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## Numerical Analysis of Thick Reinforced Concrete Slabs under Impact Loading

Analyse numérique des dalles épaisses en béton armé soumises à des chocs

Numerische Analyse dicker Stahlbetonplatten unter Stoßbelastung

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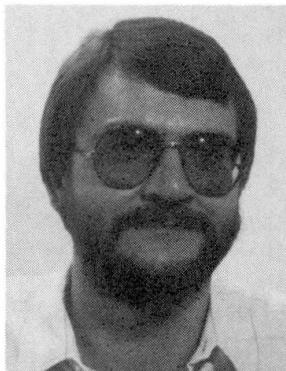
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### SUMMARY

The local response of impact loading on reinforced concrete structures is not yet clarified. The pure existence of FE-programs does not fulfill the demands of a design engineer. He has to know the constitutive input and he has to understand the influence of the different design parameters. Only this knowledge creates a reasonable design basis. Therefore, by means of an own developed FE-program which devotes special attention to the constitutive assumptions, parametric studies have been carried out to create such an understanding of the physical behaviour.

### RÉSUMÉ

Le comportement local sous l'effet de chocs dans les structures en béton armé est mal connu. La simple existence de logiciels ne permet pas de répondre à toutes les questions de l'ingénieur projeteur. Celui-ci doit connaître les lois constitutives des matériaux et comprendre l'influence de différents paramètres du projet. Seule cette connaissance permet d'avoir une base raisonnable pour l'étude du projet. Un programme de calcul appliquant la méthode des éléments finis et tenant compte des lois constitutives des matériaux a été réalisé et comparé avec diverses études paramétriques pour créer une base solide du comportement physique.

### ZUSAMMENFASSUNG

Über die lokalen Vorgänge bei Stoßbelastungen auf Stahlbetonbauteile besteht noch weitgehend Unklarheit. Auch wenn prinzipiell entsprechende EDV-Programme existieren, ist dem entwerfenden Ingenieur damit heute meist nur wenig gedient. Er wird sehr oft die stoffgesetzlichen Annahmen, wenn er sie überhaupt kennt, kritisieren müssen und er braucht ausgewertete Parameterstudien, die das komplexe ineinander greifen der verschiedensten Einflüsse klar erkennen lassen. Nur so erhält er eine Entscheidungsbasis für seine Entwürfe. Es wurde deshalb mit einem eigenen FE-Programm unter besonderer Berücksichtigung von stoffgesetzlichen Vorgaben sowie eigenen und fremden Versuchen Parameterstudien durchgeführt, um eine solide Ausgangsbasis zu schaffen.



## 1. THE PROBLEM

Impact loading is caused by the collision of at least two masses. For the civil engineer the special case where a moving mass - projectile - hits a structure is of dominant interest. The impact of a vehicle against a bridge column, a falling rock on a road shelter or an aircraft crash on the containment of a nuclear power plant represent such a situation.

Until now for the design of impact endangered reinforced concrete structures such as slabs and shells not many reliable procedures are available. Especially the common rules to treat local response, i.e. punching, are very sensitive with regard to the necessary amount of stirrups, and do neither include the influence of support conditions - contact of missile at the support or in the interior slab region -, nor the influence of bending reinforcement or the shape of the load-time-function.

Tests demonstrated clearly that there is a great influence of these mentioned parameters which is not fully understood. Slabs under nearly equal test configurations showed a quite different response, some being nearly undamaged, others being completely punched after a slight change of parameters (comp. [1],[2]). Although during the last years in a series of experimental and analytical investigations the impact problem has been treated (see e.g. [3],[4]), we still do not know which parameters influence the local behaviour in what way. For a safe and economic design however a detailed knowledge of material and structural behaviour is of considerable practical importance.

Therefore by means of an own developed FE-model which has been carefully checked and compared with experimental results to assure the validity of this analytical tool parameter studies on thick slabs have been performed. Thickness and span of the slabs, amount of bending and shear reinforcement, load distribution on the impacted area, shape of the load-time-function and support conditions were varied. The computational model and some typical results of this investigation will be presented .

## 2. COMPUTATIONAL MODEL

With regard to the available space just the main topics of the computational model will be presented . A detailed description is given in [5].

