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## **Stress-Strain Based Inelastic Earthquake Response Analysis of Reinforced Concrete Frame Structures**

Analyse de la réponse sismique inélastique des structures en béton armé

Unelastisches Erdbebenverhalten von Stahlbetonrahmentragwerken

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### **SUMMARY**

This paper presents a method of stress-strain based inelastic earthquake response analysis of RC frame structures with varying axial forces. The material non-linearity of a RC element is evaluated by using a so-called 'fiber model' based on stress-strain relation. The accuracy of constitutive laws is examined by comparing of analytical and experimental moment-curvature relations. Earthquake response is calculated by step-by-step integration of the equation of motion. The method gives the most accurate estimation of inelastic earthquake response of RC structures.

### **RÉSUMÉ**

Cet article présente une méthode d'analyse, basée sur le rapport entre la contrainte et la déformation, de la réponse sismique inélastique des structures en béton armé, analyse qui tient compte des forces axiales variables. La non-linéarité matérielle d'un élément en béton armé est évaluée avec un modèle appelé 'modèle fibre', reposant sur le rapport contrainte-déformation. La précision de ce rapport est estimée à partir de la comparaison des relations moment-courbure obtenues de façon analytique et expérimentale. La présente méthode assure la plus haute précision de l'évaluation de la réponse sismique inélastique des structures en béton armé.

### **ZUSAMMENFASSUNG**

Im vorliegenden Beitrag wird eine Methode der auf der Spannungs-Dehnungs-Beziehung beruhenden nicht-linearen Erdbeben-Respons-Analyse von Stahlbetonrahmentragwerken bei variierenden Axialkräften gegeben. Die stoffliche Nichtlinearität eines Stahlbetonelements wird durch den Gebrauch eines sog. 'Faser-Modells' erfasst, das auf der Spannungs-Dehnungs-Beziehung basiert. Die Genauigkeit der konstitutiven Gesetze wird durch den Vergleich zwischen analytischen und experimentellen Momenten-Krümmungs-Beziehungen bestätigt. Die Methode liefert die genaueste Einschätzung des nichtlinearen Erdbeben-Respons der Stahlbetontragwerke.



## I INTRODUCTION

With advance of construction technology and increase of transportation, large RC structures, such as cable-stayed PC girder bridges, arch bridges and high-rise buildings, have become feasible and practical, however, seismic safety of the structures are highly focused because of their social and economical importance.

Among them, seismic safety of RC frame towers of large cable-stayed bridges and RC columns of high-rise buildings is one of the most critical design problems in Japan. These structural members shall bear not only large bending moment but also high and time varying axial forces due to earthquake response of structures.

Recently, in designing RC structures the limit state design concept is becoming reasonable and popular in which the material non-linearity of an RC element needs to be considered. In the previous structural analysis of RC frame structures, material non-linearity were often assessed on the assumption of constant axial forces. In the case of statically indeterminate RC frame structures, however, the change of axial forces is an important factor to evaluate the safety of the whole structure. It is extremely complex and difficult to take account of variational axial forces in the conventional modeling[1] [2] of RC beam elements.

In this paper, a method of stress-strain based inelastic earthquake response analysis of RC frame structures with varying axial forces is proposed. Accuracy of the stress-strain models of concrete and steel is examined from the comparison of analytical and experimental moment-curvature relation of the cross section. A few numerical examples of earthquake response of RC structures are presented.

## II NON-LINEAR ELEMENT STIFFNESS

### 1. Hysteretic rules of concrete and steel

Based on past experimental studies of stress-strain relations of structural materials under generalized cyclic loading, the stress-strain models for concrete and steel fibers are formulated including main parameters influencing these relations.

The Muguruma-Watanabe's rule[3] is modified to represent stress-strain relations of confined concrete fiber elements. In the model, concrete confinement levels[4], tension stresses, failure in tension, plastic strains, crush of concrete, compressive failure and stiffness degradation are described with nine different paths, five of which are previous path history dependent.

The Meneggoto-Pinto's rule[5] is adopted to represent stress-strain relation of steel fiber elements, which includes Baushinger effect, plastic strain and isotropic strain hardening for arbitrary strain history.

