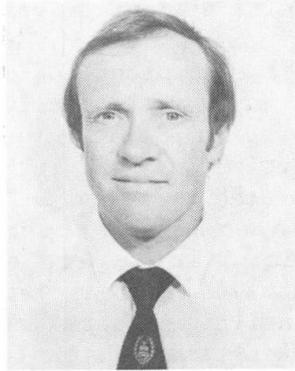# Ultimate Shear Design from Non-Linear Finite Element Analysis
Calcul à l'effort tranchant ultime à partir de méthodes non-linéaires aux éléments finis
Schubfestigkeitsberechnung mittels nicht-linearer Finiter Elemente

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SUMMARY

This paper briefly describes certain aspects of ultimate shear evaluation with respect to current non-linear finite element codes. Consideration is given to apparent pitfalls in the direct application of the finite element results to a general design approach.

RÉSUMÉ

Cet article décrit succintement quelques aspects de calcul à l'effort tranchant ultime par rapport aux méthodes courantes, non-linéaires, d'éléments-finis. L'accent est mis sur certains cas où l'utilisation de méthodes aux éléments-finis, comme outil général de calcul, n'est pas satisfaisante.

ZUSAMMENFASSUNG

Der folgende Artikel beschreibt gewisse Aspekte der Schubfestigkeitsberechnung mittels der Methode der nicht-linearen Finiten Elemente. Es wird dabei auf offensichtliche Probleme hingewiesen, die bei der Anwendung der Resultate in allgemeinen Entwurfsberechnungen auftreten können.

1 INTRODUCTION

Finite element analysis in its early form was based primarily on a linear-elastic stiffness approach. As such it was recognised by reinforced concrete designers as defining certain service conditions fairly well but not defining ultimate limit states satisfactorily. However, it can be used successfully in ultimate design applications because of maintenance of overall equilibrium of the structure and suitable use of code resistance models.

A number of current non-linear finite element programs are making significant progress in addressing this short-coming, and are being used with increasing frequency in research and design in reinforced and prestressed concrete, even in evaluation of ultimate limit states. Typically, steel reinforcement is represented by truss elements having an elastic-plastic material model, or a rebar option, with similar properties, can be incorporated in the concrete elements. The concrete is represented by 2- or 3-dimensional solid elements having a material model with a non-linear compressive stress-strain relationship resulting ultimately in crushing, and cracking in tension at a suitably low level of tensile strain. Various techniques, such as tension stiffening, are used to aid the modelling of localised effects. The composite material is thus modelled by linking the two basic material models in the formation of the finite element mesh.

It would appear that the flexural ultimate limit state is well defined by this form of combination of the material models, but the shear ultimate limit state, most usually associated with abrupt strain-weakening, appears to be difficult to predict in terms of this approach. Indeed, detailing for shear from all types of finite element analysis results appears to present persistent difficulties for reinforced concrete designers.

A reason for the difficulties experienced with existing material models in non-linear finite element codes is that cracking *per se* does not necessarily indicate an ultimate limit state in reinforced concrete, even for structural elements unreinforced for shear. The extent of the cracking, however, might be a guide to evaluating a shear ultimate limit state. Some proposals as to possible avenues which could be explored in solving this problem are presented in this paper.

2 ASPECTS OF SHEAR BEHAVIOUR

Shear failure should not precede flexural failure in reinforced concrete in general, and particularly where ductility is necessary in terms of the design concept of the structure. It is thus important to be able to monitor the ultimate shear capacity of the structure (in terms of general non-linear finite element code results) relative to the ultimate flexural capacity, especially for structural members unreinforced for shear or lightly reinforced for shear. Structures of this type are relatively common in reinforced concrete construction, and are well within the scope of current codes of practice[1,2,3].

