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**Autor:** Lenkei, Peter  
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## Deflections of Concrete Structures under Repeated Service Loads

Déformations des constructions en béton armé sous charges de service répétées

Verformungen von Stahlbetonbauten unter wiederholten Betriebslasten

### Peter LENKEI

Professor  
Hungarian Inst. Build. Science  
Budapest, Hungary



Peter Lenkel, born 1933, obtained his structural engineering degree from the Moscow Institute of Construction. For seven years he was involved in structural design. Now in a research institute, he is responsible for structural research. He holds the Chair for Studies in Reinforced Concrete at the Technical University of Budapest.

### SUMMARY

On the basis of the author's experiments – which are compared with a wide range of published data – the influence of several factors (strength of materials, characteristics of repeated load cycle, etc.) on the deflections of reinforced concrete beams is shown. A method of calculation with a code provision is proposed to determine the increase over the static deflections in function of the repeated load parameters, i.e. number of repetitions, ratio of maximum load to the ultimate static load, characteristics of the repetition cycle.

### RÉSUMÉ

L'effet de plusieurs facteurs – résistance des matériaux, caractéristiques du cycle de chargement répété, etc. – sur les déplacements des poutres en béton armé est présenté sur la base d'essais de l'auteur et comparé aux indications données dans la littérature. Une méthode de calcul avec les conditions de réglementation est proposée pour évaluer l'augmentation des déplacements statiques en fonction des paramètres du chargement répété, c'est-à-dire nombre de répétitions, rapport de la charge maximale à la charge limite statique, caractéristiques du cycle de répétition.

### ZUSAMMENFASSUNG

Anhand eigener Versuche, deren Ergebnisse mit denjenigen aus einer umfassenden Literaturdurchsicht verglichen wurden, wird der Einfluss mehrerer Faktoren (Materialfestigkeit, Belastungsspektrum, usw.) auf die Verformung von Stahlbetonbalken aufgezeigt. Der Beitrag enthält auch einen Vorschlag für die Bestimmung der Durchbiegungszunahme infolge wiederholter Last gegenüber der Durchbiegung unter statischer Last. Der Einfluss verschiedener Parameter wie Anzahl Lastzyklen, Verhältnis der Maximallast zur Bruchlast und Belastungsspektrum wird berücksichtigt.



## 1. INTRODUCTION

In the last decades the self weight of the concrete structures is decreasing due to the higher quality of materials used. At the same time the nature of actions on structures are changing too, much more repetitive actions are expected to act during the life time of the structure. These both effects are resulting in bigger importance in determining deflections at serviceability limit state under repeated loads.

A great part of the service loads are not permanent but changing, usually stochastically. Modelling this stochastic change by a regular repeated load is a good approximation and at the same time it is the easiest way of investigation too.

The deformations of r. c. structure subjected to repeated load depend on deformations of the constituent materials and of their connections, namely, on the bond deformations. According to experimental data the short time and creep deformation of the compressed concrete block mainly influences the deformations of r. c. elements. The deformations of the reinforcing steel have much less influence, because the steel is in the elastic range and the fatigue rupture of steel is usually a rigid one. At repeated service load level the bond deterioration does not take place yet, but the bond deformations should be accounted.

## 2. THE EXPERIMENTS

### 2.1 The experimental program

A series of 24 simply supported one span beams (Fig. 1.) were tested in 3 groups of 8 beams to determine the deformational response of r. c. beams to frequently repeated load. The span ( $L = 3,2$  m), the loading geometry (two forces at 600 mm from midspan), the cross-section ( $b \cdot h = 150 \cdot 300$  mm), the reinforcement ratio (tension  $\sim 0,8\%$ , compression  $\sim 0,24\%$ ), the selected concrete cube mean strength ( $f_{cc} = 30$  MPa) and the tension steel mean strength ( $f_{sy} = 500$  MPa;  $f_{st} = 600$  MPa), the sinusoidal form and the frequency of the load repetition (4–5 Hz) were kept constant.