The adopted hysteretic rules of concrete and steel are schematically illustrated in Fig.1 and Fig.2. Details of the rules are presented in the reference[7].

## 2. Computation of non-linear element stiffness

Non-linear stiffness of a beam element is computed in the following algorithm based on the beam theory.(See Fig.3) Three degrees of freedom(See Fig.4) are set at both end of a beam element.

- 1) To divide a beam element into some parts(Sub elements) along the axis line.
- 2) To compute axial strain  $\varepsilon_a$  and curvature strain  $\phi$  at each dividing point from displacements at the ends of the element with the assumption that axial force is constant and bending moment varies linearly in the element.
- 3) To divide a section(Interface elements) at each dividing point into some parts (Fibers).
- 4) To compute strain  $\varepsilon_i$  at the location  $y_i$  of each Fiber using the assumption that a section remains flat.(Eq.(1))

$$\varepsilon_i = \varepsilon_a + \phi y_i \quad (1)$$

- 5) To obtain tangential stiffness  $E_i$  corresponds to  $\varepsilon_i$  at each Fiber using the aforementioned hysteretic rules.
- 6) To compute sectional stiffness  $[K_{i.e.}]$  by integrating  $E_i$  over the Interface element.(Eq.(2))

$$\begin{Bmatrix} \Delta M \\ \Delta N \end{Bmatrix}_S = [K_{i.e.}] \begin{Bmatrix} \Delta \phi \\ \Delta \varepsilon_a \end{Bmatrix}_S, \quad \text{where } [K_{i.e.}] = \begin{bmatrix} \sum A_i E_i y_i^2 & \sum A_i E_i y_i \\ \sum A_i E_i y_i & \sum A_i E_i \end{bmatrix} \quad (2)$$

- 7) To compute stiffness of Sub elements using the assumption of constant axial force and linearly varying bending moment in the element.
- 8) To compute stiffness of a beam element by integrating stiffness of Sub elements over the total length of the element.

