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# Redistribution of Inner Forces in Hyperstatic Reinforced Concrete Structures

## Redistribution des forces internes dans des structures hyperstatiques en béton armé

## Umlagerungen von inneren Kräften in statisch unbestimmten Stahlbetontragwerken

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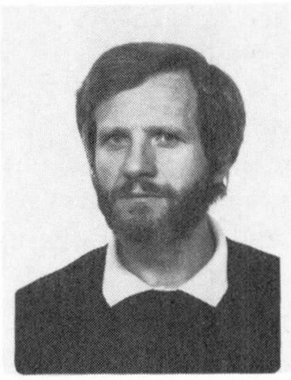

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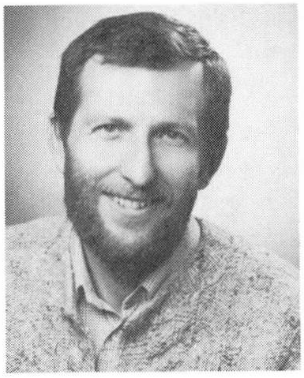

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**SUMMARY**
The redistribution behaviour of an inner tunnel wall subjected to injection pressure is studied. From the calculation using the FEM several conclusions concerning the carrying behaviour and redistribution can be drawn. The FEM provides a model which explains the arising damage very well. Furthermore, the results agree with those of comparable experiments. As a comparison an estimate is carried out on the basis of beam theory supposing plastic hinges.

**RÉSUMÉ**
L'étude traite du comportement et de la redistribution des efforts sur la surface intérieure d'un tunnel par suite de pressions d'injection. Sur la base de calculs à l'aide de la méthode des éléments finis, plusieures conclusions sont tirées sur la résistance et la redistribution des efforts. La méthode des éléments finis fournit un modèle qui permet également d'expliquer les dommages naissants. Les résultats concordent avec ceux d'expériences comparables. Une estimation est faite sur la base de la théorie de la poutre supposant des rotules plastiques.

**ZUSAMMENFASSUNG**
Am Beispiel einer durch Injektionsarbeiten überbeanspruchten Tunnelinnenschale werden die Umlagerungsvorgänge studiert. Aus der Nachrechnung mit Hilfe der FEM können einige Schlüsse auf das Tragverhalten und den Verlauf der Umlagerungsvorgänge gezogen werden. Die Nachrechnung liefert ein gutes Modell zur Erklärung der aufgetretenen Schäden, ebenso zeigt sich eine gute Übereinstimmung mit Ergebnissen aus vergleichbaren Versuchen. Zum Vergleich wird eine Abschätzung auf der Basis der Stabtheorie unter Annahme von plastischen Gelenken durchgeführt.

## 1. DESCRIPTION OF THE SITUATION

Water seepage was observed to occur over large regions of a tunnel between slurry trench walls, shown in figure 1. In order to rectify this the leaking regions were impregnated with waterproofing agent by the PU-method. During the injection work the appearance of horizontal cracks in the region about the middle of the wall was observed upon which the injection work was stopped. The assesment of the damage showed that cracks of up to 1.6mm in width had occured over a length of 30m and that the wall had bowed out to up to 17mm. The largest crack widths correlated approximately with the largest deflexion. The uncompressed wall had an average accuracy of $\pm$3mm. The damage can be assumed to be caused by exeeding the permissable injection pressure combined with the growing of a gap between the inner wall and the slurry trench wall.

## 2. THE PROBLEM

What was aimed for was:

.) a more exact explanation of the damaging process and cracking as well as a value for the highest injection pressure.
.) an estimation of the damage sustained by concrete and reinforcement on the outer side of the wall which is obscured from view.
From the calculations one could expect to be able to draw conclusions concerning the redistribution process.

The chosen example seemed especially advantegeous due to the following:
.) The calculation procedure for an existing situation with unambiguous loading history limits the width of the spectrum of possible interpretations.
.) The different reinforcement strengths and effective heights do not correspond to the stress resultants occuring under elastic conditions and consequently can be expected to cause large redistributions of the inner forces.