The likelihood of shear failure preceding flexural failure is dependent on geometric and material properties of the structural member. This concept is

elegantly portrayed by Kani's[4] valley of diagonal failure, for mild steel, 300mm deep beams unreinforced for shear, subjected to 2-point loading, as shown in Figure 1. These valleys can be considerably larger for other structural situations, such as punching shear in flat plates or use of high yield flexural reinforcement, and also continue to increase in size with the depth scale of the member, which is a major cause for concern in structures of large scale. The valleys are eliminated by the inclusion of shear reinforcement as necessary for the required probability of ductile flexural failure.

Special caution is thus required when evaluating non-linear finite element analysis results, as it is possible that shear failure is preceding flexural failure and only the flexural ultimate limit state is being identified automatically.

Some tests for the imminence of shear failure are proposed, and consideration is also given to comparisons between finite element results and observed test behaviour.

3 STRUCTURAL MEMBERS UNREINFORCED FOR SHEAR

Laboratory tests for shear failure have been conducted widely, usually with stiff testing machines, and there is some agreement that these indicate that if average shear stress is the measure of shear performance, then the major parametric trends can be represented by the curves shown in Figure 2, with an average coefficient of variation between 12 and 15%. These curves are in effect an alternate depiction of Kani's valleys. The form of these curves presents difficulties in representing ultimate shear failure in the material model for the finite element code. The problem is unlikely to be fully resolved in terms of the finite element combination of the two materials (concrete and steel) while current models for shear remain empirical and based on a "combination of materials" approach with a combined partial resistance factor. The difficulty in modelling is also caused by the difference in spread of cracks in the general finite element code results and the observed diagonal crack precipitating the shear failure. The shear deformation in the shear sensitive zone (shear arm or "D region" [5]) of the structural member is concentrated to a large extent in the diagonal shear crack the instant before shear failure. This is evident in the photograph of typical diagonal cracking in Figure 3 (in which strain softening has not yet commenced). These observations also apply to members of larger scale.

A number of non-linear finite element analyses[6,7] and a parallel laboratory and field test programme have been undertaken by the writer in a series of parametric, comparative studies on shear resistance. It appears from the finite element code results that the larger distortions caused by cracking are spread over all the cracked elements, straining in a manner consistent with the constitutive model laws and the definition of plane stress for the studies considered. In tests, distortions are concentrated in a discrete diagonal crack just prior to shear failure. Thus while accumulated distortions within the shear arm are reasonably well predicted by the finite element results, specific distortions within the mesh are not representative, as indicated by comparing the distortion at ultimate in Figure 3 with the finite element distortions for comparable loading in Figure 4.

However, without making major changes to the constitutive relationships for existing material models, there is an approach which is proposed as an possible measure in the development of a procedure for assessing the imminence

of diagonal shear failure, for general design from non-linear finite element analysis. If all the finite element distortion is assumed to be concentrated at the geometric centre of the shear arm (or shear sensitive zone), then peak shear resistance is indicated by the attainment of an absolute value for this distortion, measured normal to the observed or envisaged diagonal crack at failure, while bending reinforcement remains elastic, such as shown in Figures 4 and 5. The components of resistance to shear on this crack must be greater than or equal to the applied shear force differential across the crack. This approach results in curves which closely match those in Figure 2. This appears to be applicable for shear arm to effective depth ratios from 0,4 to 2, which is a reasonable range for most practical problems. Shear distortions can be assumed to predominate in this range while bending reinforcement remains elastic. If bending reinforcement becomes plastic during the incremental solution procedure, deformations abruptly become large, and the (desired) flexural ultimate limit state has been attained. The absolute deformation has to be calibrated by test, but preliminary evidence suggests that a cumulative deformation of the order of 1,3 to 1,5mm yields likely values for mean, ultimate shear resistance. It is believed that this value is linked to both a measure of the extent of propagation of the diagonal crack (or splitting energy), and the absolute magnitude of diagonal crack at which mean aggregate interlock is totally destroyed (for most ranges of practical concretes)[8]. Statistical variability and the use of an appropriate partial resistance factor obviously also require consideration before adopting the approach in a design application. This procedural check, although generally iterative, is fairly easily built into a post-processing subroutine (without resorting to writing a dedicated constitutive material model for shear in reinforced concrete).

