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Serviceability Limit State of Deflections in Reinforced Concrete Elements

Vérification de l'état limite de déformation de poutres en béton armé

Nachweis der Verformungen von Stahlbetonbauteilen im Gebrauchszustand

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SUMMARY

A general proposal to check serviceability limit state of deflections in reinforced concrete beams is presented, including : allowable deflections, deflection control using allowable span-to-depth ratios, simplified formulae and remarks on using an integral general method.

RÉSUMÉ

Une méthode générale est proposée pour la vérification de l'état limite de déformation de poutres en béton armé. Elle tient compte des déformations admissibles, des flèches admissibles suivant le rapport portée-hauteur, de formules simplifiées et de considérations pour l'usage de la méthode intégrale générale.

ZUSAMMENFASSUNG

Ein allgemeines Verfahren für den Nachweis der Verformungen von Stahlbetonbalken unter Gebrauchslasten wird präsentiert. Es berücksichtigt die zulässigen Verformungen und die Ueberprüfung der Durchbiegung in Funktion der zulässigen Schlankheiten. Vereinfachte Formeln und Betrachtungen werden verwendet.



1. GENERAL PROPOSAL.

This proposal has been derived from a parametric study [1] on the different variables involved in the phenomenon using a general method.

Different theoretical models have been analyzed and the CEB [2] proposed model has been adopted because of the following reasons:

- It permits the calculations of instantaneous and long-term deflections of statically determined and indetermined elements with good accuracy [1].
- It permits to take into account the principal parameters.
- It proposes a simple moment-curvature relationship and allows further simplified derivations.

1.1 Allowable deflections.

In Fig. 1 proposed allowable deflections are shown. Three different deflections must be controlled depending on the reasons for their limitation.

For those elements not supporting or attached to partitions or another construction likely to be damaged by large deflections, total deflections (f_{tot}) must be considered.

For those elements supporting or attached to partitions or another construction likely to be damaged by large deflections, three deflections must be considered: total deflection, incremental deflection after partitions are constructed (f_{act1}) and incremental deflection after partitions are constructed without considering live load deflections (f_{act2}).

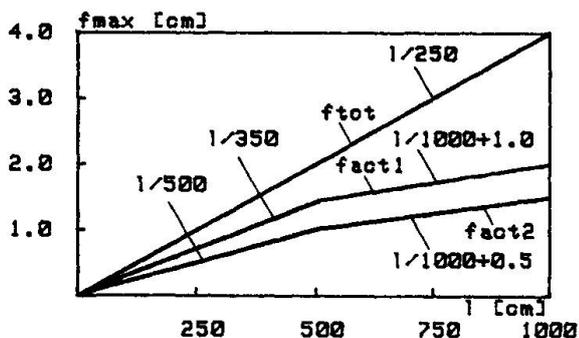


Fig. 1 Allowable deflections.

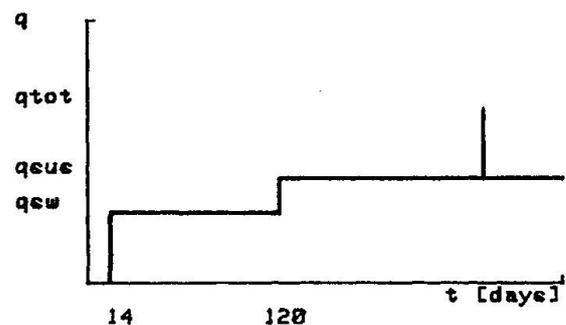


Fig. 2 Load history

1.2 Allowable span-to-depth ratios.

In order to avoid computing deflections, allowable span - to - depth ratios are proposed.



$$l/d = < 16. \alpha_3 / (\alpha_1 \cdot \alpha_2 \cdot \alpha_4 \cdot \alpha_5) \tag{1}$$

Coef.	f_{tot}	f_{act1}	f_{act2}
α_1	$0.33 \cdot \alpha + 0,80$	$0.46 \cdot \alpha + 0,72$	$2.85 \cdot \alpha - 0,71$
α_2	$0.17 \cdot \varphi + 0.56$	$0.19 \cdot \varphi + 0,52$	$0.24 \cdot \varphi + 0,41$
α_3	$0.90 + 3.82/Q$	$1.10 + 0.78/Q$	$Q \leq 19.6 \text{ KN/m} : 1.21 + 5257/Q^4$ $Q > 19.6 \text{ KN/m} : 1.07 + 0.01 \cdot Q$
α_4	1.0	$l \leq 5 \text{ m} : 1.0$ $l > 5 \text{ m} : 1.0 + 0.09 \cdot (l-5)$	$l \leq 5 \text{ m} : 1.0$ $l > 5 \text{ m} : 1.0 + 0.08 \cdot (l-5)$

Table 1

$\alpha = g/(g + q)$: permanent to service load ratio.

φ = creep coefficient.

$Q = g + q$: service load. (KN/m)

α_5 : correcting coefficient.

