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Autor: Vestroni, Fabrizio / Capecchi, Danilo
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Aspects of the Application of Structural Identification in Damage Evaluation

Problèmes d'application de l'identification structurelle dans l'évaluation des dommages

Ueber die Anwendung der strukturellen Identifikation für die Beurteilung von Strukturschäden

Fabrizio VESTRONI

Prof. of Dynamics of Structures
University of L'Aquila
L'Aquila, Italy



Fabrizio Vestroni, born 1945, received his civil engineering degree at the University of Rome, where he was research assistant. He became professor of Structural Engineering in 1986. He is author of several papers on nonlinear dynamics of elastic and hysteretic structures, earthquake engineering, linear and nonlinear structural identification.

Danilo CAPECCHI

Research Assistant
University of L'Aquila
L'Aquila, Italy



Danilo Capecchi, born 1948, graduated as a civil engineer from the University of Rome, Italy. For ten years he was employed by the Italian Nuclear Energy Committee Commission (ENEA). Since 1984 he is Researcher at the University of L'Aquila. His activities concentrate in linear and nonlinear dynamic analysis and structural identification.

SUMMARY

Many important features of the actual behaviour of a large civil engineering system can be revealed by experimental tests. In this paper the fundamental role of structural identification in processing the measured data to obtain a maximum amount of information about the state of a structure is discussed. The attention is focused on the aim of using response measurements to evaluate structural damage; changes of modal quantities are taken into account along with parametric physical model.

RESUME

De nombreuses caractéristiques du comportement réel d'un ouvrage important peuvent être connues au moyen d'essais. Dans ce mémoire on discute le rôle fondamental de l'identification structurelle dans l'utilisation des données expérimentales pour obtenir le maximum d'informations sur l'état de la structure. En particulier on se réfère au problème de l'évaluation du dommage structural et de sa distribution.

ZUSAMMENFASSUNG

Wichtige Eigenschaften des wirklichen Verhalten grösserer Strukturen können durch experimentelle Versuche ermittelt werden. Die vorliegende Arbeit erörtert die fundamentale Rolle, welche der strukturellen Identifikation bei der Auswertung von Messdaten im Hinblick auf die höchstmögliche Information über den Zustand der Strukturen zukommt. Besondere Beachtung findet ferner die Benutzung experimenteller Messungen zur Beurteilung von Strukturschäden.



1. INTRODUCTION

In recent times great advances have been made in the technology and the design procedures of structural systems; very important, complex structures have been built and many others are now under construction.

Notwithstanding the great improvement in analytical techniques for predicting the response of structures and for assessing their safety, the need is increasingly felt for experimental knowledge of the real response of the structure, even limited to particular external conditions.

The results of experimental analysis can be utilized directly to compare specific response quantities - static or dynamic - with those ones analytically predicted and in some instances it is possible to assess the extent to which the real structure reflects the designed structure. But, in the case of large systems mainly, the conditions which can be analyzed are quite particular and limited in number.

Better use can be made of the experimental results by referring to a suitable interpretative model: system identification is the most correct and convenient way of relating experimental and analytical results [1, 2]. It is only quite recently that system identification has been applied to structural engineering. The aims of structural identification are several; models and techniques thus differ depending on the aims, though these have mainly been:

- a) to obtain a mathematical model derived solely from experimental results [3];
- b) to improve a structured mathematical model by adjustment of the prior values of its parameters [4].

Of late the role of structural identification has been extended to furnish information on the damage state of the structure. This new objective has emerged as a result of general interest now prevalent in evaluating the safety of existing buildings and in developing of a correct policy of periodical checks on structural integrity [5, 7].

Damage accumulates continuously in structural systems during their life under service loads and environmental conditions. In order to assure system safety and serviceability it is possible to plan for periodical experimental tests or continuous monitoring of meaningful dynamic response quantities so as to reveal the possible occurrence of damage and to quantify the extent.

