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Dynamic Response of Softening Concrete Frames

Réponse dynamique de portiques en béton armé ramollissants

Dynamisches Verhalten von Betonrahmentragwerken mit Entfestigung

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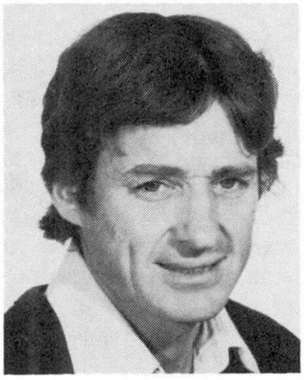

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SUMMARY

A method is presented to include flexural softening in the analysis of multi degree-of-freedom unbraced plane frame structures. The element model has finite length hinges which follow a degrading stiffness and softening hysteresis model. An example of a two storey frame is subjected to the factored El Centro 1940 SOOE earthquake ground motion. The maximum load factor for a given ductility and the ductility requirement for a given load factor are both sensitive to the softening slope.

RÉSUMÉ

La méthode proposée permet d'inclure le ramollissement dû à la flexion dans l'analyse des portiques plans non-contreventés à plusieurs degrés de liberté. Le modèle de l'élément a des articulations d'une longueur limitée que se conforment à un modèle d'hystérésis de rigidité décroissante et de ramollissement. On soumet le spécimen d'un portique à deux étages aux tremblements de terre semblable à ceux d'El Centro 1940 SOOE intensifié. Le facteur de charge maximum pour une ductilité donnée et la ductilité requise pour un facteur de charge donné sont tous deux sensibles à l'inclinaison de la courbe de ramollissement.

ZUSAMMENFASSUNG

Eine Methode zur Erfassung von Biegeentfestigung in knotensteifen Rahmentragwerken mit vielen Freiheitsgraden wird vorgestellt. Das Elementenmodell hat Gelenke endlicher Länge mit abnehmender Steifigkeit und Entfestigungshysterese. Ein zweistöckiger Rahmen wird als Beispiel den El Centro 1940 SOOE-Bodenbewegung ausgesetzt. Es zeigt sich, dass Lastfaktor und Duktilität empfindlich auf die Entfestigungsbeziehung reagieren.

1. INTRODUCTION

Softening is the name used herein for the loss of moment capacity of a reinforced or prestressed section at advanced curvature. Softening is less likely to occur where critical sections have been carefully detailed for extended plasticity. On the other hand, tests show that softening often occurs earlier, and is more pronounced, at joints, when the shear/moment ratio is high, or when substantial axial load is also present, as for prestressed members. Softening may become a factor of considerable significance when very high strength concretes are used with high proportions of high yield strength steel.

The unidirectional moment-curvature (M-ϕ) curve has been approximated by an elastic-plastic-softening trilinear model. Softening was considered to take place over a finite hinge length and the implications for static collapse and shakedown loads were examined [3].

The response of a single degree-of-freedom softening frame to simple unidirectional dynamic loads has also been studied [7]. It was found that a critical softening parameter or slope at which collapse will occur may be identified for each type of loading and depends on the severity of the applied load as represented by the ratio of maximum applied force to maximum resistance (or by energy of impulse to maximum elastic strain energy), on the plastic plateau length (ductility), on any limit to the softening region, and on duration of load.

Computation of the full response of concrete frame structures to severe reversible, repeated and dynamic loads, such as induced by the ground motions of a strong earthquake, requires consideration of the plasticity, softening and hysteresis characteristics of the most severely stressed locations. This is a formidable task, both analytically and in collecting sufficient data on which to predict behaviour over a reasonable range of variables.

An economical method is presented herein for the dynamic analysis of multi degree-of-freedom plane frame structures with softening hinges.

2. TECHNIQUE OF ANALYSIS

2.1 Member model

A flexural element of length L, shown in Fig. 1, is assumed to have discontinuity or hinge lengths at each end as shown. The reference flexural rigidity of this element is EI. The hinge lengths AB and CD have flexural rigidities aEI and bEI respectively, where a and b are negative in the softening region.

