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Design Techniques for Continuous Deep Beams Using Finite Element Modelling

Projet de poutres continues de grande hauteur au moyen d'un modèle des éléments finis

Entwurfstechniken für hohe Durchlaufträger mit Hilfe von Finite Element Modellen

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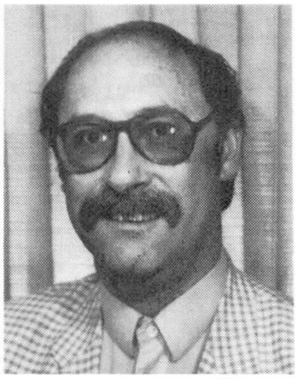

Dr. D.V. Phillips obtained his Ph.D. from the University of Swansea, Wales in 1972 and then worked at the University of Nairobi, Kenya until taking up his present appointment in 1976. His major research interests are in the analysis and design of structural concrete, and in computing methods, particularly finite element techniques.

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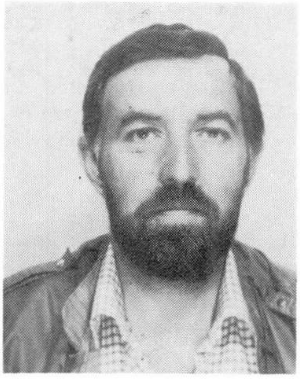

Dr. D.R. Green has been active in research since 1966, working on shearwalls, deep beams and slab structures. He is the author of many papers in these fields and is the co-author of a finite element analysis and design package which has been commercially successful throughout Europe.

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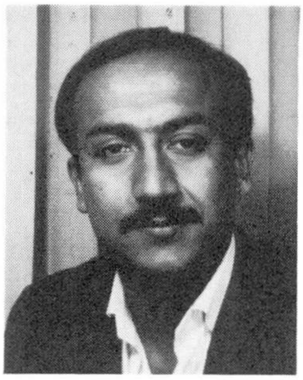


Ghous Khaskeli graduated from M.U.E.T., Jamshoro, Pakistan in 1983 and received an M.Eng. Degree from Sheffield University, U.K. in 1984. Since 1985 he has been a research student in the Department of Civil Engineering, Glasgow University where he has been working on the design and analysis of reinforced concrete continuous deep beams for his Ph.D. degree.

SUMMARY

A lower bound direct design method for the design of reinforcement in continuous deep beams is described. A linear-elastic finite element plane stress model is used to determine a stress field in equilibrium with the applied design load. This stress field is employed to calculate orthogonal reinforcement ratios throughout the beam. The resulting design is checked at serviceability and ultimate limit states using non-linear finite element techniques for structural concrete and by large scale experimental tests.

RÉSUMÉ

Une méthode directe de projet pour le calcul de l'armature dans des poutres continues de grande hauteur est décrite. Un modèle linéaire et élastique à l'aide des éléments finis est utilisé pour déterminer un champ de contraintes en équilibre avec la charge de projet appliqué. Ce champ de contraintes est employé pour calculer les ratios d'armature orthogonales tout au long de la poutre. Le projet résultant est contrôlé pour l'aptitude au service et à l'état ultime en utilisant des techniques non-linéaires d'éléments finis pour le béton armé, et à l'aide d'essais expérimentaux à grande échelle.

ZUSAMMENFASSUNG

Eine Untergrenzenberechnungsmethode für hohe Stahlbetondurchlaufträger wird beschrieben. Lineare FE-Berechnungen bestimmen den Gleichgewichtsspannungszustand. Hieraus werden orthogonale Bewehrungen berechnet. Der Entwurf wird für den Gebrauchs- und Versagenszustand mit Hilfe nichtlinearer FE-Berechnungen überprüft.

1. INTRODUCTION

In recent years various proposals have been made for the design of reinforcement for in-plane forces based on the lower bound limit state approach where a stress field in equilibrium with the ultimate load is used in conjunction with an appropriate yield criterion. Such a stress field can be obtained by any suitable procedure such as a linear elastic finite element analysis. Reinforcement is then provided so that the combined resistance of the steel and concrete at every point is equal to or greater than the applied stress.

In theory, by satisfying equilibrium and yield exactly at every point simultaneously, the entire structure will become a mechanism at ultimate load. Practical considerations, such as reinforcement being provided as discrete bars, make it impossible to achieve this idealised behaviour. Also the theory gives no guarantee that serviceability behaviour will be satisfactory. However, if verified as an acceptable design process, the following advantages ensue:
- analysis and design becomes one continuous process which is suited to automatic computation,
- steel is used economically as the design equations are based on minimising steel requirements, although this will be affected by the convenience of fabrication,
- excessive ductility demands are minimised by aiming for most parts of the structure to yield simultaneously at a particular ultimate load. The difference between the load at which yielding starts and the ultimate load is kept at a reasonable level which should prevent excessive cracking at working load.