The employment of a systematic and rational approach for processing experimental results is always recognized to be very useful, as in the following two extreme case:

- a) when structures and/or environmental conditions are very complex and any damage must be detected indirectly, since a complete care inspection of all members cannot possibly be performed
- b) when the loss of integrity of the structural elements is so evident that it can be ascertained by visual inspection and local damage can be

described also quantitatively, but correlation with the global state of the structure is desirable and an updated model consistent with the new state has to be produced in order to redevelop the calculations under the design loadings.

This paper examines the methods for using experimentally measured data typically referred to in structural identification with a view to their possible application in assessing the damage. Various conditions are considered and attention is centred around methodologies for evaluating structural damage by analyzing modal quantities as functions of time. The use of a parametric physical model is proposed as an interpretative 'robust' model to register changes in frequencies and eigenvectors; related quantities are selected to identify the site of damage in the structure and the extent thereof.

2. RESPONSE QUANTITIES AS DAMAGE INDICATORS

Various conditions can occur in which it is necessary to evaluate structural damage. In certain cases, in the absence of experimental information and visual inspection, knowing the intensity of external loading, the decrease in load carrying capacity can be predicted analytically by means of mechanical models derived from the theory of structures or by expert systems based on knowledge of the behaviour of the class of structure concerned.

In other cases the damage is detected by visual inspection and it is possible to infer a measure of its extent by adopting different available techniques which mainly provide information on the amount of repair work to restore the original integrity rather than furnish an evaluation of the loss of the load carrying capability as a property on its own.

As regards indicators for measuring damage if experimental results are available, the main important difference is bound up with whether these results are relevant to the phenomenon at the time when the damage occurs or whether they are relevant to periodical tests with damage occurring when the structure was not being monitored.

The first case is the typical when the structure is subjected to rare exceptional loading of intensity greater than the mean value expected, for example strong ground motion, which results in several members being stressed beyond the elastic limit. From the recorded acceleration responses it is possible to obtain reliable displacement histories and identify the hysteretic behaviour of a group of generalized one-dimensional systems. Knowledge of these response quantities permits quantification of the extent of the damage revealed by inelastic behaviour, if a measure criterium is established.



By referring to a hysteretic response, several damage functions have been introduced; these are based on maximum normalized deformation, stiffness degradation, cumulative inelastic deformation and dissipated energy. Most of these indices were selected mainly on the base of test results of isolated members or simple assemblages. Hence there are many uncertainties involved when they are applied to complete structural system. This is also due to the circumstance that only very limited data are available on entire structures which have experienced inelastic response under controlled conditions, such as monitored real structures subjected to violent external loadings, experiments on full-size structural systems in the real world or on structural models of various scales in the laboratory [8].

Different damage parameters and different approach must be used when there are no histories available for the time during which the damage occur or when the damage is associated with accumulated phenomena over the course of time - such as local overstress, materials decay, fatigue effects, modifications of boundary conditions - rather than with a particularly strong excitation. Information on the overall state of a structure can be obtained by using the change in modal quantities as the damage index. The effectiveness of such indexes has been discussed by many authors and it is now under study [8 - 12].

By comparing low-level vibration tests before and after the damaging event marked correlations has been observed between the change of structure period and the overall damage suffered during an earthquake [12]. Similar correlations are being sought for other structures, isolated elements, buildings, bridge and monuments as a result of various different damaging events. The correlation is less clear with respect to the earthquake case since damage is not distributed according to the loading pattern but is frequently restricted to just a few parts of the structure and more accurate experimentation is needed to detect it.

Other quantities to be considered in damage detection are the natural modes, which generally appear to be less effective than frequencies, as it is more expensive to measure them. However, by appropriate manipulation the modifications in the mechanical characteristics of the structure can be located and - to some extent - quantified. This is discussed in greater detail ahead.