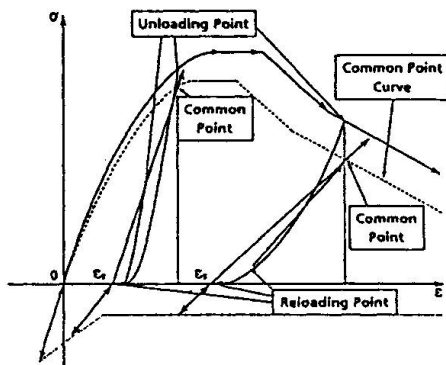


Fig.1 A hysteretic rule of concrete

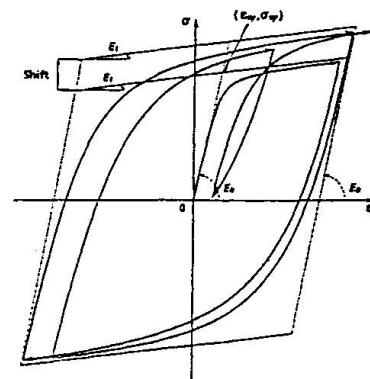


Fig.2 A hysteretic rule of steel

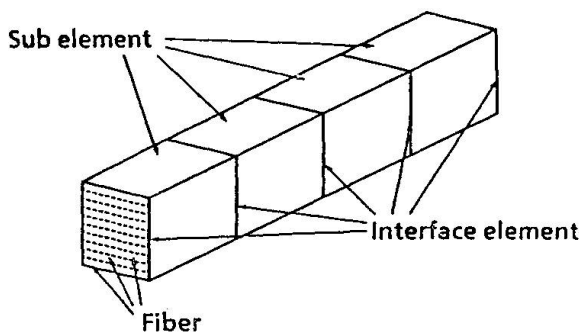


Fig.3 Division of a beam element

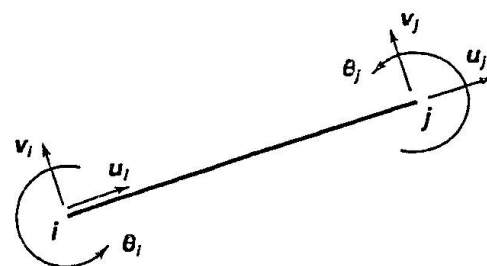


Fig.4 Degrees of freedom of a beam element



### III COMPARISON OF EXPERIMENTAL AND ANALYTICAL MOMENT-CURVATURE RELATIONS

#### 1. Results of hybrid loading tests

In order to obtain experimental results for the hysteretic behavior of RC elements under bending and time varying axial loads, a number of tests with the newly developed hybrid loading system of earthquake response (named as HYLSEER shown in Fig.5) have been conducted, where earthquake response is calculated by a digital computer adopting the real hysteretic restoring force of elements which is directly measured from a loading actuator, of which details are found in the reference [6] [7] [8].

In the experiment, reinforced concrete specimens with cross section of  $150 \times 150$  mm and total length of 2090 mm are used. The longitudinal reinforcement consists of  $4 \times \phi 12.7$  mm steel bars. The lateral reinforcement of spiral tie hoops ( $\phi = 5$  mm) is adopted with pitches of 60 mm and 90 mm. The cross section and its fiber model is shown in Fig.6. The hysteretic moment-curvature relations of the critical cross section of one of the tested specimens is plotted in Fig.7 for time intervals,  $t = 0 \sim 10$  sec,  $t = 10 \sim 20$  sec and  $t = 20 \sim 30$  sec. In this case, N-S component of El-Centro record obtained during the Imperial Valley earthquake (5-18-1940) in the U. S. is used as an input earthquake ground motion. The time varying compressive axial force is controlled to be proportional to the restoring force of the specimen and plotted in Fig.8.

Generally, quite unique features of hysteretic loops due to time varying axial forces are found. In the moment positive side where axial force is increased, high but deteriorating restoring force is found due to spalling of concrete. On the contrary, in the moment negative side where axial force is decreased, low but non-deteriorating restoring force is found.

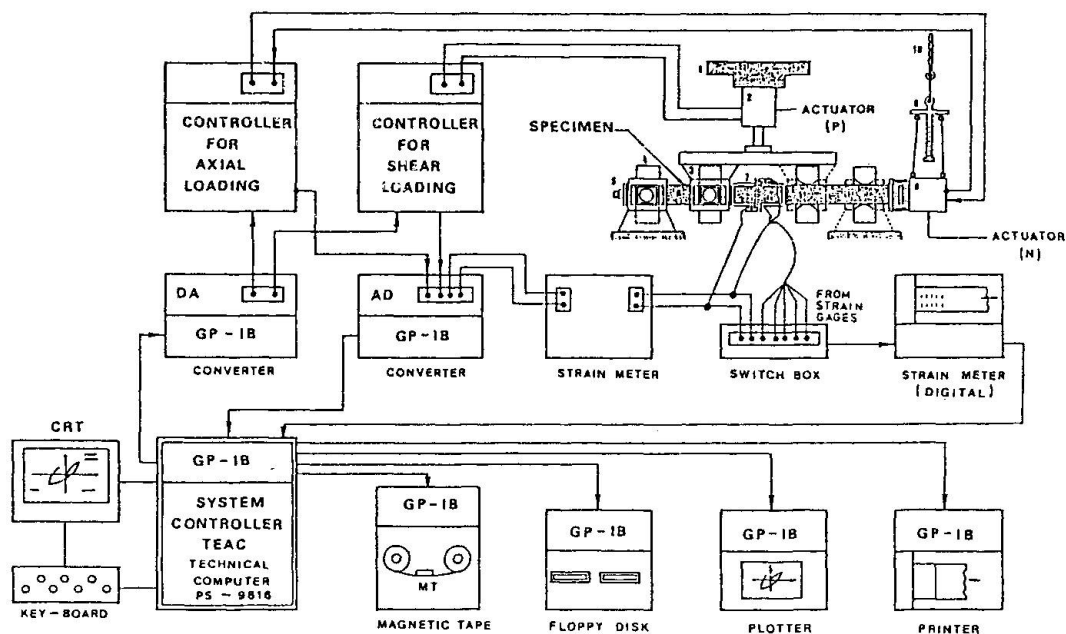


Fig.5 Diagram of the Online-Hybrid System

## 2. Analytical results from stress-strain model

Analytically calculated hysteretic moment-curvature relation is plotted in Fig.9 for the same time interval as shown in Fig.7. From the comparison of the experimental and analytical results, it is evident that complete pattern as well as corresponding moment and curvature levels are in good agreement even for this specific case where input curvature and axial force histories are given in quite irregular form. The empirically verified agreement encourages further coming stress-strain based dynamic analysis of RC frame structures.

