Zeitschrift: IABSE reports = Rapports AIPC = IVBH Berichte

Band: 54 (1987)

Artikel: Tension stiffness model under reversed loading including post yield range

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DOI: https://doi.org/10.5169/seals-41961

Tension Stiffness Model Under Reversed Loading Including Post Yield Range

Modèle de rigidité sous charges cycliques au-delà du domaine d'écoulement

Zugsteifigkeit von Stahlbeton unter Wechsellast im Fliessbereich

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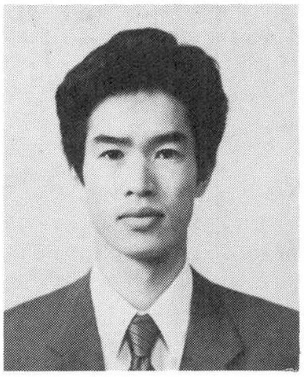

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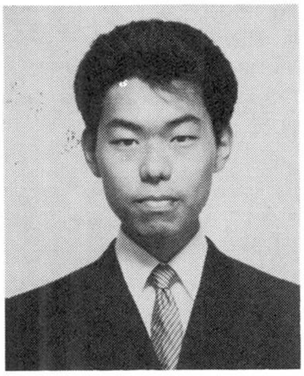

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SUMMARY
A method for calculating tensile stiffness of a reinforced concrete element is proposed by referring to the test results. This method is applicable for reversed cyclic loading, including the post yield range of steel bars. In the post yield range, the average stress of a steel bar is obtained from the average strain by modeling the stress distribution along the bar.

RÉSUMÉ
Une méthode est proposée pour le calcul de la rigidité à la tension d'un élément en béton armé sur la base de résultats d'essais. Cette méthode peut être appliquée pour des charges cycliques inversées, et va au-delà du domaine d'écoulement des barres d'armature. Au-delà du domaine d'écoulement, la contrainte moyenne d'une barre d'armature est obtenue à partir de la déformation moyenne, par modélisation de la distribution des contraintes le long de la barre.

ZUSAMMENFASSUNG
Eine Methode zur Berechnung der Zugsteifigkeit von Stahlbeton wird vorgeschlagen und an Versuchsresultaten überprüft. Die Methode ist für Wechselbelastung anwendbar, einschliesslich des Nachfliessverhaltens der Bewehrung. In diesem Bereich erhält man die mittlere Stahlspannung aus der mittleren Dehnung durch die Betrachtung der Stahlspannungsverteilung entlang des Stabes.

1. INTRODUCTION

In the smeared crack model, the tension stiffness of a reinforced concrete element is usually obtained by superimposing the average stiffness of concrete and that of reinforcing bars, and the stiffness of the bars in concrete is usually modeled as same as that in the air. However, this modeling is not applicable for the post yield range of steel as indicated in this paper. The first object of this research is to obtain the average stress strain relationship in the post yield range of a bar in concrete.

Many models have already been proposed for the average tension stiffness of cracked concrete under monotonic loading [1,2]. However, any tension stiffness models of concrete under reversed cyclic loading have not yet proposed. Formulation of the tension stiffness under reversed cyclic loading is the second object of this research.

Uniaxial tension-compression tests were carried out using prismatic reinforced concrete specimens. When the bar starts to yield at a crack in reinforced concrete element, the yielding does not extend into concrete so quickly due to the existence of bond. Generally, both yielded and non-yielded portions exist along the bar. This makes difficult to determine the average steel stress directly from the average steel strain in the post-yield range. Therefore, the appropriate method to determine the average steel stress from the average steel strain has to be established.

The stress distribution of a bar along its axis was measured to obtain the average steel stress. The relationship between the average stress and average strain of a bar was predicted by modeling the stress distribution curve of the bar. The average stress strain relationship of concrete under reversed cyclic loading was also formulated referring this test results.

2. TESTS

The important parameters for the tension stiffness of a reinforced concrete element are concrete strength, reinforcement ratio, yield strength of steel and curing conditions. Combining these factors, six specimens were tested under tensile cyclic loading and one specimen was tested under reversed cyclic loading as shown in Table 1.