### 2.1 Finite Element Strategy

The nonlinear finite element analysis is carried out with an incremental solution strategy. After each time step equilibrium iterations are performed and the stiffness matrix is updated. Time integration is done by Newmark's scheme.

### 2.2 Discretisation

In practical cases it is sufficient to use an axisymmetric model for the examination of local response because maximum local straining mostly occurs in the first impact phase. At this time there is still little influence of the boundary conditions. The slab does not 'know' whether it is rectangular or circular. However three-dimensional stress and strain states have to be considered.

The discretisation of the circular slab in a radial cross-section was done by 4-, 5- or 6-node isoparametric axisymmetric elements describing concrete as well

as reinforcement. Inertia effects are included using a lumped mass matrix. A typical FE-mesh used in the parameter study is shown in Fig. 3. The external load is applied as uniform pressure load on a massless fictitious load-distribution-plate which is modeled with linear-elastic material ( $E = 210000$  MPa) just above the impact zone. Contact elements, transferring only compressive stresses, connect this plate with the concrete structure. This plate is just a tool to avoid that load is applied to a node which in case of a material failure has no stiffness. In such a situation the load has to be distributed to other nodes. Furthermore the stiffness of the fictitious plate influences the stress distribution in the contact area, thus giving an impression of the consequences of different projectile rigidities.

### 2.3 Constitutive Assumptions

The three-dimensional constitutive model and failure criterion of OTTOSEN [6] served as a starting basis for the description of the material behaviour of a concrete element. Characteristics as dilatation under compressive stresses close to failure and softening behaviour in the postfailure region are included. As the original model is only designed for monotonic loading, the capability to describe un- and reloading with a Young's modulus parallel to the initial value was added (Fig. 1). To improve equilibrium iterations in the incremental solution algorithm the secant formulation of OTTOSEN was extended to a tangential formulation.

A smeared crack model was used to allow cracks to open and close. Special constitutive matrices for cracked elements have been developed. Tensile stresses perpendicular to a crack cannot occur, whereas shear stresses can be transmitted depending on the actuell crack width. The main effects of high strain rates are included additionally. This was done by a calibration of OTTOSEN's failure envelope with modified parameters at every time step.

For the reinforcement an elastic-plastic behaviour is assumed. A tension-stiffening effect, taking into account the concrete locally surrounding a rebar and stiffening it, was considered (Fig. 2).

## 3. PARAMETER STUDY

In a first step a reference slab, representing the lower bound of the nowadays used slabs in German nuclear power plants, was analysed with the following data:

### Geometry (comp. Fig. 3)

-  $d = 1,50$  m,  $r = 10,80$  m,  $a = 1,50$  m,  $d_L = 0,10$  m

### Concrete

-  $E_b = 30000$  MPa,  $\nu = 0,20$ ,  $f_c = 35$  MPa,  $f_t = 0,14 \cdot f_c$

### Bending reinforcement

- BST 1100/1300,  $as_1 = 60$  cm $^2$ /m (bottom face),  $as_2 = 50$  cm $^2$ /m (top face)

### Stirrups

- BST 420/500,  $as = 66$  cm $^2$ /m $^2$ .

Two different loading functions  $F(t)$  were considered (Fig. 4): first LF 1, the original function according to the German regulations for an airplane crash and second LF 2, a modification of that function with a steeper slope in the second ascend.

Using LF 1 the deformations develope as shown in Fig. 5. Due to the relative slow load ascend an overall bending deformation similar to a static loading can be observed. The highest tensile stresses occur first at the bottom face in near of the axis of symmetry forming nearly vertical orientated cracks (comp. Fig. 6).



During the load history more and more cracks develop. The inclination of these cracks increases with the distance from the center of the slab. After nearly 40 ms a punching cone has formed which is connected essentially by stirrups with the remaining slab. At this time the stresses in the shear reinforcement are just below the ultimate stress of 500 MPa. They are plotted in Fig. 7. For comparison the dashed line indicates the assumed punching cone with an angle of  $\alpha = 32,5^\circ$  according to the German regulation KTA 2203. One can recognize that nearly no stressed stirrup crosses this line and that an angle  $\alpha \approx 45^\circ$  would be much more realistic. The stress distribution of the bending reinforcement is shown in Fig. 8. Obviously the circumferential direction is more stressed than the radial one. After 100 ms nearly every circumferential rebar reaches its yield stress.