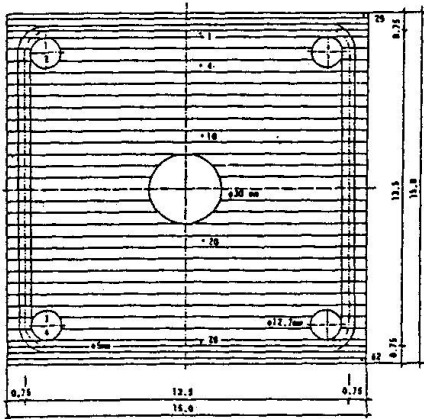


Fig.6 Cross Section Fiber Model

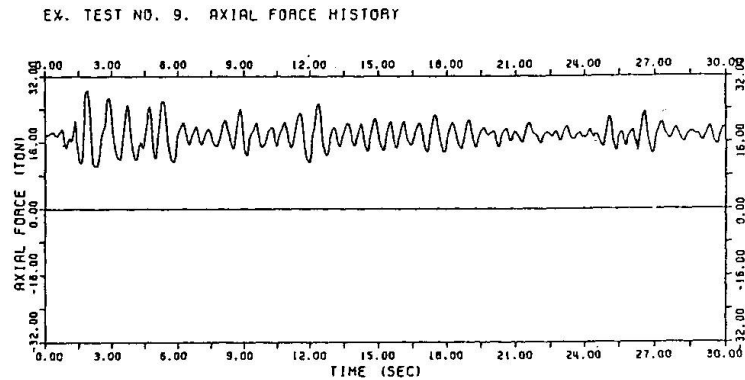


Fig.8 Time Varying Axial Force of the Specimen

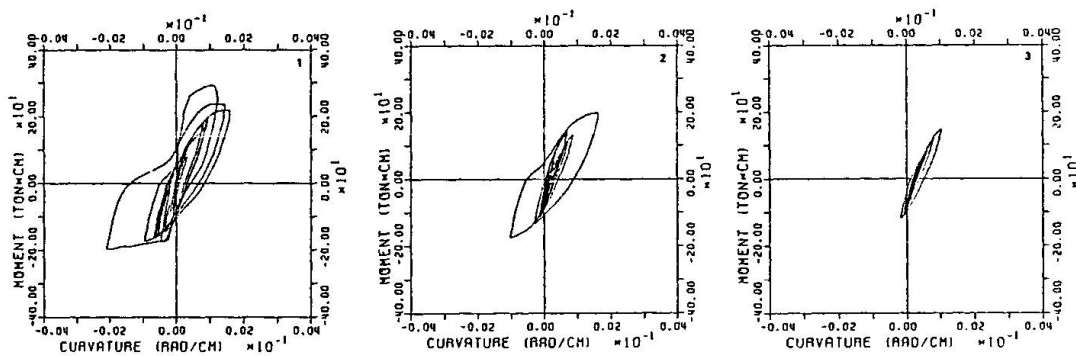


Fig.7 Experimental Moment-Curvature Relation of the Specimen in Every 10 Seconds of Earthquake Response