Table 1 Outline of test specimens

<table>
<tr><th>Specimen No.</th><th>f'c (MPa)</th><th>p (%)</th><th>Steel Bar</th><th>Curing condition</th></tr>
<tr><td>1</td><td rowspan="2">45</td><td rowspan="3">1.0</td><td rowspan="4">SD50</td><td>in Water</td></tr>
<tr><td>2</td><td rowspan="6">in Air</td></tr>
<tr><td>3</td><td rowspan="4">25</td></tr>
<tr><td>4</td><td rowspan="3">0.6</td></tr>
<tr><td>5</td><td>SD70</td></tr>
<tr><td>6</td><td rowspan="2">SD30</td></tr>
<tr><td>7 *</td><td>29</td><td>1.0</td></tr>
</table>

f'c : Compressive strength of concrete
p : Reinforcement ratio
* : Reversed cyclic loading test

The dimensions of the specimens are shown in Fig.1. The strains of a bar were measured and converted to the stresses by using the stress strain relationship. In order to measure the strain distribution along the bar, foil resistance strain gauges having length of 5mm were attached on the opposite faces at an interval of 10 ribs.

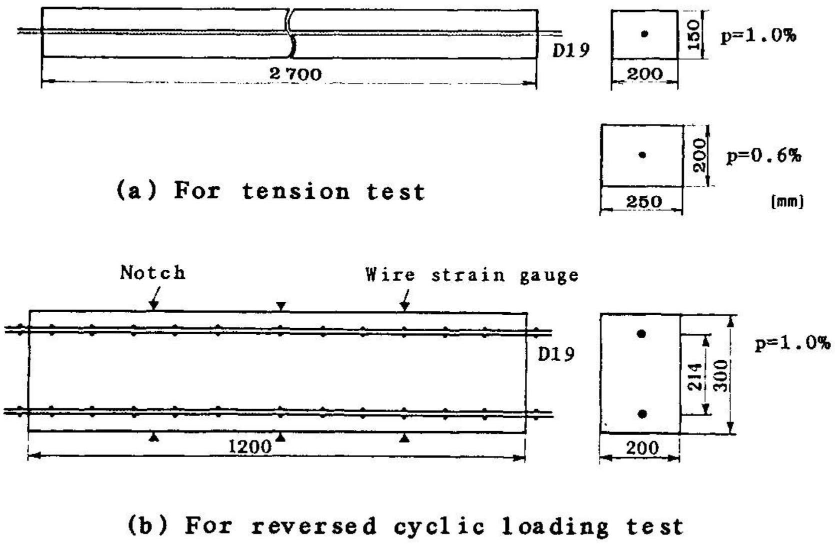

Fig.1 Specimens

Table 2 gives the properties of steel bars. The measured stress strain curves of steel bars are plotted in Fig.2. The solid curves in the figure were used in the analysis. These curves are obtained from the following equations.

$$\sigma = E_s \varepsilon \qquad \text{for } \varepsilon < \varepsilon_y \qquad (1)$$

$$\sigma = f_y \qquad \text{for } \varepsilon_y < \varepsilon < \varepsilon_{sh} \qquad (2)$$

$$\sigma = f_y + (1-e^{(\varepsilon_{sh}-\varepsilon)/k})(1.01f_u-f_y) \qquad \text{for } \varepsilon > \varepsilon_{sh} \qquad (3)$$

where
$k = 0.032(400\text{MPa}/f_y)^{1/3}$
σ : stress (MPa)
ε : strain.