These hinges (AB and CD) are the only portions to undergo softening deformations. Points A and D are the only points which may be plastic hinges. The central portion BC has only reversible elastic deformations. In adopting this model it is assumed that bending moment maxima occur at the ends of elements. This normally corresponds to reality, but will always be true if node positions are chosen appropriately.

2.2 Model for hysteresis

The hinge lengths ℓ_{p_1} and ℓ_{p_2} have the idealised hysteretic M-ϕ behaviour shown in Fig. 2, where M is the maximum moment in the hinge length. This is a modification to include softening of the model suggested by Clough [1] for elastic-plastic behaviour.

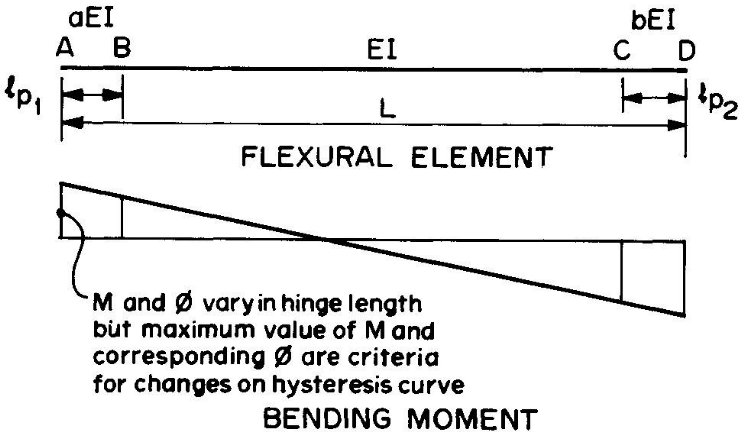

Fig. 1. Plastic-softening hinges at end of flexural element.

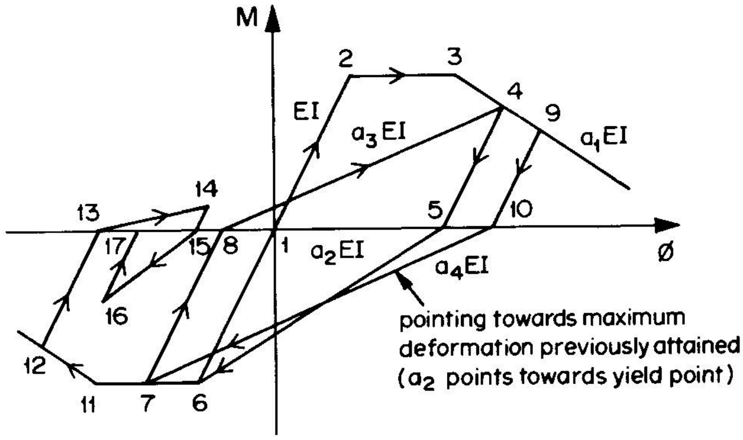

Fig. 2. Hysteresis model for plastic-softening hinges.

The M-ϕ path may be defined as follows: 12-elastic, gradient EI; 23-plastic till specified rotation capacity reached; 34-softening till deformation reversal, gradient a_1EI; 45-unloading with elastic slope to zero moment; 56-reduced elastic slope to yield point in reverse bending; 67-plastic till deformation reversal before specified rotation capacity reached; 78-unloading with elastic slope to zero moment; 84-reduced elastic slope to previous highest deformation point reached with this sign of bending; 49-further softening till deformation reversal; 910-elastic unloading; 107- as 84; 711- as 23; etc.

After reaching 17 the two possible paths could be 17,14 and 17,9. In this case the path with the higher slope is followed, i.e. 17,14 is chosen [6] and then 14,9 is followed.