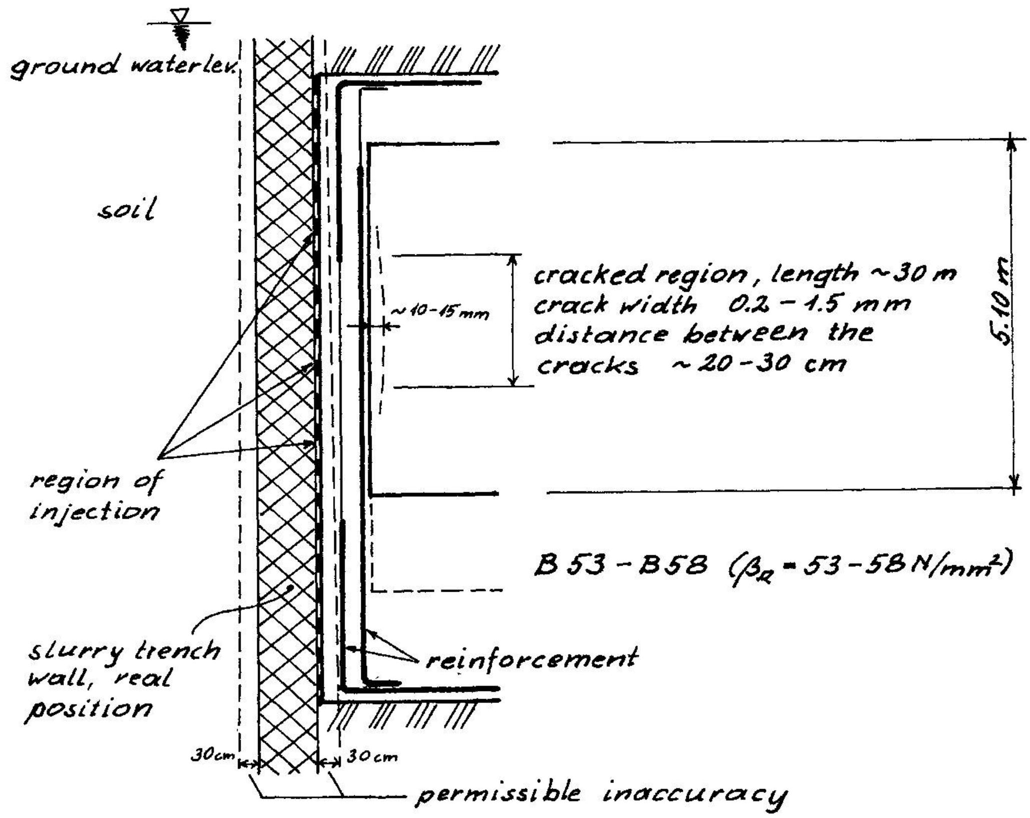



Fig. 1 Cross section of tunnel with cracked region

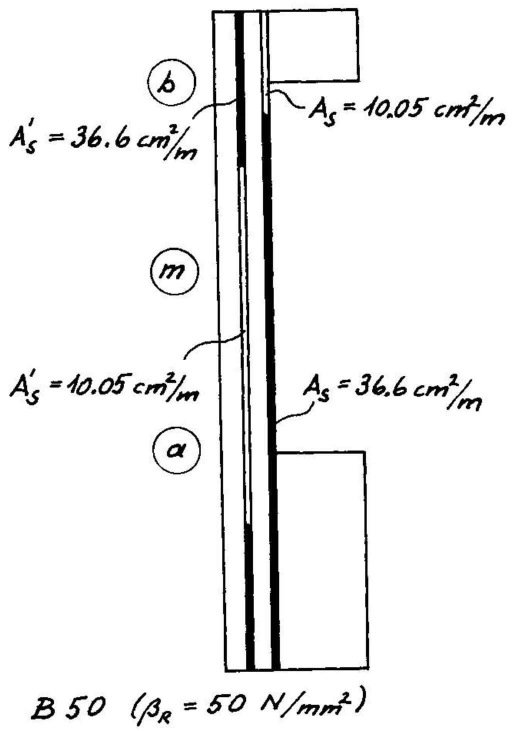



Fig. 2 Cross section with reinforcement

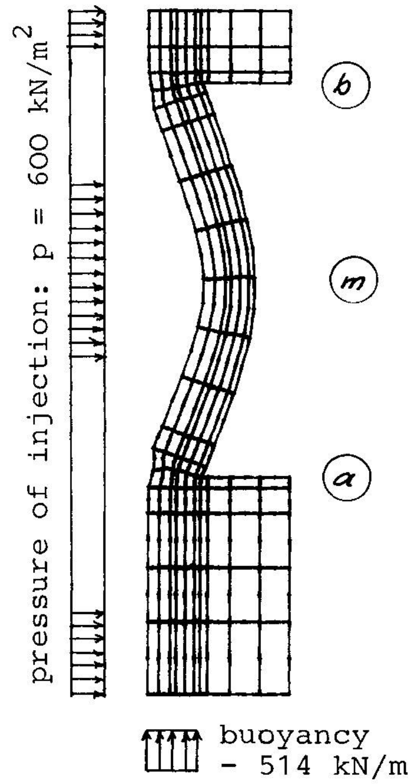



Fig. 3 Ultimate load: FE-mesh and deformation

## 3. FINITE ELEMENTE ANALYSIS

### 3.1 Fundamentals

There are a great number of numerical models used for reinforced concrete (see e.g. /1,2/).
The material model for reinforced concrete used in the present work is based on the following assumptions:

a) Restrictions of crack width and crack spacing require a minimum reinforcement distributed over the whole structure. This reinforcement is imagined to be smeared. Consequently reinforced concrete is treated as a compound material.

b) In compression the material is mathematically described by a 3-D plastic-isotropic hardening model based on a formulation of Shareef and Buyukozturk /3/ (fig. 4a)

