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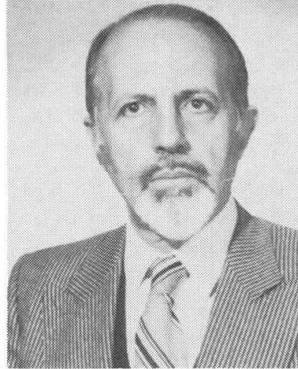
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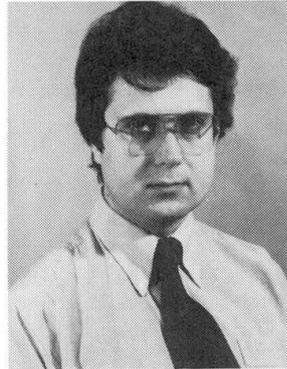
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**A Model for Local Crack Behaviour**  
Modèle de comportement à proximité des fissures  
Ein Modell für lokales Rissverhalten

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#### SUMMARY

A physical model is proposed for the study of the phenomenon of interface shear transfer in cracked RC. By accounting separately for the contribution of all shear resisting mechanisms at the crack interface, the model can predict both the monotonic and the reversed cyclic response of cracked RC interfaces subjected to a system of in-plane forces.

#### RÉSUMÉ

Un modèle physique est proposé pour l'étude du phénomène de transfert des efforts tranchants dans le béton armé fissuré. Prenant en compte séparément la contribution de tous les mécanismes de résistance à l'effort tranchant dans la zone de fissures, le modèle prédit aussi bien la réponse monotonique que cyclique dans les zones de fissures du béton armé soumis à un système de forces dans un plan.

#### ZUSAMMENFASSUNG

Ein physikalisches Modell zum Studium der Schubübertragung in Rissen im Stahlbeton wird vorgeschlagen. Dadurch, dass die Beiträge aller Schubübertragungsmechanismen getrennt erfasst werden, kann das Modell das statische wie zyklische Verhalten von Stahlbeton im Scheibenspannungszustand vorhersagen.



## 1. INTRODUCTION

The predominantly non-linear behaviour of R/C members has led many researchers into using the Finite Element Method as a versatile tool to assist in understanding their response. Any accurate R/C finite element analysis must consider the influence of non-linearities. For R/C members, the most pronounced material non-linearity is cracking.

The development of a computer program which accounts automatically for crack formation and propagation and which includes the shear transfer mechanisms across cracks has been undertaken at NTUA [1]. The Discrete Cracking approach has been used, since it was felt that it provides a means of incorporating in the analysis the complex characteristics of the crack interface shear transfer mechanisms and, at the same time, allows the detailed study of the phenomena of crack formation and propagation.

Modelling of the restraining action of the reinforcing bars crossing a crack and of their contribution to the phenomenon of shear transfer at the crack interface is achieved by means of a specially developed "shear transfer" element. Its stiffness has been derived on the basis of the results of a physical model for local crack behaviour. The essence of the model and its unique ability to simulate the interaction between all the shear resisting mechanisms at the crack interface were first presented in Ref. [9].

## 2. SHEAR TRANSFER IN R.C. CRACKS

The shear transfer mechanisms at a R/C cracked interface are identified in Fig. 1. With increasing shear displacements, the wedging action at the crack faces tends to push apart the concrete blocks on each side of the crack. Unless this dilatancy is restrained, very little shear can be transferred. In R.C. cracks the restraint is provided by external normal stresses (if compressive) and by the reinforcement crossing the crack.

From Fig. 1 it becomes apparent that because of the wedging action the reinforcement is tensioned while, at the same time, compressive forces are developed at the crack faces. This plying action of the reinforcement not only restrains crack opening but also enables the development of the friction mechanism at the faces of the crack. In addition, the imposed shear displacements activate the dowel mechanism of the bars which cross the crack. Thus in R.C. cracks, shear transfer results from the combined action of both, friction and the dowel action mechanisms.