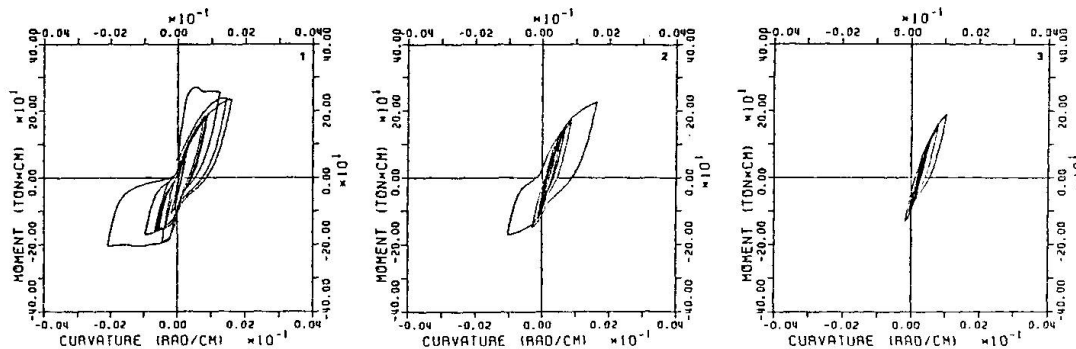


Fig.9 Analytically predicted Moment-Curvature Relation of the Specimen in Every 10 Seconds of Earthquake Response



#### IV DYNAMIC RESPONSE ANALYSIS

Since the afore-mentioned non-linear stiffness is time-variant, the basic equation of motion of a system is given in the following load incremental form using the assumption that stiffness is time-invariant in a short time step[9].

$$[M]\{\ddot{\Delta u}\} + [C]\{\dot{\Delta u}\} + [K]^t\{\Delta u\} = \{\Delta R\} \quad (3)$$

where  $[M]$  : the mass matrix,  
 $[C]$  : the damping matrix,  
 $[K]^t$  : the stiffness matrix at time  $t$ ,  
 $\{\ddot{\Delta u}\}$  : the incremental acceleration vector,  
 $\{\dot{\Delta u}\}$  : the incremental velocity vector,  
 $\{\Delta u\}$  : the incremental displacement vector,  $\{u, v, \theta\}$  for each node,  
 and  $\{\Delta R\}$  : the incremental earthquake force vector.

There are two main numerical problems to solve non-linear equations, that is, a problem of iteration and a problem of an unbalanced force. While a problem of numerical iteration should be mainly discussed on computing time and numerical accuracy, a problem of an unbalanced force should be also discussed including methodology how to compute it. The varying stiffness method is popular to consider an unbalanced force, though it generally requires a great amount of computing time. The equivalent external force method(or the initial stiffness method) is also becoming popular because it requires less computing time than the former method, however there is another problem that an impulse due to an equivalent external force is often generated in response.

The varying stiffness method with iteration is employed in this study because varying stiffness is computed at every time step according to the afore-mentioned algorism. However, from the practical view point, no iteration is performed in the following numerical examples.

In a computer program developed in this study, the consistent mass matrix and the lumped mass matrix can be employed alternatively as the mass matrix. Rayleigh damping matrix is used as the damping matrix. And Newmark's method and Wilson's  $\theta$  method are available as a time integration scheme. The RC members can be alternatively processed as linear or non-linear elements.



## V NUMERICAL EXAMPLES

### 1. A column model

To examine the effect of non-linearity, numerical examples of non-linear analysis(case-B) of a column model(10 elements, 11 nodes)(Fig.10) are compared with that of linear analysis(case-A). And the results in case-B are also compared with that of another non-linear analysis(case-C) in which the non-linearity of an RC element is assessed by  $M-\phi$  hysteretic relation. The magnified El-Centro earthquake acceleration of which maximum value is 600(gal) is employed as an input ground motion. The time duration is 10.0(sec). The computing time increment is 0.005(sec). The structural and material constants used in this example are summarized in the following. The consistent mass matrix and Wilson's  $\theta$  method are employed.

