An alternative to miner's rule for cumulative damage calculations?

An Alternative to Miner's Rule for Cumulative Damage Calculations?

Une alternative à la loi de Miner pour calculer le cumul du dommage?

Eine Alternative zur Miner'schen Regel für die Berechnung der Schadens-Akkumulation?

Tim GURNEY
Chief Research Engineer
The Welding Institute
Cambridge, England

Stephen MADDOX
Principal Fatigue Consultant
The Welding Institute
Cambridge, England

Tim Gurney, born 1929, read Mechanical Sciences at Cambridge University where subsequently, he also obtained his PhD. Following four years with a firm of Consulting Engineers, he joined The Welding Institute, where he has worked primarily on fatigue of welded structures.

Apart from three years' industrial consultancy at Southampton University, Stephen Maddox has been with The Welding Institute since graduating in Civil Engineering in 1966. His research field is fatigue of welded structures, including design recommendations and the application of fracture mechanics, for which he received his PhD.

SUMMARY

Based upon an analysis of the available variable amplitude fatigue test results for welded joints, it is shown that, in some circumstances, Miner's rule may be unsafe and that the Area rule may be a better alternative. This applied particularly for short block length loading and wide band loading. However, further work is required to confirm these findings.

RÉSUMÉ

Sur la base d'une analyse des résultats d'essais de fatigue à amplitude variable sur des joints soudés, on peut montrer que, dans certains cas, la loi de Miner n'est pas du côté de la sécurité, et que la «règle des surfaces» serait une meilleure alternative. Ceci s'applique en particulier quand la contrainte maximum se produit fréquemment et pour des chargements à bande étendue. Cependant, des études supplémentaires doivent être faites pour confirmer ces conclusions.

ZUSAMMENFASSUNG

Basierend auf einer Analyse der erhältlichen Versuchsresultate zur Ermüdung von Schweissverbindungen unter variabler Amplitude wird gezeigt, dass unter gewissen Umständen die Miner-Regel auf der unsicheren Seite liegt und deshalb die Anwendung der «Oberflächen-Regel» angebracht erscheint. Dies trifft besonders auf «short block length-» und «wide band loading»-Spektren zu. Um diese Beobachtungen zu bestätigen, sind jedoch weitere Forschungsarbeiten unabdingbar.

1. INTRODUCTION

In service, the great majority of structures which are liable to suffer fatigue failure are subjected to variable amplitude loading. For historical reasons, however, and particularly on account of the capabilities of fatigue testing machines, most of the experimental data, certainly for welded joints, has been obtained under constant amplitude loading. The assessment of the fatigue life of structures therefore requires the use of some sort of cumulative damage rule and Miner's [1] linear damage rule is the one most commonly used.

As far as is known, the first time that this rule appeared in a Standard for the design of welded joints was in 1962 when it was introduced into BS 153, Steel Girder Bridges. At that time such an action was probably quite unjustified; it was really a simple 'act of faith'. After all, Miner had originally proposed his rule for defining the life to crack initiation of unnotched specimens of aluminium alloy. There were no sound reasons for assuming that it would be equally applicable to the life to rupture of severely notched members in steel, where it was known that the great majority of the life consisted of crack propagation. It was only much later that it was shown by Maddox [2] that Miner's rule could be derived by fracture mechanics for any specimen/joint involving crack propagation to failure. Since then, however, Miner's rule, or a simpli-rule based on Miner's rule, has appeared in several National Standards.

The obvious problem is, "Is it right?" In order to try to answer that question an extensive literature survey has recently been carried out to gather together and analyse all the available test results for welded steel specimens subjected to variable amplitude loading, either of the programmed or random variety, in air. This included recalculating the values of $\Sigma\frac{n}{N}$ for every test specimen so as to ensure that all the results were defined in the same manner; unbroken specimens were ignored.

It was found that numerous test series gave mean values, and particularly lower limit values, of $\Sigma\frac{n}{N}$ which were less than 1.0. This has sometimes been taken as proof that Miner's rule is unsafe. However, that is not necessarily true, since one could reasonably expect the scatter in variable amplitude test results to mirror that found in constant amplitude results. In other words, if a specimen would have given a low life under constant amplitude loading, it seems reasonable to assume that it would also be expected to give a low life under variable amplitude loading. Furthermore, to a first approximation, it also seems reasonable to assume that the amount of scatter would be the same with the two different types of loading.