c) The hardening function follows closely the parabola of the Austrian Standards - ÖNORM. (fig. 4b). For uni-axial compression ideal plasticity is assumed after 2‰ total strain.

d) Below tensile strength the model behaves linear elastically. If one or both principal stresses reaches the tensile strength cracking is assumed. From then on no additional load can be applied until the reinforcement is able to carry the load (fig. 4c).
Above 2.4‰ total tensile strain, again ideal plasticity assumed.

e) Concentrated reinforcement, e.g. as may occur in beams or panels, is modelled by additional bar elements or plane elements and a bi-linear stress-strain relationship for steel.

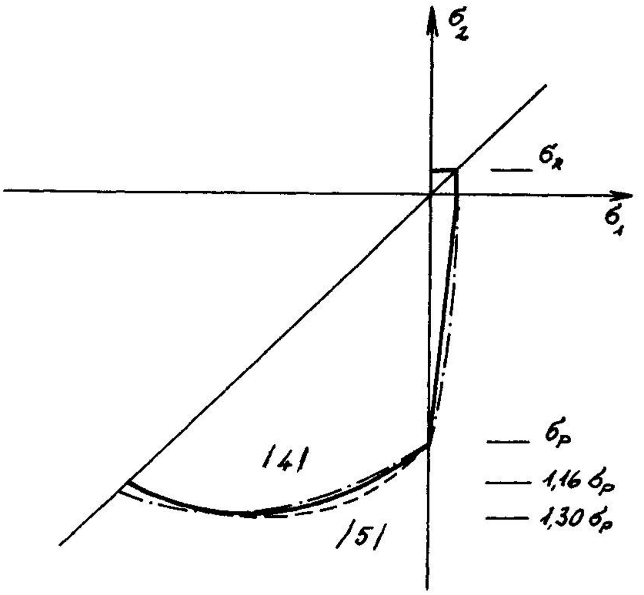



Fig. 4a Fracture envelope in the $\sigma_1 - \sigma_2$ plane

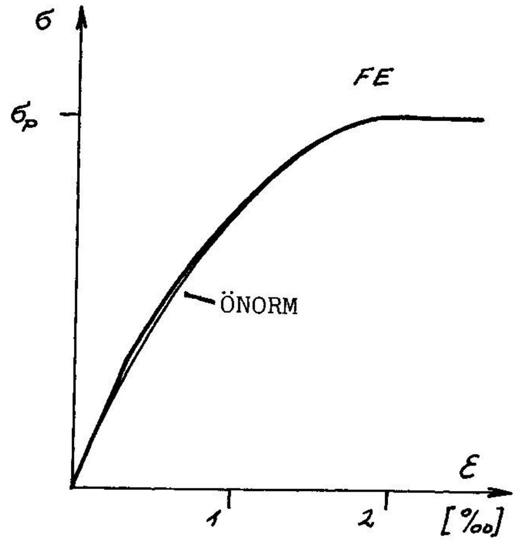



Fig. 4b Stress-strain diagramm of the FE-model and of ÖNORM under uni-axial compression

