments. Also in this case it can be noticed that the numerical simulation interprets fairly well the trend of the experimental data.

Examining fig. 4 it can be noticed that the scattering in the numerical results is greater for the lower stress ranges than for the higher ones. This is also in agreement with the experimental evidence.

It is important to notice that, from both figs. 3 and 4, independently on the stress range level, it is evident a greater scattering in the experimental

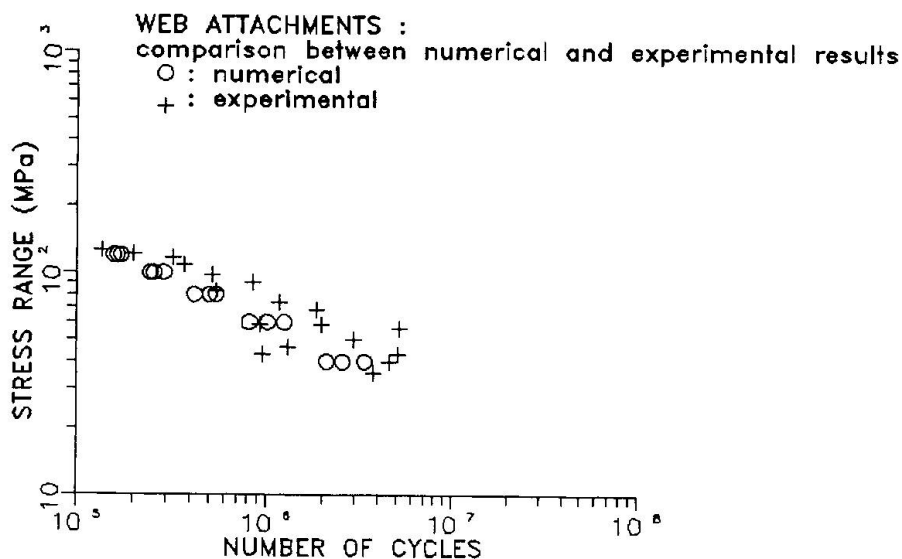


Fig. 4

data than in the numerical ones. This is probably due to two main reasons:

- the actual initial defect size is greater than that assumed in the numerical simulation. This fact might be easily corrected by generating a new sample of initial defects, with a larger mean value and standard deviation;
- the presence of residual stresses due to the welding process influences relevantly the fatigue life of actual welded components, as widely demonstrated and discussed in the literature [19,20]. Of course, the statistics of the residual stress distributions are characterized by large standard deviations from the mean value, depending on such factors as, for example, the plates' thickness and the welding process. In the numerical



model, at the present state, the presence of residual stresses in the joints is completely disregarded, thus reducing the possible causes of randomness, and the scattering of the results.

From examining both figures 3 and 4, however, it can be concluded that the numerical model is capable of predicting the fatigue life of web attachments and coverplated beams under constant amplitude loading in fairly good agreement with the experimental results.

## 2.2 Variable Amplitude Loading

### 2.2.1 Influence of the Loading History

In order to verify the model's capacity to interpret the loading spectrum's influence on fatigue life, a plate similar to the one shown in fig. 5 is first examined.

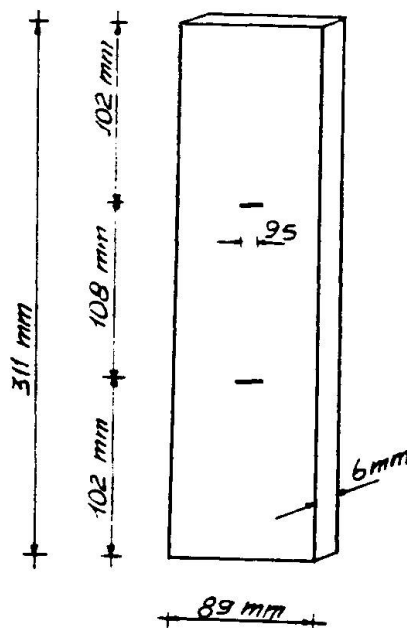


Fig. 5

A set of random loading histories, with a Rayleigh type distribution of the variable amplitude loading cycles are generated, with stress ranges  $S$  ranging from 5 MPa to 13.5 MPa.