Although no detailed analysis has been made of the scatter in constant amplitude results, inspection of a number selected at random suggests that in a single S-N curve the overall scatter can be represented by a factor of between 2 and 3 on life. For comparison Haibach [3] proposed that his 'standard curve' should have corresponding factors of 3 at 10^5 and 4 at 10^6 cycles.

For a factor of 2.0 the corresponding expected range of values of $\Sigma\frac{n}{N}$ becomes about 0.7 - 1.4, while for a factor of 3.0 it is about 0.6 - 1.8. Thus any values of $\Sigma\frac{n}{N}$ greater than about 0.6 - 0.7 can reasonably be regarded as normal, provided that in a test series there are also some corresponding 'high' values. Thus, for Miner's rule to be proved safe one should expect a series of variable amplitude fatigue tests to give a mean value of $\Sigma\frac{n}{N}$ approximately equal to 1.0 (or more) but with individual results down to about 0.6. In fact it was found that many test series gave values which were considerably lower, thereby suggesting that Miner's rule certainly is not safe under all conditions.

Fig. 1 shows the same data expressed in terms of the 1292 individual test results. The values extend from 0.16 to 9.12 but it should be noted that three test series for which the <u>mean</u> values of $\Sigma\frac{n}{N}$ were in the range 12.3 - 15.7 have been ignored. These were so out of line with the other results that it was assumed that some error must have occurred in reporting the results. Results

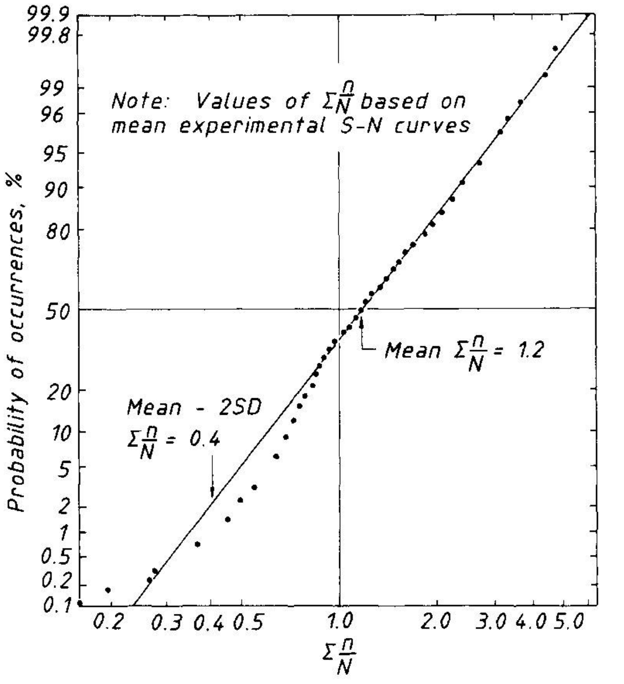

Fig.1 Summary of published values of $\Sigma\frac{n}{N}$

obtained using spectra in which all cycles pulsated downwards from a constant maximum stress have also been omitted. As before, the results are based upon the corresponding mean experimental S-N curve extrapolated linearly downwards.

Although the individual values of $\Sigma\frac{n}{N}$ varied widely, the mean and mean minus 2 standard deviations (SD) values were approximately 1.2 and 0.4 respectively. It is therefore tempting to argue that Miner's rule, with $\Sigma\frac{n}{N}$ = 1.0, is a satisfactory design approach, since the low value of 0.4 is counterbalanced by the fact that design is normally based upon the mean minus 2 SD design curve and not the mean curve. In the British design rules the difference in life between the two curves, represented by the ratio $\frac{\text{mean - 2 SD life}}{\text{mean life}}$ at any given stress, is as follows:

Class	D	E	F	F2	G
Ratio of lives	0.381	0.315	0.366	0.350	0.437

Thus, in all cases except Class G, the reduction in life between the two curves is more than 0.4 so that, basing the calculation on the mean minus 2 SD curve would be safe for more than 97.7% of situations.

This is, however, a fallacious approach. The reduction in life between the mean and mean - 2 SD design curve only takes account of the scatter found in constant amplitude tests, caused by such variables as differences in local joint geometry. It does not allow for any extra variability in life, if such exists, due to variable (rather than constant) amplitude loading. It is therefore necessary to consider the influence of the type of loading separately and not to assume that it is necessarily covered by using the mean - 2 SD design curve.

