

**Zeitschrift:** IABSE reports = Rapports AIPC = IVBH Berichte  
**Band:** 57/1/57/2 (1989)  
  
**Artikel:** Crack widths and deformation behaviour of reinforced concrete structures  
**Autor:** Günther, Gerd / Mehlhorn, Gerhard  
**DOI:** <https://doi.org/10.5169/seals-44217>

### **Nutzungsbedingungen**

Die ETH-Bibliothek ist die Anbieterin der digitalisierten Zeitschriften auf E-Periodica. Sie besitzt keine Urheberrechte an den Zeitschriften und ist nicht verantwortlich für deren Inhalte. Die Rechte liegen in der Regel bei den Herausgebern beziehungsweise den externen Rechteinhabern. Das Veröffentlichen von Bildern in Print- und Online-Publikationen sowie auf Social Media-Kanälen oder Webseiten ist nur mit vorheriger Genehmigung der Rechteinhaber erlaubt. [Mehr erfahren](#)

### **Conditions d'utilisation**

L'ETH Library est le fournisseur des revues numérisées. Elle ne détient aucun droit d'auteur sur les revues et n'est pas responsable de leur contenu. En règle générale, les droits sont détenus par les éditeurs ou les détenteurs de droits externes. La reproduction d'images dans des publications imprimées ou en ligne ainsi que sur des canaux de médias sociaux ou des sites web n'est autorisée qu'avec l'accord préalable des détenteurs des droits. [En savoir plus](#)

### **Terms of use**

The ETH Library is the provider of the digitised journals. It does not own any copyrights to the journals and is not responsible for their content. The rights usually lie with the publishers or the external rights holders. Publishing images in print and online publications, as well as on social media channels or websites, is only permitted with the prior consent of the rights holders. [Find out more](#)

**Download PDF:** 20.01.2026

**ETH-Bibliothek Zürich, E-Periodica, <https://www.e-periodica.ch>**

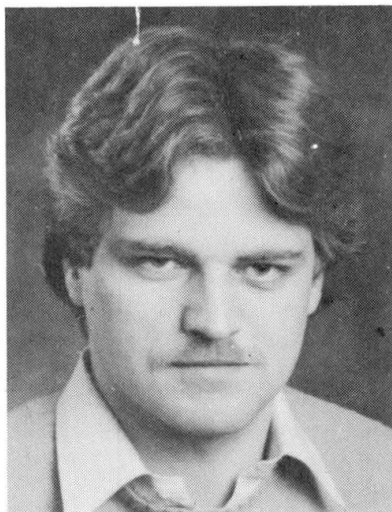
## Crack Widths and Deformation Behavior of Reinforced Concrete Structures

Largeur de fissures et comportement en déformation de structures en béton armé

Rissbreiten und Verformungsverhalten von Stahlbetontragwerken

**Gerd GÜNTHER**

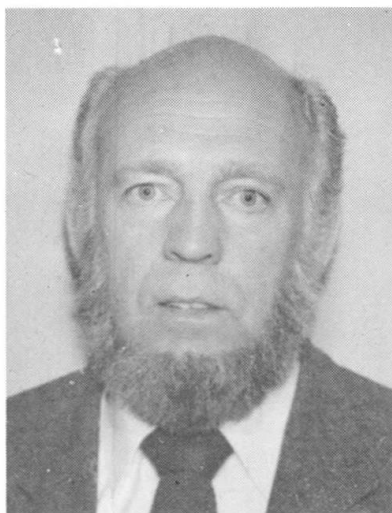
Gesamthochschule Kassel  
Fachgebiet Massivbau  
Kassel, Fed. Rep. of Germany



Gerd Günther, born in 1957, 1985 Dipl.-Ing., University of Kassel since 1985 Assistant in the Division of Reinforced Concrete of the University of Kassel.

**Gerhard MEHLHORN**

Gesamthochschule Kassel  
Fachgebiet Massivbau  
Kassel, Fed. Rep. of Germany



Gerhard Mehlhorn, born in 1932, apprenticeship as steel construction worker, 1959 Dipl.-Ing., TH Dresden, 1959-1965 Design Engineer, 1965-1983 TH Darmstadt, Dr.-Ing. 1970, since 1972 Professor of Civil Engineering, 1982 Visiting Professor University of Toronto, since 1983 Professor of Civil Engineering at University of Kassel, since 1984 also Consulting Engineer at Kassel (until 1984 at Darmstadt).