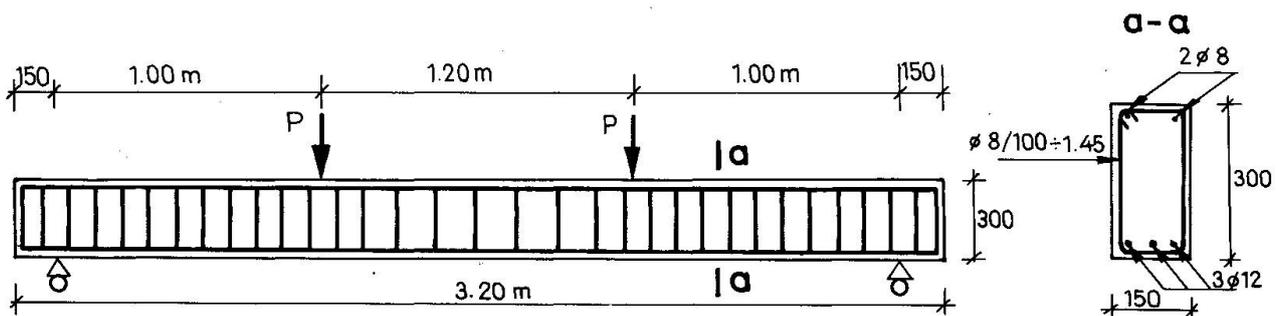


Fig. 1. The test beams

The variables were the pulsating load cycle parameters i. e. the ratio of the maximal load to the static ultimate load – 3 variables ( $\gamma = P_{max} / P_{ult} = 0,35; 0,6$  and  $0,85$ ) and the amplitude of the maximal and minimal load ( $\varphi = P_{min} / P_{max} = 0,1$  and  $0,5$ ). The static ultimate load  $P_{ult}$  was determined as mean value of the static rupture load of 3 beams for each group. The maximum number of repetitions ( $n$ ) was  $1,28 \times 10^6$ , but some beams of high maximum load level ( $\gamma$ ) and big load amplitudes ( $\varphi$ ) failed at a lower number of repetitions (Table 1.). Altogether 15 beams were tested under repeated load and 9 beams under static load.

The measurements of the material deformations, deflections and crack widths were tak-

en after  $1,25 \times 10^3$ ,  $2,5 \times 10^3$ ,  $5 \times 10^3$ ,  $10^4$ ,  $2 \times 10^4$ ,  $4 \times 10^4$ ,  $8 \times 10^4$ ,  $1,6 \times 10^5$ ,  $3,2 \times 10^5$ ,  $6,4 \times 10^5$ ,  $1,28 \times 10^6$  cycles, using intermediate static loading procedure. The cycle parameters of the dynamic load were controlled automatically.

$\varphi = \frac{P_{min}}{P_{max}}$	$\gamma = \frac{P_{max}}{P_{ult}}$	0,35	0,60	0,85
0,1		3 beams	3 beams *	4 beams *
0,5			2 beams *	3 beams

\* In one beam steel fatigue failure due to mechanical defect (not shown in Fig.2. and 3.)

Table 1. The test variables

At the age of 119-149 days the test beams were subjected to repeated loading, the mean concrete prism strength at this time varied between 33,1 and 35,9 MPa. Altogether 9 beams were tested under static and 15 beams under repeated loading procedure.

### 2.2 The experimental results

At the investigated service load levels a very good bond was registered between tension steel and concrete. The measured concrete and steel deformations, the crack pattern showed the predominating influence of the concrete compression block deformations on the beam deflections.

The ratios of deflections measured after  $n$  cycles of repetition ( $a_n$ ) to the deflection measured after the first load cycle ( $a_1$ ) are shown in Fig.2. as function of the number of repetitions.

The experiments showed increased deformations under repetitive loads of service level. The increase depends on the load cycle parameters namely the deflection increase is

- directly proportional with the number of load repetition  $n$ ;
- directly proportional with the relative load parameter  $\gamma = \frac{P_{max}}{P_{ult}}$
- directly proportional to the load amplitude, i. e. inversely proportional to the amplitude parameter  $\varphi = \frac{P_{min}}{P_{max}}$

It must be mentioned that high relative load level ( $\gamma = 0,85$ ) coupled with high ( $\varphi = 0,1$ ) and sometimes with lower ( $\varphi = 0,5$ ) load cycle amplitudes may result in fatigue failure ( Fig.2. ).

The irreversible parts of the deflections after  $n$  cycles ( $a_{n,t}$ ) in function of the  $n$  number of repetition are shown on Fig.3. The comparison with the previous figure makes it evident that the deflections increases consist of reversible and irreversible parts.

Both part are due to the sum of respective concrete, steel and bond deformations. The reversible part of the deflection is made up from elastic deformations ( both linear and non-linear), and the measured irreversible part mainly that of the plastic deformations. The time dependent (viscous and creep) deformations were largely out of scale of the reported experiments, because of two reasons: the test beam were relatively mature at testing and the loading procedure lasted only about four days for each beam.