The change in damping too can provide information on structural damage, but though a possible relationship between the latter and the observed index modifications has been sought the results have been unsatisfactory. This is due not least to the fact that for undamaged structures and low-level vibrations the damping factor varies with amplitude; moreover its evident deviation seems to be affected only by marked modifications in the structure.

The feeling today is that the mere use of modal parameters - mainly



frequencies and modes - can furnish only general information on the damage state of the structure; the deviation indicate that damage has occurred but not its local extent or underlying causes thereof. Effectiveness could be improved through combined use of a careful mathematical approach and sophisticated experimental techniques that can reveal even small changes in modal parameters. Moreover the measurement of modal quantities - which has been developed considerably in mechanical and aeronautical engineering applications - can now be more easily performed for civil structures too because improved experimental equipment enables good results to be obtained even through very low-level vibrations, as for example the forced response to environmental forces.

3. IDENTIFICATION OF PHYSICAL MODELS

A closer examination is now made of the more general case related to evaluation of the damage state of a structure by analysing changes in dynamic response; though the procedure is clear from a conceptual point of view several operative aspects need to be dealt with in greater detail. Since comparison of the response of damaged and undamaged structure has to be developed through a mathematical model, the first step in the procedure is to establish a model of structural system in the virgin state. For civil system, particularly for large structures, the measured data are quite limited since the response in all the principal degrees-of-freedom cannot be recorded; the same is true for the external forces applied. It would thus appear advisable to adopt a physical interpretative model, for instance a finite element model, which makes use of all prior information on the mechanical behaviour of the system, while uncertainties on some assumptions in the model description concern only the values of a number of physical parameters [13, 14]. The latter are determined in such a way as to minimize the difference between measured and predicted modal quantities.

Let $h(x)$ be the function which relates the vector $(n \times 1)x$ of parameters with the observed response quantities z . The following relation between z and x is assumed:

$$z = h(x) + n \quad (1)$$

where n is vector noise, assumed to be stochastic gaussian with x_0 the mean value and covariance $\bar{\Sigma}_n$ independent of x . Due to the presence of errors both in the mathematical model and in the experimental data, there is little sense in forcing the model to match the data. It is more correct in this context to follow a Bayesian approach according to which the best estimate \hat{x} of parameters x is that which maximizes the probability of occurrence of x given measured quantities \bar{z} . The value of \hat{x} is furnished by the minimum of the function::

$$l(x) = [\bar{z} - h(x)]^T \bar{\Sigma}_n^{-1} [\bar{z} - h(x)] + (x - x_0)^T \bar{\Sigma}_n^{-1} (x - x_0) \quad (2)$$



which plays the role of the objective function in the problem. It is made up of two terms, the first takes into account the difference between measured z and predicted $h(x)$ quantities weighted by the inverse of the covariance matrices Ξ_h^{-1} .

Since the response quantities depend nonlinearly on the parameters, the minimization of $l(x)$ is sought by a numerical iterative procedure. For large structures this is not a simple task, because at every iteration step the direct eigenvalues problem has to be solved.

To reduce the amount of computational effort two different techniques have been developed to obtain an approximate relationship between modal quantities and the assumed parameters around the reference solution corresponding to the base values x^0 of the parameters.

The first technique is based on achieving an approximate solution by means of an interpolation quadratic function. Each component $h_i(x)$ is approximated by:

$$h_i(x) = h_i(x^0) + \sum_1^n a_{ik} (x_k - x_k^0) + b_{ik} (x_k - x_k^0)^2 \quad i = 1, 2, \dots, m \quad (3)$$

where n is the number of parameters and m the dimension of z . The coefficients a_{ik} and b_{ik} are determined by imposing the passage through three points, the solution furnished by the model for three values of the parameters, base x_0 , upper x_u and lower x_l values [15].