The loading histories differ from one another either in the value of the stress ratio  $R$  between the minimum and maximum stress, or in the exceedance rate  $\gamma$ , i.e. the probability that the maximum cycle amplitude ( $S_{\max}$ ) exceeds a specific limit value ( $S_{\lim}$ ). Stress ratios  $R=0.5$  and  $0.7$ , and exceedance rates  $\gamma=0.1\%$ ,  $1\%$  and  $5\%$  were considered. Furthermore, for fixed  $R$  and  $\gamma$  values, three loading histories are randomly generated, each one different from the other only for the sequence of the stress ranges. Each loading history consists of a block of 5,000 cycles, repeatedly imposed on the simulated specimen until collapse situation is reached; in any case the simulation was

interrupted after a maximum of 50,000 repetitions (equivalent to  $250 \times 10^6$  cycles).

The results obtained are presented in fig. 6, where the simulated fatigue lives ( $N$ ) are plotted against the root mean cube (equivalent) stress range,

$$S_{\text{eq}} = (\sum_1 \alpha_1 S_1^3)^{1/3} \quad (3)$$

By examining fig. 6 it can be noticed how the model can interpret the effect of the stress ratio  $R$ , on the fatigue life. In fact, for each exceedance rate level, the numerical model predicted shorter endurance for those specimens subjected to stress histories associated with the higher  $R$  ratios ( $R=0.7$ ).

The effect of the exceedance rate on fatigue life is also evident, in fact, for a fixed  $R$  ratio value, decreasing  $\gamma$  results in longer endurance.

By examining fig. 6 it can also be noticed that fatigue life is strongly influenced not only by the stress ratio  $R$  and the exceedance rate  $\gamma$ , but also by the loading sequence. In fact, for  $R=0.5$  and  $\gamma=0.1\%$ , certain cases did not reach critical conditions after  $250 \times 10^6$  cycles, and are plotted in fig. 6 as run-outs.

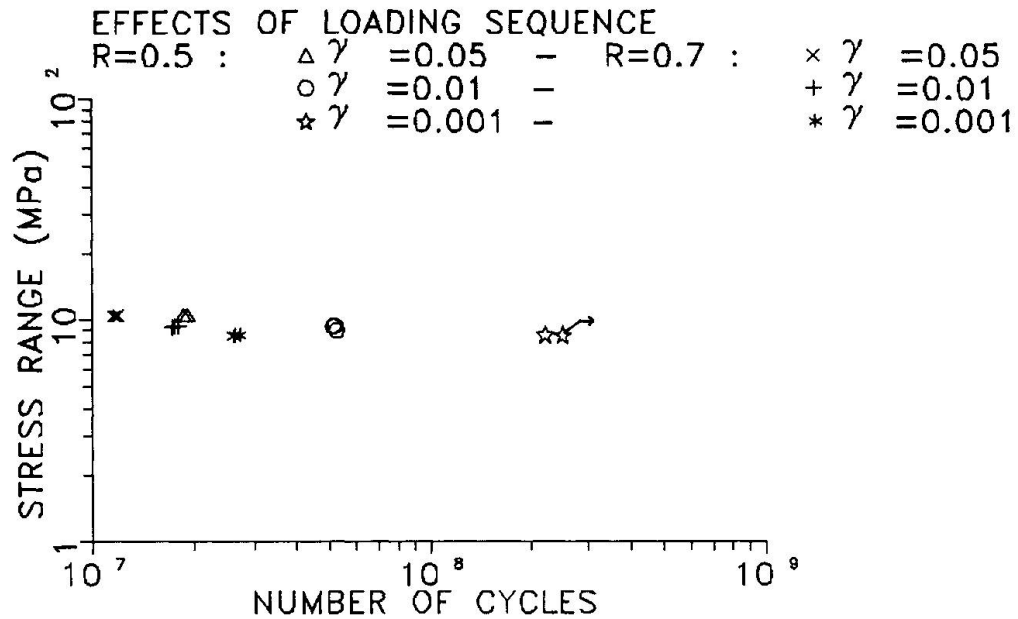


Fig. 6

It can be concluded that the numerical results are in good agreement with the physical reality, though it is not possible to make comparisons with the experimental results reported in [9], because these were obtained by varying in each case the stress range, both in order to compensate the effects due to the increase in crack size and to get as close as possible to the threshold value.

