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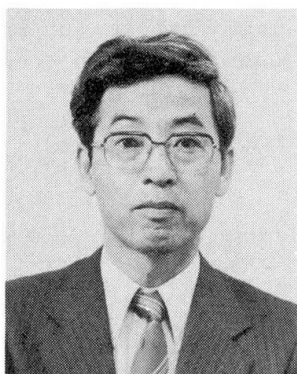
## Reinforced Concrete Plate Element Subjected to Cyclic Loading

Elément de dalle en béton armé soumis à des charges cycliques

Stahlbetonplattentragwerken unter Wechsellast

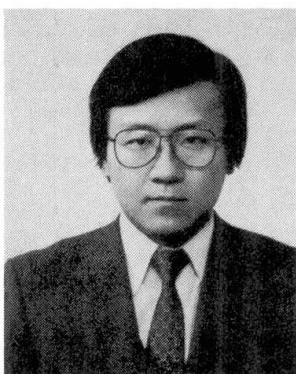
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### SUMMARY

A comprehensive model has been developed to analyze the behaviour under reversed cyclic loading of reinforced concrete plate elements, with the reinforcement uniformly distributed. The model comprises tension, compression and shear models of cracked concrete, and a model for reinforcement in the concrete; and also a so called smeared crack model is incorporated. The applicability of this model has been verified by comparing the predictions with published experimental results of reinforced concrete panels.

### RÉSUMÉ

Un modèle global a été développé pour étudier les comportements d'éléments de dalles en béton armé, avec des barres d'armature distribuées uniformément, sous l'effet de charges cycliques alternées. Le modèle comprend des cas de tension, compression et cisaillement de béton fissuré, ainsi qu'un modèle de renforcement du béton; il est aussi envisagé un modèle de fissuration homogénéisée. La valeur de ce modèle a été vérifiée en comparant les prévisions avec les résultats expérimentaux publiés pour des panneaux en béton armé.

### ZUSAMMENFASSUNG

Ein umfassendes Modell zur Analyse des Verhaltens von Betonflächentragwerken mit gleichmäßig verteilter Bewehrung unter Wechselbelastung wurde entwickelt. Das gesamte Modell enthält Zug-, Druck- und Schubmodelle sowie ein Modell für Stahlbeton und behandelt auch verteilte Rissbildung (smeared crack). Die Anwendbarkeit des Modells wurde an Vergleichsrechnungen mit veröffentlichten Ergebnissen an Stahlbetonscheiben überprüft.



## 1. INTRODUCTION

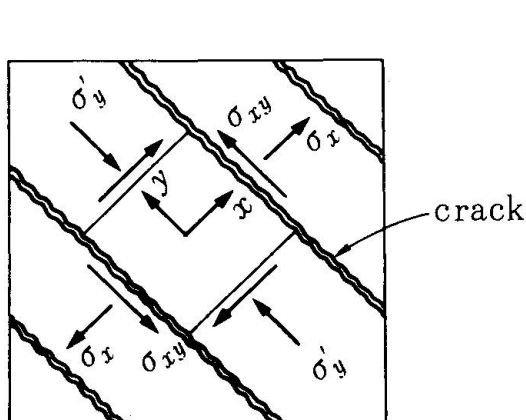
The mechanical behaviors of reinforced concrete members under cyclic or reversed loading are being clarified by many experimental works. However, very few works have been done for analytical prediction of these behaviors [1] - [5], mainly because of the difficulty in modeling the cracked concrete appropriately. Due to the same reason none of the contestants was able to predict the behaviors for monotonic loading in the Collins' Competition [6]. Before predicting the behaviors of a reinforced concrete member, the behaviors of an element of that member should be predicted accurately.

For analysis of a plate element having distributed reinforcement, the smeared crack model is appropriate. We have been trying to develop material models [7] that are capable of accurately describing the behaviors of cracked concrete, and these models stood tests of comparison against experimental results on panels conducted by Vecchio and Collins or by Aoyagi and Yamada. The present model has been developed out of these models so as to include the cases of reversed cyclic loading.

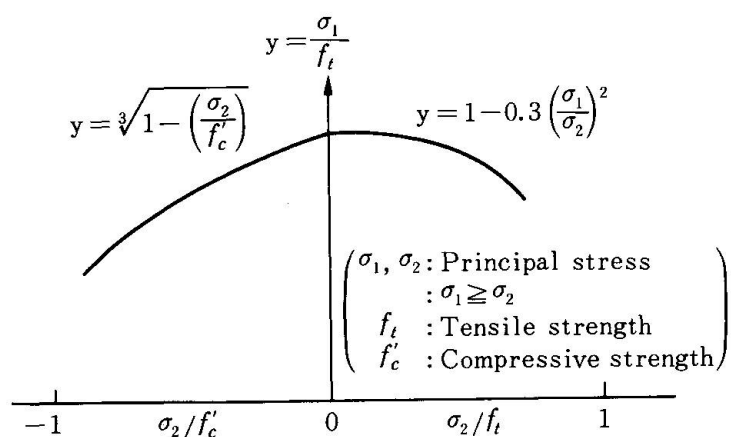
## 2. MATERIAL MODELING

### 2.1 General

Reinforced concrete plate elements are often presented as a superimposition of concrete and reinforcement. In the concrete part, taking the y axis in the direction of cracks and the x axis normal to it, the constitutive laws of cracked concrete can be constructed with comparative ease in terms of tensile stiffness along the x axis and compression along the y axis as shown in Fig.1. The coordinates for the reinforcement is conveniently taken in the directions of the two reinforcement axes, and the constitutive equations for reinforced concrete can be obtained by superimposing the two elements by coordinate conversion. However, it is to be remembered that, because these rules are for the concrete and the reinforcement as element of a reinforced concrete, they can be different from those that are for isolated reinforcement or for isolated concrete.



**Fig.1** The coordinates for cracked concrete



**Fig.2** Cracking criteria of concrete under biaxial stress states [8] [9]

## 2.2 Cracking of Concrete

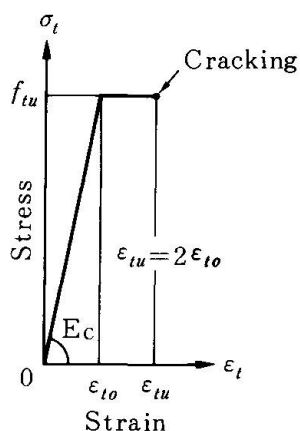
The cracking has been assumed to occur in the concrete when the principal tensile strain has reached the ultimate tensile strain. Here, the ultimate tensile strain is generally 0.01 to 0.03 %, but for the sake of simplicity, it was set to twice of the principal strain at the time when the stress acting on concrete has reached the tensile fracture envelope. This is to predict the occurrence of cracking with certain degree of accuracy even in the cases of nonuniform stress distribution.