Table 2 Properties of steel bars

Steel bar	SD30	SD50	SD70
Diameter of bar D, mm	19.5	19.5	19.5
Young's modulus Es, GPa	190	190	190
Yield strength fy, MPa	350	610	820
Tensile strength fu, MPa	540	800	910
Initial strain of strain hardening εsh, %	1.65	1.40	0.60

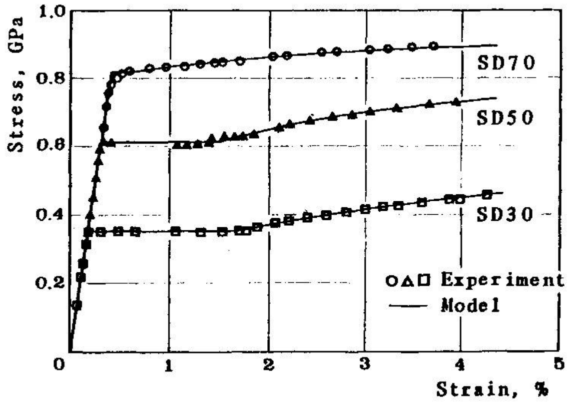

Fig.2 Stress-strain relationships of steel bars

The testing apparatus for tension loading is shown in Fig.3. The friction between the specimen and the floor was prevented by setting rollers under the specimen. The testing apparatus for the reversed loading is shown in Fig.4. Tensile force was applied to the bar and compressive force was applied to both the bar and concrete using loading plates as shown in the figure.

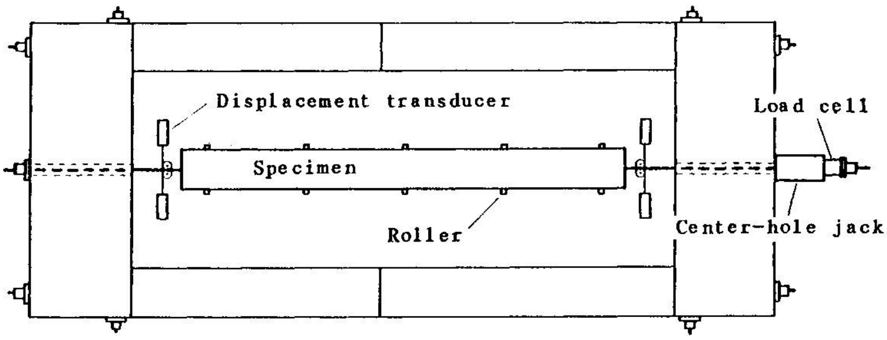

Fig.3 Testing apparatus for tension

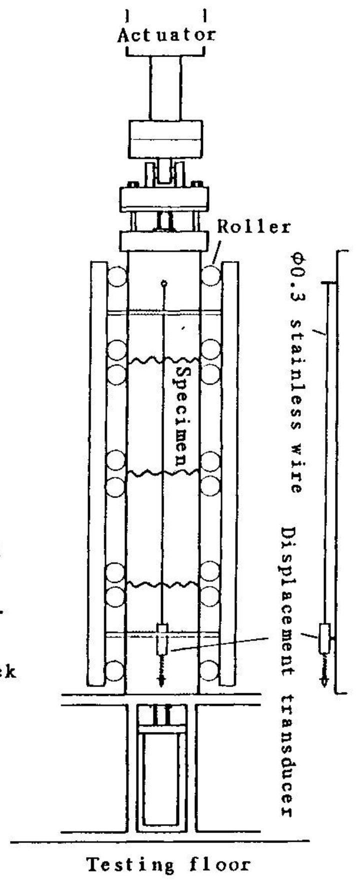

Fig.4 Testing apparatus for reversed loading

In addition to strains, the applied force and the total elongation were measured. The total elongation was obtained from the relative displacement of the both end of the specimen. In the tension loading test, the stainless wires had been adhired to the opposite surfaces of the bar at the end faces of the concrete. The wires were pulled out of the specimen and connected to electrical displacement meters. In the reversed cyclic loading test, the stainless wires were extended from the central point on the opposite faces of concrete block to displacement meters fixed at those points at the other end as shown in Fig.4.

3. AVERAGE STRESS-STRAIN RELATIONSHIP IN POST YIELD RANGE

3.1 Test results

The typical strain distributions along the bar as well as the stress in post yield range are shown in Fig.5. The stresses at the measured points were obtained from the strain measurement and the stress strain relationship shown in Fig.2. The stress distribution was drawn by using the 2nd order polynomial curve fitting the three neighboring points. The average stress was calculated by integrating the stress along the specimen.