## 3. PHYSICAL MODEL FOR LOCAL CRACK BEHAVIOUR

Natural concrete aggregates have an irregular shape and are randomly orientated within the cement paste. Because the strength of aggregates usually exceeds that of the hardened cement paste, cracks intersect the cement paste but run along the surface of the embedded larger aggregate particles, Fig. 2. Hence in modelling the topology of the proposed model, a "saw tooth" idealization appears to be adequate, at least for the purposes of setting up the constitutive equations of the model.

At this stage, no prediction needs to be made with regard to the magnitude of the angle  $\alpha$ , the length and the regularity of the crack teeth. The model can be calibrated with respect to these parameters, on the basis of available experimental results, and refinements can be made to portray their influence on the computed crack behaviour. For example, recent experimental results obtained at NTUA [2] indicate that a realistic estimate for the magnitude and the variation of the crack

angle  $\alpha$  under monotonic loading can be obtained from the relation :

$$w = k * s^{2/3} \quad , (k = 0.6 \text{ to } 0.7) \quad \text{Eq. 1}$$

Only local crack behaviour is to be simulated by both, the proposed "physical" model and the corresponding "shear transfer" finite element. Within the context of the Finite Element Method, shear transfer along the length of a discrete crack spanning through concrete elements, can be simulated by the inclusion of a number of "shear transfer" elements connecting the nodes at opposite faces of the crack line [1], [6].

On a local basis and depending on whether the opposite crack faces come into contact or not, the corresponding cracked regions will be termed in the following as "one - side closed" and "fully open", respectively.

### 3.1 Locally Fully Open Crack

Considering the cracked concrete section shown in Fig. 3(a), to an imposed magnitude of crack slip  $s$  corresponds a crack width  $w$ . If the two concrete blocks are assumed to displace as rigid bodies without rotation, then, all points along the upper crack face displace from their original (before cracking) positions along a vector parallel and equal to the vector  $AA'$ . The same is true for two points along the reinforcing bar, one at each face of the crack.

Following a kinematic analysis, vector  $AA'$  can be analysed into components parallel and transverse to the reinforcing bar axis, Fig. 3(a). Hence, the net bar elongation  $w'$  can be expressed in terms of the crack opening displacement  $w$  and the imposed shear displacement  $s$  as :

$$w' = w * \sin\theta - s(\tan\alpha * \sin\theta + \cos\theta) \quad \text{Eq. 2(a)}$$

and the net bar dowel displacement  $s'$  as :

$$s' = w * \cos\theta - s(\tan\alpha * \cos\theta - \sin\theta) \quad \text{Eq. 2(b)}$$

Knowledge of these steel displacements allows the determination of the corresponding bond  $B$  and dowel  $D$  forces developed by the reinforcement at the face of the crack (Fig. 3(b)) by means of formalistic sub-models that have been developed on the basis of extensive theoretical and experimental work [2].

A particular characteristic of this model is that to each shear slip displacement  $s$  corresponds a whole range of possible crack width  $w$  values, Fig. 4. However, out of these displacements there exists a unique one for which the overall static equilibrium of the element (see also Fig.5) :

$$f(\Delta V, \Delta N) = 0 \quad \text{Eq. 3}$$

is satisfied. The algorithm of Fig. 6 presents the successive steps for the computation of response of the proposed crack model to imposed shear displacements. Details can be found in [1] and [3].

### 3.2 Locally One-Side Closed Crack

In the case when the value of  $w$  required for equilibrium is equal to  $2s * \tan\alpha$ , the crack becomes "one-side closed". Now and because of the wedging action of the crack faces, the upper concrete block slides along the crack teeth, Fig. 7(a).



Following a kinematic analysis, the bar pull-out can be computed as :

$$w' = s(2\tan\alpha\sin\theta - \cos\theta) \quad \text{Eq. 4(a)}$$

and the dowel-displacement as :

$$s' = s(2\tan\alpha\cos\theta + \sin\theta) \quad \text{Eq. 4(b)}$$

In Fig. 7(b) the external and internal forces for the case of an "one-side closed" crack are identified. Comparison of this model with the "fully open" one reveals that in the former, two additional actions, namely, friction and concrete contact forces participate in shear resistance.