### SUMMARY

In this contribution the results of tests are shown. With the tests the increase of crack widths and deformations in dependence on load duration and number of load cycles was investigated. Especially considered are the consequences of the initial stress state caused by shrinkage on the crack width development. Besides simple tension and bending tests biaxial compression-tension tests were also carried out on reinforced concrete panels.

### RÉSUMÉ

Grâce à des essais, l'augmentation des largeurs de fissures et des déformations dépendant de la durée et de la valeur de la charge devait être clarifiée. Les conséquences de l'état d'auto-contraintes créé par le retrait sur l'évolution des largeurs de fissures sont prises en considération. Outre les essais de traction et de flexion, des essais biaxiaux compression-traction ont été réalisés.

### ZUSAMMENFASSUNG

Mit Versuchen sollte geklärt werden, wie gross die Verformungs- und Rissbreitenzunahmen in Abhängigkeit von der Belastungsdauer und der Anzahl der Lastwiederholungen sind. Die Auswirkungen des durch Schwinden verursachten Eigenspannungszustandes auf die Rissbreitenentwicklung wurden besonders berücksichtigt. Neben den reinen Zug- und Biegeversuchen wurden auch biaxiale Druck-Zugversuche an Stahlbetonscheiben durchgeführt.



## 1. INTRODUCTION

The durability of reinforced concrete structures mainly depends on crack widths and deformations. The protection against corrosion of the reinforcement is lost when crack widths are large and the stability is endangered. With too large deformation increases owing to long-term loads considerable usage restrictions of the building may result. In this contribution the results of about 120 tests on crack spacing, crack widths, and deformation behavior of reinforced concrete structures under monotonically increasing, cyclic, and long-term loads are presented.

## 2. CRACK SPACINGS

The average and maximum crack spacings of the reinforced concrete panels [1, 2], the centrally and eccentrically reinforced tension specimens [3, 4, 5], and the reinforced concrete beams [6] are presented in table 1. A dependence of the crack spacing on the concrete compressive strength and on the related rib area was not observed.

Owing to repeated loads and permanent loads no new cracks occurred (not considering a few exceptions) with constant top load. As a result of unintentional eccentricities with the regularly and centrally reinforced tension specimens the cracks separated the concrete only partially at initial loading. Thus, the cracks still developed with repeated and permanent loads.

The crack development which occurred with the eccentrically reinforced tension specimens (see Fig. 3) and the reinforced concrete beams are to be traced back to the decrease of concrete tensile strength owing to repeated loads.