As for the fracture envelope, those that have been proposed with emphasis on cracking under biaxial loading, one due to Niwa for compression-tension [8] and the other to Aoyagi and Yamada for tension-tension [9], have been adopted as shown in Fig.2. According to Maekawa, the fracture envelopes for monotonic loading are hardly affected by the loading path [10]. Another assumption has been that the concrete remains perfectly elastic until cracking as shown in Fig.3.

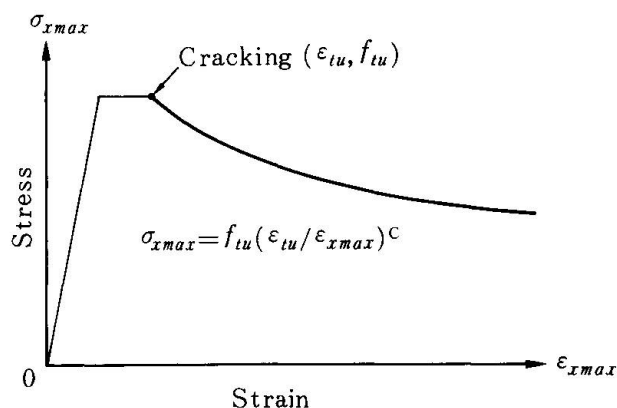
## 2.3 Tension Stiffening Model

Onset of cracking in reinforced concrete means that the concrete has lost its load carrying capacity at the plane of crack, and there the entire tensile force is borne by the reinforcement. In the portion between two cracks, however, concrete still bears up a part of the tensile force owing to the bond stress that transfers the tensile force from reinforcement to concrete. Therefore, reinforced concrete develops, even after it has been cracked, a higher stiffness than that of an isolated reinforcement. This increase of stiffness owing to concrete is generally known as the tension stiffening.

In order to accomodate this phenomenon, various proposals, such as a method that assumes either bond slip relation [11] – [12] or bond stress distribution [13], one that assumes maximum strain average strain relation for reinforcement [14] – [15], one that assumes directly the stress strain relation for reinforced concrete [16] – [17], and one that assumes average stress average strain relation for concrete [18] – [19], have been advanced. Judging the method that presupposes an average stress average strain relation for concrete to be most convenient to apply to the plate elements, we have selected the one that is shown in Fig.4 [7]. The parameter  $c$  that is included in this model is to represent the bond, and a value of 0.4 is appropriate for ordinary deformed bars, while 0.2 fits well to the cases of welded mesh that Vecchio and Collins employed.



**Fig.3** Stress strain relation of concrete for tension

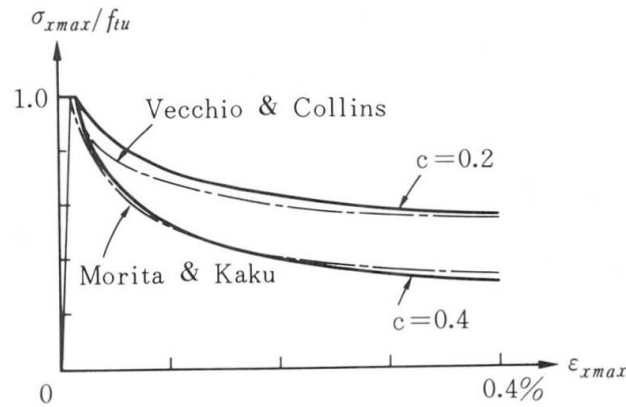


**Fig.4** Tension stiffening model [7]



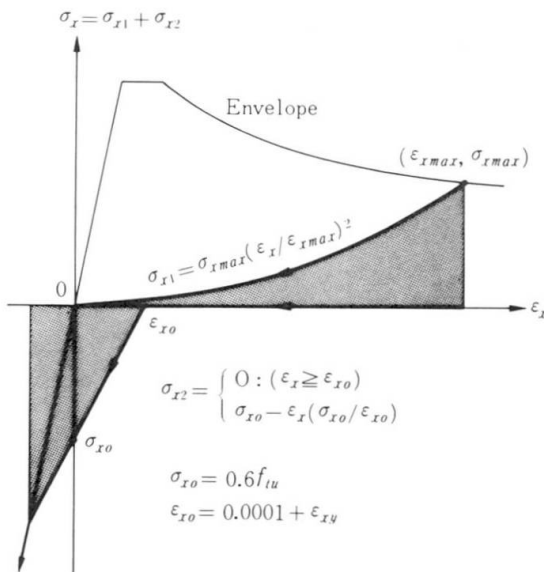


Values according to this model are compared with proposals by Morita and Kaku [20], and Vecchio and Collins [21] as shown in Fig.5.

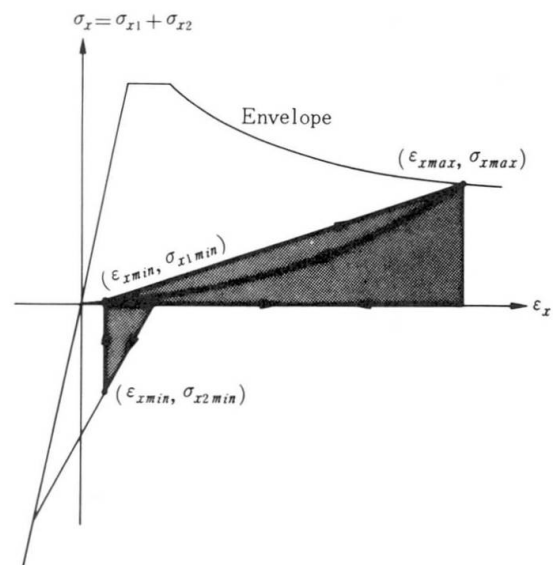


**Fig.5** Tension stiffening model compared with the proposals by Morita and Kaku [20] and Vecchio and Collins [21].

The tension stiffening model on unloading and reloading has been developed on the basis of the experimental results of Tamai and Shima [22]. In the model, the stress acting on concrete was assumed to consist in the stress transferred from reinforcement by bond and that transferred by contact of crack surface and these two were individually modeled. The stress arising from bond was presented by a quadratic curve connecting the envelope and the origin for unloading, and by a straight line extending to the envelope for reloading as shown in Fig.6 and 7. As the contact of crack surfaces takes place the faster the larger the shear displacement along the crack, the tensile strain at onset of contact was taken to be a function of the shear strain. Unloading after contact was made to proceed elastically at a slackened stiffness, whereas the contact stress was made to become zero on reloading as shown in Figs.6 and 7.