Because of the dependance of friction stresses on both the normal stresses acting on the interface and the magnitude of the imposed frictional slip, the magnitude of the friction force  $F$  can be determined by means of an appropriate frictional constitutive law (see e.g.[7]).

As shown in the algorithm of Fig. 8, for the computation of the response of an "one-side closed" crack model, a trial-and-error numerical procedure is followed for the determination of the values of friction force  $F$  and concrete contact force  $R$ , which fulfill the overall static equilibrium of the crack element.

#### 4. LOCAL CRACK MODEL FOR MESH REINFORCED CONCRETE

Based on the same principles and techniques, a similar model has been developed [1] [4] for the study of the response of cracked interfaces in concrete elements reinforced with a rectangular mesh, Fig. 9.

#### 5. NUMERICAL APPLICATIONS

For the application of the proposed physical "Local Crack" model to the study of both, monotonic and reversed cyclic shear transfer along cracked interfaces, a computer program was developed. For any given increment of slip  $\Delta s_i$ , the program automatically determines whether the crack is "fully open" or "one - side closed". Using the algorithms of Figs. 6 and 8, the shear force  $V_i$  corresponding to the imposed crack slip  $s_i = s_{i-1} + \Delta s_i$  is computed. By means of this program, the response of cracked concrete elements reinforced with steel bars and subjected to monotonic or reversed cyclic loading histories was studied.

The dramatic increase in shear carrying capacity of a R.C. element (of length 200 mm and width 100 mm, reinforced with two parallel bars) as a result of the increase in bar inclination  $\theta$  from 45 to 135 degrees is shown in Fig. 10(a). Both the stiffness and the shear carrying capacity increase abruptly after closing of the crack (i.e. one-side closed model) and the engagement of the much stiffer friction mechanism. The improved control of crack width as the inclination of the bar increases is shown in Fig. 10(b). On a qualitative basis, the results of Fig. 10 compare very favourably with Walraven's experimental results (8).

Axial force plays a major role in determining crack behaviour, Fig. 11, since, depending on its magnitude and direction, participation of the friction mechanism in carrying a significant portion of the externally imposed shear may be accelerated, delayed or even completely annihilated (e.g. for large tensile forces). The significance of bar diameter is shown in the same figure. Larger bar diameter implies larger participation of the bond and the dowel mechanisms in resisting the imposed shear at the cracked interface.



For an element reinforced with a single bar, reversed cyclic loading resulted to a severe reduction in stiffness as early as the second half of the first cycle, while the response became very soft at low load levels, Fig. 12(a). Since the stiffness of all involved mechanisms reduces with cycling, a continuous increase of crack width was observed, Fig. 12(b).

Less pronounced appeared to be the influence of bar inclination on the shear carrying capacity of a cracked element (of length 200 mm and width 100 mm) reinforced with two layers of mesh reinforcement, Fig. 13. Nevertheless, as  $\theta$  increased, better control of crack opening was achieved. Here again, significant appeared to be the influence of both the bar diameter and the axial load, Fig. 14.

## 6. CONCLUSIONS

The physical modelling of a reinforced crack, supported by detailed sub-models of the intervening shear transfer mechanisms, is a versatile analytical tool in describing the structural behaviour of particular interfaces (e.g. precast connections) and of cracks through R.C. elements in general.

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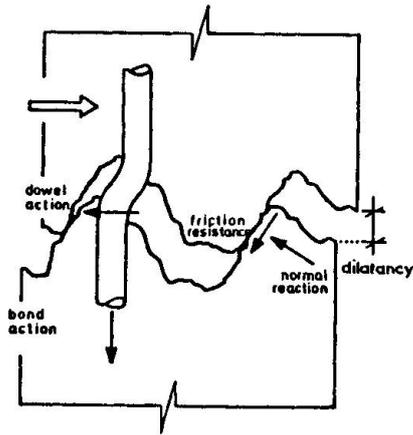


Fig. 1: Shear transfer mechanisms at the crack interface.