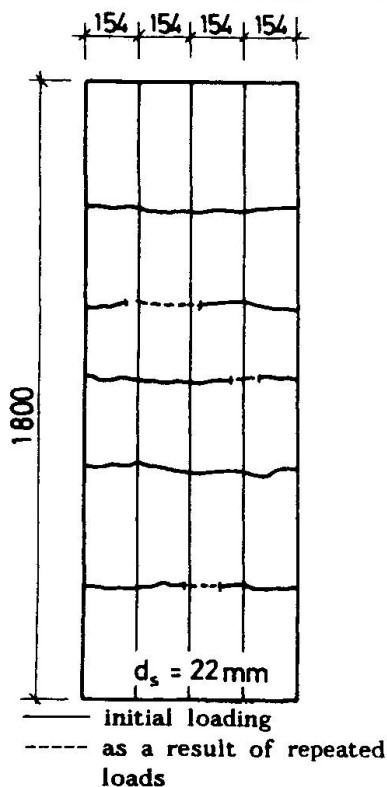


Fig. 1 Crack pattern of intended centrally reinforced specimen S 22

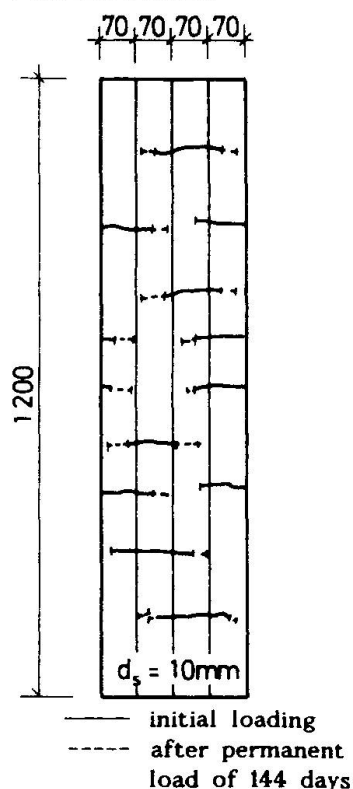


Fig. 2 Crack pattern of intended centrally reinforced specimen DAUER 4

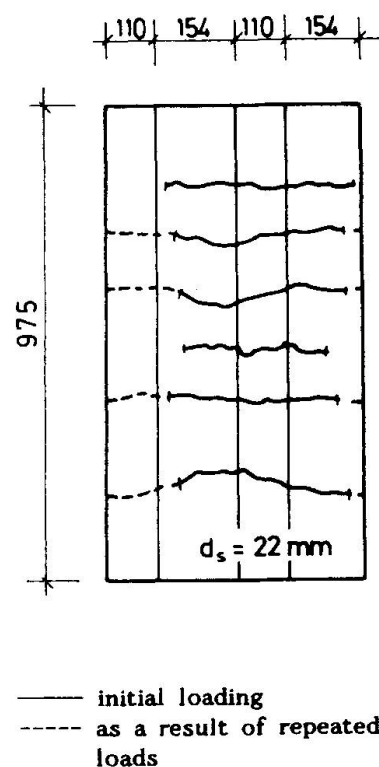
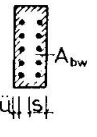
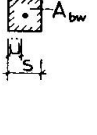
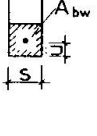


Fig. 3 Crack pattern of eccentrically reinforced specimen E 11

In the crack patterns of the reinforced concrete panels [2] without transverse compression (see Fig. 4) and with  $0.6 \beta_c$  transverse compression (see Fig. 5) no dependence of the amount of cracks on transverse compression can be seen.