**Fig.6** Tension stiffening model for unloading [22]



**Fig.7** Tension stiffening model for reloading [22]

## 2.4 Compression Model of Cracked Concrete

For the compressive stiffness of concrete in the crack direction (y direction), modified Maekawa's model was used. In the Maekawa model[10], the initial elastic modulus  $E_o$ , the fracture parameter  $K_o$ , and the compressive plastic strain are introduced to define the envelope for compressive stresses and compressive strains, so that unloading and reloading can be conveniently described as shown in Fig.8. Since neither the fracture parameter nor the plastic strain changes inside the envelope, the internal curves are represented by straight lines of slopes  $E_o K_o$ , meaning that no energy is dissipated in the unloading and reloading. Although this is untrue, this assumption may well be tolerated for its merit of highly simplifying the calculation, provided that the energy consumed in compressive deformation of concrete is sufficiently small compared with the total expenditure of energy. Actually, modeling of increasing internal damages under cyclic loading is necessary to reasonably describe the energy consumption in the unloading and reloading process. For this purpose, a model has been completed recently by the present authors, which will be reported in due time.

The compressive plastic strain has been formulated, as shown in Fig.9, in terms of a single value function of the maximum compressive strain that the body had sustained in the past the strain right on the envelope. Furthermore, this formula has been experimentally demonstrated to be applicable even to the cases where cracks are present in parallel to the direction of compression [23].

The fracture parameter  $K_o$ , which is to represent the extent of the internal damage of concrete, has also been assumed to be a function of the maximum compressive strain as in the case of plastic strain as shown in Fig.10. However, the fracture parameter is affected by the cracks running in parallel to the compression direction. This is clarified by a compression testing of cracked reinforced concrete cylinders as shown in Fig.11 [23].

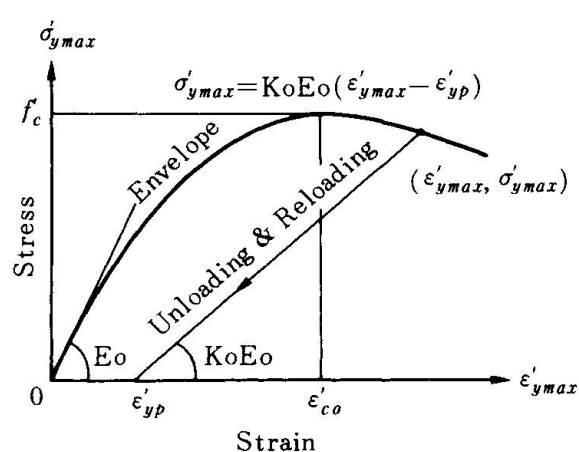


Fig.8 Maekawa model for compression [10]

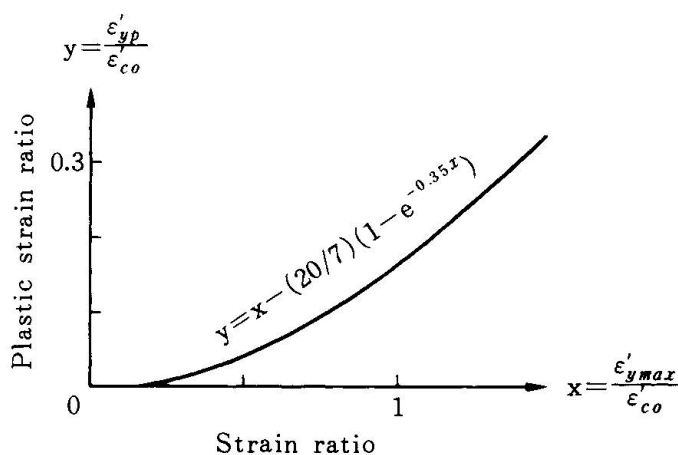
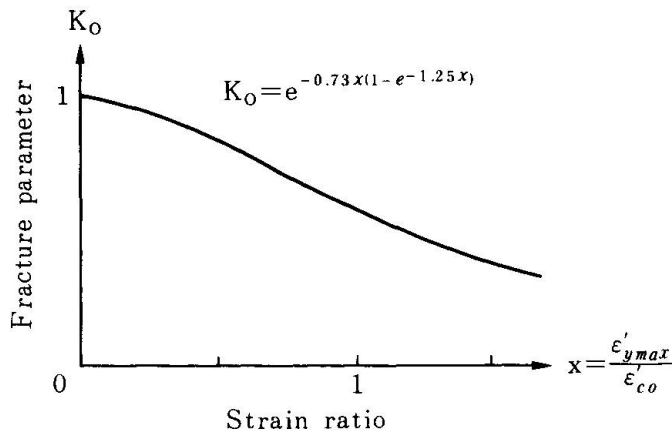
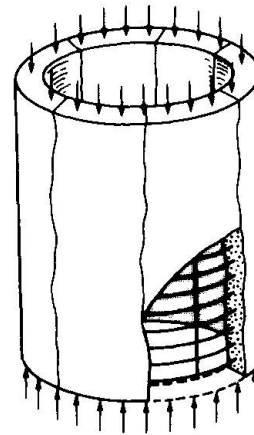


Fig.9 Plastic strain in terms of a function of the maximum compressive strain [10]



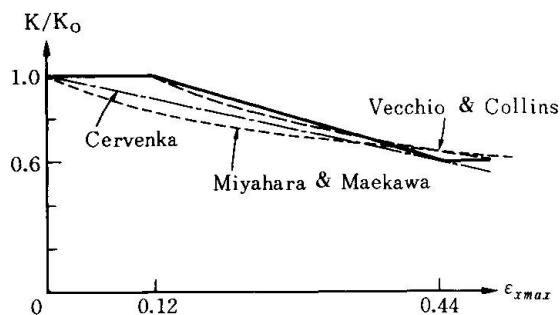
**Fig.10** Fracture parameter for uncracked concrete [10]