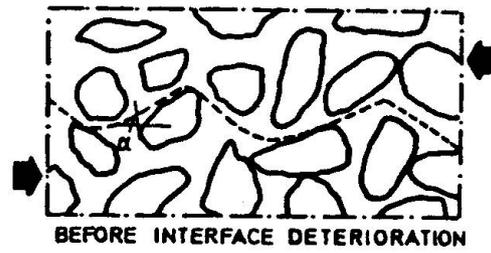
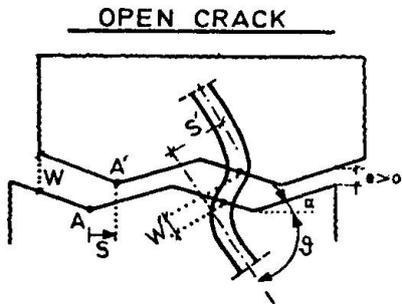


Fig. 2: Formation of the "concrete teeth"



KINEMATIC CONDITIONS:  $W = f(W, S, \vartheta)$   
 (a)  $S = f(W, S, \vartheta)$

FORCE EQUILIBRIUM:

$$f_h(D, B, V, N, \vartheta) = 0$$

$$f_v(D, B, V, N, \vartheta) = 0$$

Fig. 3: "Open" crack model

- EXTERNAL FORCES  $N, V$
- MOBILISED RESISTANCES  $D = D(S')$   
 $B = B(W)$

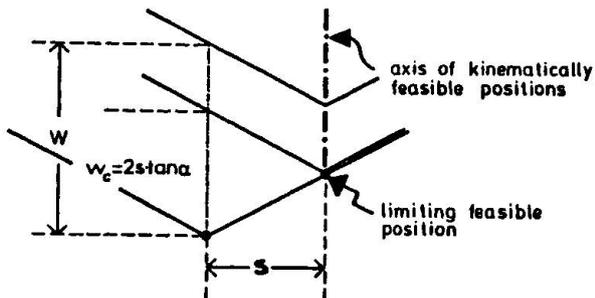
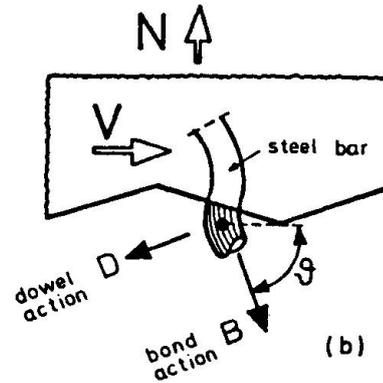


Fig. 4: Feasible equilibrium positions for the "Open" crack model

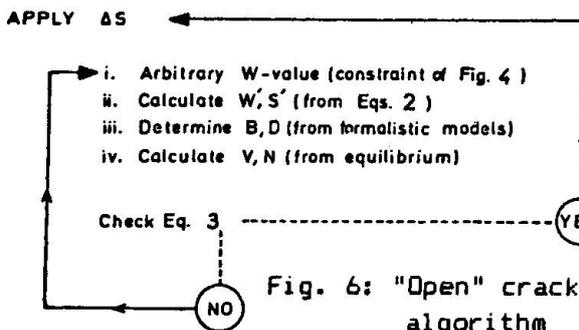
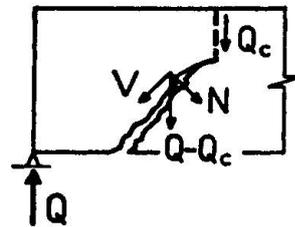