2. INFLUENCE OF TYPE OF LOADING ON FATIGUE LIFE

The following information about the influence of the type of loading on the value of $\Sigma\frac{n}{N}$ is based heavily on the results of work at The Welding Institute [4,5] over the last six or seven years. All the work involved longitudinal non-load-carrying fillet welds, either on the edge or surface of the stressed plate, because of the known tendency of such joints to give very little scatter; it was therefore hoped that it would be easier to define any trends which might emerge. It is convenient to consider the results more or less in chronological order.

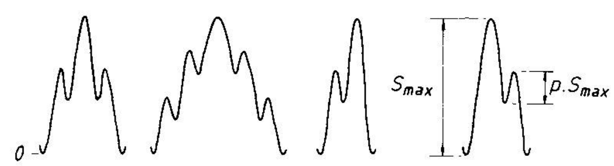

Fig.2 Typical loading in initial tests

The initial tests [4] were carried out with very simple stress histories, consisting essentially of constant amplitude loading with one or more excursions applied on each stress cycle (Fig. 2). With this type of loading there are few real problems

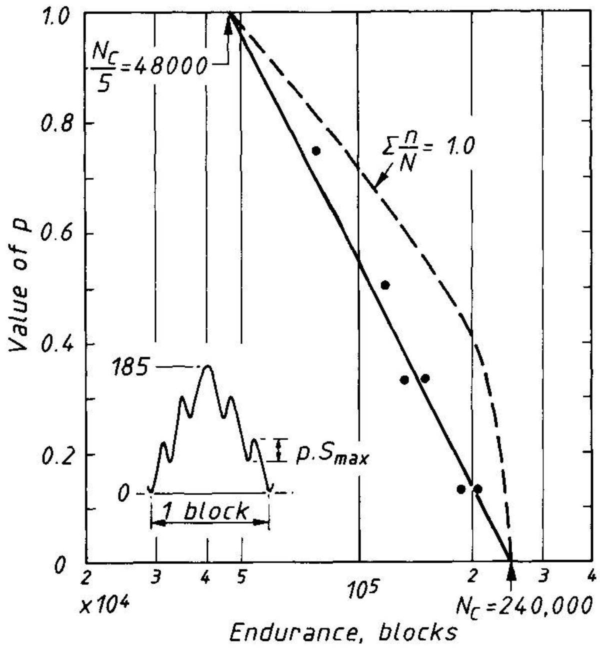

Fig.3 Typical results obtained using simple stress sequences

with stress counting, although in all cases the reservoir method was used. On the other hand there is certainly a potential stress interaction problem and the expectation was that the tensile peak of the main constant amplitude cycle would tend to introduce compressive residual stresses at the crack tip and thereby make the smaller cycles less damaging. As a result of this stress interaction it was anticipated that the life would always be greater than that predicted by Miner's rule – in other words it was expected that $\Sigma\frac{n}{N}$ would be greater than 1.0. In fact the opposite was found to be true.

A typical set of results is shown in Fig. 3. They relate to specimens tested under a main constant amplitude cycle of 185 N/mm² at R = 0 with four subsidiary cycles of magnitude p. In this context p represents the ratio between the stress range of the excursions and of the main cycle. Thus, using this nomenclature, p can clearly vary from 0 to 1.0 and at the two extremes (p = 0 or p = 1.0) the loading degenerates to constant amplitude. At p = 0 the four subsidiary cycles disappear, so that the life under the complex load cycle merely becomes N_c, the constant amplitude life under the peak stress range. At p = 1.0 all the cycles (one main and four subsidiary) have the same magnitude, so that the life in 'blocks' (where a block is defined as consisting of the main constant amplitude cycle and all its associated subsid- iary cycles) becomes $N_c/5$. It will be seen from Fig. 3 that there is an excellent linear relationship between p and Ln (N_B), where N_B is the life measured in 'blocks', joining these two end points. The relationship between p and N_B is therefore

$$N_B = N_c\ (1 + v)^{-p}$$

where v is the number of subsidiary cycles per block (4 in Fig. 3).

For comparison, the lives required to give $\Sigma\frac{n}{N}$ = 1.0 are also indicated. As noted above, it is clear that the actual values of $\Sigma\frac{n}{N}$ were consistently less than 1.0, except at the two limits (p = 0 and p = 1.0).

The same general form of behaviour was found to occur with each of the loading blocks shown in Fig. 2. It was also found with as-welded specimens subjected to an alternating, rather than pulsating tension, main cycle and with stress relieved specimens under a pulsating tension cycle. Anomalies were found, however, with stress relieved specimens subjected to an alternating main cycle, particularly when the subsidiary cycles were at a different mean stress so that they were either fully tensile or fully compressive. This problem of the influence of mean stress in stress relieved specimens remains to be fully resolved.