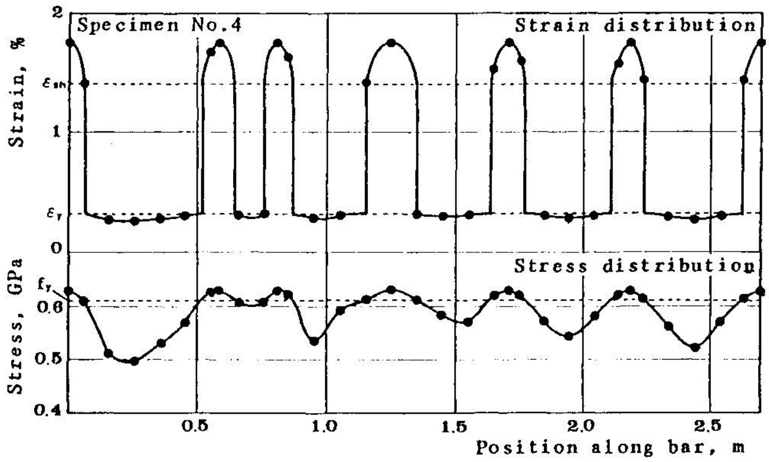

Fig.5
Distributions of strain and stress of steel bar in post yield range of Specimen No.4

The strain is in the elastic range or in the post yield range according to whether the stress is smaller or larger than the yield strength. The steel strain should jump up to the strain hardening range when the stress becomes to yield. This is because the stresses at both sides of an infinitesimal length at yielding location of the bar in concrete should be different due to the existence of bond.

The measured load and the average strains are plotted in Fig.6. The load carried by the bar is calculated from the average stress of the bar. The load carried by concrete is obtained by subtracting the load carried by the bar from the total load. In Fig.6, the average stress strain curve thus obtained is also shown for the steel bar and concrete respectively.

The concrete carries the tensile load even after the bar has yielded and there is hardly any tendency of sudden reduction of tension stiffness. The concrete resists the load roughly 10% of its tensile strength when the average strain was exceeded 2%. This means that the stiffness of bars in concrete shows higher stiffness than in the air for even post yield range. The other specimens also showed the similar tendency.

The average stress strain curve of a bar in a reinforced concrete element has the following distinction from that in the air.

1) The yield point is lower.
2) No yield plateau exists.

When a bar at cracked sections starts to

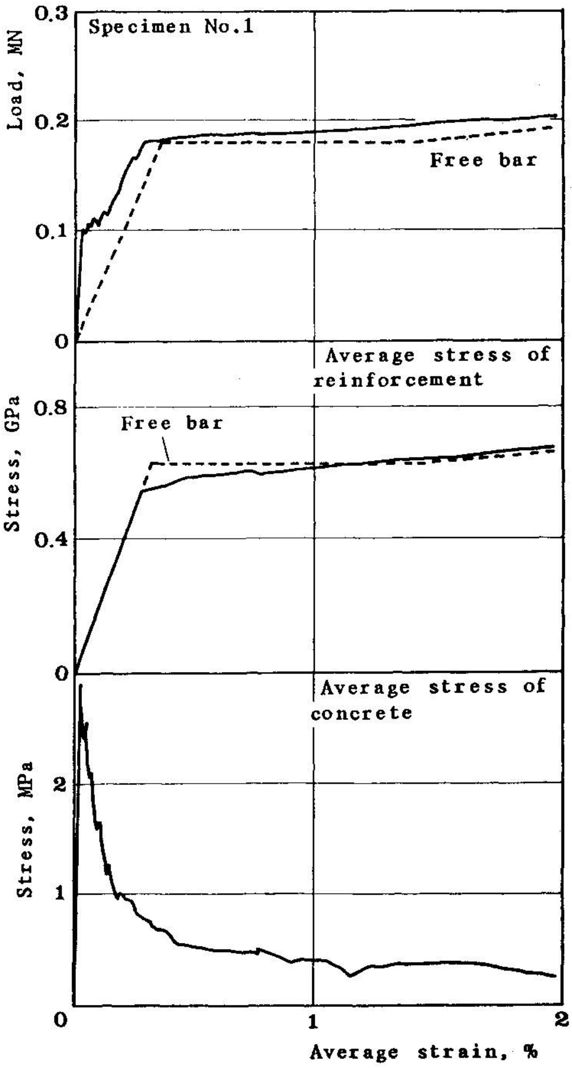

Fig.6 Load vs average strain and average stress-strain relationships of steel bar and concrete