According to the investigations of [2], the concrete deformations, due to the lower level of the cyclic load ( $P_{min}$ ) - which in deterministic term is the permanent load - can be expressed as usual creep deformations. The concrete deformations due to the cyclically changing load above this level  $P_{min}$  are the vibro-creep deformations, which are developing much faster in time than the usual static creep deformations. And these vibro-creep deformations depend largely on the cyclic load parameters  $\gamma, \varphi$

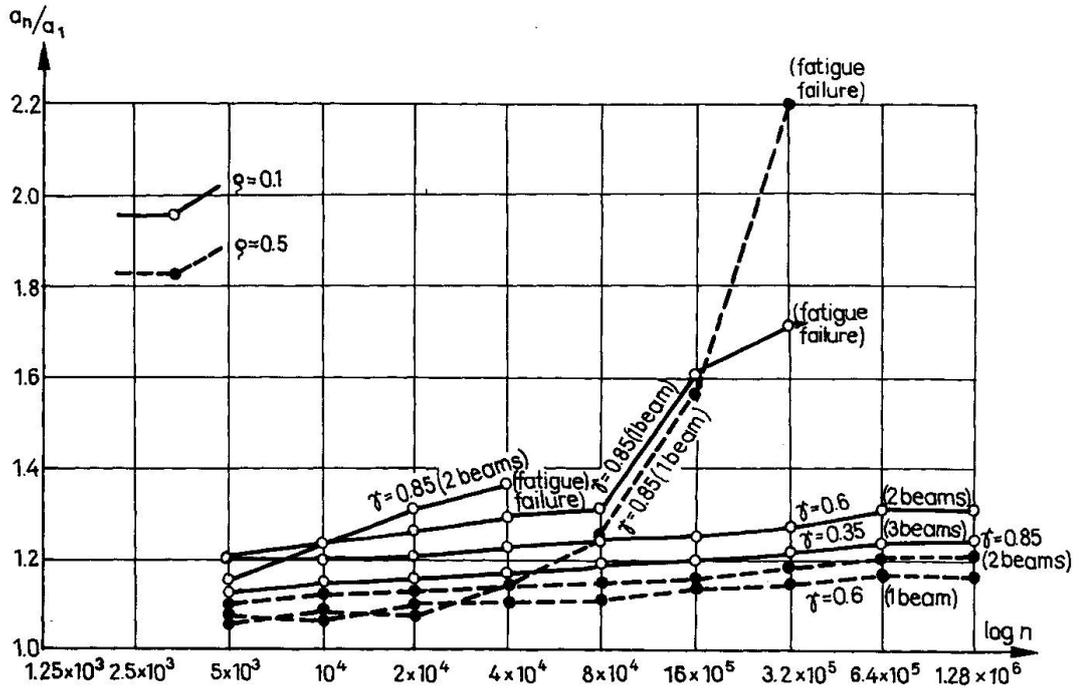


Fig. 2. The ratios of the deflections measured after n cycle ( $a_n$ ) to the deflections measured after the first load cycle ( $a_1$ )

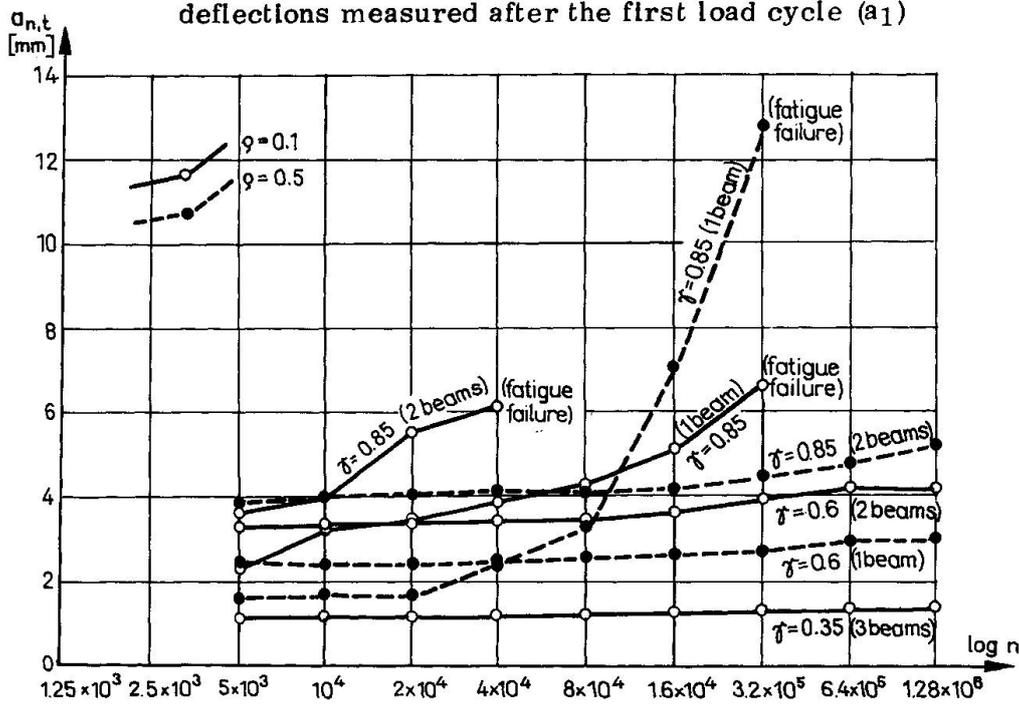


Fig. 3. The irreversible parts of the deflections ( $a_{n,t}$ )