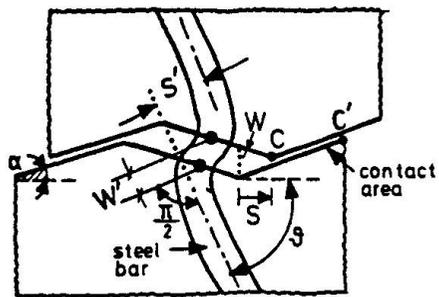
Fig. 6: "Open" crack algorithm



$$f(\Delta V, \Delta N) = \Delta V - \Delta N = 0$$

Fig. 5: Differential relationship for the equilibrium of internal forces along an inclined crack

**ONE SIDE CLOSED CRACK**



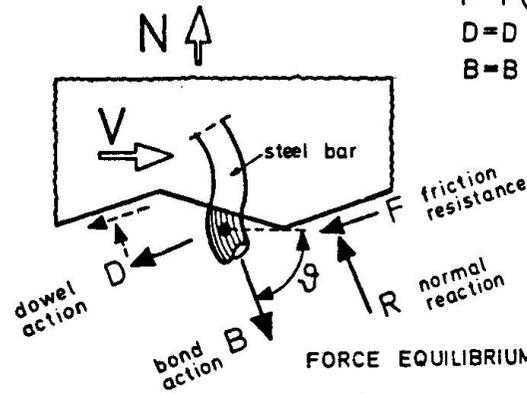
KINEMATIC CONDITIONS:

$$\begin{aligned} \tilde{w} &= s(2 \tan \alpha \cdot \sin \vartheta - \cos \vartheta) \\ \tilde{s} &= s(2 \tan \alpha \cdot \cos \vartheta + \sin \vartheta) \end{aligned}$$

(a)

- EXTERNAL FORCES N, V
- MOBILISED RESISTANCES R

$$\begin{aligned} F &= F(R, S) \\ D &= D(S') \\ B &= B(W) \end{aligned}$$



FORCE EQUILIBRIUM:

$$\begin{aligned} (b) \quad f_h(D, B, F, R, V, N, \vartheta) &= 0 \\ f_v(D, B, F, R, V, N, \vartheta) &= 0 \end{aligned}$$