yield, the steel stress between the cracks should be less than the yield strength. This causes the average stress strain relationship of steel to have lower yielding point than that of the bar in the air. Immediately after the bar yields at cracks, the strain jumps up to the strain hardening range as shown in Fig.5. This causes no yield plateau in the average stress strain curve of steel in reinforced concrete element. After the strain at cracked sections has entered the strain hardening range, the strain of the bar between cracks gradually enters the strain hardening range as long as the applied load increases. This phenomenon brings the gradual increase of the average stress in post yield range. When the strain becomes large, the average stress strain curve approaches that of the bar in the air. This is because the larger the steel strain, the lower the bond stress and the smaller the tension stiffness of concrete.

3.2 Modeling

The average stress of a bar in concrete can be obtained from the stress distribution along the bar and the average strain can also be obtained from the strain distribution. The stress and the strain of a bar can be converted each other by using the stress strain relationship so that the determination of the stress distribution or the strain distribution is enough for predicting the average stress strain relationship of the bar in concrete.

The bond stresses at a cracked section and at the center of a segment are zero and the bond stress is the differential of the stress distribution curve. Therefore, the stress distribution curve should have a form such that the slopes of the tangent at the cracks and the center of segment are zero. The cosine is one of the functions which agree with the above conditions. Experimental results indicated that the stress distribution could be shown by cosine curve independent on the strength of the bar or concrete and on crack spacings which were varied with drying shrinkage or reinforcement ratio.

The calculate procedure to obtain the average stress strain relationship of a bar in concrete after yielding is given in Fig.7. The accuracy of the cosine curve model for stress distribution can be verified by the comparison of the analytical and the experimental results as shown in Fig.8. In this calculation, the average stress strain relationship of concrete in tension obtained from the experiments was used.

In the analysis of reinforced concrete, the average stress strain relationship of concrete shall be given. The following equation is one of the tension stiffness model of concrete [3].

$$\sigma c = f_t \left(\frac{\varepsilon_{tu}}{\varepsilon}\right)^c \qquad (4)$$

where

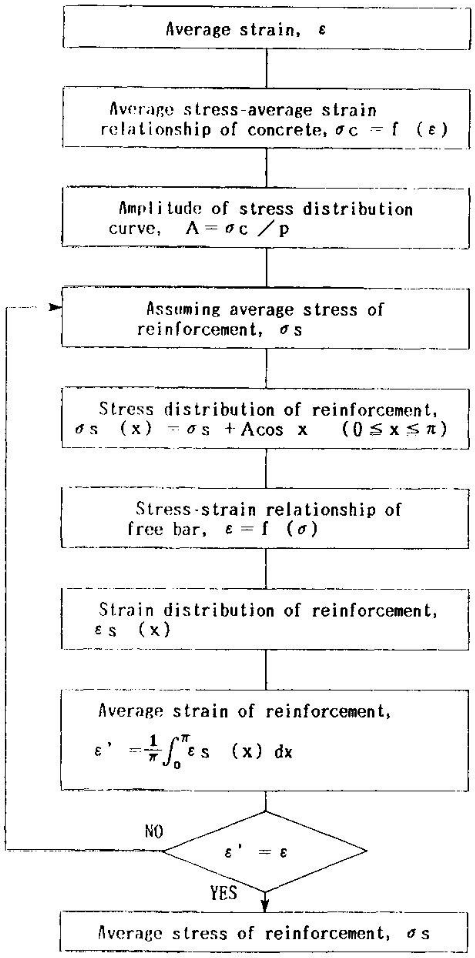

Fig.7 Calculate procedure to obtain average stress-strain relationship of steel bar in post yield range

