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Discrete Element Method and Beam Dynamics, an Application of TILLY

Méthode des éléments discrets et dynamique des poutres, une application de TILLY

Methode der diskreten Elementen und dynamische Analyse von Trägern, eine Anwendung von TILLY

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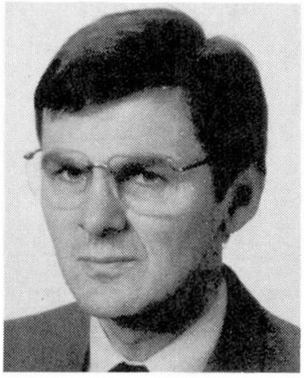

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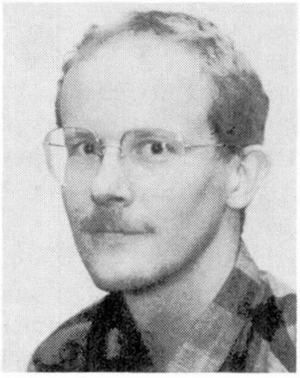

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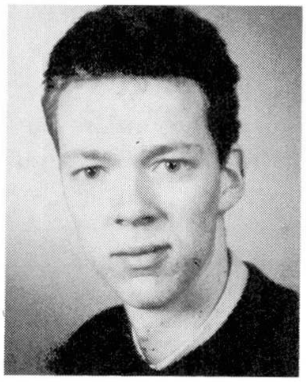

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SUMMARY

A discrete element method (DEM) is applied for the dynamic response of a reinforced beam. The motives to do so are explained and an example is discussed. A comparison with the results of a test and a finite element analysis shows the possibilities and limitations.

RÉSUMÉ

Une méthode des éléments discrets est appliquée pour déterminer le comportement dynamique d'une poutre en béton armé. On explique les motifs pour cette application et on présente un exemple. Une comparaison entre les résultats d'un essai et d'un calcul à l'aide de la méthode des éléments finis montre les possibilités et les limites.

ZUSAMMENFASSUNG

Eine Methode mit diskreten Elementen wird auf das dynamische Verhalten eines Trägers aus Stahlbeton angewandt. Der Anlass wird erklärt und ein Beispiel wird besprochen. Ein Vergleich zwischen den Ergebnissen eines Versuchs und einer Berechnung zeigt die Möglichkeiten und Grenzen dieser speziellen Methode der Finiten Elemente.

1. WHY AND WHEN A SIMPLE MODEL

In this paper the name "discrete element method", in short DEM, is used for a mechanical model, which is a composition of undeformable rigid finite elements, which are connected by deformable lumped springs and dampers. Lumped masses can be applied which correspond with the degrees of freedom of the model. The spring behaviour can be defined in a free way, such that nonlinearities and time dependency is involved. Simular options are included for the dampers. The name of the program is TILLY, which is composed of the first characters of the following list of specifications:

* Transient and static analysis: Both dynamic and rheologic transient processes have to be simulated by the model
* Incremental loading and initial strains: The load and initial strains are always applied step–wise in time; even for static calculations one has to introduce one or more time steps.
* Linear and nonlinear behaviour: In due time both material nonlinearity and geometrical nonlinearity will be covered. Material nonlinearity may be plasticity and fracturing, hardening and softening.
* Lumped masses, springs and dampers: In due time to be extended to elements of two and more generalized deformations.
* Young and aging materials: Material properties can be constant in time, but also dependency of time must be included. Material stiffness and damping data may increase in time (for instance: young concrete) or may deteriorate (for instance: damaging by cyclic loading).