and n similarly as deflections in the reported herein experiments.

According to [3] "A repeated load has a similar influence on the bond as on the deformation and failure behaviour of unreinforced concrete . . ." The reported test beams showed a comperable behaviour.



### 3. LITERATURE COMPARISONS

The problem of r. c. beam deflections under repeated loading has attracted a lot of investigators. The experimental results are largely alike in respect of the main influencing parameters  $\gamma$ ,  $\varrho$  and  $n$ , but the proposed methods for calculating deflections differ according to the codes and habits operating in a specific country and according to the authors' interests too.

#### 3.1 The proposed methods of calculating

The simplest and earliest method [1] assumes, that the cyclic load should be substituted by the maximal load ( $P_{\max}$ ) and the amplitude and the number of repetition should be converted into the duration of that load. This complicated conversion is problematic, due to the insufficient test data.

Another simple method [4] proposes to multiply the deflection under short-time maximal load ( $P_{\max}$ ) by a function of the load repetition number e.g. logarithm of  $n$ .

An accurate but complicated method [5] accounts for all the influencing factors, parameters of the load cycle separately in determining the deflections.

In [7] the deflections are determined as summ of the deflections caused by  $P_{\min}$  as a long time effect and from the deflections caused by the variable part of the load. The problem is that these two parts are interconnected.

The investigations in [6] and the method proposed by this paper determine the deflections increments as function of the complete set of load cycle parameters.

#### 3.2 The available test data

The parallel to the reported herein tests [6] carried out on beams of bigger cross sections and different reinforcement ratios, showed similar tendencies under repeated load deflections, and the registered relative deflections increases were in the same range.

The experiments in [4] were carried out on small and large beams without shear reinforcement in the constant moment region and with reinforcement ratios 0,8 - 1,0 %. The results of experiments showed considerable influence of the amplitude on the deflections increase, which were depending on the number of repetition being as maximum 35 % for  $n = 10^6$  and 80 % for  $n = 10^8$ .

Box beams up to  $10^5$  repetitions were investigated in [7]. The reinforcement ratios varied between 1 and 3 %. The increase of the measured deflections after repetition is not given, but reevaluating the results showed a bigger value (70 %) than in other tests. This could be the result of the comparatively small thickness of concrete walls.

In [8] T beams were tested up to  $3,6 \cdot 10^7$  cycles. The reinforcement ratio was  $\sim 1$  %, the  $\gamma \approx 0,85-0,9$ ,  $\varrho < 0,1$ . The final deflection increase was in the range of 50 % over the initial deflections.

### 4. RECOMMENDATIONS FOR CALCULATIONS

The available test data supported the two main hypotheses.

- the load cycle parameters  $\gamma = \frac{P_{\max}}{P_{\text{ult}}}$ ;  $\varrho = \frac{P_{\min}}{P_{\max}}$  and  $n$  have predominant influence on the deflection increase.
- the increase itself are in the range of 5-80 % in the domains investigated.



On this basis the following Table 2. is proposed for practical calculations and/or for codes of r. c. structures.

Table 2.

$\bar{q}$		0,4		0,6		0,8	
		0,1	0,5	0,1	0,5	0,1	0,5
number of repetition $n$	$2 \cdot 10^4 - 10^5$	0,15	0,075	0,2	0,1	0,3	0,15
	$2 \cdot 10^6$	0,3	0,15	0,4	0,2	0,6	0,3
	$10^7$	0,6	0,3	0,8	0,4	1,2	0,6

The numerical values in the table indicate the deflection increases over the static deflections of r. c. beams subjected to cyclic bending (e. g. if  $\bar{q} = 0,6$ ,  $\bar{q} = 0,5$  and the number of repetition  $n$  is  $2 \cdot 10^6$  than the total deflection will be 1,2 time the static one). The intermediate values are to be determined by interpolation.

This proposed method is rather simple, but it is very effective as a first approximation. The advantage of the method is, that can be used for all kinds of static deflection calculations.

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