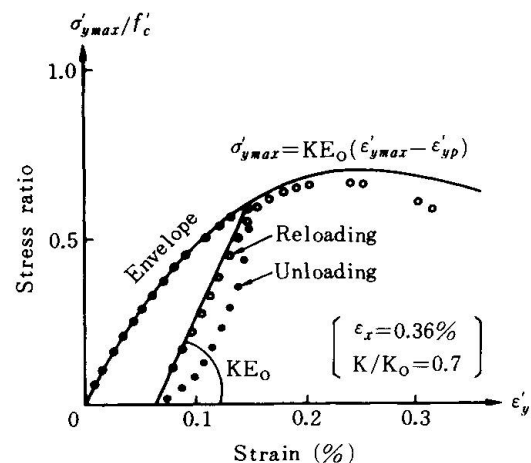
**Fig.11** Compressive test specimens [23]

It has been pointed out by Vecchio and Collins that the apparent compressive strength and stiffness are decreased by cracks. Since these phenomena may be attributed mainly to stress relaxation taking place in the vicinity of cracks, we have elected to take a ratio of the fracture parameter  $K$  and that of uncracked concrete  $K_o$  as a function of the tensile strain in the direction normal to cracks. Namely, the parameter  $K$  was set to be the same as  $K_o$  for the tensile strain less than 0.12 %, be constant at 0.6  $K_o$  for the tensile strain of over 0.44 %, and to change linearly in the intervening range as shown in Fig.12. For comparison, the strength ratios of the proposals of Vecchio and Collins [21], Cervenka [24], and Miyahara and Maekawa [23] are shown in the Fig.12.

The values obtained by the modified Maekawa model are compared with the experimental values due to Miyahara and Maekawa that include the cases of unloading and reloading with the tensile strain constant as an example. It will be seen in Fig.13 that agreement is quite satisfactory except for the lack of bulge in the unloading curve.



**Fig.12** Relationship between the ratio ( $K/K_o$ ) and tensile strain



**Fig.13** Modified Maekawa model versus experimental result [23]

To accomodate the plastic strain that is incurred in concrete when subjected to a compressive stress, the stress strain curve was shifted by that much amount of the compressive plastic strain toward the compressive stress as shown in Fig.14. Here, the tensile elastic modulus was assumed to be  $E_oK$ , the same as for compressive stress, and the tensile strength to be  $Kf_t$ . These have been verified experimentally.

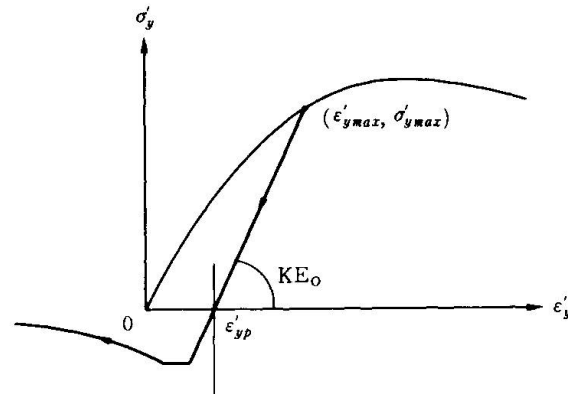


Fig.14 Constitutive laws of cracked concrete from compressive region to tensile region

## 2.5 Shear Transfer Model

Though many investigations have been done on the mechanism of shear transfer along cracks, the majority is concerned only with monotonic loading. We have developed a contact density model that applies well to cyclic loading, assuming each crack to consist of numerous contact planes oriented in various directions as illustrated in Fig.15. By giving appropriate distribution profiles to the orientations of the contact microsurfaces and to their effective areas, relation among the shear displacement, the crack opening and the shear stress has been formulated.

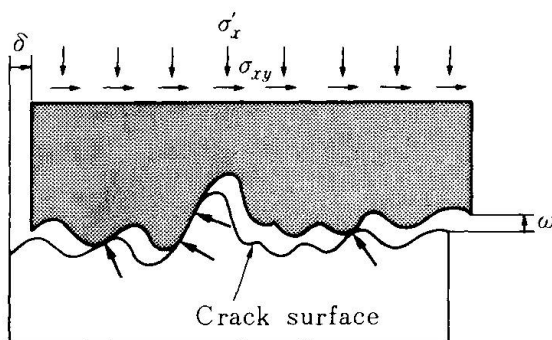


Fig.15 Shear transfer mechanism

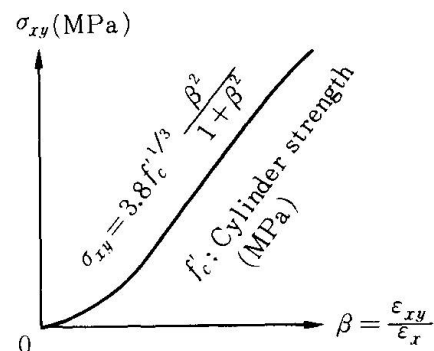


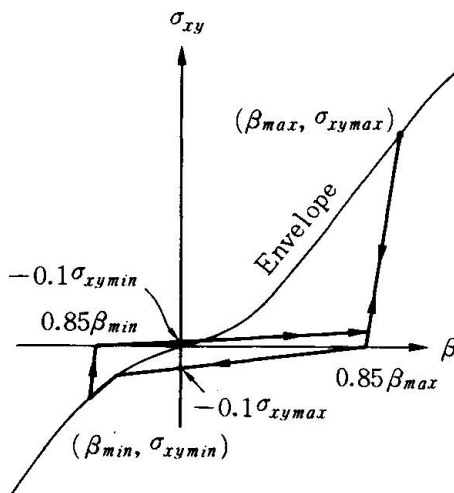
Fig.16 Shear transfer model



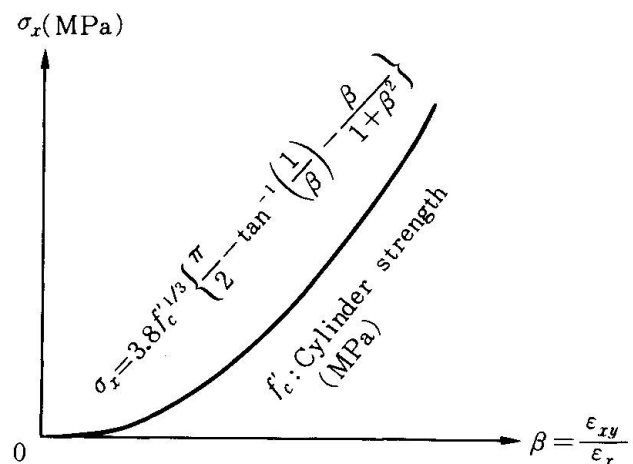
This model is described by an integral equation with regard to the directions of contact planes, and is able to predict the shear transfer behavior for any loading paths. When the loading is monotonic, in particular, this integral equation has an analytical solution, in which the shear stress is given by a uniquely determinable function of the ratio between shear displacement and the crack opening as shown in Fig.16. This function was considered to represent the envelope for the shear transferred stresses. Where, the ratio gives the orientation of a plane situated at a boundary between contacting surfaces and non contacting surfaces.