Reference is made to [1] and [2] for more details. The reasons to develop and apply such a program are manifold. An important consideration has been the immense computing time which is involved in the use of finite element programs for dynamic analysis in combination with nonlinear behaviour. A complete run on a high speed computer demands many man weeks and cpu hours. The state of the art makes clear, that major changes are needed to place these tools at the disposal of the profession. One way to achieve that, is to use supercomputers and specially designed finite element machines. A remarkable improvement is expected from these new facilities, but it is to be doubted, that practizing engineers will apply them. Research engineers are more likely to profit from these new tools.
Another way to serve the profession is to simplify the model. The engineer does not require exact data. He will be satisfied with approximating engineering models. For him the only requirement is, that researchers have proved by the use of their advanced models, that the course model is sufficiently appropriate.
We conclude, that two parallel activities are needed: continuing research on advanced finite element models (FEM) and the development of approximating course models. The discrete element model (DEM) is an attempt for such a simplified engineering tool. Another reason to start the development of TILLY has been the wish to involve as much graduate students as possible in the development and use of numerical models. Large and powerful finite element packages restrict the number of students, which can participate. Computing time is limited and also the number of specialistic supervisors. A more simple model can run on the PC's of the students themselves and other supervisors can be involved as well.

2. THE FALLING BEAM PROBLEM

To demonstrate the use of TILLY, the falling beam has been calculated which was analyzed earlier using the Dutch finite element package DIANA [3]. The reinforced beam is one of a series which has been tested in Switzerland [4]. In this example the bending and shear behaviour is modelled, so rotational and shear springs are the obvious discrete elements. The reader should keep in mind, that only a specific application of TILLY is discussed. Other spring types and compositions of discrete elements can be used in other cases.

The beam of reinforced concrete has the following dimensions: the total length is 8.15 m, the depth of the beam is 0.3 m and the width 0.4 m. The span l between the two supports is 7.85 m. The support at the right hand end is a hinge. The beam was elevated at the left hand end over the height h = 3.75 m and then was dropped. The left hand support is a shock absorbing one and has a progressive spring stiffness. For TILLY the average stiffness k = 6800 N/m has been chosen. The reinforcement percentage is constant over the beam span. Both the top reinforcement and the bottom reinforcement are 0.56 %. For the rest, the following strength and other data apply: specific density $\rho = 2500$ kg/m³, $f_s = 650$ N/mm² (steel), $f_{ct} = 4.8$ N/mm² (concrete), $E_s = 210000$ N/mm².

From the test result it is known, that the shock absorber is compressed over about 6 cm at the very start, when the falling beam gets in contact with the shock absorber (t = 0), but ignorable compression occurs at later time. The maximum displacement at midspan is 0.69 m at time t = 0.164 s. This displacement is mainly due to plastic bending deformation. The final state is the result of a complex history. The beam cross- section starts to become a plastic hinge near to the shock absorber and this plastic region then moves in midspan direction. So, at maximum deflection the deformation state is more or less to be compared with the failure mechanism for static loading, but it takes some time and it requires some other intermediate limit states to achieve the final state.

For the rest, we do not need a computer at all to explain the ultimate deflection. A simple calculation by hand already provides a fair estimate of the maximum displacement $\hat{w}$ at midspan and the time $\hat{t}$ at which it occurs. If one neglects the early limit states, one can assume a rigid plastic behaviour of the beam with a plastic hinge at midspan. The triangular speed distribution with $v_0 = \sqrt{3gh}$ at the shock absorber immediately transforms in another triangular distribution with the maximum speed $\hat{v} = 3/4\, v_0$ at midspan. This yields a rigid plastic model with one degree of freedom. In this model a mass M with initial momentum P is decelerated by a plastic force F_p and accelerated by a gravity force F_g. The equivalent mass is $M = 1/3$ ml in which m is the mass per unit length, the equivalent momentum P at $t = 0$ is $P = M\hat{v}$, the equivalent force from dead weight is $F_g = 1/2$ mgl in which g is the acceleration due to gravity and the equivalent yield force $F_p = 4M_p/l$, in which M_p is the full plastic moment of the cross- section. The resulting force on the mass M is $F = F_g - F_p$. The time $\hat{t}$ at maximum deflection is calculated from $\hat{t} = P/F$, the acceleration of the mass from $\hat{a} = -\hat{v}/\hat{t}$ and the maximum displacement $\hat{w} = \hat{v}\hat{t} - 1/2\, \hat{a}\hat{t}^2$. Applying all available data, it is found that $\hat{t} = 0.16$ s and $\hat{w} = 0.62$ m, which is close to the test results $\hat{t} = 0.164$ s and $\hat{w} = 0.69$ m.