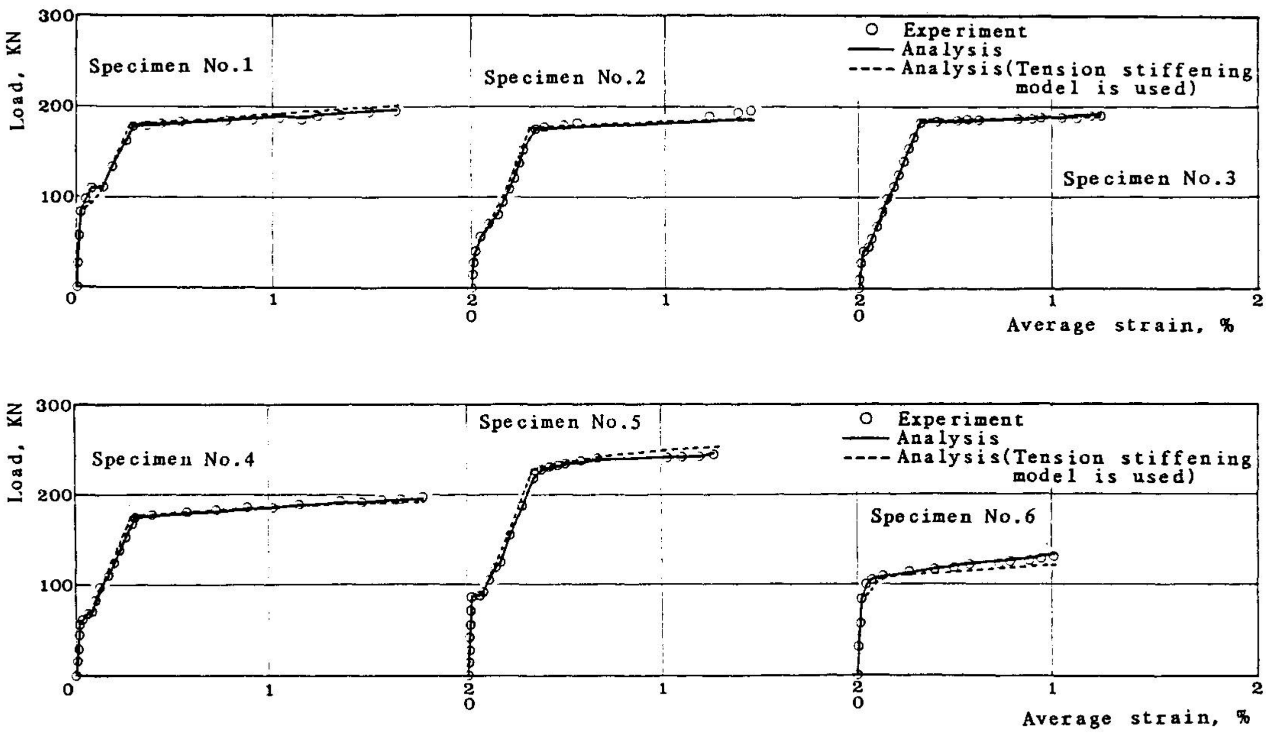

Fig.8 Comparison of analytical and experimental results of load-average strain curve

f_t : tensile strength of concrete
ε_{tu} : cracking strain equal to 0.02%
c : coefficient depending on bond characteristic, 0.4 for deformed bar.

For simplicity this equation is assumed to be applied to post yield range. The analytical results of load average strain relationship using eq.(4) are illustrated by the broken curves in Fig.8. The analytical results agree fairly well with the experimental results.

4. TENSION STIFFNESS UNDER REVERSED CYCLIC LOADING

4.1 Average stress average strain relationship of concrete

The stress in concrete under reversed cyclic loading is produced by the bond action and the contact of the crack. Therefore, the average stress of concrete shall be divided as follows.

$$\sigma c = \sigma cc + \sigma cb \qquad (5)$$

where
σc : average stress of concrete
σcc : average stress of concrete produced by contact of the crack
σcb : average stress of concrete produced by the bond action.