The parameters a and b (Fig. 1) follow the hysteresis shown in Fig. 2. At each stage of the (separate) deformations of the end hinges, the stiffness matrix can be formed for each straight line segment of the M-ϕ curve by using the corresponding 'a' values (Fig. 2) for a and b. For example, if hinge AB is softening and following the path 3,4 then a = a_1; at the same time hinge CD can be following the path 10,7 which gives b = a_4.

2.3 Stiffness Matrix

The stiffness matrix for a prismatic flexural element with a finite softening hinge length has previously been developed [2]. The following technique allows the generalisation of this formulation to non-prismatic members.

Six relevant degrees-of-freedom of element AD are shown in Fig. 3. The element is considered as an assembly of AB, BC and CD. Actions and displacements are related in AB by

$$\begin{Bmatrix} A1 \\ A3 \end{Bmatrix} = \frac{a\ EI}{\ell_{AB}} \begin{bmatrix} k'_{ii} & k'_{ij} \\ k'_{ij} & k'_{jj} \end{bmatrix} \begin{Bmatrix} D1 \\ D3 \end{Bmatrix} \qquad (1)$$

where the stiffness coefficients k'_{ii}, k'_{ij} and k'_{jj} depend on the shape of the member. For prismatic members $k'_{ii} = k'_{jj} = 4$ and $k'_{ij} = 2$. a is negative when the hinge is softening.

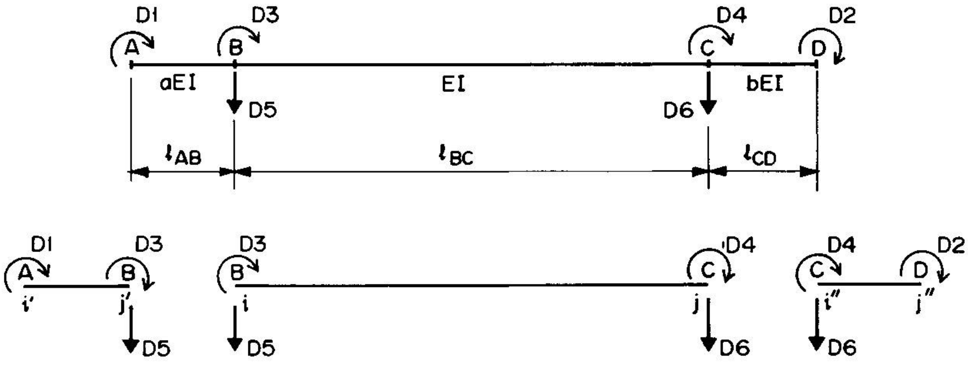

Fig. 3. Assembly of flexural element.

The stiffness Eq. (1) is modified to include the displacement D5 and then assembled with similar equations for BC and CD. Noting that actions A3 = A4 = A5 = A6, the assembled stiffness equation is

$$\begin{Bmatrix} A1 \\ A2 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} = \frac{EI}{\ell} \left[\begin{array}{c|c} S_{ee} & S_{ce}^{T} \\ \hline S_{ce} & S_{cc} \end{array} \right] \begin{Bmatrix} D1 \\ D2 \\ D3 \\ D4 \\ D5 \\ D6 \end{Bmatrix} \tag{2}$$

Using the method of condensation the matrix equation can be reduced to a 2 × 2 form

$$\begin{Bmatrix} A1 \\ A2 \end{Bmatrix} = \frac{EI}{\ell} [S_e] \begin{Bmatrix} D1 \\ D2 \end{Bmatrix} \tag{3}$$

where $$S_e = S_{ee} - S_{ce}^{T} S_{cc}^{-1} S_{ce} \tag{4}$$

The stiffness matrix to include displacement degrees-of-freedom at A and D is related to the flexural stiffness matrix in the normal way.

The structure stiffness matrix is then assembled from the element stiffness matrices S_e and is modified each time a different stage is reached on the hysteresis curve for any hinge.