3. THE DISCRETE ELEMENT MODEL

The beam has been divided in 21 elements. The length of the end elements is 0.275 m and all other elements are 0.4 m in length. Four different models have been used. One difference between the 4 models is the number of springs (model A and model B) and another difference regards the nonlinear characteristics of the spring (type I and type II). Model A contains rotational springs only. Fig. 1 shows this model and makes clear, that 1 degree of freedom occurs between the rigid elements at the position of

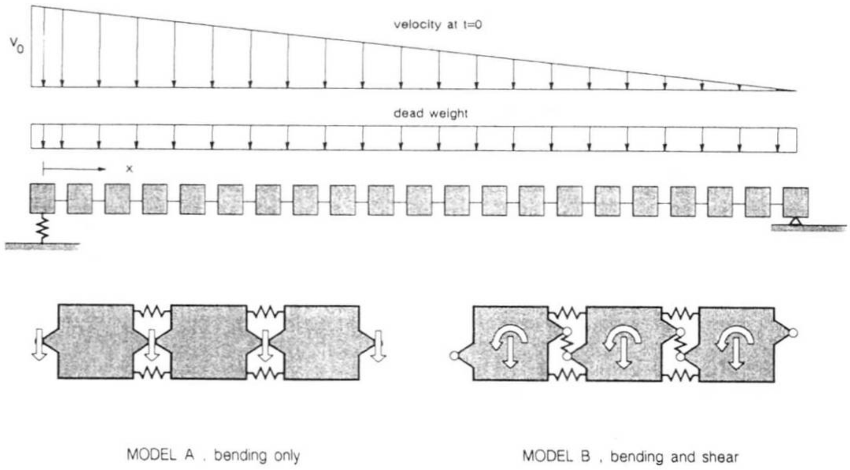

Fig. 1 Two different discrete element models

each rotational spring. The degrees of freedom are vertical displacements. The mass is lumped at the same position. This model takes into account the inertia of translation, but neglects the inertia of rotation. Also shear deformation is ignored. This model predicts moments and displacements but does not provide any information on shear forces. Model B uses both rotational springs and shear springs (fig. 1). Now the degrees of freedom are defined in the mid of the rigid elements and 2 ones occur per element, a vertical displacement and a rotation. This model can account for bending deformation, shear deformation, inertia of translation and inertia of rotation. The results are displacements, bending moments and shear forces. The extra information for the shear forces is got at the cost of more computing time. The number of degrees of freedom is doubled, which makes the computing time increase by a factor 1.5 to 2.

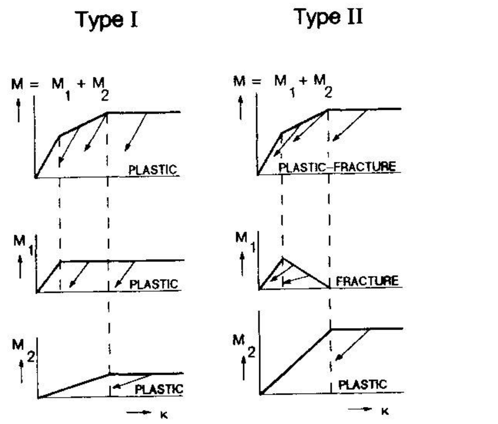

Fig. 2 Decomposition of constitutive model in two components. Left: two elastic–plastic springs representing the plastic model. Right: one elastic–fracture spring representing the plastic–fracture model.