| Specimen                                      |                                                                                     | $d_a$<br>(mm) | $\bar{u}$<br>(mm) | $s$<br>(mm) | reinforcement                                                              | number of<br>specimens | crack spacing<br>(mm) |                   | calculated average<br>crack spacing (mm)                |                 |
|-----------------------------------------------|-------------------------------------------------------------------------------------|---------------|-------------------|-------------|----------------------------------------------------------------------------|------------------------|-----------------------|-------------------|---------------------------------------------------------|-----------------|
|                                               |                                                                                     |               |                   |             |                                                                            |                        | $a_m$                 | $a_{max}$         | $a_m = 2 (\bar{u} + 0,1 \cdot s) + 0,1 \frac{d_a}{\mu}$ | $1,7 \cdot a_m$ |
| reinforced concrete panels                    |    | 10x8,5        | 10                | 50          | ribbed bars with<br>transverse reinforcement<br>ribbed bars<br>ribbed mats | 2<br>5<br>2            | 134<br>136<br>107     | 185<br>200<br>150 | 105                                                     | 178             |
|                                               |                                                                                     | 10x5,5        | 11                | 50          | ribbed bars with<br>transverse reinforcement<br>ribbed bars<br>ribbed mats | 2<br>2<br>1            | 126<br>103<br>102     | 200<br>200<br>125 | 147                                                     | 249             |
|                                               |                                                                                     | 5x16,0        | 42                | 100         | ribbed bars                                                                | 2                      | 140                   | 250               | 183                                                     | 311             |
|                                               |    | 1x10,0        | 30                | 70          | ribbed bars                                                                | 10                     | 128                   | 210               | 135                                                     | 230             |
|                                               |                                                                                     | 1x10,0        | 40                | 90          |                                                                            | 2                      | 127                   | 220               | 200                                                     | 340             |
|                                               |                                                                                     | 1x16,0        | 32                | 80          |                                                                            | 3                      | 115                   | 160               | 131                                                     | 223             |
|                                               |                                                                                     | 1x16,0        | 48                | 112         |                                                                            | 2                      | 183                   | 240               | 219                                                     | 372             |
|                                               |                                                                                     | 1x16,0        | 64                | 144         |                                                                            | 2                      | 333                   | 390               | 322                                                     | 547             |
|                                               |                                                                                     | 1x22,0        | 44                | 110         |                                                                            | 20                     | 174                   | 310               | 180                                                     | 306             |
|                                               |                                                                                     | 1x22,0        | 66                | 154         |                                                                            | 27                     | 290                   | 450               | 300                                                     | 511             |
|                                               |                                                                                     | 1x22,0        | 88                | 198         |                                                                            | 16                     | 359                   | 630               | 442                                                     | 752             |
|                                               |                                                                                     | 1x28,0        | 56                | 140         |                                                                            | 2                      | 290                   | 300               | 229                                                     | 390             |
|                                               |                                                                                     | 1x28,0        | 84                | 196         |                                                                            | 2                      | 333                   | 450               | 382                                                     | 650             |
|                                               |                                                                                     | 1x28,0        | 112               | 252         |                                                                            | 2                      | 417                   | 530               | 563                                                     | 957             |
| eccentrically reinforced<br>tension specimens |  | 1x22,0        | 44                | 110         | ribbed bars                                                                | 8                      | 156                   | 280               | 180                                                     | 306             |
|                                               |                                                                                     | 1x22,0        | 66                | 154         |                                                                            | 4                      | 285                   | 425               | 300                                                     | 511             |
| reinforced concrete beams                     |  | 1x22,0        | 44                | 150         | ribbed bars                                                                | 3                      | 244                   | 300               | 214                                                     | 363             |
|                                               |                                                                                     | 2x22,0        | 44                | 75          | ribbed bars                                                                | 2                      | 162                   | 250               | 151                                                     | 256             |
|                                               |                                                                                     | 3x22,0        | 44                | 54          | ribbed bars                                                                | 2                      | 113                   | 185               | 131                                                     | 223             |

Tab. 1 Crack Spacings

In accordance with the approach in the CEB/FIP requirements [7] for the calculation of the average crack spacing  $a_m$  the following equation resulted for ribbed bars and mats:

$$a_m = 2 (\bar{u} + k \cdot s) + 0,1 \frac{d_a}{\mu} ;$$

$k = 0,1$  for ribbed bars



This yields a satisfactory agreement with the experimentally determined crack spacings (see table 1) for design of buildings. The effective zone of the reinforcement was established as in Fig. 6 according to Gergely and Lutz [8] as with the eccentrically reinforced tension specimens no dependence of the crack spacing on the height of the specimen was stated.

Furthermore a differentiation between tensile and flexural load could be neglected with respect to the CEB/FIP requirements so that it can generally be calculated with  $k = 0,1$  for ribbed bars. The average crack spacings multiplied by 1.7 are in approximate accordance with the largest crack spacings of the experiment (see table 1).

With smooth mats the bond strength is generally transferred by the transverse reinforcement only. Cracks always form at the transverse reinforcement because of the smaller concrete cross section in this area. Forces can be transferred into the concrete until only two transverse reinforcement bars are left between two cracks. Thus the maximum crack spacing corresponds to the double spacing of transverse reinforcement. With this spacing only one transverse reinforcement bar is left between two cracks and no more forces can be introduced into the concrete.

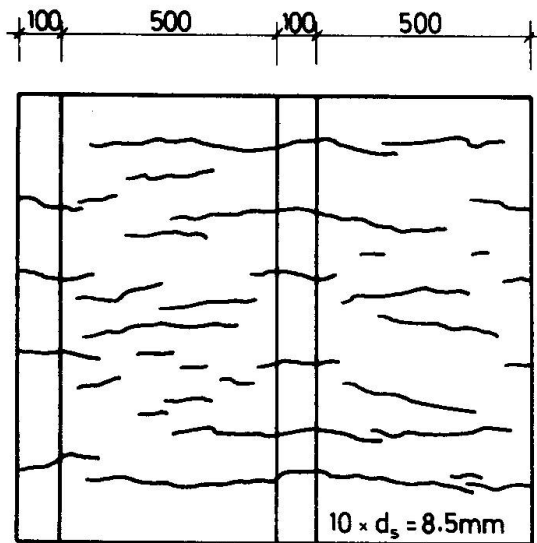


Fig. 4 Crack pattern of reinforced concrete panel B 19 ( $Q = 0$ )