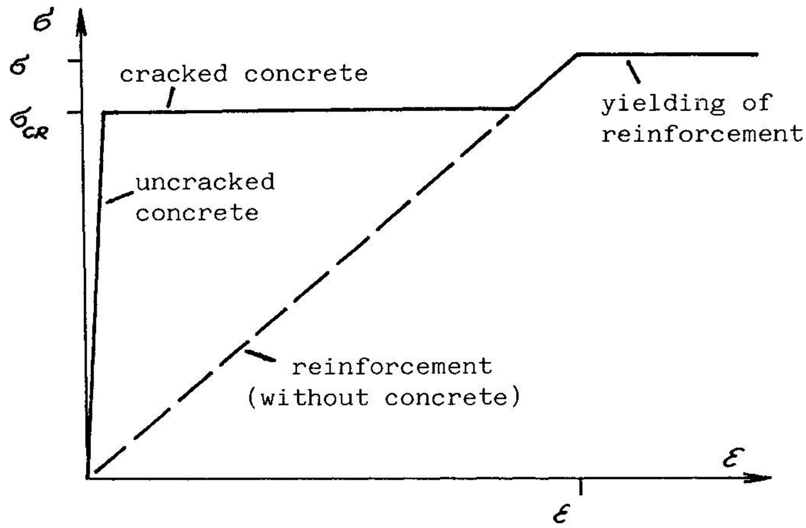
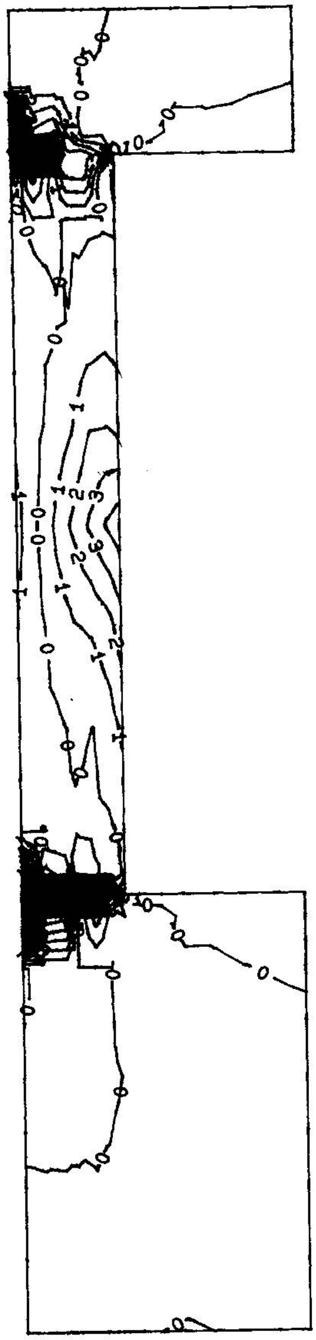



Fig. 4c Stress-strain diagram of reinforced concrete under uni-axial tension

The model parameters are: the comprehensive and tensile strengths of concrete and the directions, cross-section areas and yield stresses of the reinforcement. The model has been tested on a series of simple examples with comparison to some well known experiments /4,6/. Also, a number of RC-beams have been analyzed. The collapse load as well as the type of failure agreed excellent with experience and analytical calculations according to the Austrian Standards. Details are described in /7/.

## 3.2 Results of the calculations

The finite Element calculations were carried out using the experiences from /7/. Summarizing, there are no extra restraints for the mesh layout necessary due to the nonlinear material behaviour. The chosen mesh is shown in fig. 3. The material parameters are for concrete: $\sigma p$=37,5 N/mm$^2$ , $\sigma cr$=3,75 N|mm$^2$ ; for steel $\sigma f$=510 N|mm$^2$ .



b) Strains $\varepsilon by$

a) cracked regions

Fig. 5 Status of overloading deformation at "m": 14.9 mm

According to the FEM calculation the injection pressure at collapse is slightly greater than 600 kN/m$^2$ .

At this load the largest strain on the inside of the wall is 2.1‰, which would have caused crack widths of about 0.4 to 0.6 mm with the observed distances of 20 to 30 cm between the cracks. Conversely, from the observation of the largest crack widths of 1.0 to 1.5 mm one can conclude that the largest strains at these points are about 5‰. Therefore, during the injection process the pressure must briefly have been sustancially higher. Due to the movement of the wall the volume of the gap increased and the pressure was relieved. However, this caused the reinforcement to yield and so the cracks in the highly strained tensile regions of the concrete opened wide.