Though shear transferred stresses in the unloading and reloading can be calculated by this contact density model, it is not too well suited for analysis of reinforced concrete plate elements, because of the trouble of numerical intergration. When numerical integrations were actually conducted for many different paths, it was found that no great errors would be incurred even if the shear transferred stress were expressed uniquely by the ratio between the shear displacement and the crack opening. On the basis of this fact, it was decided that the shear transferred stress should move along a straight line that passed through the envelope point for the onset of unloading and the corresponding plastic point as shown in Fig.17.

Although this model is for a single crack, the average shear stiffnesses for elements containing multitude of cracks in a certain distribution can be presented by the average strain that is given by the relative displacement across a crack divided by the spacing to the next crack. In these cases, the ratio between the shear displacement and the crack opening coincides with that of shear strain and tensile strain, and the shear stress acting on an element containing cracks may be evaluated solely by the shear strain and the tensile strain regardless of the crack spacing. The compressive stresses arising from shear transfer and acting normal to the crack plane are presented in terms of these average strain ratio as shown in Fig.18.



**Fig.17** Shear transfer model for unloading and reloading



**Fig.18** Compressive stress acting on the crack surface due to shear transfer

## 2.6 Steel Reinforcement Model in Concrete

As mentioned earlier, the tensile stiffness of a reinforced concrete may be represented by a superimposition of the tension stiffening due to concrete and the tensile stiffness the steel reinforcement develops. However, the constitutive laws for reinforcement need be modeled taking the bond to concrete into account, so that it should be different from that for an isolated steel reinforcement. Moreover, the stress acting on a reinforcement in between two cracks is not uniform because of the bond, taking on a maximum at the plane of crack as shown in Fig.19.

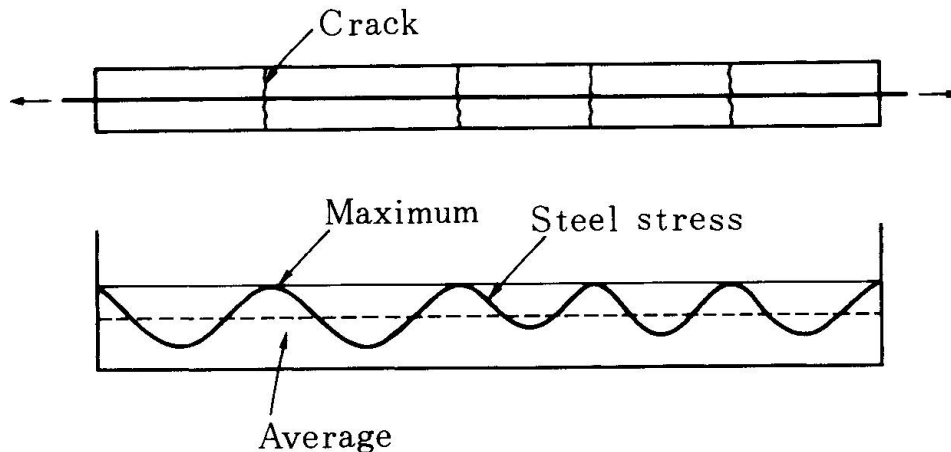
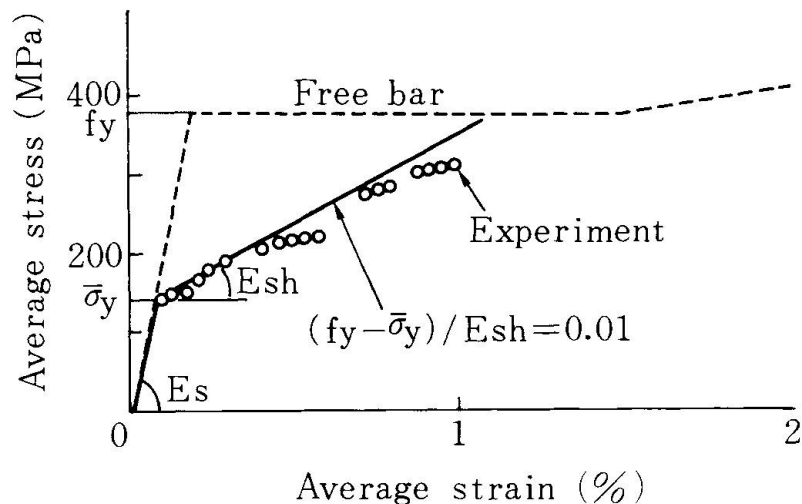


Fig.19 Stress distribution of reinforcement in cracked concrete

As a reinforcement may be treated as elastic body until the stress acting on it reaches the yield strength of that reinforcement at the plane of concrete crack, the relation between the average stress and average strain is elastic as in the case of isolated reinforcement. Once the stress reaches the yield strength and the reinforcement yields, however, the stiffness that represents the average stress average strain relation suffers a rapid decrease. It is important to recognize that yielding of reinforced concrete as such takes place when the stress at the plane of concrete crack has reached the yield strength, not when the average stress in a reinforcement has become equal to the yield strength of isolated reinforcement. This difference is clearly shown in the uniaxial tensile testing of reinforced concrete element conducted by Tamai et al. as shown in Fig.20 [25]. It is also to be noted that the plastic strain shelf, which is often seen in the stress strain curve of isolated steel reinforcement, is not present in the cases of reinforced concrete, but the stress starts to increase monotonically with strain as in the strain hardening stage of isolated reinforcement.