If one assumes (even if there is no obvious reason for it) that the behaviour indicated in Fig. 3 will also occur when there are subsidiary cycles of several different magnitudes associated with each main cycle, instead of only one, it is easy to deduce that the expected life (in blocks) can be written as

$$N_B = N_c\ e^{-Area}$$

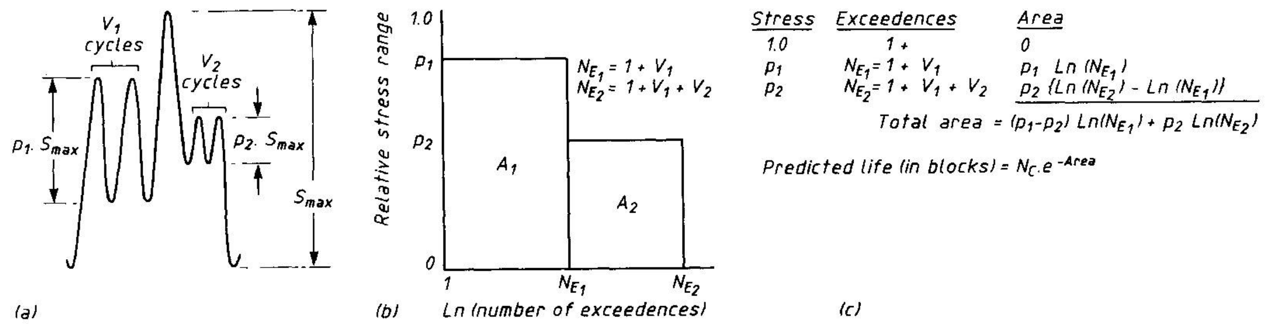

Fig.4 Example of life calculation using the area rule

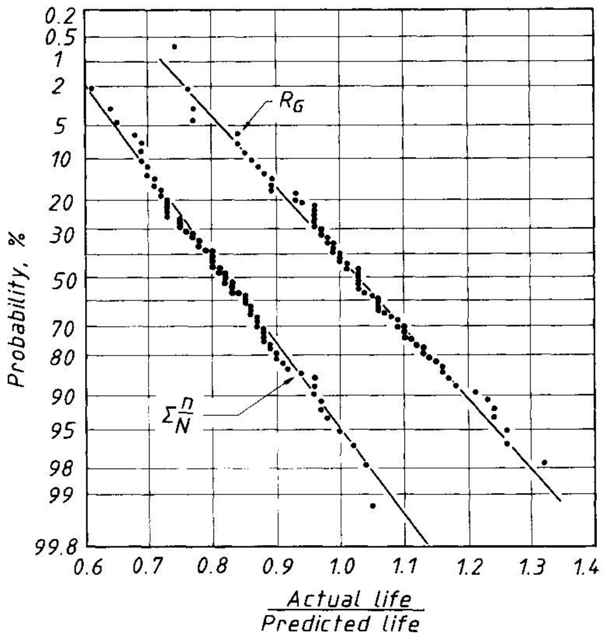

Fig.5 Comparison of Miner's rule and area rule for short blocks

where 'Area' is the area under the p v. Ln N_E exceedence diagram for the loading spectrum, N_{E_i} being the number of exceedences of stress p_i per block. A simple example is shown in Fig. 4. Like Miner's rule, the 'area rule' at least has the merit of simplicity. The problem is, 'Is it any better?' In order to make the comparison it is convenient to express experimental results in terms of the ratio R_G (= actual life/life predicted by the area rule). This is directly comparable to the value $\Sigma\frac{n}{N}$ for Miner's rule.

A summary of all the results obtained for very simple loading of the type shown in Fig. 2, but ignoring the results for stress relieved specimens under alternating loading (where special considerations apply) is shown in Fig. 5. Clearly for those types of loading the area rule is superior to Miner's rule.

The next step was therefore to compare the two under more complex load spectra and the initial tests [5] involved Rayleigh distributions of stress ranges of various block lengths (and hence various clipping ratios). In all cases the individual cycles forming a block were applied in random order at R = 0 and the block was then repeated, in the same order, until failure occurred.