#### • Structural constants

$$A = 1.0\text{m}^2 \quad h = 10.0\text{m}$$

$$A_s = 0.030\text{m}^2$$

#### • Material constants

$$\sigma_{ck} = 4000.0\text{t/m}^2$$

$$\sigma_{cu} = 2000.0\text{t/m}^2$$

$$\sigma_{tk} = 100.0\text{t/m}^2$$

$$\sigma_{sy} = 30000.0\text{t/m}^2$$

$$\varepsilon_{cy} = 0.0020$$

$$\varepsilon_{cr} = 0.0024$$

$$\varepsilon_{cu} = 0.0100$$

$$\varepsilon_{cl} = 0.0210$$

$$\varepsilon_{sy} = 0.0010$$

$$\gamma_c = 2.0\text{t/m}^3$$

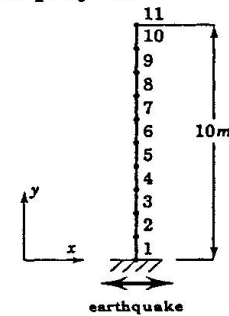


Fig.10 A column model

Fig.11 and Fig.12 show the acceleration response of the top-end point in case-A and in case-B respectively. Fig.13 shows the displacement response of the same point in case-B. Fig.14 shows the displacement response of the same point in case-C. Fig.11, Fig.12 and Fig.13 show that the frequency of acceleration response in case-B becomes lower than in case-A, the maximum value of acceleration in case-B is smaller than in case-A(86.6%) and the maximum value of displacement in case-B is bigger than in case-A(134.5%). These results are explained by the effect of material non-linearity, that is, the degradation of the system. Fig.14(case-C) shows a good agreement with Fig.13(case-B). Hence,  $M-\phi$  model(case-C) may be satisfactory when there is no change in axial force.

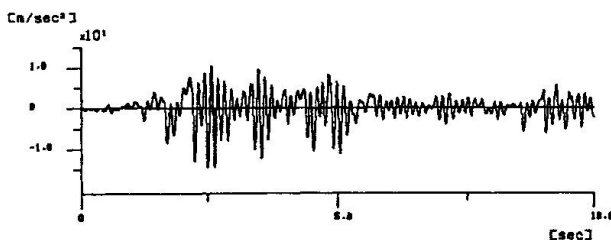


Fig.11 Acceleration response in case-A

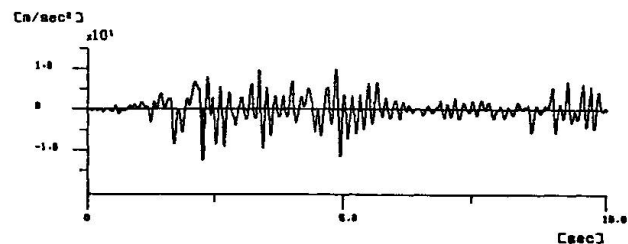


Fig.12 Acceleration response in case-B

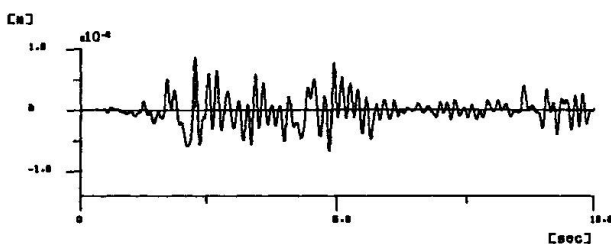


Fig.13 Displacement response in case-B

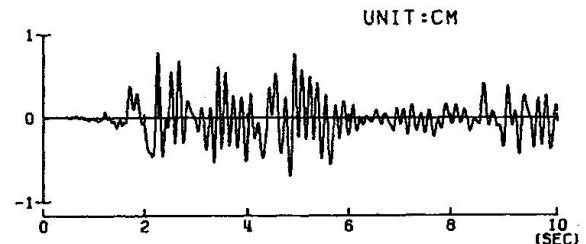


Fig.14 Displacement response in case-C





## 2. An RC tower model of a cable-stayed PC girder bridge

Numerical examples of an RC tower model(24 elements, 25nodes)(Fig.15) of a cable-stayed PC girder bridge of which span length is more than 200 meters are computed.

The input earthquake motion is the same as in the first numerical example. The duration time is 6.0 (sec). The computing time increment is 0.005(sec). The material constants and others used in this example are summarized in the following.