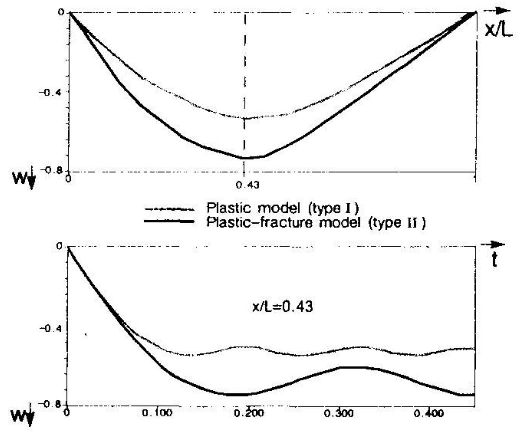

Fig. 3 Results for model 1 (rotational springs only). Spring type I is far more stiff than spring type II which models fracturing.

Fig. 2 is helpful to explain the similarity and the difference between the two rotational spring types I and II. They have in common, that a moment–curvature relation is used which consists of three branches, one for the uncracked state, one for the cracked state and one for the limit state at yield of the reinforcement. In both models the tri–linear behaviour is simulated by a fraction model of two parallel springs, which have different stiffnesses and fail at different stress levels. In both models an ultimate value can be specified for the plastic curvature (deformation capacity). Type I is a fraction model of two elastic–plastic springs. This means, that unloading always takes place with the stiffness of the branch for the uncracked state. In type II one elastic–plastic spring is used and one elastic–fracture element. This last one represents the softening at the first loading cycle due to cracking in the tensile zone. In concrete research this contribution sometimes is mentioned tension stiffening. It is not present any more after full loading has been applied. This type II is closer to reality than the approximation of type I. Also the unloading stiffness is better. For the shear spring in model B only one stiffness type is applied. Both rotational type I and rotational type II have been combined with an elastic shear spring.
To be honest, at the time the runs were made, the softening spring did not exist yet. At that time this spring was built from five separate parallel elastic springs which fail in a brittle way at different ultimate strengths. The result is practically the same, but the needed CPU time is higher.

4. RESULTS

The difference between the models A and B is not noticeable. Almost the same displacements and bending moments are found. The most important extra of model B proves to be that also shear forces can be calculated. The difference between the spring types I and II, however, is notable. Fig. 3 shows the results for model A (only rotational springs). Spring type I is far more stiff. The maximum displacement is smaller and the natural frequency after unloading is roughly two times higher. The result of type II is closer to the test results than type I. From here we only show data for type II.

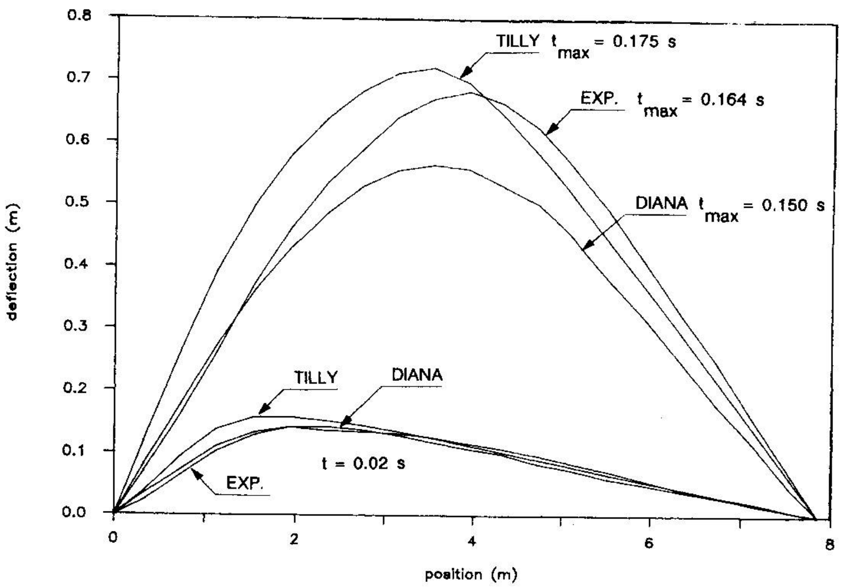

Fig. 4 Deflection lines for two times (t=0.02 s and t_{max}). Comparison of test, DIANA and TILLY (model B, type II)