4 STRUCTURAL MEMBERS REINFORCED FOR SHEAR

The parametric trends shown in Figure 2 for members unreinforced for shear maintain some influence for structural members lightly reinforced for shear, but rapidly disappear as the link reinforcement ratio increases. Once the "transition" phase is passed, the truss analogy becomes fully applicable for vertical link reinforcement[9] and finite element modelling of the truss concept is feasible. In this type of "dedicated" model of the concrete, the ultimate limit state is identified by either plastic yield of the links or flexural steel, or crushing of the concrete in the compression flange or web, and all these modes can be detected by the current "general" material models. However, this approach does not make use of direct modelling of the structure with what could be perceived as the more appropriate solid elements, and is applicable only to members having a reasonably high link reinforcement ratio. Tests indicate that an increase in "spread" or smearing of diagonal cracks across the shear arm is observed with increasing link reinforcement ratio, a trend which is also difficult to simulate with existing combinations of material models.

In the event that plane stress solid elements are used with truss element vertical links and flexural reinforcement, excessive shear distortions of the concrete elements occur between links. The links traversing the shear arm thus appear to be virtually ineffective in the finite element analysis, whereas tests on members lightly reinforced for shear indicate that the vertical links reach yield near the centre of the diagonal shear crack which precipitates ultimate shear failure, prior to peak shear resistance being attained. Horizontal links in members having steeply inclined diagonal shear cracks at ultimate, such as corbels, are better modelled by the general material models,

because the finite element distortions match the observed test behaviour more closely in this case. Axial tension or compression (prestress) affecting shear is also well modelled in this case.

The transition zone for structural members very lightly reinforced for shear with vertical links appears to be an area where modelling is particularly difficult. Either a simulated sloping link can be used or a revised constitutive material model for the composite solid finite element must be derived, as for the layered plate bending elements dedicated to flexural evaluation of reinforced concrete slabs. The advantage of automatic application of a general material model is, however, lost in these approaches.

5 APPLICATIONS

Notwithstanding the manipulation that is required from general finite element codes, the approaches outlined briefly in this paper have been used to aid in the derivation of a design model for shear in reinforced concrete, particularly in deep slab applications. The following are examples of structures for which the writer investigated various shear ultimate limit states:
- 20m high reinforced concrete retaining wall, with wall thicknesses up to 2000mm, for diamond mine washing plant.
- 2000mm deep silo floor slabs for 35m high rock silos for gold mining applications.
- Investigation of alternatives for transfer level of high-rise hotel structure.

6 CONCLUSION

Reinforced concrete designers should exercise some caution in using general non-linear finite element codes, as not all ultimate limit states might be identified automatically. This is attributable to the trend for state-of-the-art non-linear finite element analyses to be "dedicated" in many of their formulations, probably being more aligned to research than general design at present.