The results, in terms of $\Sigma\frac{n}{N}$, are shown in Fig. 6(a). They show two interesting features:

1. a clear tendency for $\Sigma\frac{n}{N}$ to increase with increasing block length
2. an equally clear tendency for $\Sigma\frac{n}{N}$ to decrease as the stress magnitude is decreased.

In other words, at least for this particular type of spectrum, the trends suggest that Miner's rule would become unsafe at short block lengths, with the critical block length increasing as the applied stresses decrease.

In contrast, the results expressed in terms of the area rule were all safe (Fig. 6(b)), although their trend suggested that it might well become unsafe at very long block lengths (approx. 10^6 cycles). On the other hand, the great accuracy of the prediction for the very simple type of loading obviously did not carry through to these particular tests, since R_G obviously rose to a peak at a block

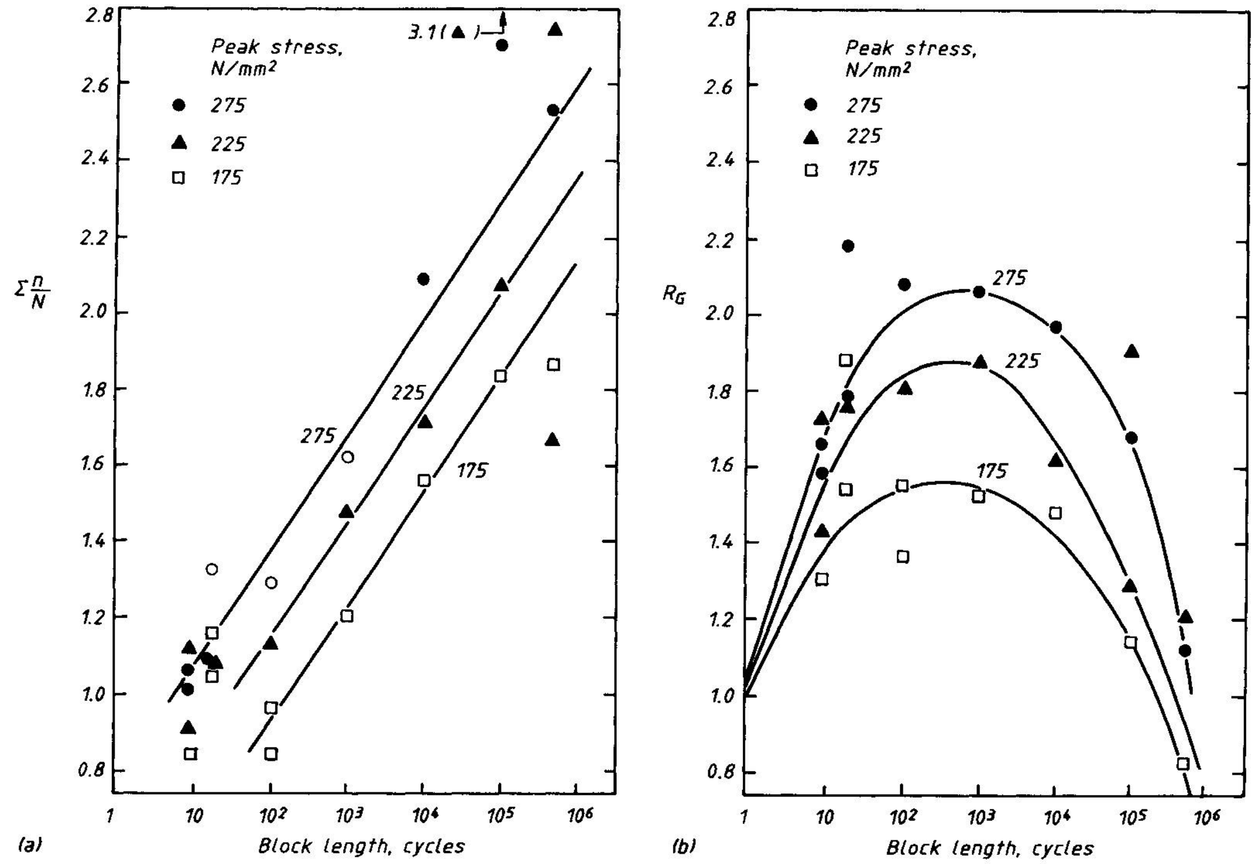

Fig.6 Influence of block length on (a) $\Sigma\frac{n}{N}$ and (b) R_G for Rayleigh spectra

length of about 10^3 cycles before reducing again; equally, like the value of $\Sigma\frac{n}{N}$, R_G was obviously also a function of the stress magnitude.