In order to asses the damage caused by overloading calculations using the FEM were continued up to a load of 650 kn|m$^2$ , i.e. just beyond the ultimate load. With a deflexion of 14.9 mm a strain of 4.4‰ occured in the middle of the innerside of the wall. Figure 5 shows the occuring strains and the cracked zones. The reinforcing steel is so heavily strained in the crossestions "a", "m" and "b" that a further increase in stress is no longer possible. However, the largest strain of 13.6‰ in "a" is much lower than strains that have been measured in comparable experiments. In the crossection "b" the concrete is strained about 2‰. This is the highest level ever reached, which is also within permissable limits.

The redistribution process: In figs. 6 and 7 the process of deflexion, the concrete stresses and the steel stresses are plotted versus the applied load.

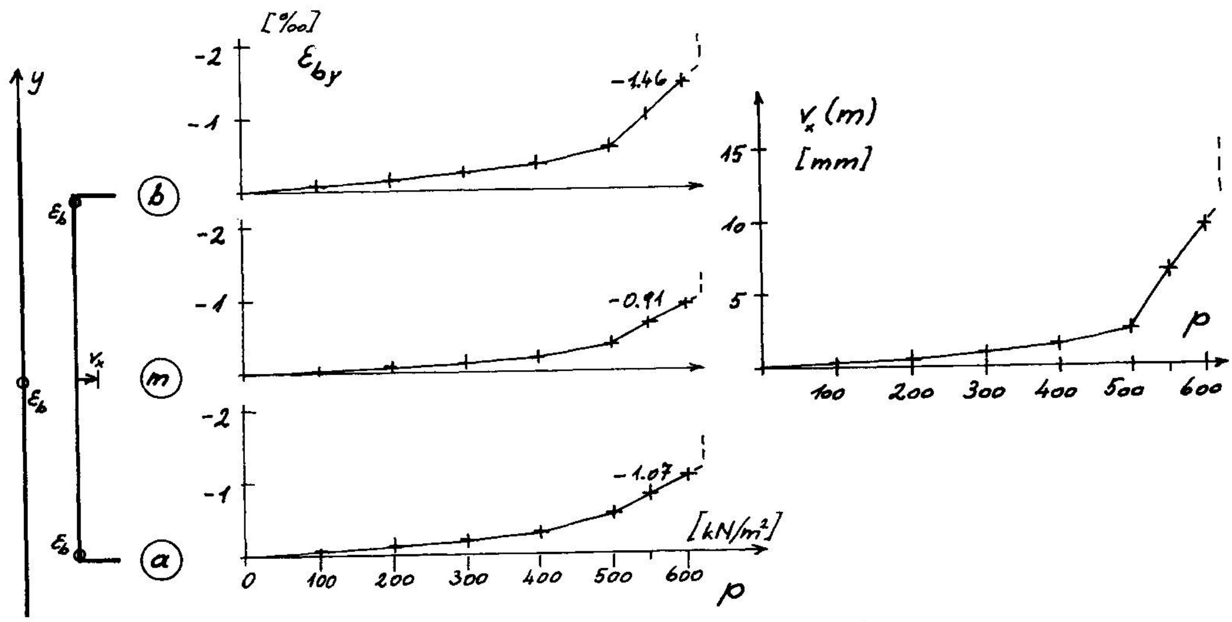



Fig. 6 a) Characteristic results on the regions of compression b) Deflexion of "m"