Verification for practical reinforcement details can be provided by non-linear finite element modelling of the resulting designs, backed up by large experimental tests. Although the direct design philosophy is applicable to a wide range of continuum structures, this paper is concerned with its application to the design of continuous deep beams.

2. DIRECT DESIGN EQUATIONS FOR IN-PLANE FORCES

The design equations adopted here for in-plane actions were originally presented by Neilsen [1] using the yield criterion for orthogonal tension reinforcement,

$$(N_x^* - N_x)(N_y^* - N_y) - N_{xy}^2 = 0 \qquad (1)$$

where N_x^* and N_y^* are yield ultimate strengths, and N_x, N_y and N_{xy} are the in-plane stress resultants in the concrete at any point. Clark [2] later developed this approach to cover compression steel and skew reinforcement. The design equations are derived assuming:
1) the reinforcement is placed symmetrically about the middle surface of the section and is in two non-orthogonal directions (Fig.1) and carries only uniaxial stress in the original bar directions so that kinking and dowel actions are neglected,
2) bar spacing is small in comparison with the overall structure dimensions so that reinforcement can be considered in terms of area per unit length rather than as individual bars,
3) concrete has zero tensile strength, exhibits the square yield criterion shown in Fig.2(a) and is perfectly plastic, whilst reinforcement exhibits perfect elastic-plastic behaviour with a yield stress of f_s in tension and f_s' in compression, as shown in Fig.2(b),
4) that proper detailing and choice of section prevents instability and bond failures,
5) in-plane stresses are resisted by a combination of concrete and steel stresses (Fig.3).

These assumptions lead to design equations for nine possible combinations of reinforcement which are shown schematically in Fig.4. In each direction tension, compression or no reinforcement may be required. When tension reinforcement is provided $\sigma_1 = 0$ because the concrete is cracked and when compression reinforcement is provided $\sigma_2 = f_c$ in order to make optimum use of concrete. Direct solutions are possible in all cases except when tension-tension or compression-compression reinforcement is provided. In these cases an additional equation is available by minimising the total reinforcement, i.e.

$$\frac{\partial(\rho_x + \rho_y)}{\partial(\tan\theta)} = 0 \qquad (2)$$

It is also necessary that $\tau_{xy} < 0.5 f_c$ in order to prevent shear failure.

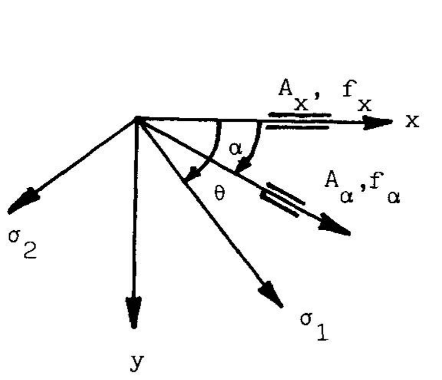
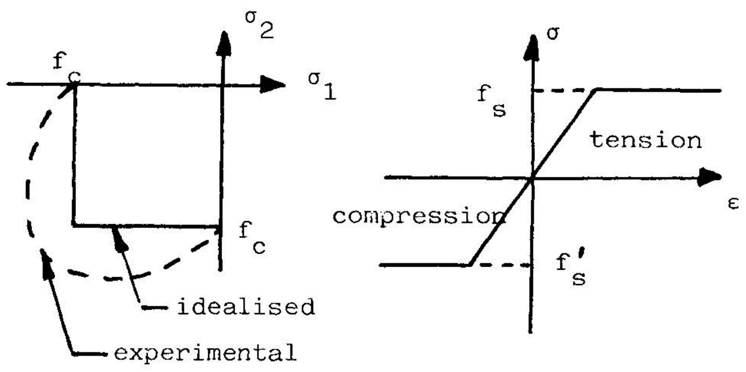