The second technique is based on an asymptotic expansion of the solution in the Taylor series up to the second order; each component $h_i(x)$ is expressed as:

$$h_i(x) = h_i(x_0) + \Xi_j \epsilon_j h_{ij} + \Xi_j \Xi_1 \epsilon_j \epsilon_1 h_{ij1} + 0(\epsilon^3) \quad (4)$$

where ϵ_j is the increment of the parameter x_j and the unknown coefficients h_{ij} , h_{ij1} are obtained by solving successive linear systems derived according to a perturbative scheme up to second order [16].

In both cases an approximated model is obtained, reliable in the description of the relation between response and parameters in the neighbourhood of assumed value x_0 . The relation is expressed in closed-form which allows x to be minimized by making use of efficient minimization algorithms that require knowledge of the value of the objective function and of derivatives.

By substituting eq. (3) or (4) into eq. (2) to express $h(x)$ and by denoting with

$$H_1 = \left(\frac{\partial h(x)}{\partial x} \right)_{x=x_1} \quad (5)$$

the sensitivity matrix of the problem, the minimization of $l(x)$ is obtained by a recursive formula:

$$\hat{x}_{i+1} = \hat{x}_i + (\Xi_x^{-1} + H_1^T \Xi_n^{-1} H_1)^{-1} \{ H_1^T \Xi_z^{-1} [z - h(\hat{x}_i)] + \Xi_x^{-1} (x_0 - \hat{x}_i) \} \quad (6)$$

derived according to a modified Gauss-Newton procedure.

4. USE OF PARAMETRIC MODELS IN DAMAGE EVALUATION

The parametric physical model, which has been identified to describe the behaviour of the structure in its original undamaged state, is taken as a reference to evaluate the modifications of structural properties, mainly, through an identification process following periodical tests [6].

In principle changes in all three modal quantities, frequencies, mode shapes and damping ratio can provide information on damage but actually only the first two appear really suitable for the purpose. If reliable data on changes in frequencies and in eigenvectors too are available, they can be useful for localizing the damage; it can be shown that for each mode u_r the distribution of elastic energy V can be written:

$$V_r = \frac{1}{2} u_r^T K u_r = \frac{1}{2} \sum_i \left(\sum_j u_{jr} K_{ji} \right) u_{ir} = \frac{1}{2} \sum_i \left(\sum_j k_{ij} u_{jr} \right) u_i = \frac{1}{2} \sum_i f_{ir} u_{ir} \quad (7)$$

where the contribution at each node i is indicated. Since the mass is assumed to be constant and it is frequently diagonal, any deviation in the elastic energy can be more conveniently ascertained from the kinetic energy, since $V_r = T_r$; it follows:

$$T_r = \frac{1}{2} \omega_r^2 u_r^T M_r u_r = \frac{1}{2} \omega_r^2 \sum_i m_{ii} u_{ir}^2 \quad (8)$$

where, unlike eq.(7), the only unknown quantities are the measured data ω_r and u_{ir} . It can be argued from eq. (8) that the analysis of the variation of the kinetic energy distribution among different degrees-of-freedom will indicate where the stiffness decrease is concentrated. Theoretical and numerical applications would enlighten the limits and effectiveness of this procedure.

Another problem is evaluation of the amount of damage; in this case it is necessary to calculate the quantities which are assumed to be related to the damage. They could be individual coefficients of the stiffness matrix, various coefficients which affect the stiffness matrix of subsystems of the structure, and also some physical parameters which describe the behaviour of a certain number of 'critical' elements.

This is a typical complex inverse problem which does not always have a unique solution; a priori information based on experience is necessary to provide some constraints for the problem. Within this context it is important to make a preliminary analytical study on the sensitivity analysis of meaningful parameters with respect to certain vibration properties which change significantly with damage. The approximate techniques previously outlined are very useful for performing a simple economic sensitivity analysis.



5. OPTIMAL CHOICE OF OBSERVED QUANTITIES

Structural identification of a structure is based on a number of observed response quantities which are selected by the investigator. The number is usually limited to minimize the cost of instrumentation, but for given figure the problem is that of fixing the optimal location for sensors so as to obtain the best estimate of the unknown parameters [17, 18].