3 COMPUTER PROGRAM

An element subroutine library has been developed for the model described above, and added to the computer program DRAIN-2D to enable softening deformations to be handled. DRAIN-2D is a general purpose computer program for the dynamic analysis of inelastic plane structures [4]. The program requires the following information for all possible hinge locations in addition to the usual data input:

- Plastic rotation capacity.
- Softening slope and the maximum allowable reduction of moments through softening.
- Hinge length ratio.

4. EXAMPLE

The same two storey frame example used previously to highlight the effects on static collapse loads of plastic hinges [5]and of softening hinges [3] is used in this paper. Dimensions and loads are shown in Fig. 4. Member stiffness, strength and hinge data are given in Table I. The vertical loads are assumed to be dead loads, hence the floor masses are obtained by dividing the vertical loads shown by $g = 9.81\ ms^{-2}$.

A damping factor $\beta_o = 6.366 \times 10^{-3}$, proportional to the original elastic stiffnes, was assumed. This is approximately 5% critical damping, with a natural period of 0.4 sec.

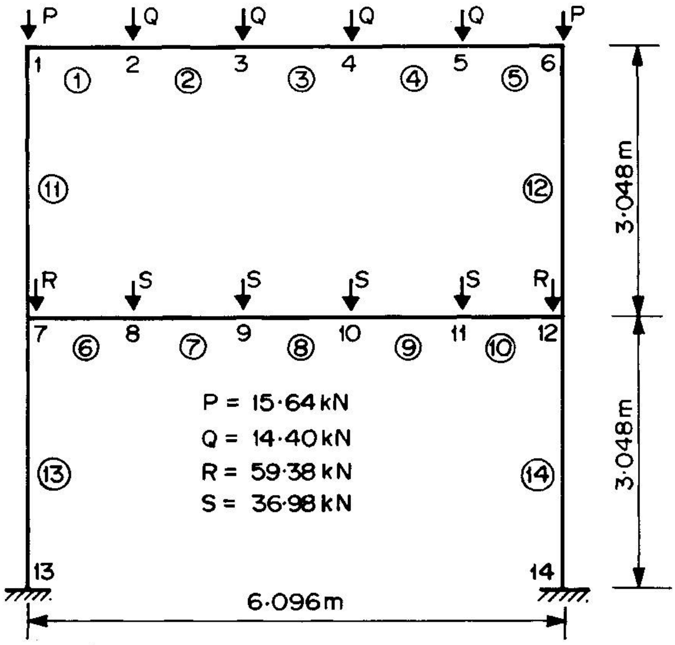

Fig. 4. Dimensions and loads for two-storey example.

The El Centro 1940 SOOE ground motion, multiplied by a factor λ, was applied to the two-storey frame. The factor λ can be compared to the load factor for static loading conditions.

The structure with hinge properties shown in Fig. 5 was subjected to the ground motion with $\lambda = 1.39$.

The yield status of the structure at various times is shown in Fig. 6. The moment-curvature path followed by the hinge at node 12, member 10 is shown in Fig. 7 with the corresponding event numbers marked.

TABLE I
DATA FOR EXAMPLE FRAME

Member No.	I $(m^4)\times10^{-4}$	M_p^+ (kN m)	M_p^- (kN m)	L (m)	$h_i = \ell_p/L$
1,5	5.99	62.1	92.6	1.22	1/6
2-4	5.99	92.6	62.1	1.22	1/6
6,10	41.14	186.3	220.1	1.22	1/6
7-9	41.14	275.4	186.3	1.22	1/6
11,12	15.22	162.6	162.6	3.05	1/12
13,14	15.22	189.7	189.7	3.05	1/12

E = 24,800 MPa.