Fig.1 Principal concrete stress and directions of reinforcement



(a) concrete (b) steel

Fig.2 Design yield criteria for concrete and steel

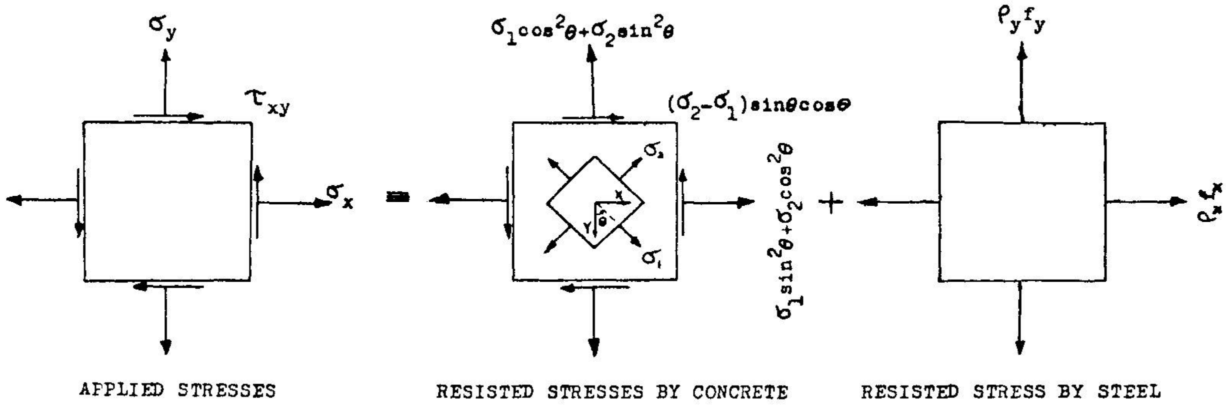


Fig.3 Combination of resistant stresses

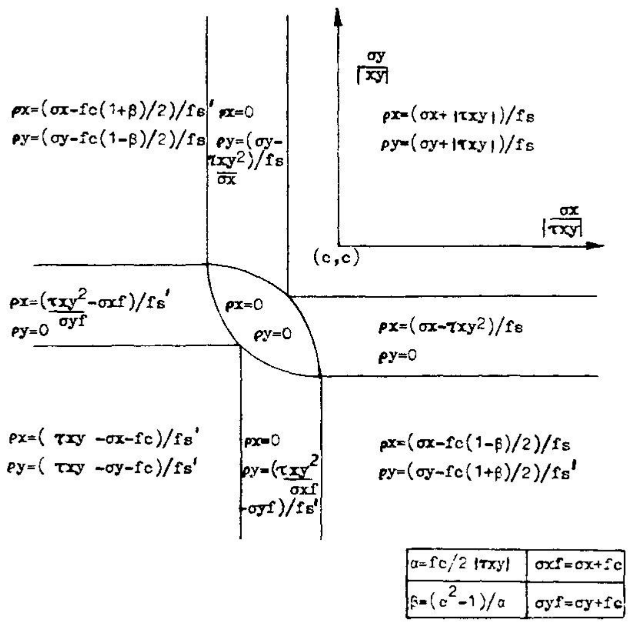


Fig.4 Design equations for reinforcement

3. DIRECT DESIGN OF CONTINUOUS DEEP BEAMS

Rogowsky and MacGregor [3] and Ricketts and MacGregor [4] have carried out a number of tests on continuous deep beams which has provided useful information on their behaviour. They used plastic truss analogies to predict the ultimate loads of their beams and used a concrete efficiency factor to achieve satisfactory agreement between the analytical and experimental behaviour. Little other experimental data has been reported and so design rules are mainly based on a knowledge of single span deep beam behaviour.

The CIRIA Guide [5], in common with many Codes of Practice, base deep beam reinforcement design on indirect methods which use moment and shear envelopes and internal lever arms which are assumed functions of beam dimensions. Appropriate distribution rules are then used for positioning the reinforcement in sagging and hogging zones, and for resisting shear. This approach, although simple to use, does not make the most effective use of the reinforcement due to the varying nature of the stress distribution in the beams. The direct design philosophy attempts to overcome this by using an actual equilibrium stress field.

The following procedure is adopted here.
1. An internal stress field in equilibrium with the design ultimate load is obtained from a linear– elastic finite element analysis of the unreinforced beam using elastic material data for concrete.
2. Reinforcement ratios are calculated using the design equations (Fig.4) with intended values of f_s, f_s' and f_c.
3. Discrete reinforcing bars are selected by taking arbitrary levels across and along the beam and calculating the average reinforcement ratios at each level. Reinforcing bars are then selected and distributed using specified bar diameters and limits on cover and spacing.