In a succeeding calculation the thickness of the load-distribution-plate was increased from  $d_L = 0,10$  m to  $d_L = 0,50$  m in order to simulate a rigid projectile. This has the effect that now due to the local deformation of the concrete slab the load is not longer distributed uniformly over the impact area but is concentrated at the boundaries of the loaded area. The calculated stirrup-stresses are plotted in Fig. 9. Again the ultimate stresses of 500 MPa are nearly reached. Similar to the 'soft' load introduction a punching cone with  $\alpha \approx 45^\circ$  can be observed. However in this case stirrups are activated just outside of the loadradius  $a$  (comp. Fig. 7). Under loading function LF 2 (Fig. 4) the slab is completely punched as well with 'soft' as with 'rigid' load introduction. Due to the steep second load ascend stirrups start cracking after ca. 38 ms. Only a cross-section of about  $a_s = 100$   $\text{cm}^2/\text{m}^2$  for the shear reinforcement could avoid a punching failure.

In a next analysis the thickness of the slab is increased to  $d = 1,80$  m. With the above given reinforcement this slab resists both loading functions. However under LF 2 the stirrups reach their ultimate load bearing capacity.

A decreasing amount of bending reinforcement leads to another failure mechanism. As already shown in Fig. 5 the whole slab is showing an overall bending deformation, an effect which is emphasized by reduction of the flexural rigidity e.g. by less reinforcement. After the first ascend of the loading function the slab shows now big vertical bending cracks. If now the second ascend follows, the already damaged slab cannot resist this further straining. Punching failure occurs with a very steep angle.

As investigations of this type can only be done by large computer programs as the one mentioned for practical design purposes a simple design model on the basis of a two-mass-system has been developed giving in principally the same results. For details of this model the reader is referred to [5].

#### 4. SUMMARY AND CONCLUSION

Many parameters influence the local behaviour of thick reinforced concrete slabs under impact. Often the response is quite sensitive to only a slight change of parameters. As result of the performed investigations it can be stated that thickness of the slab, span of the slab, support conditions, shear reinforcement, bending reinforcement, strength of concrete, shape of the loading function, magnitude of impacted area and load distribution on the impacted area all effect the punching behaviour significantly.

For design purpose a simple model consisting of a nonlinear two-mass-system is mentioned in [5].

## REFERENCES

1. Jonas, W., Rüdiger, E. et al.: Kinetische Grenztragfähigkeit von Stahlbetonplatten. Bericht zum Forschungsvorhaben RS 165, Hochtief AG, Abt. Kerntechnischer Ingenieurbau, Frankfurt, 1982
2. Eibl, J., Kreuser, K.: Abschlußbericht zum Forschungsvorhaben 'Durchstanzfestigkeit von Stahlbetonplatten unter dynamischer Beanspruchung'. Lehrstuhl für Beton und Stahlbetonbau, Universität Dortmund, 1982
3. Plauk, G. (ed.): Proceedings of the RILEM-CEB-IABSE-Interassociation Symposium on Concrete Structures under Impact and Impulsive Loading. Berlin: Bundesanstalt für Materialprüfung, 1982
4. Eibl, J. (ed.): Berichte zum Forschungskolloquium 'Stoßartige Belastung von Stahlbetonbauteilen'. Abt. Bauwesen, Universität Dortmund, 1980
5. Schlüter, F.-H.: Das lokale Verhalten dicker Stahlbetonplatten unter stoßartiger Belastung. Dissertation Universität Karlsruhe, 1987 (in preparation)
6. CEB: Concrete under Multiaxial States of Stresses - Constitutive Equations for Practical Design. Bulletin d' Information No. 156, Paris, 1983