7 ACKNOWLEDGEMENTS

M_u is the ultimate moment carried by the beam at the instant of shear failure.
M_{fl} is the ultimate flexural capacity of the beam.
p is the flexural steel ratio of the beam.
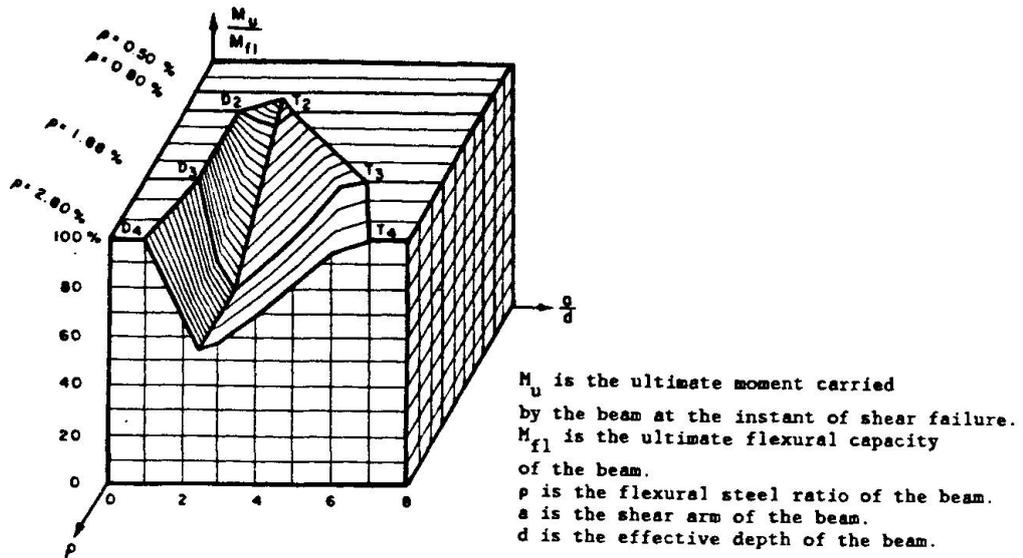a is the shear arm of the beam.
d is the effective depth of the beam.

Figure 1   Kani's valley of diagonal failure
for mild steel, 300mm deep, RC beams
subjected to two-point loading.

MEAN RESISTANT SHEAR STRESS ADJUSTMENT

CONCRETE STRENGTH (MPa)

MEAN RESISTANT SHEAR STRESS ADJUSTMENT

FLEXURAL STEEL RATIO (%)