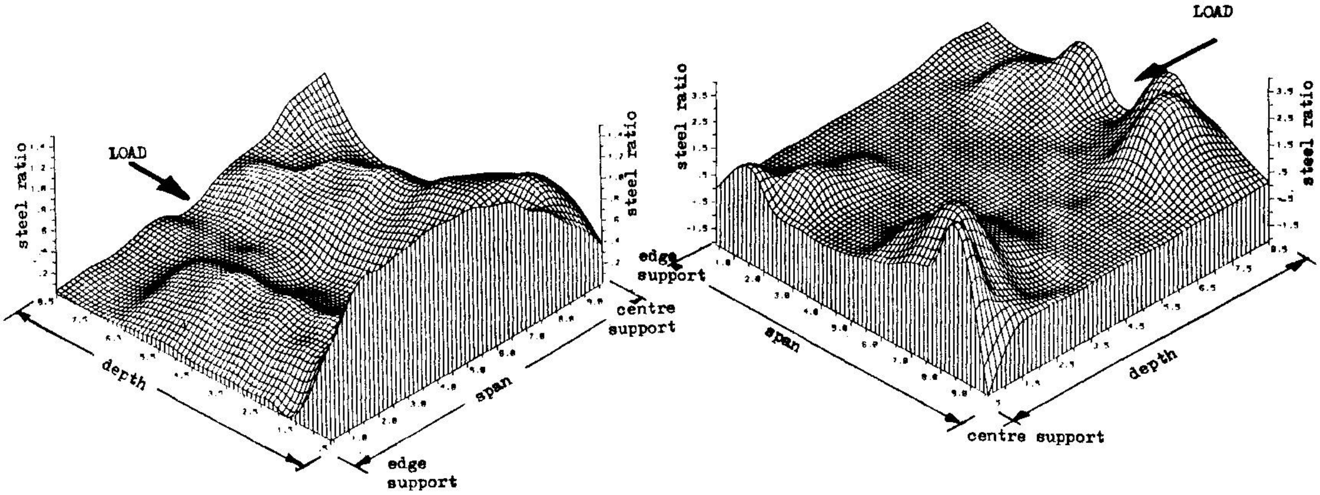
This procedure was used to design the two–span continuous beam shown in Fig.6. The beam was designed to carry a total ultimate load of 850kN as two concentrated loads acting at the centres of each span. The overall depth and length of the beam was 900 mm and 2000 mm respectively with a thickness of 100 mm. The span between the centres of support bearings was 960 mm. Hot rolled deformed high tensile reinforcing bars with a 0.2% proof stress of 501 N/mm^2 and concrete with a cube strength of 45 N/mm^2 were specified. Isometric views of the envelopes of the resulting reinforcement ratios in the horizontal and vertical directions are shown in Fig.5. The final designed beam with discrete reinforcing bars is shown in Fig.6.

4. LARGE SCALE TEST OF CONTINUOUS DEEP BEAM

The full size designed beam was experimentally tested to failure in increments of 50 kN under displacement control. Applied loads and support reactions, deflections on the underside of the beam and steel strains at each reinforcement level were automatically measured using data logging facilities. Concrete strains, crack propagation and crack widths were also monitored. Material properties were measured as follows: concrete – cube strength = 63 N/mm^2, cylinder splitting strength = 3.22 N/mm^2, modulus of elasticity = 19.5 kN/mm^2: steel – 0.2% proof stress for 6 mm and 8 mm bar = 513 N/mm^2 and 501 N/mm^2 respectively, modulus of elasticity for 6 mm and 8 mm bar = 200 kN/mm^2 and 195 kN/mm^2.

The beam failed in shear at a load of 1333 kN, over 50% higher than the design load. The serviceability load corresponding to a 0.3 mm crack width was 900 kN. Measurements indicated that most of the steel had yielded prior to ultimate conditions. Load–displacement and steel strain curves are shown in Figs.11 and 12 whilst Fig.13(b) illustrates the cracks in the beam at failure. The high experimental load can be attributed to several causes:
– the actual concrete strength was 63 N/mm^2 compared to the design value of 45 N/mm^2,
– the steel exhibited strain hardening which was not accounted for in the design equations,
– significant displacement along shear cracks was observed at higher loads causing dowel action and interface shear transfer effects which were not accounted for in the design equations,
– the reinforcement provided, particularly the shear reinforcement, was more than the design values because of practical requirements such as minimum bar size.

Notwithstanding, the design procedure produced an acceptable design with an adequate reserve of strength and satisfactory behaviour at serviceability load.



(a) horizontal steel (b) vertical steel

Fig.5 Isometric views of envelopes of reinforcement ratios (1/2 beam)