In passing, it is worth noting that these results show, quite conclusively, that there is little benefit to be gained by working in terms of 'equivalent stress' (cube root mean cube stress range) or any variety of rms stress. As can be seen from Fig. 7 the use of these parameters does not normalise the results to a single curve. Not only do results at different stress ranges with the same spectrum plot on different curves, but so do the results for different spectra (in this case Rayleigh and Laplace).

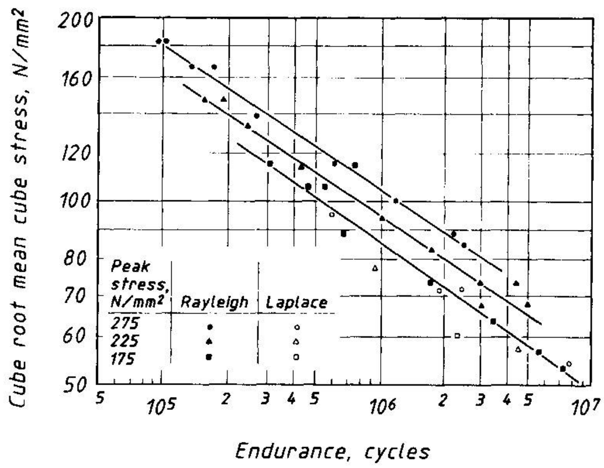

Fig.7 Results for Rayleigh and Laplace spectra

As stated previously, one interpretation of the results in Fig. 6 is that $\Sigma\frac{n}{N}$ tends to increase as block length increases, but an equally good interpretation, since clipping ratio is related to block length, is that $\Sigma\frac{n}{N}$ tends to increase as clipping ratio increases. Since they are related it is impossible, for any particular shape of spectrum, to determine which is the more relevant variable. However, by varying spectrum shape as well, it was possible to carry out tests with the same block lengths but different clipping ratios and also with the same clipping ratios but different block lengths. The spectra were based upon the 2 parameter Weibull distribution and the results again showed that both $\Sigma\frac{n}{N}$ and R_G tended to decrease as the stress level decreased; at 275 N/mm² the values were typically 19% higher than at 175 N/mm². It was therefore possible to increase the database by applying that correction and assuming that all specimens were tested at 275 N/mm².

Analysis of the results after applying that correction showed that the value of $\Sigma\frac{n}{N}$ was almost completely insensitive to clipping ratio but, as with the Rayleigh spectra (Fig. 6(a)) there was again a fairly general tendency for $\Sigma\frac{n}{N}$ to increase with increasing block length, although there were some 'outlying' results. Similarly, the influence of block length on the value of R_G appeared to be very like that shown in Fig. 6(b).

Since these results appear to show that R_G gives safe predictions of life at short block lengths but that they become unsafe at long block lengths, all the available test data from the literature were re-analysed in order to derive the values of R_G at failure. The results are summarised in Fig. 8, but it does not include those for very short block lengths, which were summarised in Fig. 5. For relatively short block lengths (up to 10^5 cycles) the results were nearly all 'safe', even with N_c derived from the mean S-N curves. If the mean - 2 SD design curves had been used, only 2 results would have been 'unsafe'. However, it is clear that, for long block lengths ($\geq 2 \times 10^5$ cycles), R_G certainly does tend to become unsafe.

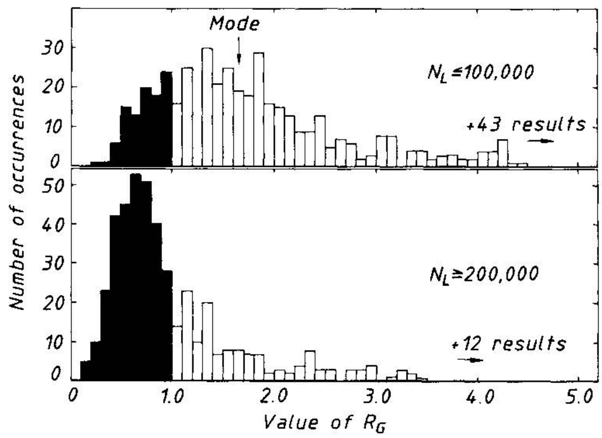

Fig.8 Summary of values of R_G

Up to this point all the results considered, except a few of the tests with very simple stress cycles (Fig. 2), involved all the cycles being applied at R = 0. The next stage therefore involved some exploratory tests to study the influence of stress ratio and mean stress. They were based upon 2 stress spectra, each with a block length of 128 cycles. The particular variants which were employed were:

a) All cycles at R = 0, random order

b) All cycles at R = -1, same order

c) All cycles with the same S_{max}, again in the same order (cf stalactites)

d) With the peak cycle at R = 0 or R = -1 but with the stress ratios of the other cycles varied within the overall range of the peak cycle (i.e. wide band loading). Again the order of the cycles was the same as before.

e) With the cycles arranged in decreasing-increasing order, as in a typical block programme test. These tests involved all cycles being at the same stress ratio (R = 0 and R = -1).