The average stresses of concrete produced by bond and contact of the crack can be determined by the following equations.

$$\sigma cb = p\ (\ \sigma s,cr - \sigma s\ ) \qquad (6)$$

$$\sigma cc = \frac{P - As\ \sigma s}{Ac} - \sigma cb \qquad (7)$$

where
  p : reinforcement ratio
  $\sigma s,cr$: stress of steel bar at crack
  σs : average stress of steel bar
  P : external load.

The relationship between the stress produced by contact of the crack and the average strain is shown in Fig.9. This relationship for the unloading path is formulated as

$$\sigma cc = Ecc(\varepsilon - \varepsilon cs) \geq 0 \qquad \text{for } \varepsilon cp < \varepsilon \qquad (8)$$

$$\sigma cc = Kc(\varepsilon) \qquad \text{for } \varepsilon \leq \varepsilon cp \qquad (9)$$

where
  Ecc : stiffness of concrete at contacting state
  εcs : strain when the crack starts to contact
  Kc : stress-strain relationship of plain concrete
  εcp : strain when the crack contacts perfectly
and that for reloading path is assumed to be according to Kc. What the crack starts to contact at the strain larger than zero results from looseness or local plastic deformation of concrete at crack surface. The strain when the crack starts to contact and the stiffness of concrete at contacting state were 0.015% and Ec/3 respectively.

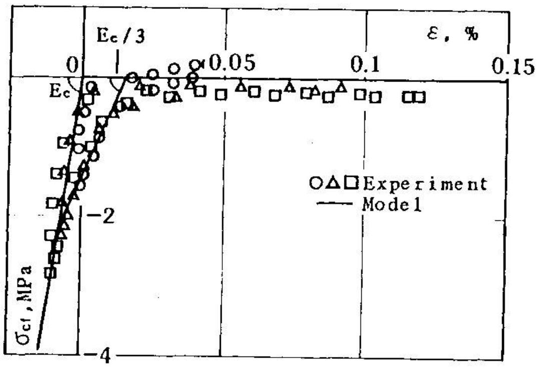

Fig.9 Relationship between σcc and average strain

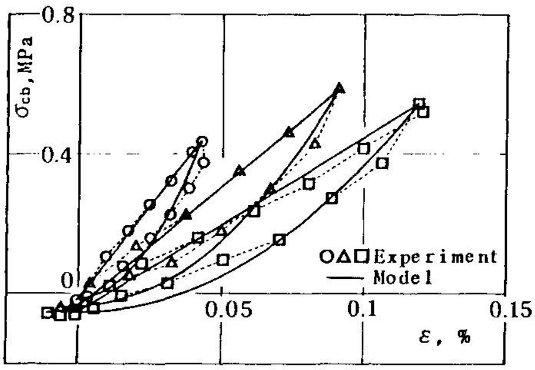

Fig.10 Relationship between σcb and average strain

The relationships between the stress produced by bond and the average strain are shown in Fig.10. The stress produced by bond is assumed to be constant when the external stress is carried by contact of the crack. This constant stress starts at a certain strain between the strain when the crack starts to contact and that when the crack contacts perfectly. This strain was assumed to be zero and the stress produced by bond at this strain was found from experimental results to be given by

$$\sigma cbo = -0.0016 \; Ec \; \varepsilon max \qquad (10)$$

where
  σcbo : stress produced by bond at zero of strain
  εmax : maximum strain produced before
  Ec : young's modulus of concrete.
The relationship between the stress produced by bond and the average strain is

formulated using this stress. The second degree polynomial curve was used for the unloading path and straight line was used for the reloading path as follows.

$$\sigma cb = \frac{\sigma cbun - \sigma cbo}{\varepsilon un^2} \varepsilon^2 + \sigma cbo \qquad \text{for unloading} \qquad (11)$$

$$\sigma cb = \sigma cbre + \frac{\sigma cbun - \sigma cbre}{\varepsilon un - \varepsilon re} (\varepsilon - \varepsilon re) \qquad \text{for reloading} \qquad (12)$$

where
εun : strain at the turning point where unloading starts
$\sigma cbun$: stress produced by bond corresponding to εun
εre : strain at the turning point where reloading starts ($\varepsilon re \geq 0$)
$\sigma cbre$: stress produced by bond corresponding to εre

The relationship between the stress produced by bond and average strain for the envelope curve in tension can be given by the average stress strain relationship of concrete under monotonic tension because the contact of crack does not occur in this state.