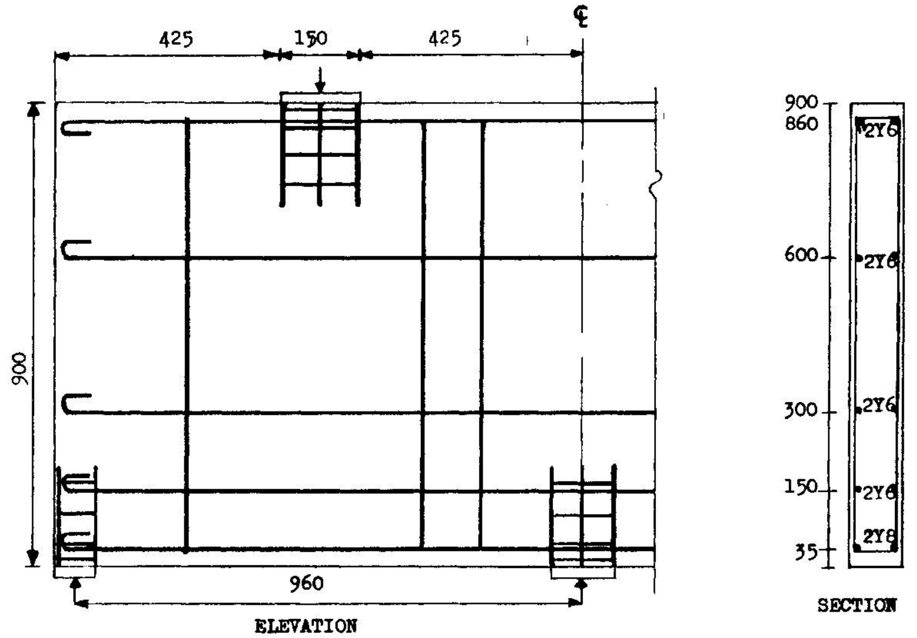


Fig.6 Dimensions and reinforcing details of designed beam (one span)

5. NON-LINEAR FINITE ELEMENT ANALYSIS

The beam was analysed using a plane stress non-linear finite element model [6] which is one of several developed at Glasgow University for analysing the non-linear behaviour of structural concrete. A brief description of the analysis follows.

5.1 Discretisation and non-linear solution procedure

Concrete was modelled by 8-noded isoparametric elements using standard Serendipity shape functions and a 3 x 3 Gaussian integration rule. Reinforcement was modelled by bars embedded anywhere within the element along lines of constant local curvilinear co-ordinates [7]. This representation assumes the bars carry uniaxial stress only and that perfect bond exists between the steel and concrete.

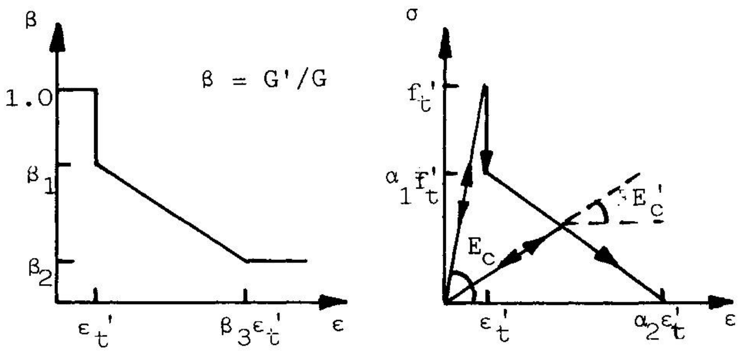
A modified Newton–Raphson incremental–iterative solution scheme was used where the stiffnesses were upgraded at the first iteration within a load increment. Load control was applied and no line searches were employed. Convergence was checked by a residual force norm and was assumed to have occured when the convergence factor was less than 2%.

5.2 Material models

Cracking was simulated by a smeared orthotropic fixed crack approach. A crack was assumed to occur in a plane perpendicular to the maximum principal stress when this exceeded the tensile strength criterion for concrete. After crack formation, the material matrix was modified to reflect the change from an isotropic to an orthotropic material and to account for the partial loss of stiffness normal and parallel to the crack planes. A second crack could occur at right angles to the first crack if the tensile strength was exceeded in this direction also. A crack was assumed to close if the strain across the crack became compressive. Post cracking behaviour was represented by laws defining a variable shear retention factor and a variable tension stiffening and strain softening factor, as shown in Fig.7. These laws were used to modify the modulii in the material matrix for subsequent stiffness and stress calculations.

Concrete behaviour under biaxial stress states was modelled by a short–term hyperelastic constitutive law which uses tangential bulk and shear modulii which vary according to the stress invariants I_1 and J_2. The relationships are shown in Fig.8. A linear form of the octahedral stress failure criterion was used to predict the biaxial ultimate strength of concrete (Fig.9).

For reinforcing steel a bi–linear uniaxial stress/strain relationship allowing for isotropic strain hardening and elastic unloading was used as shown in Fig.10.