### 2.2.2 Simulation of Web Attachments

Finally, an attempt is made to simulate some of the experimental tests on web attachments under variable amplitude loading by Fisher, Mertz and Zhong, reported in [18].

The various numerical simulations were carried out considering different initial crack depths chosen from a sample having a normal distribution, 0.1 mm mean value and 40% standard deviation.

Loading histories are generated, similar to those adopted in the experimental tests [18], and consisting of a block of 5,000 cycles, repeatedly imposed on the joint. This loading history has a probability  $\gamma=0.1\%$  that the maximum stress range exceeds the value  $S_{lim} = 31$  MPa, assumed by the AASHTO

Specifications [21] as the constant amplitude fatigue limit for this structural detail.

The comparison between the experimental data and the numerical simulation results is presented in fig. 7, where the number of cycles (N) is plotted against the root mean cube (Miner's) equivalent stress range ( $S_{eq}$ ).

A reasonable agreement between numerical and experimental results can be noticed by examining fig. 7, confirming the remarkable versatility and acceptable reliability of the numerical model [11].

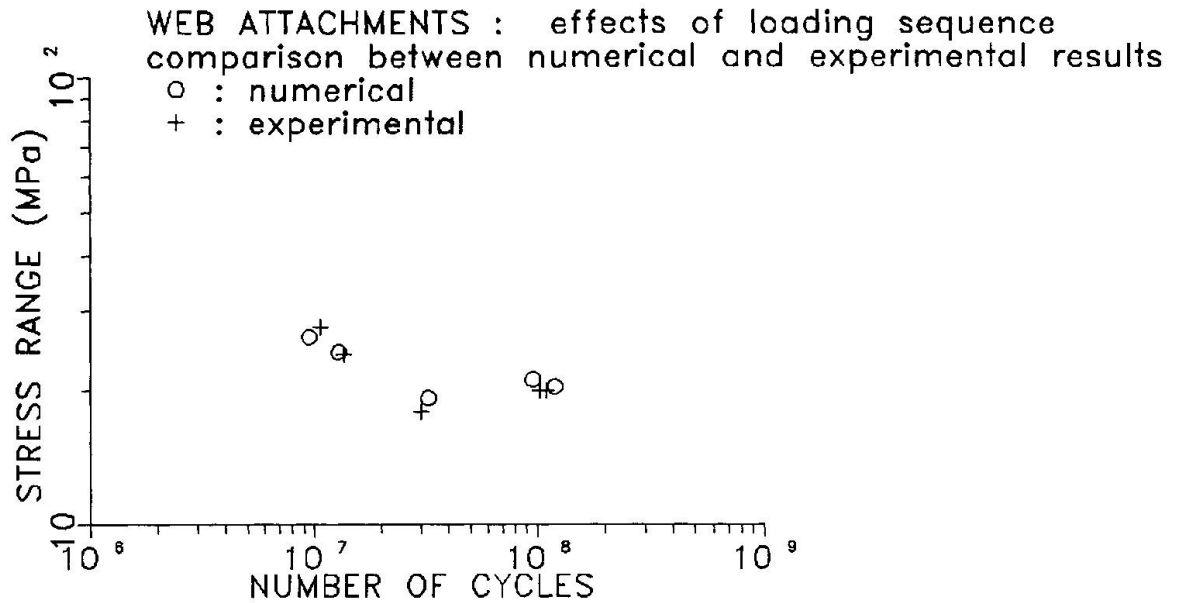


Fig. 7

### 3. CONCLUSIONS

Although the model presented in [11], in its present state, can be furtherly improved by implementing the possibility of considering for example the presence of residual stresses and/or other geometries and structural details, it nevertheless seems to be a valid starting point for a reliable numerical simulation of fatigue crack growth in structural steel details (both welded and unwelded).

The good agreement between numerical and experimental results has been shown in this paper, by comparing experimental test data with numerical results.

Once the reliability of the model has been checked on the basis of available experimental test data, attention can be focused on its possible future applications.

Two main field can be immediately identified, one regarding its use for increasing the available data base in the long life region ( $N > 50 \times 10^6$  cycles), the other aimed to the prediction of the fatigue life of a given component subjected to a known loading history.