$\alpha_5 = 1.0$ hinged-hinged

$\alpha_5 = 0.7$ hinged-fixed

$\alpha_5 = 0.6$ fixed-fixed

If steel quality is higher than 400 MPa then span-to-depth ratio must be divided by:

$$0.40 + f_{yk} / 703 \text{ (with } f_{yk} \text{ in MPa)} \tag{2}$$

This proposal considers the principal parameters involved in the phenomenon. Quality of concrete is of minor importance so therefore it can be dismissed.

The proposed formulae were developed from a parametric analysis using the general adopted method and the following hypothesis:

- To define span-to-depth ratios it is necessary to estimate the deflection and the knowledge of all data. The amount of reinforcement is important but when span-to-depth ratios are computing it is still unknown. This criteria was developed considering the reinforcement of the member as the strict one obtained from the ultimate limit state.

- This criteria has been developed for rectangular cross sections and uniform distributed loads. According to this assumption and the previous one, it is possible to define span-to-depth ratios as a function of loads instead of using the amount of reinforcement.

For other load types it is necessary to define an equivalent uniform distributed load.

For T cross sections the same criteria can be used if the neutral axis depth is in the flange.

- Fig. 2 shows assumed load history.

- Allowable deflections defined in section 1.1 have been used.



1.3 Simplified Formulas.

For computing instantaneous deflections, usual formulae for elastic deflections and an effective stiffness expressed in terms of curvature are proposed, taking into account the effects of cracking and reinforcement of the member.

$$f_i = \beta \cdot (1/r)_{\max} \cdot l^2 \quad (3)$$

$$\text{if } M < M_{cr} : (1/r)_{\max} = M / (E_c \cdot I_b) \quad (4)$$

$$\text{if } M > M_{cr} : (1/r)_{\max} = F \cdot M / (E_c \cdot I_{cr}) \quad (5)$$

$$F = 1 + (M_{cr} / M)^2 \cdot (I_{cr} / I_b - 1) \quad (6)$$

f_i : short-term deflection.

β : elastic coefficient depending on the type of load and support conditions.

$(1/r)_{\max}$: midspan maximum curvature, both for statically determined and indetermined structures.

l : span length.

M : midspan moment at stage in which deflection is computed.

M_{cr} : midspan cracking moment.

$$M_{cr} = 2 \cdot f_{ct} \cdot I_b / h \quad (7)$$

I_b : moment of inertia about centroidal axis, neglecting reinforcement

I_{cr} : moment of inertia of cracked section transformed to concrete.

E_c : modulus of elasticity of concrete. ($E_c = 5949 \cdot \sqrt{f_c}$ in MPa, according to EH-82 [3]).

f_c : compressive strength of concrete.

f_{ct} : modulus of rupture of concrete. ($f_{ct} = 0.626 \cdot \sqrt{f_c}$ in MPa, according to EH-82 [3]).

A multiplier coefficient (K_m) to estimate additional long-term deflections is proposed. This coefficient is defined graphically in Fig. 3, as a function of creep coefficient (ψ), tensile amount of reinforcement (ρ) and permanent service load ratio (α).

These values have been obtained for sections with the strictly necessary amount of reinforcement (tension and compression) according to the ultimate limit state. If the actual compression reinforcement is greater than required, K_m must be corrected by another coefficient F_r .

$$f_d = f_i \cdot K_m \cdot F_r$$

$$F_r = 1 / (1 + 0.58 \cdot \rho^* / \rho)$$

f_d : long-term deflection.

f_i : short-term deflection produced by permanent loads.

K_m : multiplier for additional long-term deflection.

F_r : correcting coefficient for superabundant compression reinforcement.

ρ : ratio of tension reinforcement. ($A_s / (b \cdot d)$)



ρ^* : ratio of superabundant compression reinforcement. ($A_c^* / b \cdot d$)
 A_s : area of tension reinforcement.
 A_c^* : superabundant area of compression reinforcement.
 n : modular ratio of elasticity. (E_s / E_c)

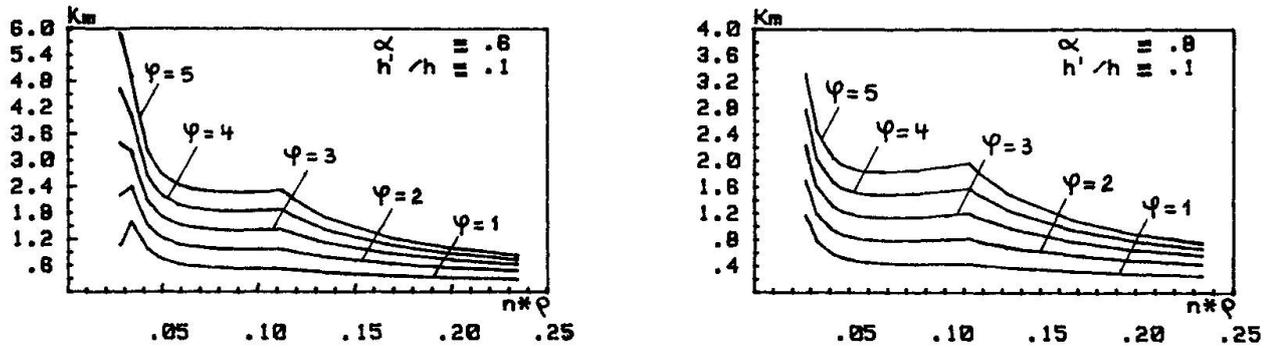


Fig.3 Multiplier coefficient K_m .