(a) Shear retention (b) tension stiffening

Fig.7 Post–cracking laws

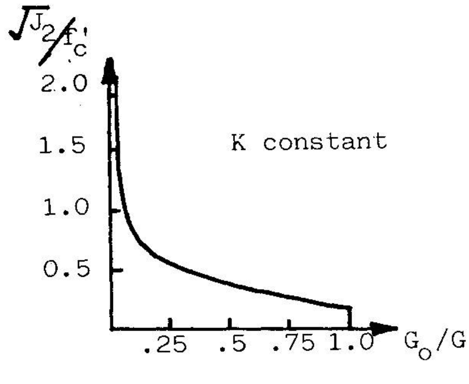


Fig.8 Hyperelastic invariant law for concrete in biaxial compression

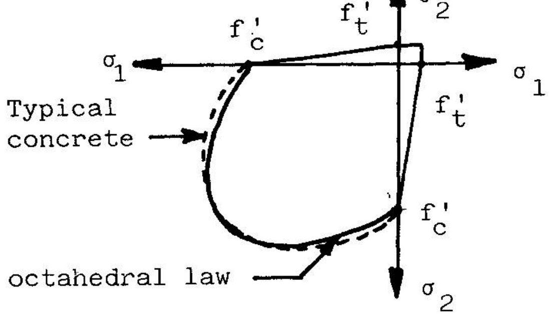


Fig.9 Failure criterion for concrete

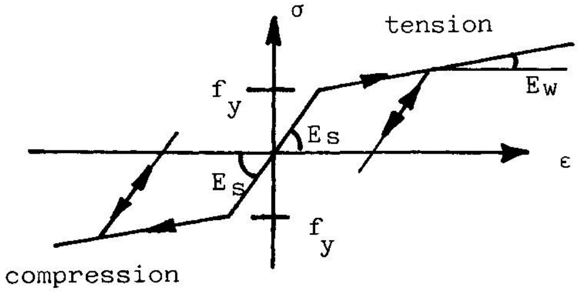


Fig.10 Stress–strain law for reinforcement

5.3 Results for the continuous deep beam

The load–deflection and load–steel strain curves are compared with the experimental results in Figs.11 and 12 where satisfactory agreement is evident. Fig.13 shows crack propagation at various load levels and compares them with the experimental crack pattern at failure. Flexural cracks first initiate at the bottom of the beam at about 300 kN, followed later by flexural cracking at the top of the beam over the central support at a load of 750 kN. At about 825 kN, shear cracks start to develop which eventually spread from the load point to the central support. Failure was by shear and crushing of concrete in the central shear span and close to the applied load. This description compares well with the experimental observations. The ultimate load predicted by the finite element analysis was 1275 kN which was 4% below the test value. A truss analogy approach [8] predicted an ultimate load of 934 kN, some 30% lower than the actual value.