Apart from (e), all variants had the same stress ranges applied in the same order so that, in terms of stress range, their rainflow (or reservoir) counts were identical.

The average values of $\Sigma\frac{n}{N}$ which were obtained are summarised in Table 1. It is obvious that there was a tendency for $\Sigma\frac{n}{N}$ to be lower under alternating than under pulsating tension loading, although the difference was small under wide band loading. Surprisingly, extremely few other tests seem to have been carried out at R = - 1 except under programmed block loading. A comparison of those results with similar ones obtained at R = 0 is shown in Fig. 9. In that case also, there was a reduction in $\Sigma\frac{n}{N}$ at R = - 1, typically of about 28%. While there is obviously a need for more information this evidence does suggest that

Type of loading	Peak stress at R = 0	R = -1
All cycles at same stress ratio, in random order	1.32	0.75
'Stalactitic' spectrum	0.55	
Decreasing-increasing block programme	1.59	0.88
Wide band (variable R)	0.80	0.75

Table 1 Values of $\Sigma\frac{n}{N}$ using different types of loading

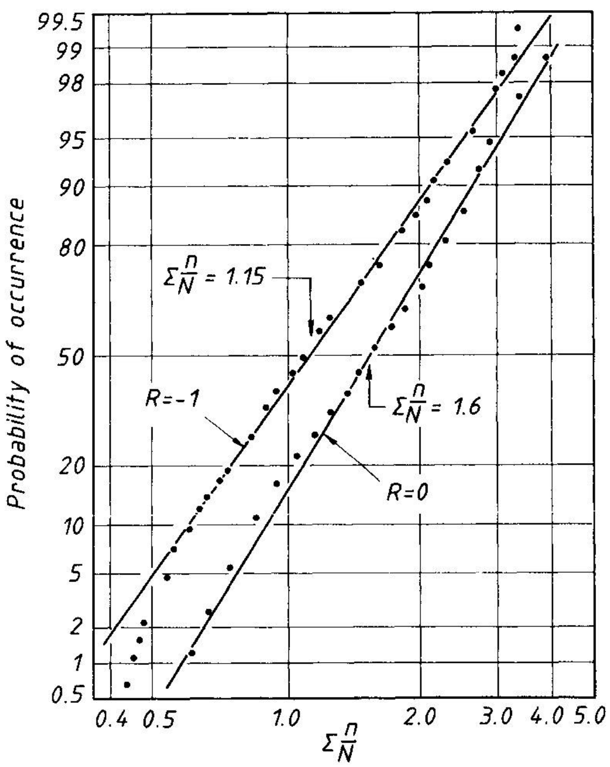

Fig.9 Effect of R on values of $\Sigma\frac{n}{N}$ for block programme tests

lower values of $\Sigma\frac{n}{N}$ are to be expected under alternating loading.

Similarly the tests in which all the cycles had the same maximum stress (what one might call a stalactitic spectrum) also gave a much lower value of $\Sigma\frac{n}{N}$, with a typical value of $\Sigma\frac{n}{N}$ = 0.55. That is very similar to the value obtained by Maddox in similar tests using the BS 5400 Axle loading spectrum. These were, incidentally, the results which were omitted omitted from Fig. 1. The original idea behind testing with this type of spectrum was that it would be an easy way to simulate the presence of high tensile residual stresses, which would be expected to be present in a large structure, in a small specimen. If correct, the results are obviously a little worrying, but it remains to be proved whether or not this method of simulation is too severe.

Turning now to the effect of wide band loading, the results were analysed using the S-N curves corresponding to the stress ratio of the peak stress ange, regardless of the fact that most of the smaller cycles were at different stress ratios. On this basis the values of $\Sigma\frac{n}{N}$ were very similar for the two stress ratios and ranged from approximately 0.5 - 1.0 with a mean value of about 0.76.