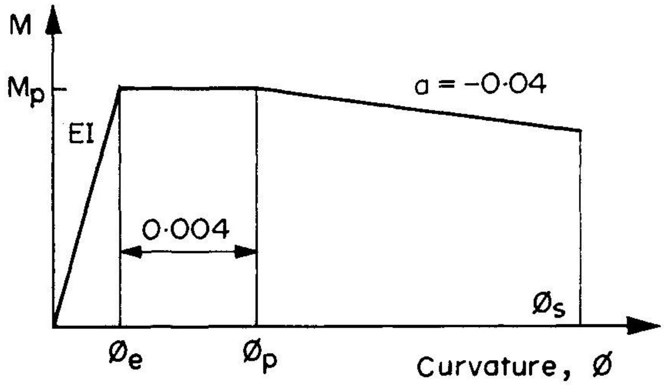

Fig. 5. Hinge properties for example frame.

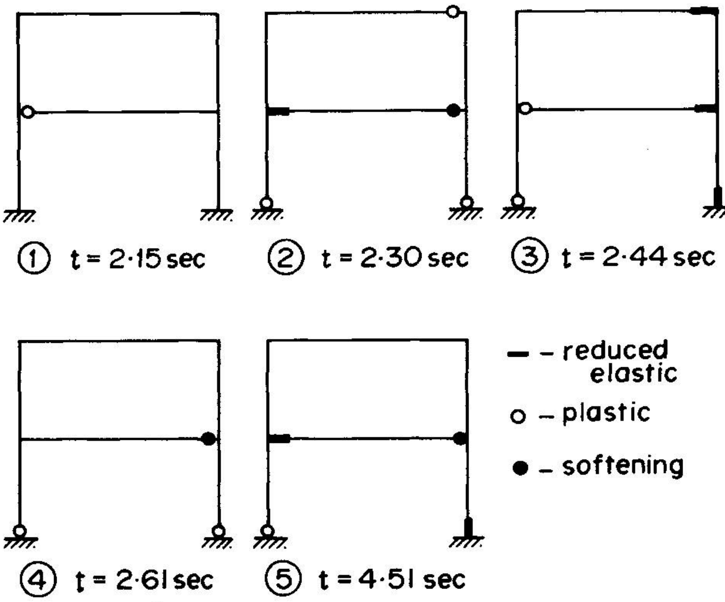

Fig. 6. Yield status of structure.

Similar analyses were performed with different hinge parameters. The various cases are summarised in Table II.

Comparing λ for cases (i) and (ii) shows the importance of including softening deformations in dynamic analysis if the alternative is a stringent limit on plastic curvature. In cases (ii), (iii) and (iv), where the maximum curvatures are limited to the same value, λ is very sensitive to the softening slope. It may also be seen from case (iv) that approximating the softening slope by a continuation of the plastic plateau may lead to significant overestimates of structural capacity.

Two further cases (v) and (vi) demonstrate the effect of softening on the maximum curvature reached. The ground motion is in both cases multiplied by $\lambda = 1.60$ as in case (iv), and the necessary curvature limits are recorded in Table II.

Cases (iv) and (v) show that softening demands significantly more ductility for the same load factor when compared to plastic behaviour. When there is no plastic-plateau, as in elastic-softening case (vi), the demand for ductility is even greater.