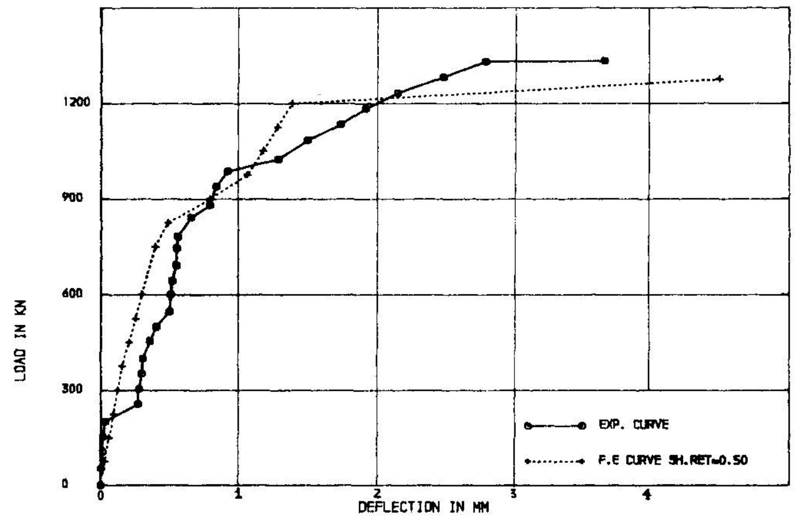


Fig.11 Comparison of experimental and finite element load– displacement curves

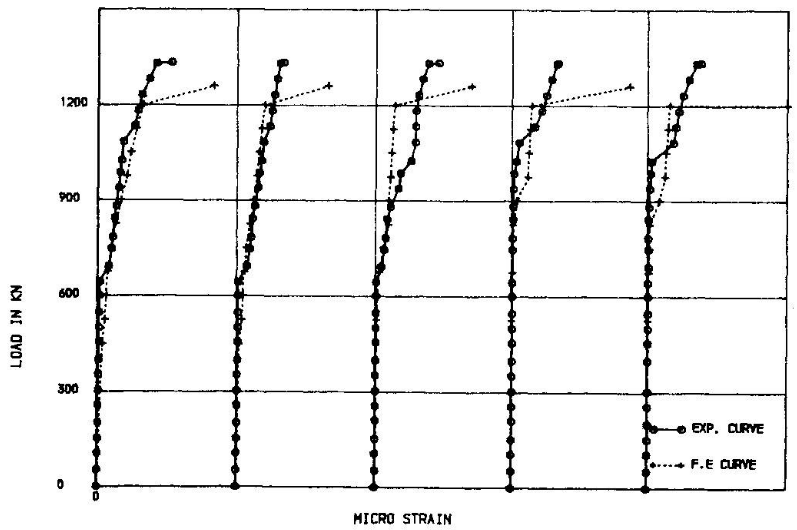
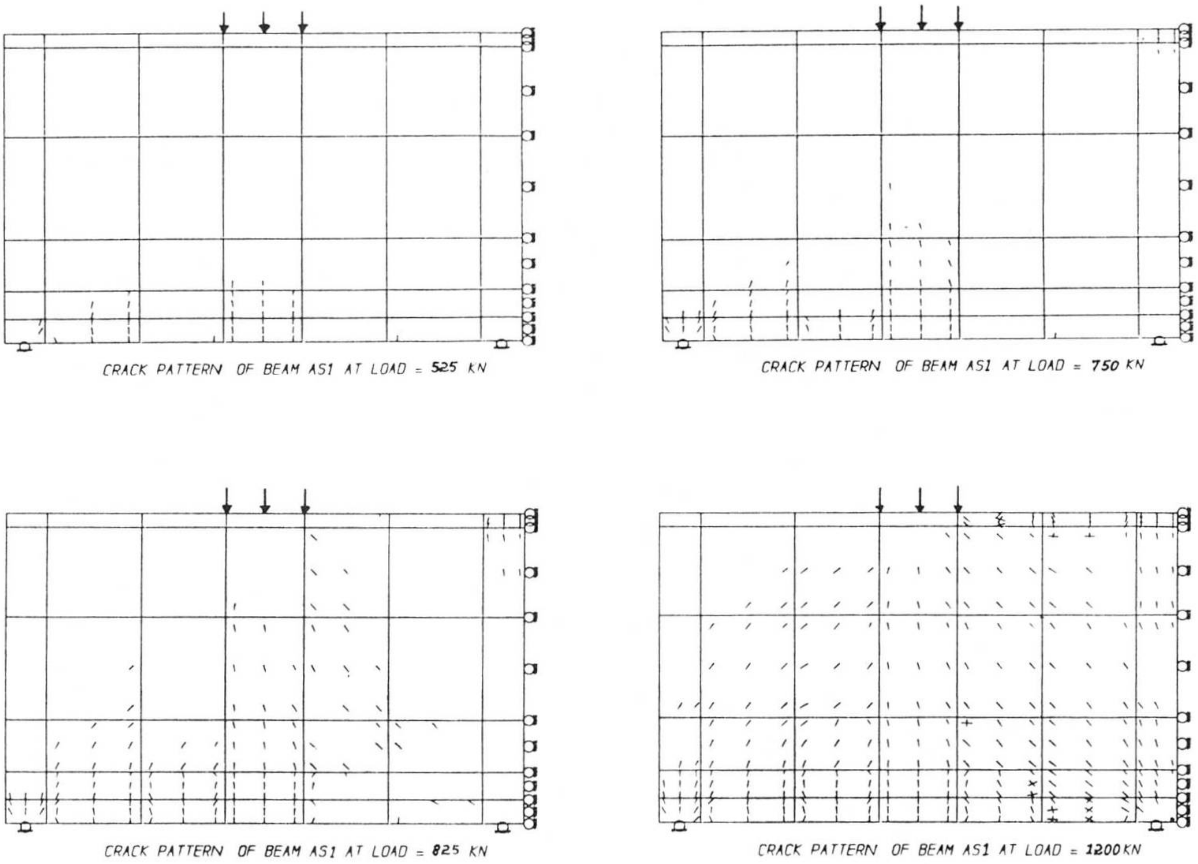
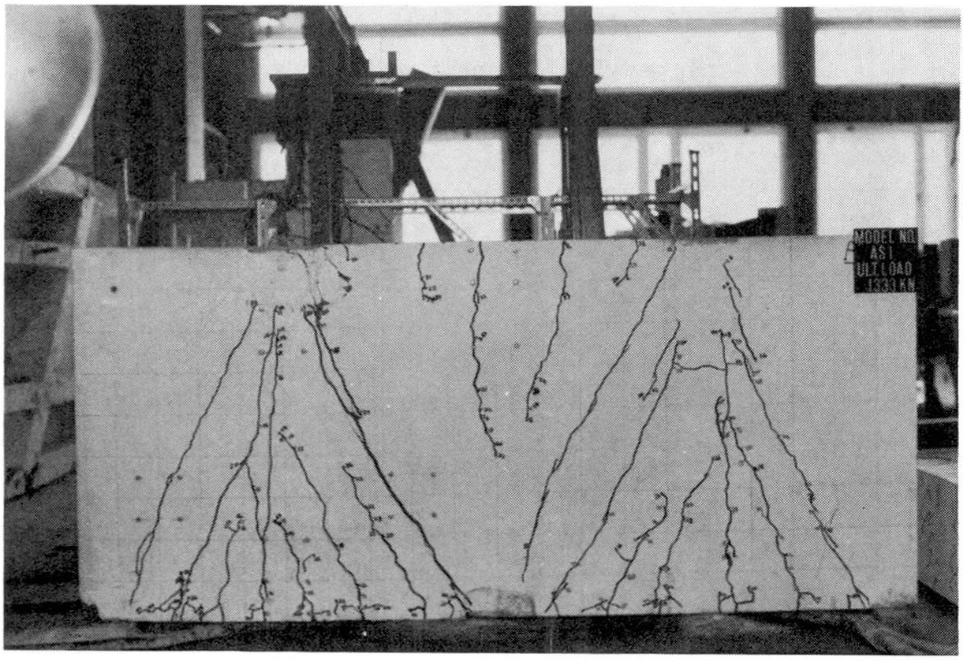