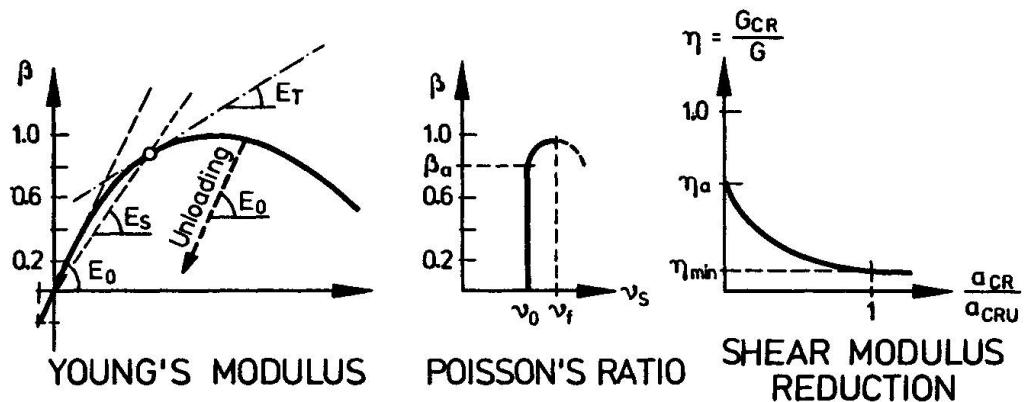


Figure 1: Modified triaxial constitutive assumptions for concrete resulting from [6]

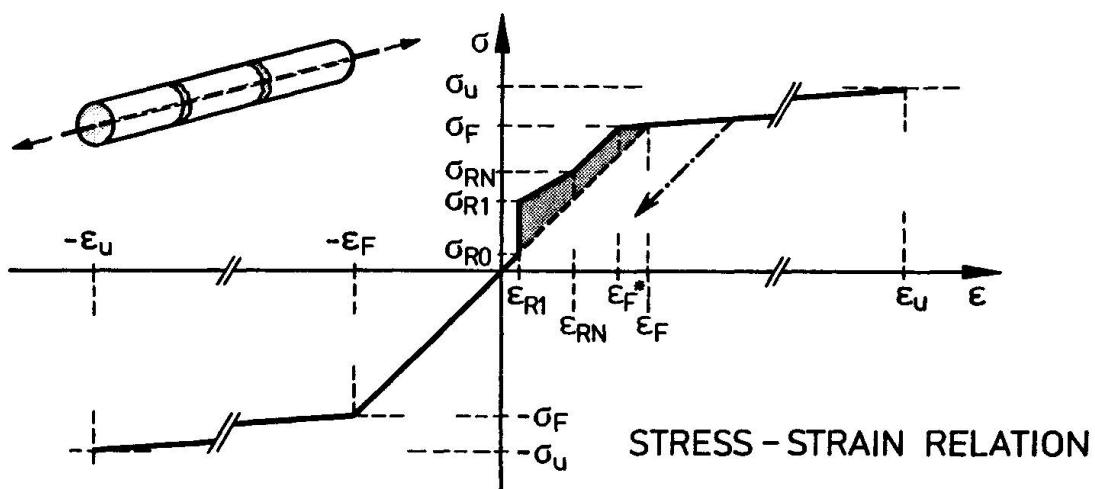


Figure 2: Constitutive assumptions for reinforcement including 'tension-stiffening'