In Fig. 4 a comparison of computed short and long-term deformations using simplified formulae and CEB method is shown. For each different structural support conditions only beams with span-to-depth ratios greater than the allowable ones have been studied. For each beam the load history represented in Fig. 2 was adopted.

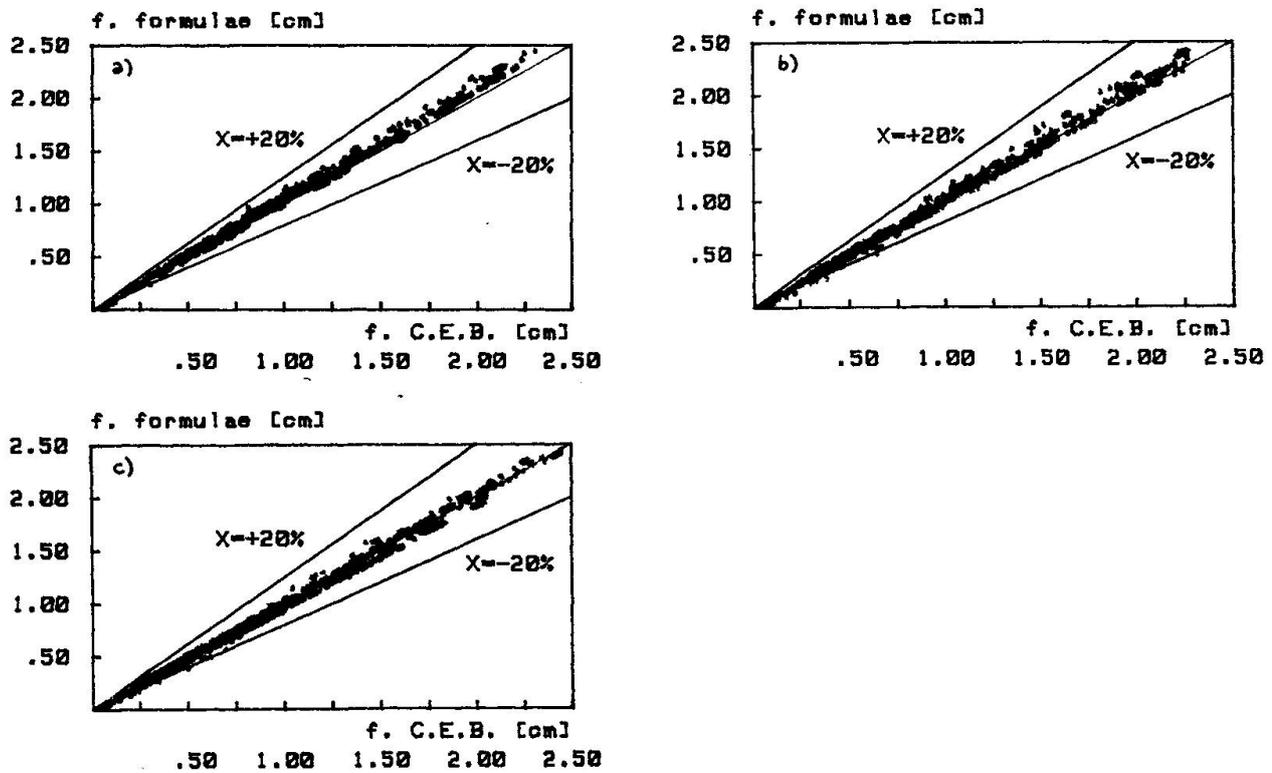


Fig.4 Comparison of computed short and long-term deformations using simplified formulae and CEB method. a) hinged-hinged beams. b) fixed-hinged beams. c) fixed-fixed beams.



1.4 General procedure.

In recent years, several proposals have arisen, regarding constitutive equations to represent both short and long term behaviour of concrete. Less work is done in the field of numerical influence of discretization of the structure on the results of non linear analysis. This problem is important when computing deflections especially for statically indetermined structures. References [4] [5] show this influence and suggest a new approach to consider it in a more accurate way.

2. FINAL CONSIDERATIONS.

More experimental research is needed to improve this proposal, especially regarding allowable deflections for members supporting non structural elements and their load history, principally during construction stage.

The authors know that the different hypothesis assumed may be submitted to revision, but consider that the general scheme of this proposal can be useful to check the serviceability limit state of deflections.

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