MEAN RESISTANT SHEAR STRESS ADJUSTMENT

a/d RATIO

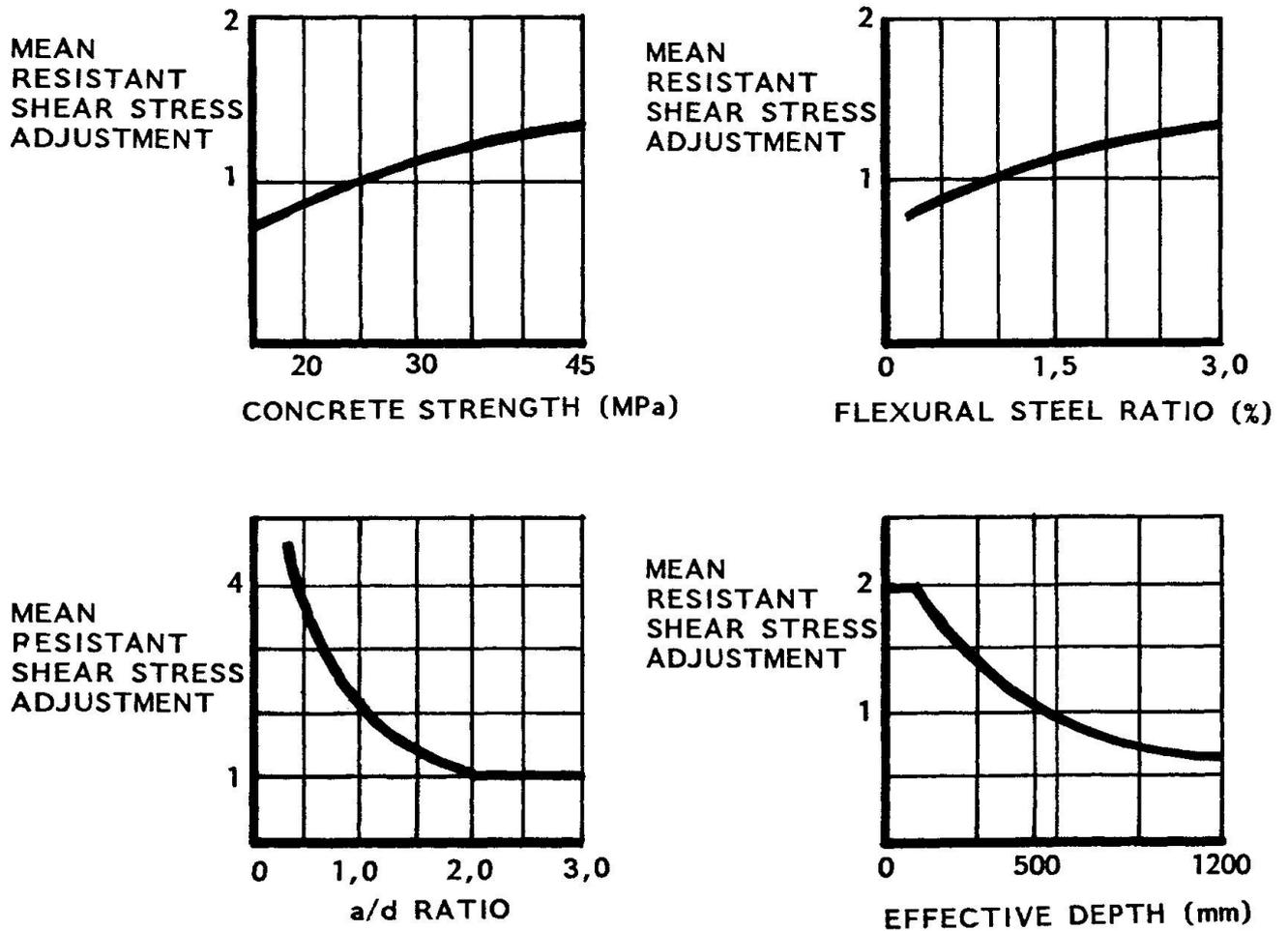MEAN RESISTANT SHEAR STRESS ADJUSTMENT

EFFECTIVE DEPTH (mm)

Figure 2   General parametric behaviour for
mean resistant shear stress in
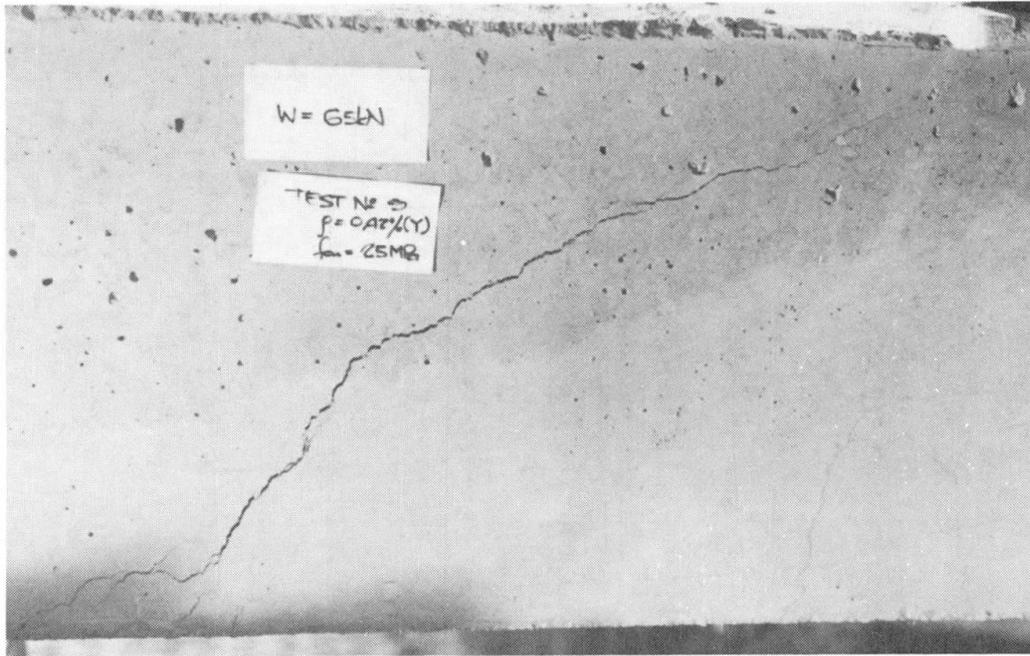members unreinforced for shear.