In the context of whether or not one should use Miner's rule to design structures subjected to such loading, this result is also a little worrying, particularly bearing in mind the numbers of specimens for which the predicted lives were safe or unsafe (see Table 2). On the other hand, the Area rule did appear to give either safe or acceptable predictions with these particular spectra. It must be remembered, however, that all these tests were carried out using spectra with relatively short block lengths (128 cycles); it has yet to be proved whether or not the results are block length dependent under wide band loading.

Finally, the opportunity was taken to obtain comparative results for the same spectra but with the individual cycles arranged in decreasing-increasing order, so as to simulate a typical block programme test. In this way it was hoped to obtain some evidence as to the likely accuracy of other block programme tests, since such results make up a very large proportion of the variable amplitude

Type of loading	No. of tests	Miner's rule		Area rule	
		>1.0	≥1.0	<1.0	≥1.0
Peak stress at R = 0					
All cycles at R = 0	4	–	4	–	4
All cycles with peak stress = S_{max}	4	4	–	1	3
Ordered block loading	7	–	7	–	7
Wide band loading	24	23	1	–	24
Peak stress at R = -1					
All cycles at R = -1	4	4	–	–	4
Ordered block loading	5	4	1	–	5
Wide band loading	29	28	1	10	19

Table 2 Summary of numbers of safe and unsafe predictions using Miner's rule and the 'area' rule

database and, as far as is known, no other directly comparable tests have been reported previously.

As can be seen from the summary of the results set out above, there was a substantial difference between the 'block programme' results and the 'random order' results at both stress ratios; typically the value of $\Sigma\frac{n}{N}$ obtained under block programme loading was 20% higher at both stress ratios. Clearly this must cast some doubt on the validity of all the earlier block programme results and it is obvious that further check tests are required for other types of spectra.

3. SUMMARY OF THE CURRENT POSITION AND FUTURE WORK REQUIREMENTS

As was shown at the beginning of this review, the available experimental evidence indicates that Miner's rule, based on the mean - 2 S.D. design S-N curves, should be a satisfactory design method in most situations. Nevertheless, there are clearly some where it may be unsafe.

The most obvious of these is when the loading involves short block lengths – in other words when the peak stress in the spectrum occurs fairly frequently. A typical example might be an overhead travelling crane working on a production process. Similarly, certain types of earth-moving plant, where the machine essentially goes through a continuous series of digging and unloading cycles, might also qualify. There are almost certainly many other examples. The evidence certainly seems to suggest that the 'area rule' would be an advance on Miner's rule for dealing with this particular type of loading.

The second major problem area seems to be the tendency for $\Sigma\frac{n}{N}$ to decrease as the applied stresses decrease. It is a trend which seems to be very evident under tensile loading when the individual stress ranges in the spectrum are applied in random order and when the peak stresses are fairly high. Much work is, however, still needed to confirm the extent to which the trend continues to lower stresses and also whether it also applies at other stress ratios and mean stresses.

If it is a general problem it would be worrying for two reasons. In the first place, almost the whole database of variable amplitude test results, which justify the continued use of Miner's rule, would be suspect. After all, nearly all the tests have, quite deliberately, been carried out at relatively high stresses in order to give shortish lives and hence reduce testing costs.

Secondly, many structures have been designed at much lower stresses, which implies that their lives might become much shorter than expected. Although it is easy to understand why tests have largely been restricted to high stresses and short lives, there is obviously a need to obtain data for more realistic stresses and longer lives.

Associated with this problem of decreasing $\Sigma\frac{n}{N}$ as stresses decrease is, of course, the question of what constitutes a 'short block length' at low applied stresses. Clearly, one would expect the critical block length to increase, but by how much will certainly need to be established.

In the light of the evidence pointing to a marked reduction in the value of $\Sigma\frac{n}{N}$ at R = -1 as compared to R = 0, and also under 'stalactitic' loading with a constant maximum tensile stress, there is also a need to investigate the influence of mean stress. In particular, it needs to be established whether stalactitic loading is a realistic simulation of the high tensile residual stresses which may occur in actual structures, and also whether the use of S-N curves related to stress range alone (i.e. ignoring mean stress) is a sensible basis for design under variable amplitude (as opposed to constant amplitude) loading.

The available evidence also suggests that there are likely to be problems under wide band loading, but it must be remembered that that evidence was obtained using short block lengths (128 cycles). It is at least possible that the problems will disappear with longer block lengths, but that is an area which also remains to be investigated.

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