It would be possible to solve this problem by trial and error techniques using different locations for sensor and adopting as the optimal solution that which gives the best parameters estimate. This procedure is quite time-consuming and the results are unsatisfactory in the absence of an efficient choice criterium.

Some of these difficulties can be overcome by adopting a method which does not call for the perform of any structural identification.

Let the measurable quantities of a structure be m , but only $p < m$ are the observed quantities collected in the vector z . The relation between z and the parameters x is then written as generalized expression of eq. (1)

$$z = S[h(x) + n]$$

where $m \times 1$ vector $h(x)$ furnishes all the measurable quantities predicted by the model while S is a $(p \times m)$ matrix which selects the p quantities z measured among the m ones observable. In the Bayesian approach the a-posteriori probability of x , given z , is:

$$p(x|z) = \text{const } e^{-\{ [z - Sh(x)]^T S \bar{\Sigma}_n^{-1} S^T [z - Sh(x)] + (x - x_0)^T \bar{\Sigma}_x^{-1} (x - x_0) \}} \quad (9)$$

The estimated values \hat{x} are such as to maximize the probability function (9) which, in the neighbourhood of \hat{x} , has a gaussian distribution with mean value \hat{x} and covariance matrix:

$$\bar{\Sigma} = [H^T S^T (S \bar{\Sigma}_n S^T)^{-1} S H + \bar{\Sigma}_x^{-1}]^{-1} \quad (10)$$

where H is the sensitivity matrix defined by eq. (5) evaluated in $x = x_0$.

The choice of measured quantities is optimal when a suitable norm of $\bar{\Sigma}$ is minimum. Since $\bar{\Sigma}$ can be ill-conditioned, several numerical difficulties are avoided by considering $\bar{\Sigma}^{-1}$. Therefore the solution is achieved by the condition:

$$\max_S \{ H^T S^T (S \bar{\Sigma}_n S^T)^{-1} S H + \bar{\Sigma}_x^{-1} \} \quad (11)$$

By assuming the trace as a norm and $\bar{\Sigma}_n$ as diagonal, then

$$\max_{g_i} \sum_{i=1}^m g_i \sum_{j=1}^n \left(\frac{\partial h_i}{\partial x_j} \right)^2 + \text{tr}(\bar{\Sigma}_x^{-1}) \quad (12)$$



where $\text{diag} \{g_i\} = S^T (S \bar{E}_n S^T)^{-1} S$. (13)

The use of eq.(12) is very simple; each contribution of the m observable quantities to E^{-1} is calculated and the set of p greatest elements is selected.

Since $h(x)$ is nonlinear, the technique outlined above is only approximate and depends on the value ascribed to x for evaluating H according to eq. (5). It was observed that the results of the optimal choice are less influenced by x ; in any case the method can be employed to select a few different solutions which are very useful to the investigator.

6. CONCLUSION

In order to have fairly reliable knowledge on the real behaviour of large structural systems it is suggested that experimental tests be performed, some just after the construction is finished and others during the service life. The purpose of the former is to check the validity of the mathematical model adopted in the design and to update this model for further investigations, while that of the latter is to detect degradation of mechanical characteristics of the structure. Within this general context the very important role of system identification is stressed in this paper. In particular attention is focused mainly on the use of parametric physical models as suitable filters for obtaining more specific information on the state of damage from the changes that occur in some quantities of the dynamic response.

Deviations in frequencies and modes are certainly the consequence of damage that has occurred elsewhere in the structure. A strongly interpretative model to a certain extent permits damage to be localized and the amount evaluated by analysing in which way deviations in response quantities can be explained by deviations in selected characteristics of the model. Two different methods of performing a simple, approximate sensitivity analysis are illustrated; these can be conveniently used to ascertain the response quantities - or the combination thereof - which are the most affected by damage. Finally, a priori criterium to define an optimal choice of the quantities to be measured - and as a consequence the location of sensors in the structure - is discussed.

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