In fig. 4 deflection lines are shown at two different times, viz. t = 0.02 s and t_{max} max at which the maximum value is reached. In both cases 3 lines are presented, namely the test result, the result of the finite element package DIANA and the TILLY result. At t = 0.02 s the largest curvature occurs between 1 and 2 meters distance from the left hand end. The DIANA result is closer to the test than the TILLY result, but TILLY does pretty well taking in account that the number of degrees of freedom is 8 times smaller (DIANA 344, TILLY 41). At the time of the maximum deflection, both TILLY and DIANA differ from the test. Near the shock absorber DIANA fits better than TILLY, but in a global sense the result of TILLY is sufficiently accurate. Fig. 5 shows a plot of the maximum displacement at x = 3.35 m versus time. TILLY and DIANA produce comparable results. The natural period of the beam after the maximum deflection is reached seems to be longer in the test than in the analysis. However, for later cycles the period in the test becomes smaller and is closer to the analysis result.

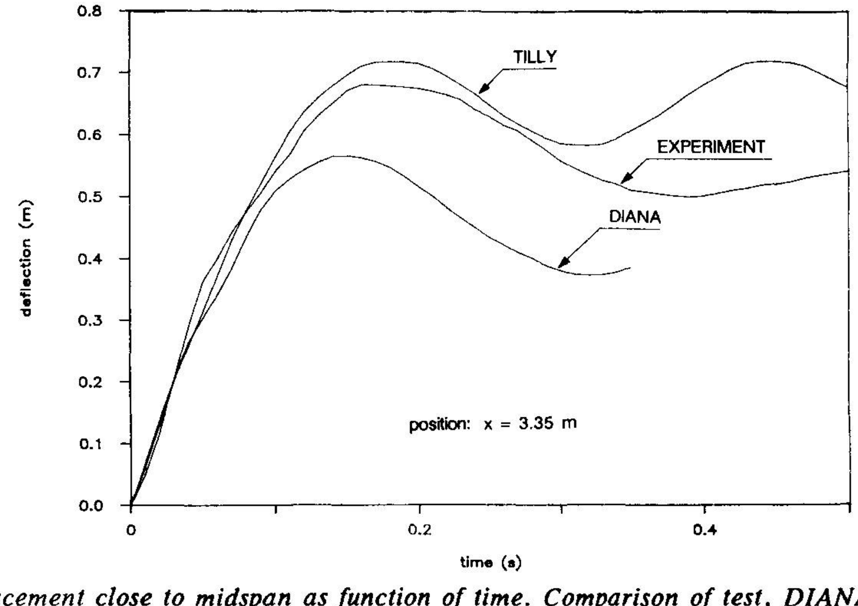

Fig. 5 Displacement close to midspan as function of time. Comparison of test, DIANA and TILLY (model B, type II)

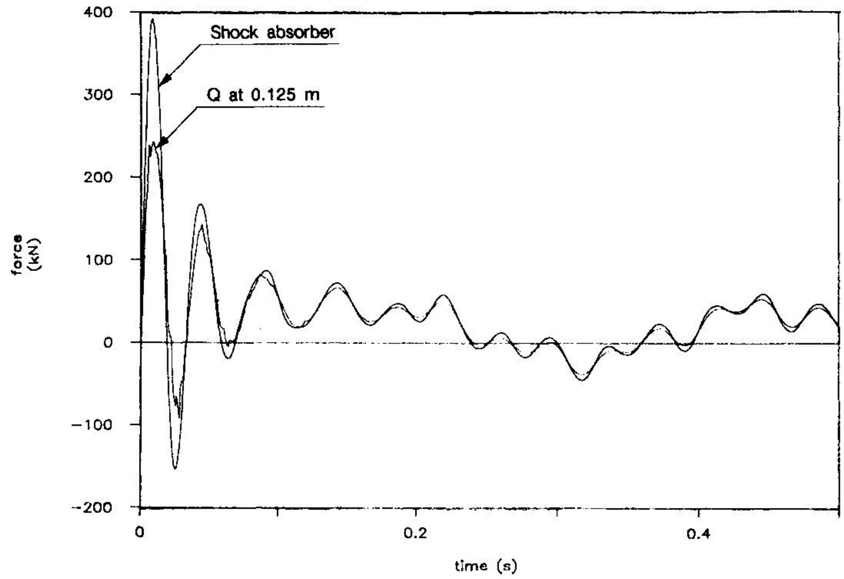

Fig. 6 Support reaction in shock absorber and shear force close to shock absorber for model B and type II.