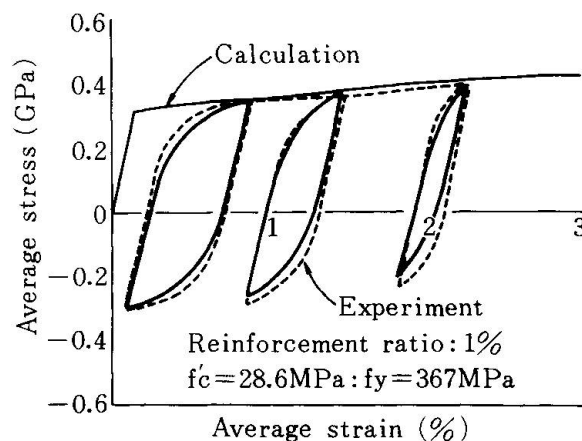
One way of determining the average stress average strain relation after yielding of reinforcement is to assume a distribution profile for the reinforcement stresses acting in between cracks. Tamai et al. assumed this profile to be sinusoidal and the tensile stress of concrete remains in the same relation as that shown in Fig.5 even in the post yielding range. Calculations conducted for many panels, with the yield strength of reinforcement, strength of concrete, reinforcement ratio, difference in reinforcement ratio between two directions, and the angle between the cracks and reinforcement axes as parameters, showed that the average stress average strain relation may well be presented by a straight line shown in Fig.20.



**Fig.20** Average stress and average strain relation of reinforcement

As this apparent strain hardening rate is affected by the reinforcement bonding property, concrete strength, reinforcement ratio, reinforcement ratio difference between two directions, and angle between reinforcement and crack, a parameter has been provided to evaluate their effects integrally. As it was found, however, that these influences may be neglected without much aggravating the precision of calculation, the results presented herein are all those that were obtained with the parameter set equal to unity.

As for the constitutive rule for reinforcement in the unloading and reloading, the Kato's model [26] was adopted for its good agreement with the experimental results. To incorporate the Kato model in the method of Tamai et al. for determining the average stress average strain relation, reinforcement stresses corresponding to strains at various places along the bar axis should be integrated over the whole length to give an average. Since comparison of this method with the one involving the average strain of reinforcements as the reinforcement strain and the average stress of reinforcements as the reinforcement stress showed a very small difference when applied to the Kato model as shown in Fig.21, the more convenient latter method has been used in the present study [22].



**Fig.21** Steel reinforcement in concrete under reversed cyclic loading compared with the experimental results [22]



### 3. ALGORITHM

The algorithm that computes out the stress strain relations for an element is shown as follows. It comprises a routine that calculates a strain increment for a given stress increment through a stiffness matrix to obtain the total strain and the stress that corresponds to it. This process is reiterated until the difference between the stress thus obtained and that given at first has become smaller than an appropriately given limit value.

The stiffness matrix is given for the first round of routine with a value that had originally been for the previous stress level. On the second round, the stiffness matrix is changed to the new one, but no further alterations are done for succeeding rounds. This practice is to take into account the large change that occurs in the stiffness matrix upon cracking of concrete and yielding of reinforcement. The stiffness matrix has been constructed using stiffnesses that are equal to or slightly larger than the tangential stiffness. For example, a zero stiffness is used for the negative stiffness that the tension stiffening model of concrete gives.

The concrete before cracking is assumed to be an elastic body, and upon occurrence of a crack, the direction of that crack is fixed in terms of the coordinates defined in Fig.1. Then, the stress is calculated from the strain in the concrete according to the constitutive rule for cracked concrete developed in section 2. In the meantime, the stress acting on a reinforcement is calculated from its strain by taking the reinforcement axes as the coordinates, and by converting the coordinates, these two stresses are superimposed each other to calculate the stress for the reinforced concrete.

On occurrence of the second crack, similar calculations are conducted in considering this crack alone. Subsequent calculations for concrete stiffness and reinforcement stiffness are done only for that crack which influences the more, though both cracks are taken into account in calculating the concrete shear stiffness.

For the unloading and reloading, material models that are suited for this process are used, meaning that the unloading and reloading of the materials are assumed to be in harmony with those of the element. This assumption is perfectly admissible in the single element analysis of the present work, but it is not without problems when many elements that are smaller than the crack spacing are involved. By making the size of element appropriately for objects that admit of applying the smeared crack model, these problems can generally be forestalled.

### 4. EVALUATIONS

As the comprehensive material models thus formulated so as to apply to reinforced concrete plate elements as described in section 2 have been developed on the results of comparatively simple experiments, applicability of the models was examined by comparing the calculations according to section 3 with the results of experiments conducted for reinforced concrete panels.