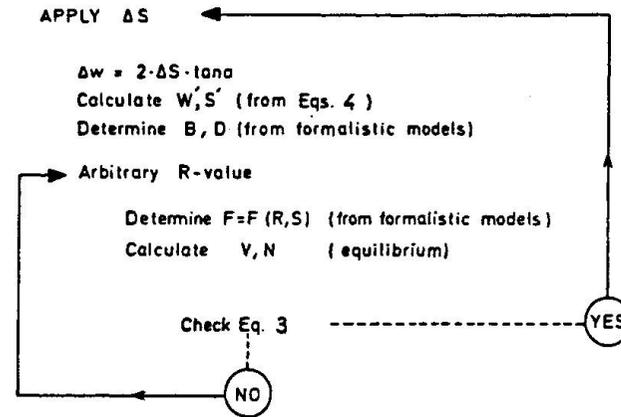


Fig. 8: "One-Side Closed" crack algorithm

Fig. 7: "One-Side Closed" crack model

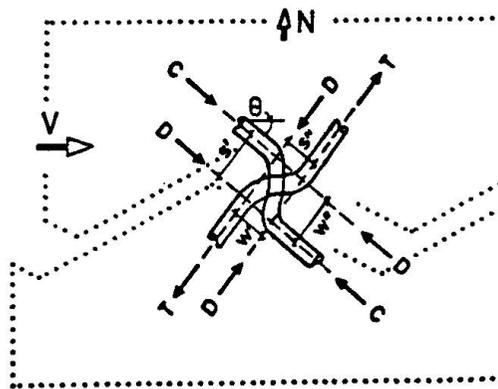
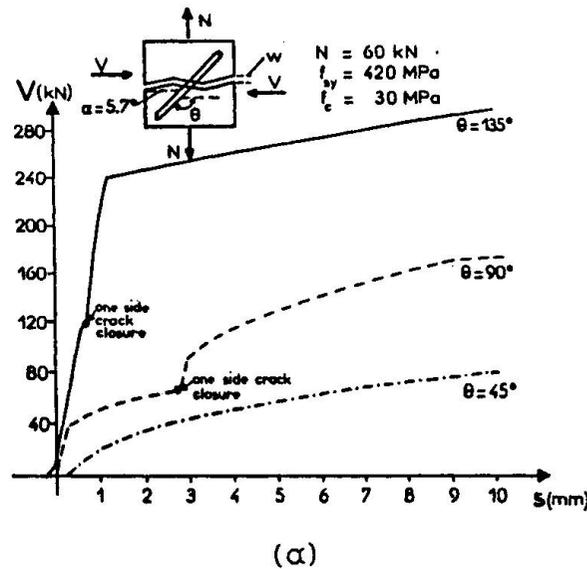
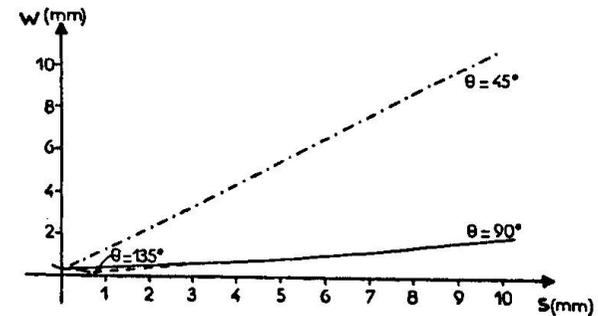


Fig. 9: Modeling of shear transfer through cracks in mesh R.C.



(a)



(b)

Fig. 10: Influence of the variation of bar inclination  $\theta$  on : (a) the response characteristics of a reinforced crack (b) the corresponding crack width  $w$

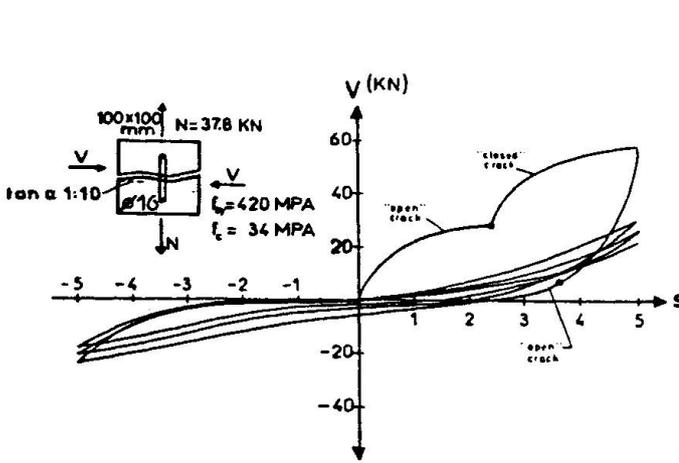


Fig. 12(a): Crack behaviour under reversed cyclic loading

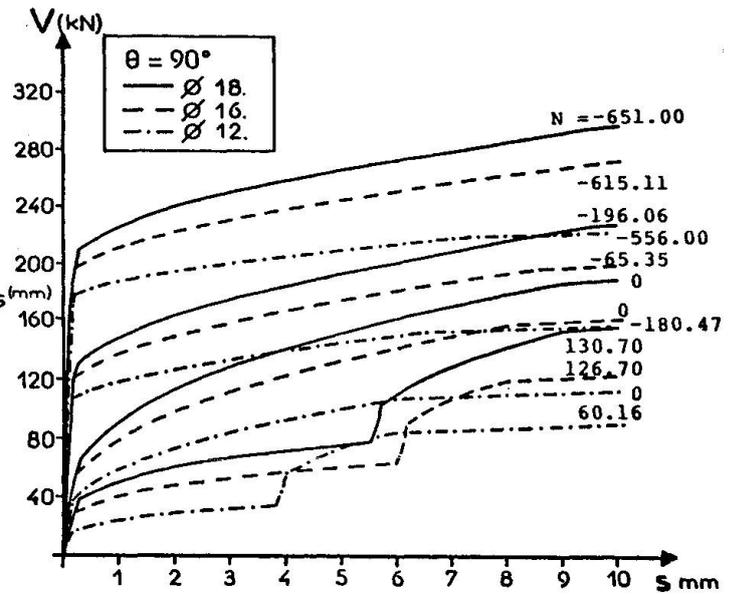


Fig. 11: Influence of the variation of bar diameter and axial force on the response characteristics of a reinforced crack