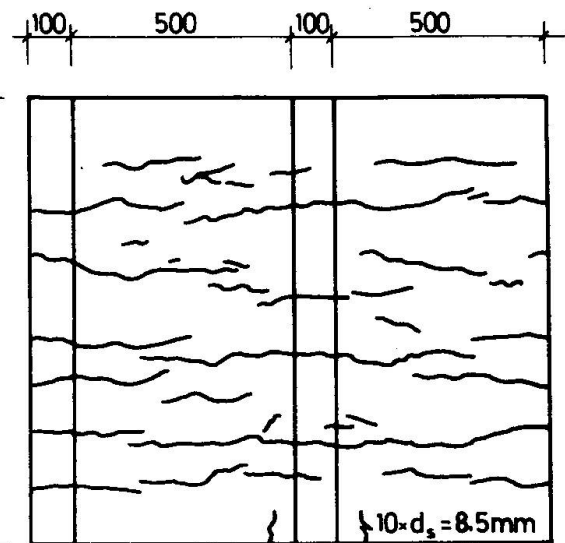


Fig. 5 Crack pattern of reinforced concrete panel B 11 ( $Q = 0,6 \beta_c$ )

### 3. CRACK SPACINGS

It was shown in [4] that the deformations of a reinforced concrete specimen mainly occur in the cracks and that the average crack  $w_m$  widths can be calculated with the equation stated in [7]:

$$w_m = \epsilon_m \cdot a_m \quad (2)$$

The maximum crack width results in approximately:

$$w_{\max} = 1,7 \cdot w_m \quad (3)$$

### 4. DEFORMATION BEHAVIOR OF CENTRICALLY AND ECCENTRICALLY REINFORCED TENSION SPECIMENS

The principal correlations between average strain over the cracks and external load related to the steel cross sectional area resulting from monotonically increasing and cyclic loading are shown in Fig. 7.

In order to describe the stress-strain behavior of centrically reinforced concrete specimens loaded in tension neglecting shrinkage of concrete the following idealization for initial loading.

$$\epsilon_m = \frac{\sigma_s}{E_s} \sqrt{1 - \left(\frac{\sigma_s^1}{\sigma_s}\right)^2} \quad (4)$$

yielded a good agreement. This idealization is also valid for reinforced concrete panels under transverse load and for eccentrically reinforced tension specimens, if the effective concrete cross section according to Fig. 6 and the central concrete tensile strength are taken as a basis for the determination of the steel stress  $\sigma_s^1$ . The largest average strain at top load which stabilizes after a certain number of repeated loads can be approximately recorded with:

$$\epsilon_{m,\max}^o = \frac{\sigma_s^o}{E_s} \sqrt{1 - \left(\frac{\sigma_s^1}{\sigma_s^o}\right)^2} \quad (5)$$

At complete unloading a residual strain of about

$$\epsilon_r = 0,2 \cdot 10^{-3} \quad (6)$$

remains in the specimen. This is independent of the number of load cycles.

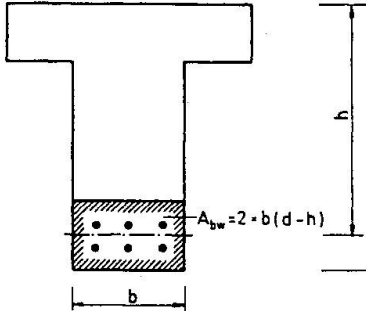


Fig. 6 Effective zone of reinforcement (acc. to [8])

It was stated with all tests that after approximately 5000 load cycles or after a 6-months permanent load no considerable deformation increases occurred.

The initial stress state caused by shrinkage results, related to the load beginning, in larger average strains compared with the specimens without shrinkage. The stress-strain behavior of specimen S07 shown in Fig. 8 makes clear that the average strains can exceed those of the pure steel.