4.2 Load-average strain relationship of uniaxial member

Before yielding of steel bar, the average stress strain relationship of the bar is $\sigma s = Es\varepsilon$ independently of the loading history. Fig.11 shows the comparison of analytical and experimental results of the load average strain relationship before yielding. The analytical result agrees with the experimental result.

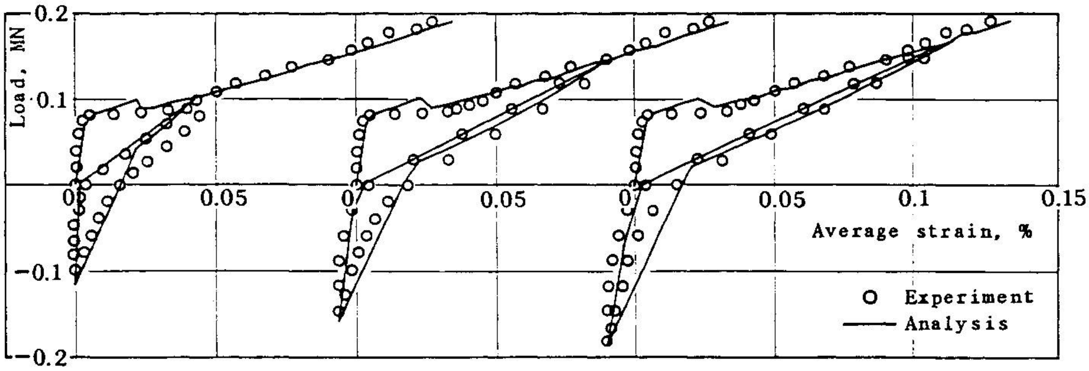

Fig.11 Comparison of analytical and experimental results of load-average strain relationship before yielding

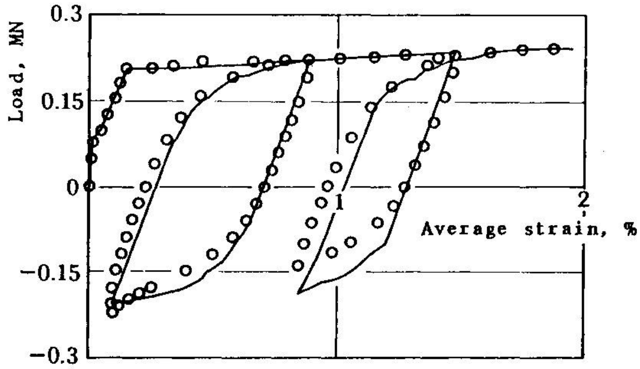

Fig.12
Comparison of analytical and experimental results of load-average strain relationship in post yield range

After yielding of steel bar, the average stress average strain relationship of the bar can be calculated in the same way as under monotonic loading if the stress distribution curve of steel bar is assumed to be cosine curve independently of the load. From the experiment, it was certified that the stress distribution curve of steel bar could be expressed by cosine curve. The comparison of analytical and experimental results of load average strain relationship in post yield range is shown in Fig.12. The conversion of stress distribution into strain distribution in the analysis was made by tracing the stress strain history by using Kato's model [4] at the points which divide half of crack spacing into 20 equal length portions. The analytical result agrees with the experimental result.

5. CONCLUSIONS

(1) A method for calculating tensile stiffness of a reinforced concrete element is proposed by referring the test results. This method is applicable for reversed cyclic loading, including the post yield range of steel bars.

(2) The average stress strain relationship of concrete under reversed cyclic loading was formulated. The average stress was divided into two components which were produced by the bond action and the contact of crack.

(3) In the post yield range the average stress of a steel bar is obtained from the average strain by modeling the stress distribution along the bar. This approach can be applied to reversed cyclic loading.

(4) The yield point of the average stress strain curve of a steel bar in a reinforced concrete element is lower than that of a bar in the air and no yield plateau exists in the curve.

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