### • Material constants

$$\begin{aligned} \sigma_{ck} &= 3400.0t/m^2 & \sigma_{cu} &= 1700.0t/m^2 \\ \sigma_{tk} &= 340.0t/m^2 & \sigma_{sy} &= 30000.0t/m^2 \\ \epsilon_{cy} &= 0.0020 & \epsilon_{cr} &= 0.0024 \\ \epsilon_{cu} &= 0.0100 & \epsilon_{cl} &= 0.0210 \\ \epsilon_{sy} &= 0.0015 & \gamma_c &= 2.35t/m^3 \end{aligned}$$

### • Damping constants

$$\alpha = 0.0 \quad \beta = 0.0318$$

### • Integration scheme(Newmark's method)

$$\delta = 1/2 \quad \alpha = 1/4$$

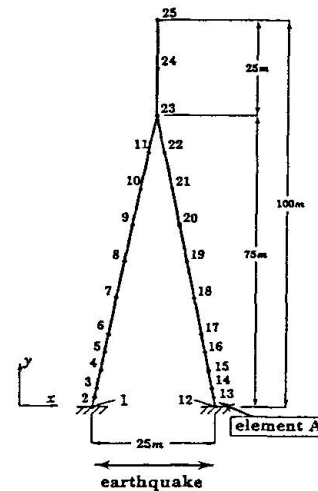


Fig.15 An RC tower model

Fig.16 and Fig.17 show the acceleration and the displacement response at the top respectively. Fig.18 and Fig.19 show the axial force( $N$ ) and the bending moment( $M$ ) response of the right-side bottom-end element(element A in Fig.15) respectively. The stress-strain relation of concrete, that of steel and  $M-N$  relation of the element are shown in Fig.20, Fig.21 and Fig.22 respectively.  $M-\phi$  relation of the element is shown in Fig.23. Fig.24 and Fig.25 show the non-stational spectra of the response in Fig.16 and that of acceleration response by linear analysis. These results show the inelastic behavior that acceleration becomes smaller(85.3%), displacement becomes bigger(320.5%) and bending moment becomes smaller(71.7%) than the results by linear analysis. As to axial force, the change of it must have a great effect on the seismic safety of the structure because the varying range of  $N$  may be up to 15000.0(ton) in the example(Fig.18) and there is a basic linear relation between  $N$  and  $M$ (Fig.22). Besides,  $M-\phi$  relation is much affected by the varying axial force(Fig.23). On the other hand, the initial sectional force due to own weight, cable tension or the initial maladjustment of structure is another important factor to the seismic safety of structure because it determines the starting point of stress-strain relation(Fig.20, Fig.21). Among them, the initial sectional force due to the initial maladjustment has a serious effect on the change of  $N$  because it is uncertain in nature and causes big initial sectional forces. Fig.24 shows the process of the degradation of the system by the non-linear analysis while Fig.25 shows no degradation, that is, the dominant frequency of the response remains unchanged.