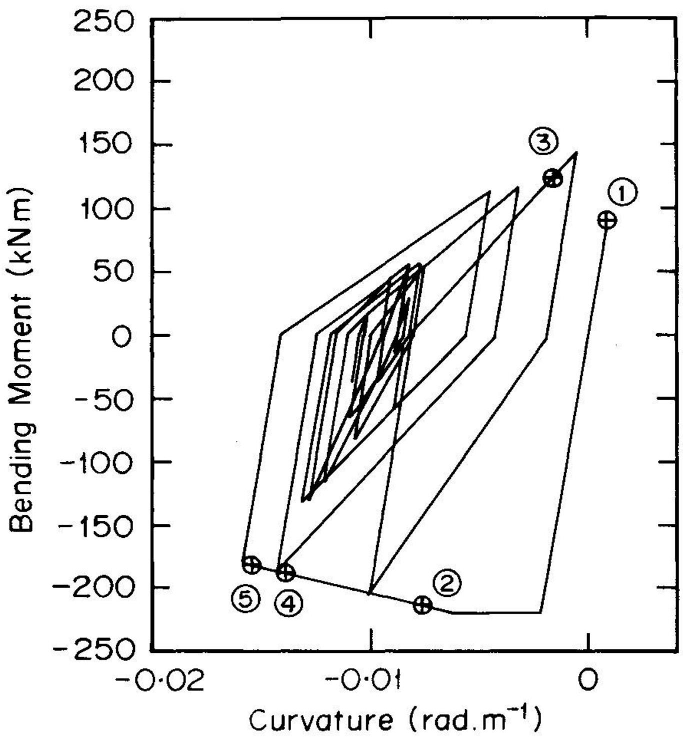

Fig. 7. Bending moment vs. curvature, member no. 10, node no. 12.

TABLE II
SOFTENING HINGE PARAMETERS AND LOAD FACTORS

Case	Type of hinge	$\phi_p - \phi_e$ rad m^{-1}	$\phi_s - \phi_p$ rad m^{-1}	Softening slope a	Maximum λ	Maximum curvature rad m^{-1}
(i)	ep	0.004	-	-	1.00	0.006
(ii)	eps	0.004	0.010	-0.04	1.39	0.016
(iii)	eps	0.004	0.010	-0.06	1.26	0.016
(iv)	ep	0.014	-	-	1.60	0.016
(v)	eps	0.004	0.018	-0.04	1.60	0.024
(vi)	es	0	0.029	-0.04	1.60	0.031

ep = elastic-plastic; ϕ_e = 0.002 rad m^{-1};
eps = elastic-plastic softening; es = elastic-softening.

5. CONCLUSIONS

- Computation of the full response of concrete frame structures to severe dynamic loads requires consideration of softening in addition to plasticity and hysteresis.

- Use of the matrix condensation technique to form the stiffness matrix for flexural elements with finite hinge lengths allows efficient analysis of softening frames with non-prismatic elements.

- The maximum load factor for a given ductility and the ductility requirement for a given load factor are both quite sensitive to the softening slope. Since the softening slope is steeper for members with significant axial load (e.g. prestressed members), this sensitivity would be of particular importance in these cases.

REFERENCES

1. CLOUGH, R.W. Effect of Stiffness Degradation on Earthquake Ductility Requirements. Report 66-16. Structural and Material Research, Structural Engineering Laboratory, University of California, Berkeley, CA., 1966.

2. DARVALL, P. LeP. Stiffness Matrix for Elastic-Softening Beams, Technical Note, Journal of Structural Engineering, ASCE, Vol. 111, No. 2, Feb. 1985, pp. 469-473.

3. DARVALL, P. LeP. and MENDIS, P.A. Elastic-Plastic-Softening Analysis of Plane Frames, Journal of Structural Engineering, ASCE, Vol. 111, No. 4, April 1985, pp. 871-888.

4. KANAAN, A.E. and POWELL, G.H. General Purpose Computer Program for Inelastic Dynamic Response of Plane Structures, Report No. EERC 73-6, Earthquake Engineering Research Center, University of California, Berkeley, CA., 1973.

5. POWELL, G.H., ORR, G. and WHEATON, R. ULARC-Simple Elasto-Plastic Analysis of Plane Frames. NISEE/Computer Applications, University of California, Berkeley, CA., 1972.

6. RIDDELL, R. and NEWMARK, N.M. Force-Deformation Models for Nonlinear Analysis. Journal of the Structural Division, ASCE, Vol. 105, No. ST12, December, 1979, pp. 2773-2778.

7. SANJAYAN, G. and DARVALL, P. LeP. Dynamic Response of a Single Degree-of-Freedom Elastic-Plastic-Softening Structure. Research Report 8/1984, Dept. of Civil Engineering, Monash University, Victoria, Australia.