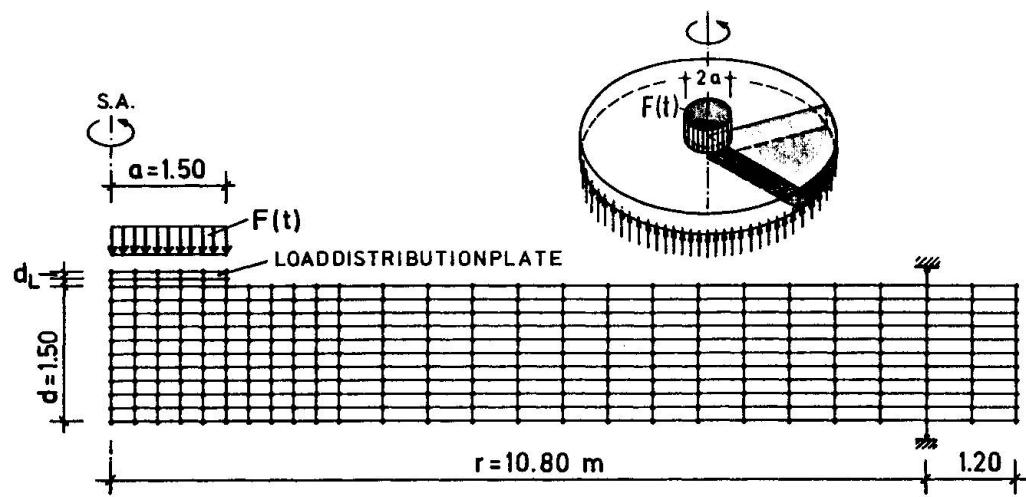


Figure 3: Discretisation of the axisymmetric slab

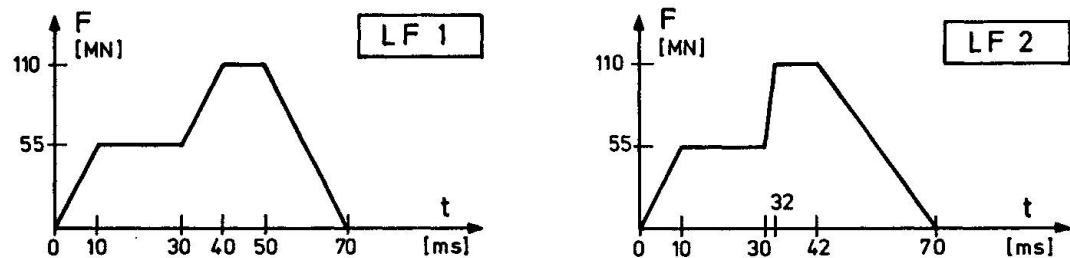


Figure 4: Applied load-time functions

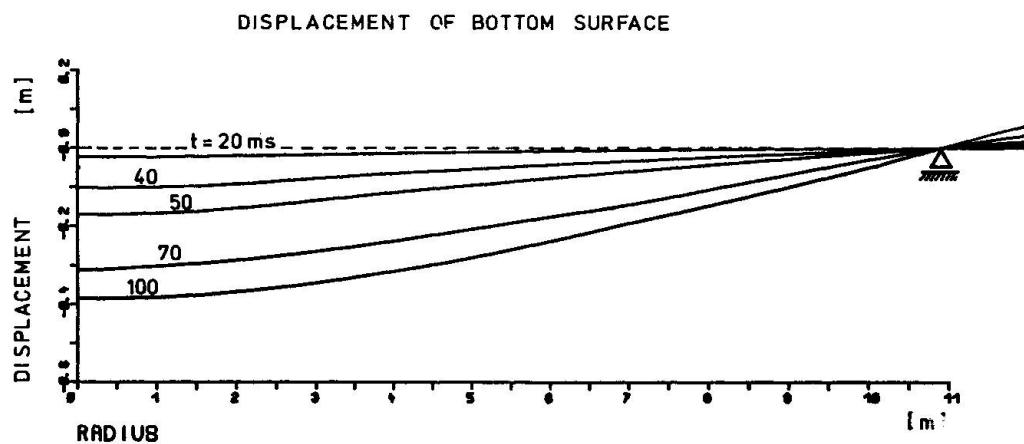


Figure 5: Deformation of the 1,50 m thick slab under LF 1 at different time steps