In case the shrinkage influence is considered for the determination of the steel stress in the cracked cross section directly after the first crack, the average strain related to the initial load can also be calculated according to Eq. (4) using  $\sigma_s^1$ ,schw

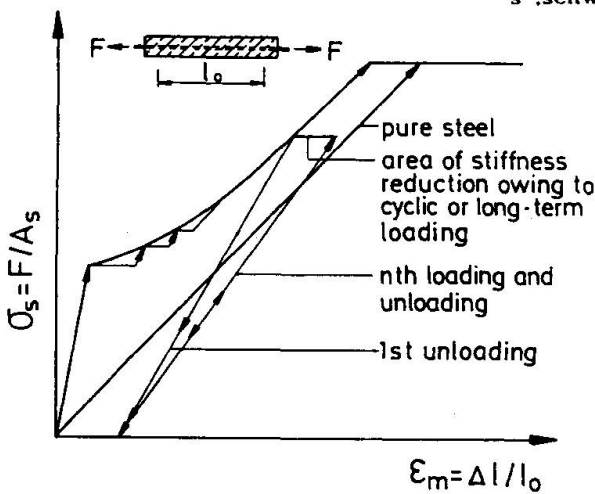


Fig. 7 Stress-strain relationships for axially loaded reinforced concrete

$$\epsilon_m = \frac{\sigma_s}{E_s} \sqrt{1 - \left(\frac{\sigma_{s,\text{schw}}^1}{\sigma_s}\right)^2} \quad (7)$$

The largest average strain increase at top load which stabilizes after a certain number of load cycles results to approximately

$$\Delta \epsilon_{m,\text{schw}}^o = \frac{\sigma_s^o}{E_s} \left[ \sqrt{1 - \left(\frac{\sigma_s^1}{\sigma_s^o}\right)^2} - \sqrt{1 - \left(\frac{\sigma_s^1}{\sigma_s^o}\right)^2} \right] \quad (8)$$

as with specimens without shrinkage.

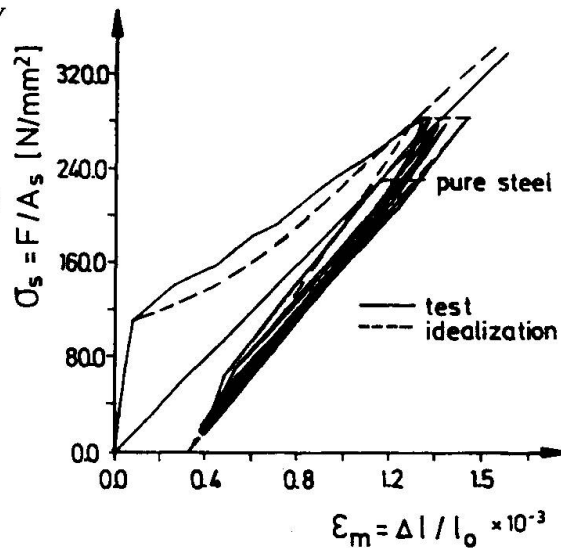


Fig. 8 Experimental stress-strain relationships of specimen S07 under shrinkage and idealization



At complete unloading of the specimens a residual strain of

$$\epsilon_r = 0,2 \cdot 10^{-3} + \epsilon_m^o + \Delta\epsilon_{m,chw}^o - \epsilon_s^o \quad (9)$$

is left.