#### 4.1 Envelopes

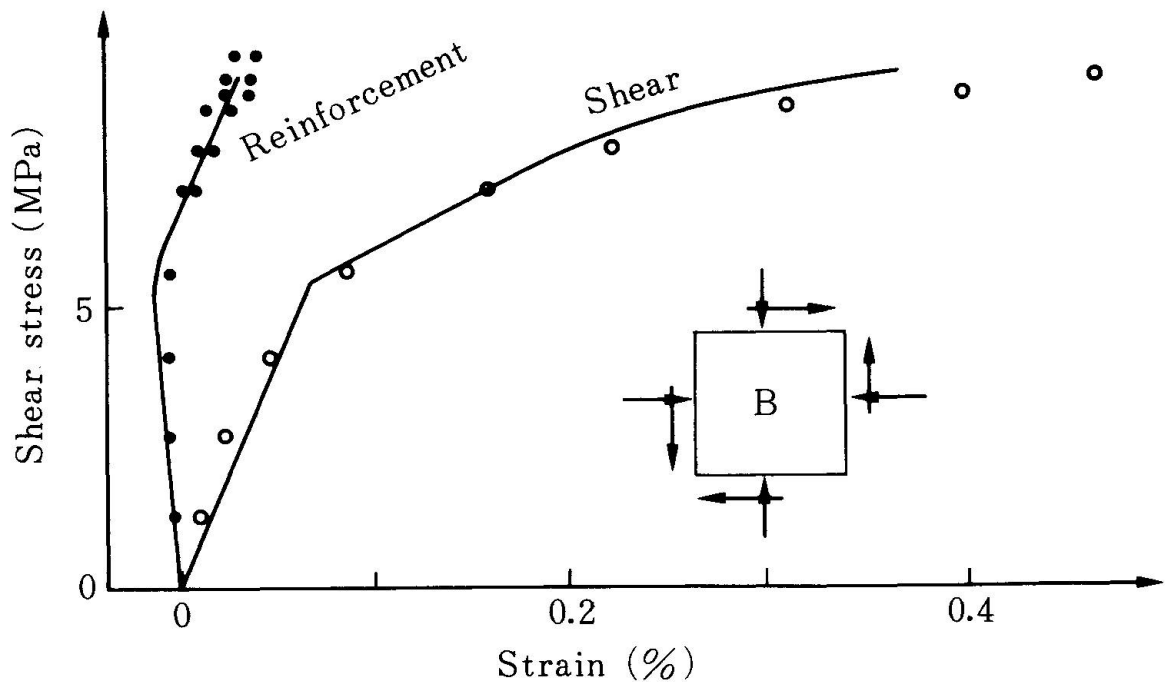
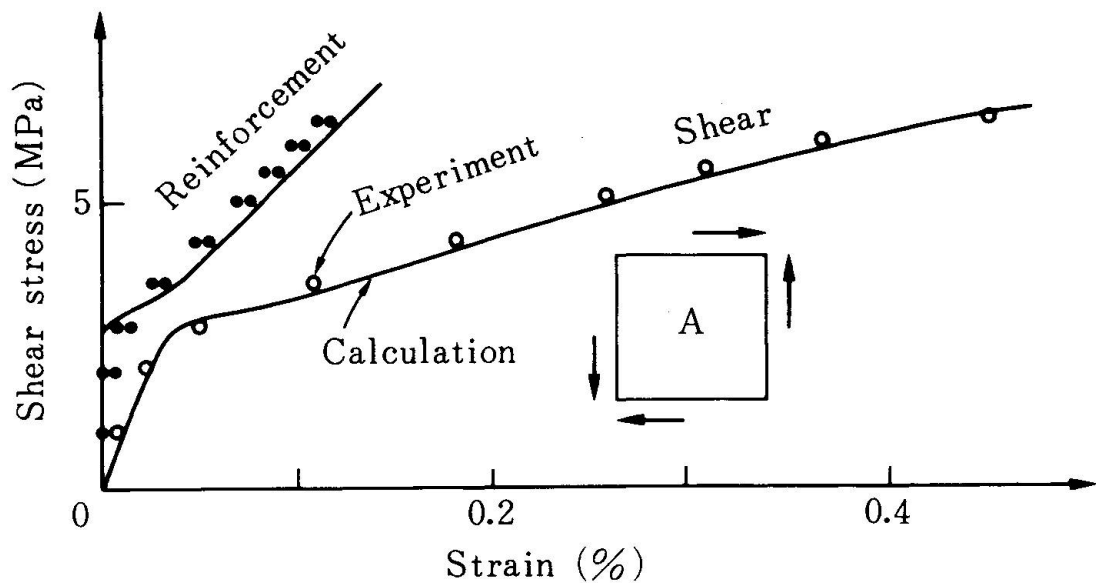
The calculated envelopes were assessed against the 17 panels from Vecchio-Collins' experiments, excluding those that the investigators themselves judged improper, and the 5 panels from Aoyagi-Yamada experiments, excluding the biaxial tensile test results. As an example, Fig.22 presents a comparison to the four panels from the Collins' competition, expect that here tensile

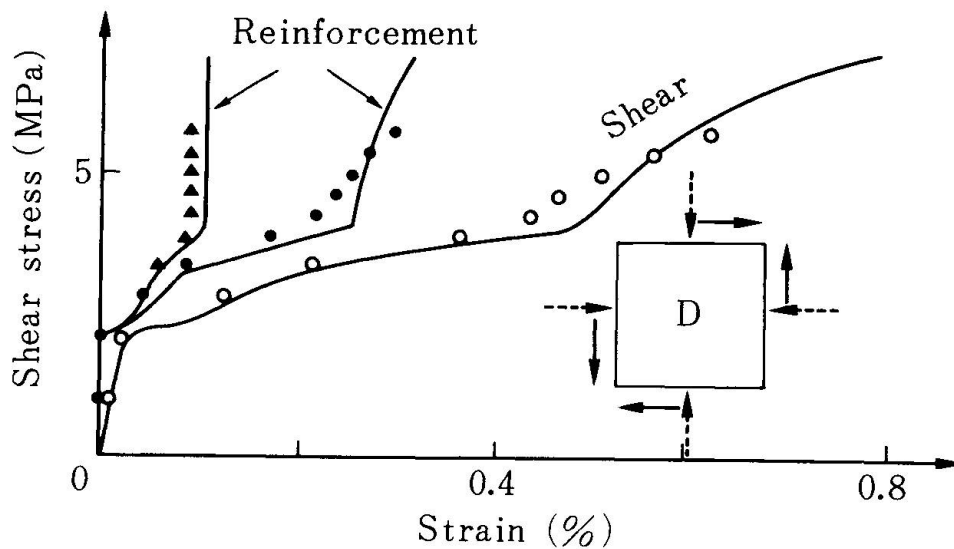
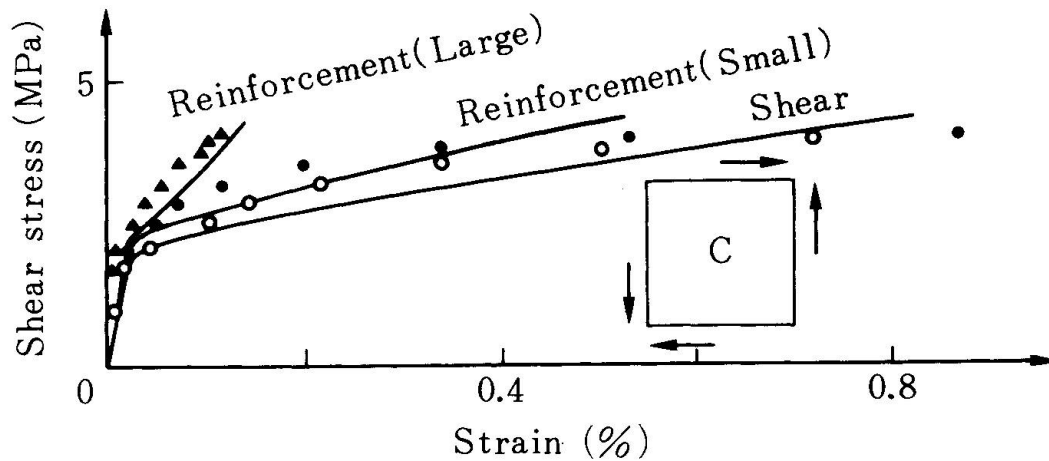




strengths of concrete that were in good accord with the experimental values were used for calculation. The agreement is quite satisfactory.