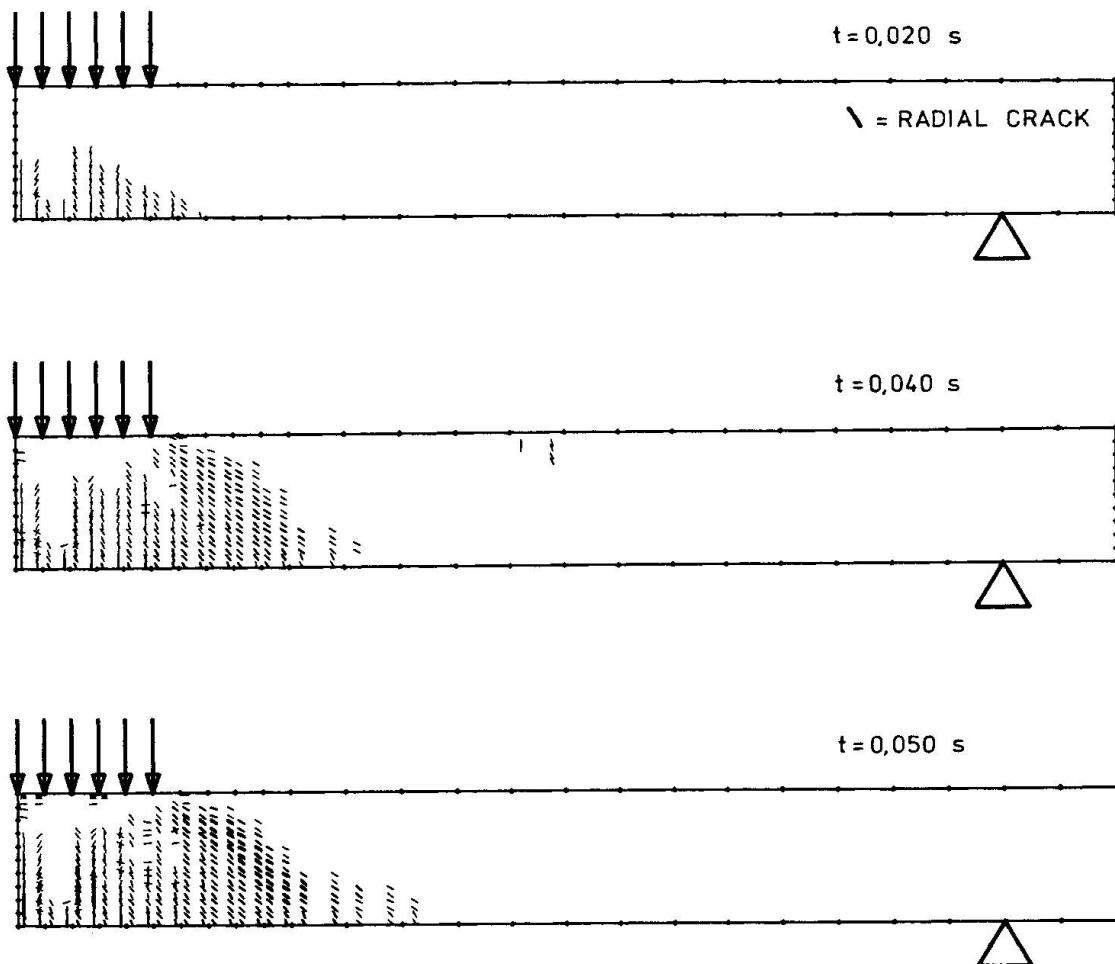


Figure 6: Developement of radial cracks of the 1,50 m thick slab

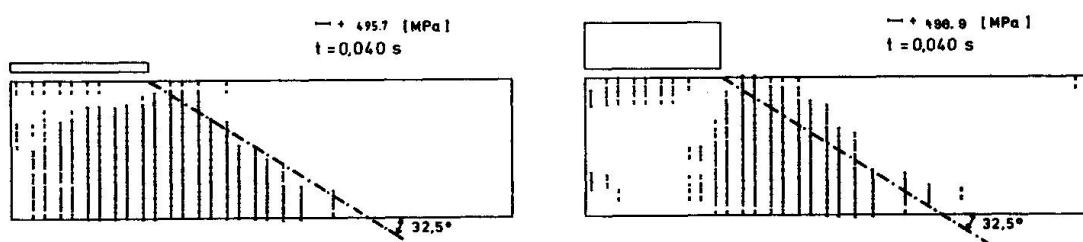


Figure 7: Stresses in stirrups at  $t = 40$  ms a) with 'soft' and b) with 'rigid' load introduction