In the first case, the model can be calibrated on a few test data either available or obtainable submitting the structural detail to constant amplitude loading, even at relatively high stress range levels. Once the model is calibrated, estimates of the fatigue life can be obtained by projection into the long endurance range. From this estimates trends can be identified rather quickly, avoiding the lengthy and expensive experimental long life fatigue tests. These will be in fact necessary only in a limited number, in order to double-check the numerical estimates obtained.

Adoption of the model for estimating the remaining fatigue life of a given component might be more difficult because it requires, in addition to the test data on which the model must be calibrated, also precise informations about the loading history experienced by the component during the service life, and an estimate of the future loading conditions. This last one is however a problem which is common to whatever procedure for estimating the remaining fatigue life of a structural detail, and does not represent an handicap of the presented numerical model.

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#### 5. REFERENCES

- [1] Keating, P.B., Fisher, J.W. "Evaluation of fatigue tests and design criteria on welded details", NCHRP rept. 286, 1986.
- [2] ESACRACK, User's Manual ESA PSS-03-209, European Space Agency, May, 1989.
- [3] Arone, R., "Influence of random overloads on fatigue crack lifetime and reliability", Engineering Fracture Mechanics, Vol. 30, n.3, 1989.
- [4] Langley, R.S., "Stochastic models of fatigue crack growth", Engineering Fracture Mechanics, Vol. 32, n.1, 1989.
- [5] Ihara, C., Misawa, T., "A stochastic model for fatigue crack propagation with random propagation resistance", Engineering Fracture Mechanics, Vol. 31, n.1, 1988.
- [6] Salivar, G.C., Yang, J.N., Schwartz, B.J., "A statistical model for the prediction of fatigue crack growth under a block type spectrum loading", Engineering Fracture Mechanics, Vol. 31, n.3, 1988.
- [7] Alawi, H., "A probabilistic model for fatigue crack growth under random loading", Engineering Fracture Mechanics, Vol. 23, n.3, 1986.
- [8] Ortiz, K., Kiremidjian, A.S., "Time series analysis of fatigue crack growth rate data", Engineering Fracture Mechanics, Vol. 24, n.4, 1986.
- [9] Ortiz, K., Kiremidjian, A.S., "Stochastic modelling of fatigue crack growth", Engineering Fracture Mechanics, Vol. 29, n.3, 1988.
- [10] Ortiz, K., "On stochastic modelling of fatigue crack growth", PhD. Thesis, Stanford University, 1985.
- [11] Castiglioni, C.A., Rossi, S., "Un modello stocastico per la stima della vita a fatica di dettagli strutturali in acciaio", Technical Report 7/89, Structural Engineering Department, Politecnico di Milano, Italy.
- [12] Newman, J.C.Jr., Raju, I.S., "Prediction of fatigue crack growth patterns and lives in three dimensional cracked bodies", VI International Conference on Fracture, New Delhi, India, 1984.
- [13] Willenborg, J., Engle, R.M., Wood, H.A., "A crack growth retardation model using an effective stress concept", AFFDL-TM-71-1-FBR, 1971.
- [14] Newman, J.C.Jr., "A crack opening stress equation for fatigue crack growth", International Journal of Fracture, Vol. 24, n.3, 1984.
- [15] Shilling, C.G., Klippstein, K.H., Barsom, J.M., Blake, G.T., "Fatigue of welded steel bridge members under variable amplitude loading", NCHRP Rept. n.188, 1978.
- [16] Fisher, J.W., Hirt, M.A., McNamee, B.M., "Effect of weldments on the fatigue strength of steel beams", NCHRP Rept. n.102, 1970.
- [17] Fisher, J.W., Yen, B.T., Klingerman, D.J., McNamee, B.M., "Fatigue strength of steel beams with welded stiffeners and attachments", NCHRP Rept. n.147, 1974.



- [18] Fisher, J.W., Mertz, D.R., Zhong, A., "Steel bridge members under variable amplitude long life loading", NCHRP Rept. n.267, 1983.
- [19] Fisher, J.W., "Fatigue and Fracture in Steel Bridges. Case Studies", John Wiley & Sons, New York, 1984.
- [20] Keating, P.B., "High Cycle Fatigue Behavior of Welded Details Under Variable Amplitude Loading", PhD. Thesis, Lehigh University, 1987.
- [21] AASHTO, "Standard Specifications for Highway Bridges", 1983.