Fig.12 Comparison between experimental and finite element load– steel strain curves



(a) Finite element analysis (1/2 beam)



(b) Experimental crack pattern at failure

Fig.13 Crack propagation in the continuous deep beam

6. CONCLUDING REMARKS

By using the elastic stress field produced by a finite element analysis a procedure has been developed whereby reinforcement in continuous deep beams can be directly designed to resist in-plane forces. The serviceability and ultimate load behaviour of the resulting design was verified by a non-linear finite element analysis and by a large scale experimental test.

Other theoretical and experimental work is continuing on designs with different span to depth ratios, and with other features such as skew reinforcement, prestressing and holes within the shear spans [9]. Although the procedure described uses a linear elastic stress field for obtaining the reinforcement ratios, any suitable equilibrium field could be used and studies are also being made using elasto-plastic finite element models with appropriate yield and flow rules.

The non-linear finite element models which have been developed are providing a better appreciation of structural behaviour at various stages of loading which has proved very useful in developing the most effective design procedures. The finite element modelling is being continuously developed and is backed up by separate experimental tests for verifying the models developed and for providing information on material behaviour.

The direct design procedures are being applied to other structural forms, including combined torsion and bending of reinforced and prestressed concrete beams [10], perforated deep beams [11], and reinforced concrete right- and skew-slabs [12],[13]. The method is proving satisfactory in these areas also. Ultimately, the objective of this work is to produce automated techniques which can be used in computer-aided design software.

REFERENCES

1. NIELSEN, M.P. "Yield conditions for reinforced concrete shells in the membrane state". Non-classical shell problems, IASS Symposium, Warsaw, 1963, pp.1030-1038
2. CLARK, L.A. "The provision of tension and compression reinforcement to resist in-plane forces", Mag. Conc.Res., Vol.28, No.94, 1976, pp.3-12
3. ROGOWSKY, D.M. & MACGREGOR, J.G. "Shear strength of deep reinforced concrete continuous beams", Structural Engineering Report No.110, Department of Civil Engineering, University of Alberta, Canada, 1985
4. RICKETTS, D.R. & MACGREGOR, J.G. "Ultimate behaviour of continuous deep reinforced concrete beams", Structural Engineering Report No.126, Department of Civil Engineering, University of Alberta, Canada, 1983
5. CIRIA, "The design of deep beams in reinforced concrete", Guide 2, Construction Industry Research and Information Association, London, 1977
6. PHILLIPS, D.V. "FECON: Nonlinear finite lement programs for structural concrete", FECON Manual, Mark 2, University of Glasgow, 1986
7. PHILLIPS, D.V. & ZIENKIEWICZ, O.C. "Finite element non-linear analysis of concrete structures", Proc.Instit,Civ.Engnrs, Part 2, Vol.61, 1976, pp.59-88
8. KUMAR, P. "Collapse load of deep reinforced concrete beams", Mag.Conc.Research, Vol.28, No.94, 1976, pp.30-36
9. KHASKHELI, G.B. "The direct design of reinforced concrete transfer girders", Ph.D Thesis, University of Glasgow, to be submitted.
10. EBIRERI, J.O. "Direct design of beams under combined bending and torsion", Ph.D. Thesis, University of Glasgow, 1985
11. MEMON, G.H. "Ultimate strength of perforated deep beams", M.Sc. Thesis, University of Glasgow, 1982
12. HAGO, A.W. "Direct design of reinforced concrete slabs", Ph.D. Thesis, University of Glasgow, 1982
13. ABDEL-HAFEZ, L.M. "Direct design of reinforced concrete skew slabs", Ph.D. Thesis, University of Glasgow, 1986