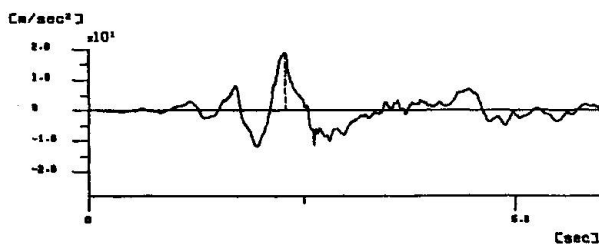


Fig.16 Acceleration response at the top

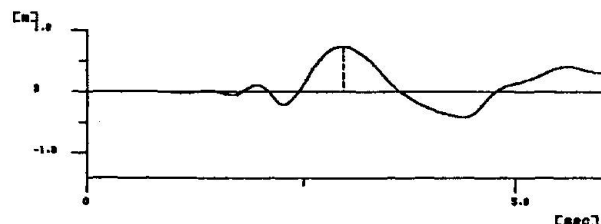


Fig.17 Displacement response at the top

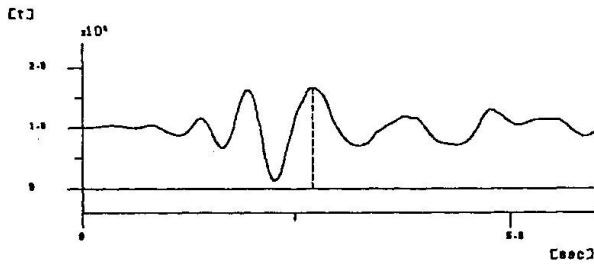


Fig.18 Axial force response at A

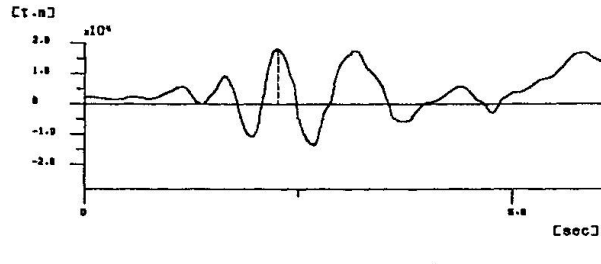


Fig.19 Bending moment response at A

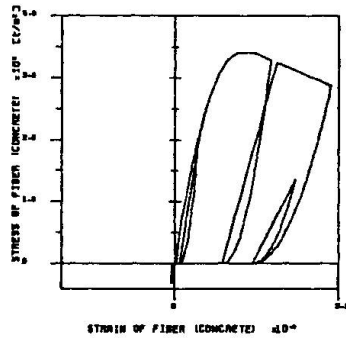


Fig.20 Stress-strain of concrete

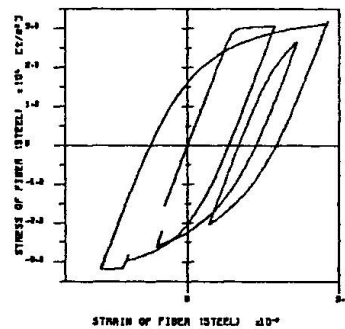


Fig.21 Stress-strain of steel

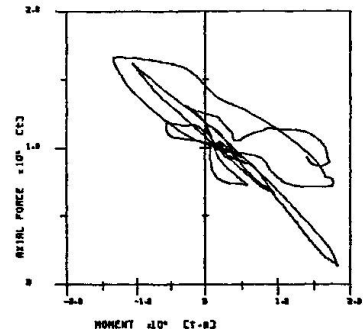


Fig.22 M-N relation

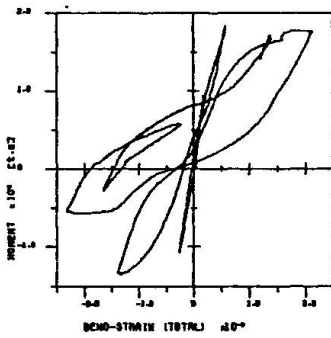
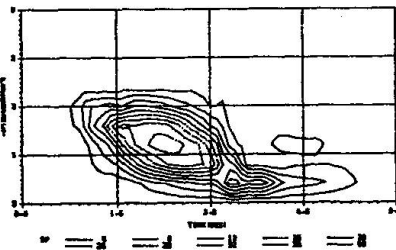

Fig.23 M- $\phi$  relation


Fig.24 Spectrum of Fig.16

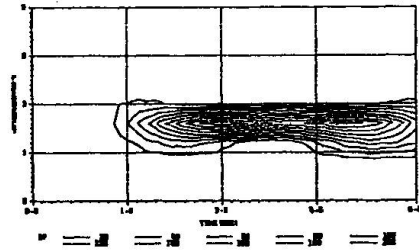


Fig.25 Spectrum of linear response

## VI CONCLUSIONS

In this study, a method of stress-strain based inelastic earthquake response analysis of RC frame structures with varying axial forces is presented, and following conclusions are obtained.

- (1) Moment-curvature relations analytically estimated from stress-strain relations used in this study agree fairly well with the experimental results of earthquake response.
- (2) A varying axial force has a nonnegligible effect on  $M-\phi$  relation of a section and the seismic response of an RC frame structures.
- (3) The strength in the axial force increasing side and the ductility in the axial force decreasing side shall be checked for earthquake resistant design of the structures
- (4) The varying range of axial force may become more than the degree of the initial axial force, depending on configuration of an RC frame structures.
- (5) The initial sectional force due to the initial maladjustment has a great effect on the seismic response of an RC frame structure.



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