## 5. NOTATIONS

|            |                                                                                                                                                                     |                            |                                                                                                             |
|------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------|-------------------------------------------------------------------------------------------------------------|
| b          | = width                                                                                                                                                             | $\sigma_s^1$               | = steel stress in concrete cross sectional area directly after first crack                                  |
| d          | = total height                                                                                                                                                      | $\sigma_s^o$               | = maximum steel stress at cyclic or long-term loading                                                       |
| h          | = height from center of gravity of tension reinforcement to compression edge                                                                                        | $\sigma_{s,chw}^1$         | = steel stress in crack cross sectional area directly after the first crack considering shrinkage influence |
| $d_s$      | = diameter of reinforcement bar                                                                                                                                     | $\epsilon_s^o$             | = steel strains at top load                                                                                 |
| $\bar{u}$  | = smallest concrete cover of reinforcement at pure tension<br>= distance between bottom side of reinforcement bar and tension edge at eccentric tension and flexion | $\epsilon_m$               | = average strain                                                                                            |
| s          | = distance of reinforcement bar (see table 1)                                                                                                                       | $\epsilon_m^o$             | = average strain at top load                                                                                |
| $A_s$      | = steel cross sectional area                                                                                                                                        | $\epsilon_{m,max}^o$       | = largest average strain owing to cyclic or long-term loading                                               |
| $A_{bw}$   | = effective concrete cross section (see Fig. 6)                                                                                                                     | $\epsilon_r$               | = residual strain in specimen after unloading                                                               |
| $A_{b,n}$  | = area of concrete (net)                                                                                                                                            | $\Delta\epsilon_{m,chw}^o$ | = strain increase owing to repeated loads at top load considering shrinkage influence                       |
| $\mu$      | = $\frac{A_s}{A_{bw}}$                                                                                                                                              | a                          | = average crack spacing                                                                                     |
| F          | = force                                                                                                                                                             | $a_{max}$                  | = maximum crack spacing                                                                                     |
| $E_s$      | = modulus of elasticity of steel                                                                                                                                    | $w_m$                      | = average crack width                                                                                       |
| $\beta_c$  | = cylinder crushing strength                                                                                                                                        | $w_{max}$                  | = maximum crack width                                                                                       |
| $\sigma_s$ | = steel stress                                                                                                                                                      |                            |                                                                                                             |
| Q          | = lateral pressure                                                                                                                                                  |                            |                                                                                                             |

## References

- [1] KOLLEGER, JOHANN; GÜNTHER, GERD; MEHLHORN, GERHARD: Zug- und Zug-Druckversuche an Stahlbetonscheiben. Forschungsbericht Nr. 1 aus dem Fachgebiet Massivbau, Gesamthochschule Kassel, 1986. \*
- [2] GÜNTHER, GERD; MEHLHORN, GERHARD: Versuche an Beton- und Stahlbetonscheiben unter ein- und zweiachialer Beanspruchung. Forschungsbericht Nr. 9 aus dem Fachgebiet Massivbau, Gesamthochschule Kassel, erscheint 1989. \*
- [3] GÜNTHER, GERD; MEHLHORN, GERHARD: Untersuchungen zur Mitwirkung des Betons zwischen den Rissen für monoton steigende und schwellende Belastung. Forschungsbericht Nr. 3 aus dem Fachgebiet Massivbau, Gesamthochschule Kassel, 1987. \*
- [4] GÜNTHER, GERD; MEHLHORN, GERHARD: Beeinflussung des Spannungs-Dehnungs-Verhaltens von Stahlbetonzugkörpern durch die Art der Rißöffnung und den durch Schwinden verursachten Eigenspannungszustand. Forschungsbericht Nr. 8 aus dem Fachgebiet Massivbau, Gesamthochschule Kassel, 1989. \*
- [5] GÜNTHER, GERD; MEHLHORN, GERHARD: Langzeitverformungsverhalten von Stahlbetonkörpern. Forschungsbericht Nr. 8 aus dem Fachgebiet Massivbau, Gesamthochschule Kassel, 1989. \*
- [6] GÜNTHER, GERD; MEHLHORN, GERHARD: Verformungsverhalten von Stahlbetonbalken unter wiederholter Belastung. Forschungsbericht Nr. 5 aus dem Fachgebiet Massivbau, Gesamthochschule Kassel, 1988. \*
- [7] CEB/FIP-Mustervorschrift für Tragwerke aus Stahlbeton und Spannbeton. Deutscher Ausschuß für Stahlbeton, 1978.
- [8] GERGELY, PETER; LUTZ, L.A.: Maximum Crack Width in Reinforced Concrete Flexural Members. In: Causes, Mechanics and Control of Cracking in Concrete, SP-20, American Concrete Institute, Detroit, 1968, S. 87-117.

\* Summary, notations, and legends to pictures both in German and in English.