Similarly good agreements have been obtained for other experimental results. For example, for the ultimate strength, the average of ratios between experiments and calculations is 0.90 with regard to Vecchio-Collins and 1.08 to Aoyagi-Yamada, while the coefficient of variation is 8.6 % and 4.8 %, respectively.





**Fig.22** Comparison of the calculations with the experimental results by Vecchio - Collins for envelope

For the shear stress or tensile stress corresponding to a certain strain, 0.002 for example, the average ratios between experiments and calculations is 1.02 for Vecchio-Collins and 0.99 for Aoyagi-Yamada, where the coefficient of variation is 6.7% and 8.9%, respectively.

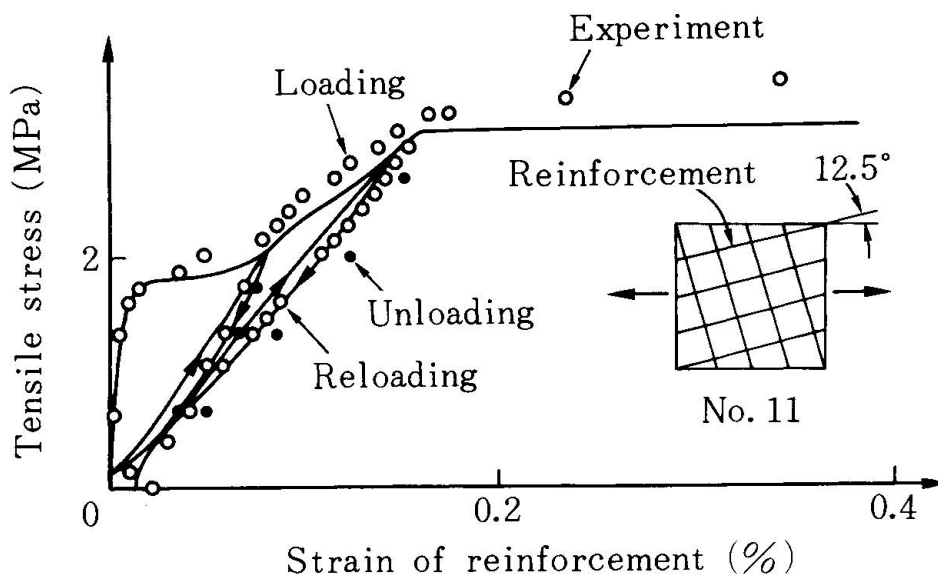
The ultimate strength for shear failure was assumed to be the stress corresponding to the shear strain along the crack plane of 0.004. The ultimate strength for compression failure was obtained when the compressive strain parallel to the crack plane exceeded that corresponding to the ultimate compressive strength of concrete. The ultimate strength for tension failure was defined as that when the average tensile strain of reinforcement reached the value of 0.01. Even though these values are admittedly in need of further examination, it was observed that small changes in those values gave rise to little changes in the ultimate strengths.



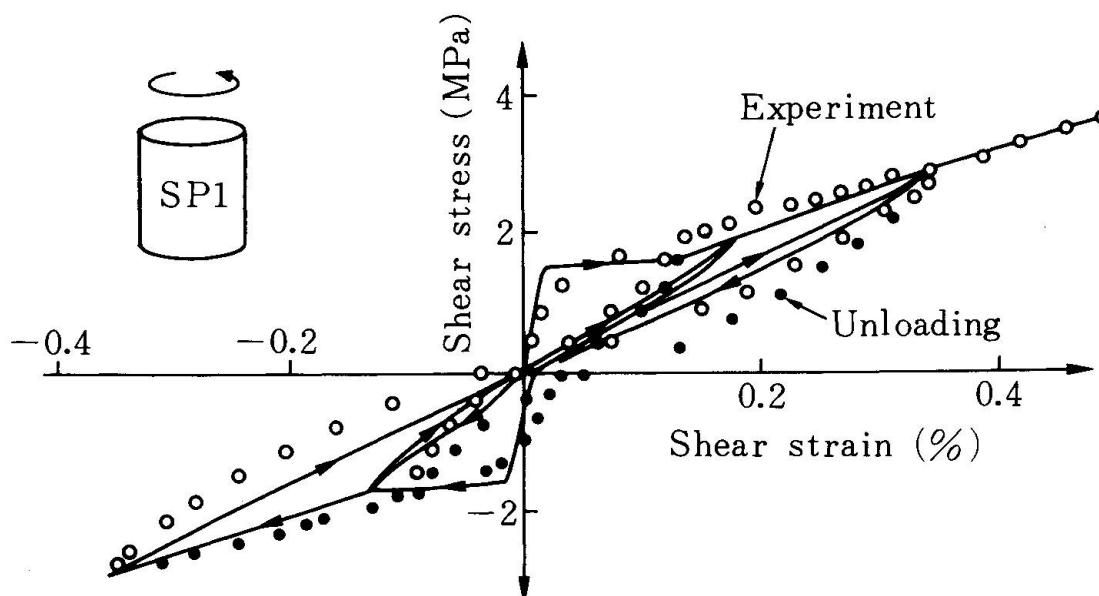
#### 4.2 Cyclic Loading

The calculations for cyclic loading were evaluated against the results of Aoyagi-Yamada experiments [9]. The stress strain relations were faithfully reproduced even for the unloading and reloading as shown in Fig.23.

The results for reversed cyclic loading were compared to Yoshikawa's results [27] on torsion experiments of reinforced concrete cylinders, where no shearing stresses were acting in the plane of crack. As may be judged in Fig.24, the calculation is capable of tracing the element behaviors. However, roundness is somewhat lacking in the calculated behaviors, arising from the insufficient bulge that was given to the material models for unloading.



**Fig.23** Comparison of the calculations with the experimental results by Aoyagi - Yamada for cyclic loading



**Fig.24** Comparison of the calculations with the experimental results by Yoshikawa et al. for reversed cyclic loading

### 5. Concluding Remarks

A comprehensive model for reinforced concrete plate element has been developed with comprising tension, compression, and shear models of cracked concrete and a model for reinforcement in the concrete. The applicability of this model has been verified by comparing with the results of experiments conducted for reinforced concrete panels.

The proposed reinforced concrete model can be applicable to predicting the envelope of a reinforced concrete panel subjected to in-plane forces fairly well. The proposed reinforced concrete model can be capable of tracing the plate element behaviors subjected to reversed cyclic loading. However, it would be necessary to use more accurate material models for unloading.

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