Fig. 6 is a time–plot for the support reaction in the shock absorber and the shear force in the beam at x = 0.125 m close to the absorber. The shear force is strongly influenced by the high frequencies of the discrete model, so dampers have been used parallel with the springs to moderate the high–frequent signals.

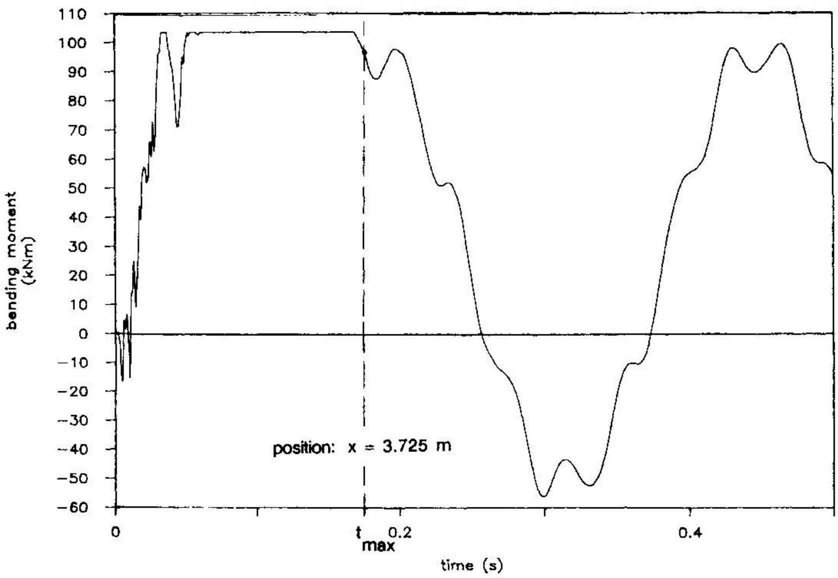

Fig. 7 Moment close to midspan as a function of time. During a long period a full plastic moment occurs.

Fig. 7 is a typical plot for the bending moment versus time for a cross–section close to the mid of the span. It can be seen, that a constant plastic moment occurs during a long time untill the maximum displacement is reached. This plot is more or less a confirmation of the assumptions for the course calculation by hand.