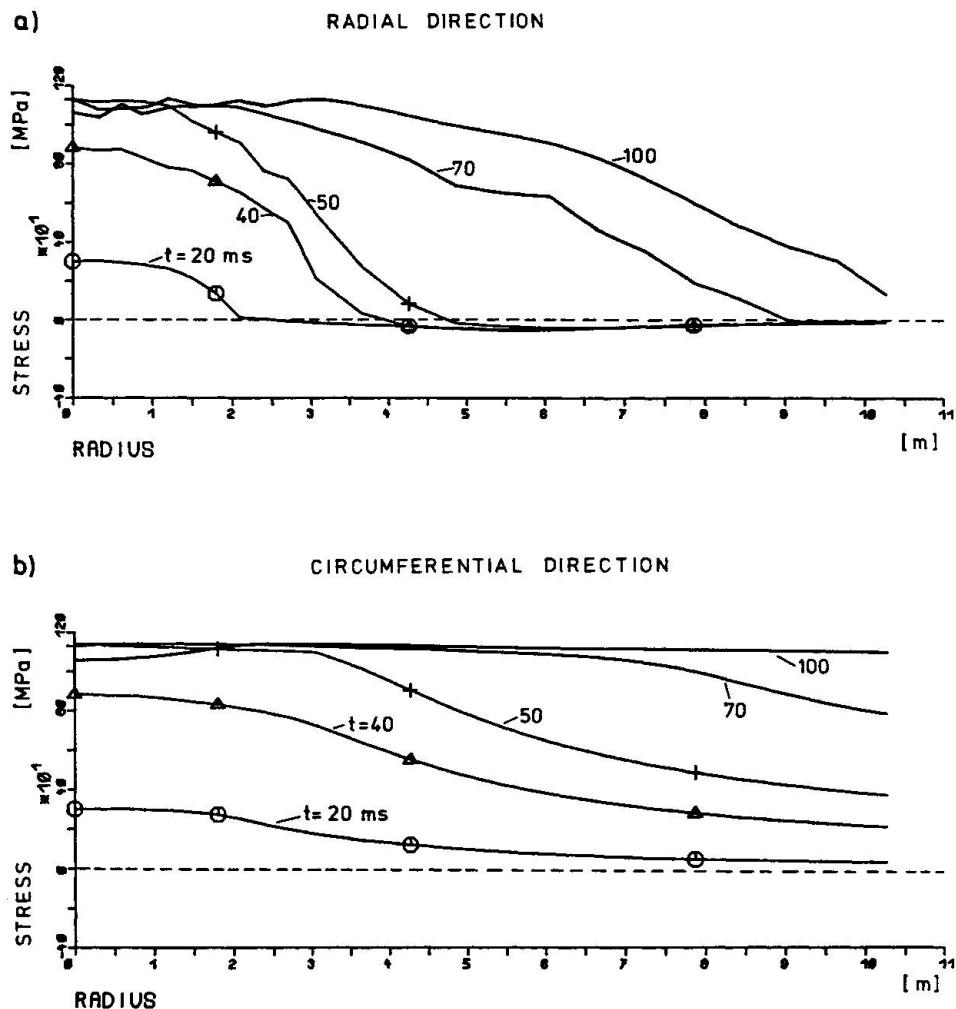


Figure 8: Stresses in bending reinforcement a) radial direction,  
b) circumferential direction

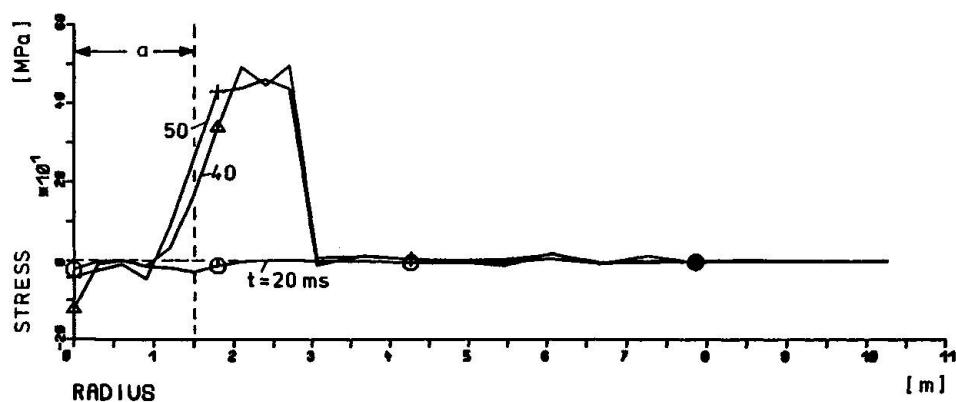


Figure 9: Stresses in stirrups with 'rigid' load introduction