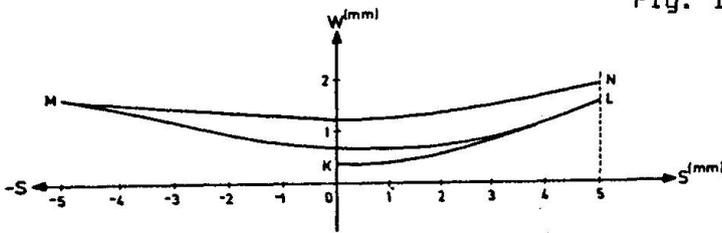


Fig. 12(b): Increase of crack width due to reversed cyclic loading.

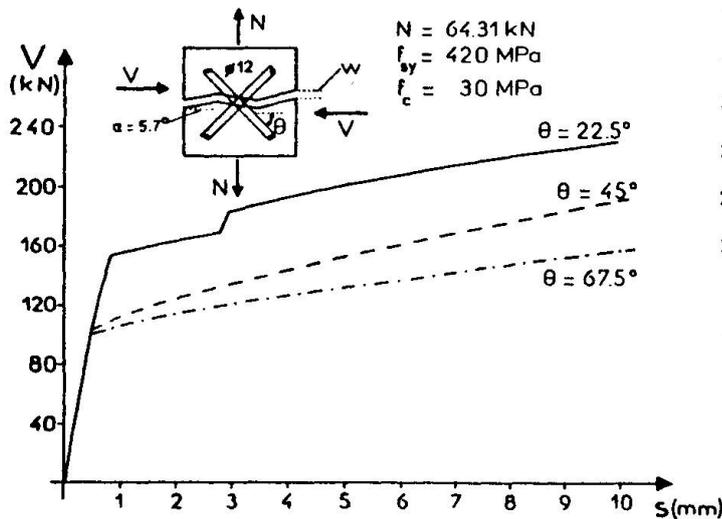


Fig. 13: Influence of the variation of bar inclination  $\theta$  on the response characteristics of mesh reinforced cracks

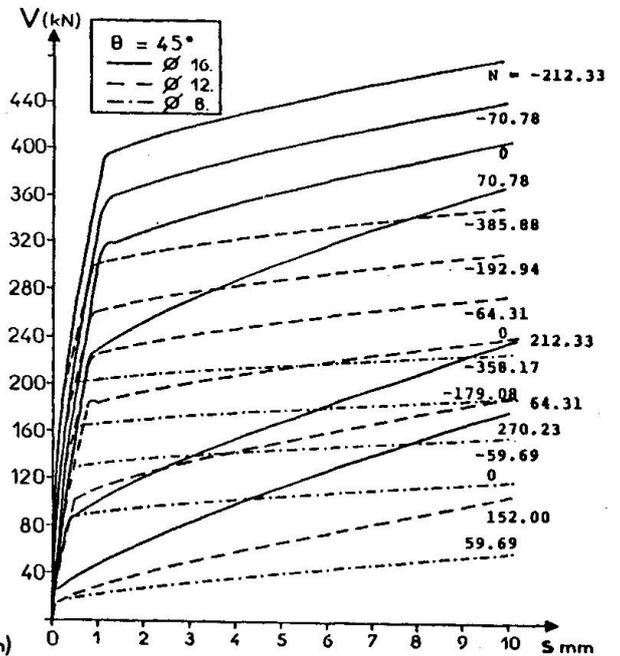


Fig. 14: Influence of the variation of axial force and bar diameter on the response characteristics of mesh reinforced cracks.