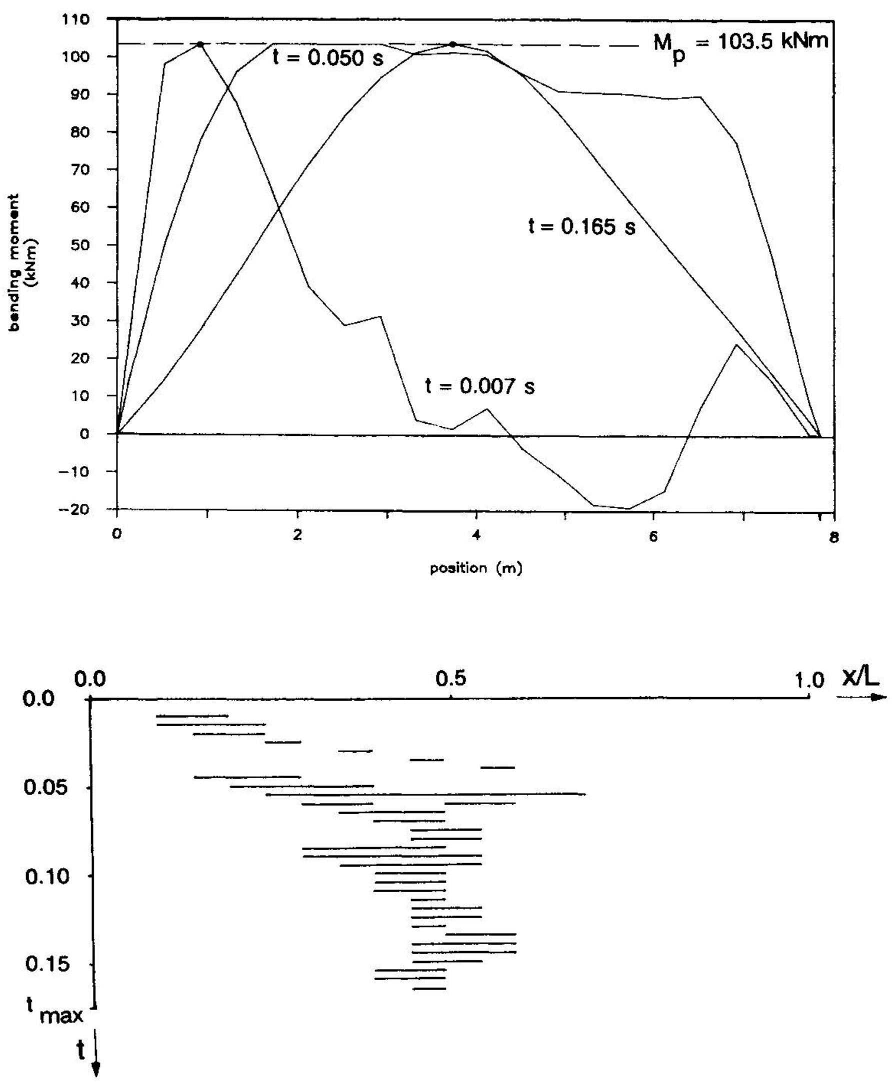

Fig. 8 Bending moment diagrams at different times and location of the plastic bending zones.

Fig. 8 shows some moment diagrams at different times. It also makes clear which part of the beam is in a plastic bending state at different times. This figure once more verifies the assumption, that the plastic zone in the mid of the span comes into being already at an early time. The shown result holds for the model in which dampers have been applied. In the TILLY run no correct model has been used for the shock absorber. It is believed that this does not influence the final results worth mentioning. However the reaction force in the shock absorber will not be too correct. In the test the time is shorter to reach the first zero value of the reaction force. At that time the beam looses contact with the absorber for a very short time. In the TILLY run a tensile force comes into being during this short time. So a more refined absorber behaviour should be modelled in order to attach much value to detailed data at early times of the calculation. This may even be an explanation why TILLY does not produce a very correct deflection line at early times. The results for later times are highly credible, even for the non- correct modelling of the absorber. After t_{max} the beam is vibrating in its natural mode around a permanent plastic deflection line, which has a displacement of about 0.65 m at midspan. Moments and shear forces oscillate then around the low values which occur statically for dead weight.

5. FINAL REMARKS AND CONCLUSIONS

The TILLY computations were done on an Olivetti M24 personal computer and require computing times of the order of 1 hour CPU. Runs for the same beam by the advanced finite element package DIANA demand in the order of 20 hours CPU on a far more powerful SEL GOULD computer. A TILLY-run is prepared and executed in a time which is expressed in a number of days. It requires weeks to complete an intensive DIANA-run. One can conclude, that the use of the course TILLY model has big advantages in all cases for which it has been proved that such a model provides sufficiently accurate results.

The program TILLY is still being developed. Other element features will be implemented in due time, for instance gap elements or contact elements. Such options are helpful to model slip of reinforcement and related phenomena. The authors are fully aware, that the applicability of models like TILLY has its own limitations. However, the engineering problems for which TILLY does apply, are sufficiently numerous to proceed. The category of simple models is another mechanism to transfer high-tech knowledge on reinforced concrete structures to the engineering profession and construction industry.

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