Figure 3 Photograph showing diagonal cracking
for beam unreinforced for shear with
a/d ratio greater than 2 just prior
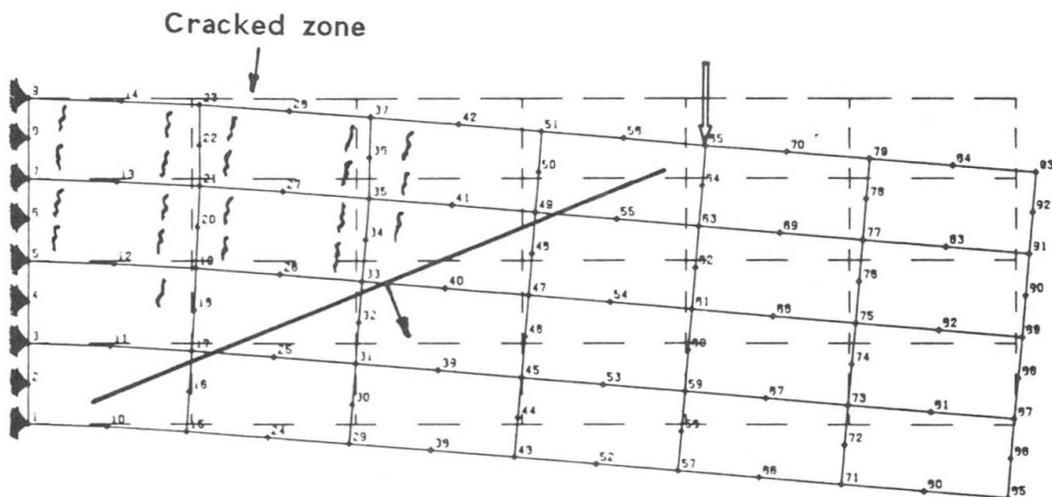to ultimate shear collapse.

Figure 4 Example of plane stress finite element
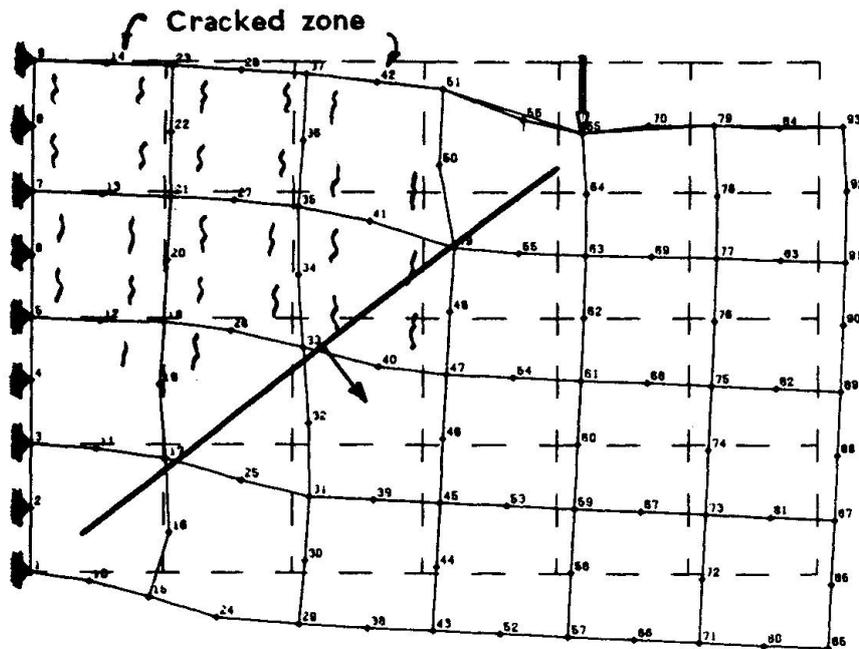mesh and distortions for a/d ratio of 2.

Figure 5   Example of plane stress finite element
           mesh and distortions for a/d ratio of 1.

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