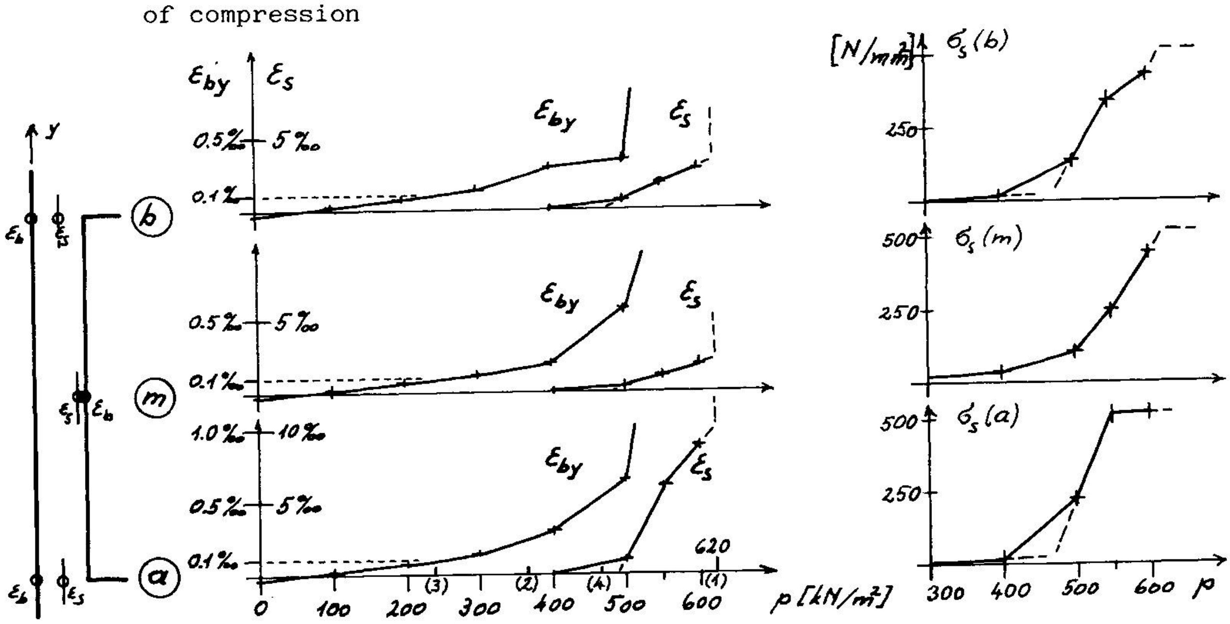



Fig. 7 Characteristic results in the tensile regions

(1) calculated ultimate load $p_u \sim 620$ kN/m² ($600 < p_u < 650$)
(2) calculated service load $p_s = 620/1.7 = 365$ kN/m²
(3) cracking ( $\varepsilon_b \geq 0.1$%o) above $p_c = 230$ kN/m² $\hat{=}$ 620.3|8
(4) forces in reinforcement increase above $p_r = 460$ kN/m² $\hat{=}$ 620.3|4

The numerical results show that from a load of 300 kN|m² onwards (just under half the ultimate) the first signs of a non-linear behaviour are evident. At the ~~three~~ most stressed points, "a", "m" and "b", the edge strains are greater than 0.1%o, at these points the concrete starts to crack.
At a loed of 400 kN/m² (this approximately represents the permissable service load) the formation of cracks is so advanced that the reinforcement in "m" starts to sustain considerable tension. This situation diverges heavily from a linear elastic analysis where the bending moments in the edges of a clamped beam are twice as high as in the middle of the beam.

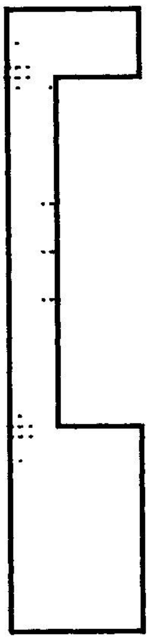


Fig. 8 cracked regions under service load

Figure 8 shows the distribution of cracked zones at service load.
At the ultimate load the inner forces are re-distributed such that they represent approximately the actual carrying capacities of the individual crossections.

Comparison with experiments: The results of the calculations correspond with the experimental results of other published sources /8,9,10/. Redistribution starts immediately with the onset of cracking of the concrete and proceeds in continuity up to the ultimate load. Even with a further increase in deformation after reaching the ultimate load the strains that occur stay well under the permissable values.
Plastic hinges in the sense of beam theory with actual rotations do not occur.

4. COMPARATIVE CALCULATION

As a control a comparative calculation was carried out based on beam theory. This was done using the assumption of plastic hinges and following /11/. The system and the individual load steps up to the occurence of a failure under kinetic conditions are shown in figure 9.

Taking the effective span as reference length we obtain a value of 590 kN for the maximum possible pressure of injection. This represents a value bearly under the load obtained via the FEM.
An important element in this method is the reaching of a value for the permissible rotation and a comparison of this with the rotation occuring in practice. In the given problem with prescribed deformation this is only possible via an estimate of the deflexion. In the given case the permissible rotation is not reached; not even if one takes into account the violation of Bernoully's Hypothesis. However, this can only be seen as a rough estimate based on the geometrics of the situation.

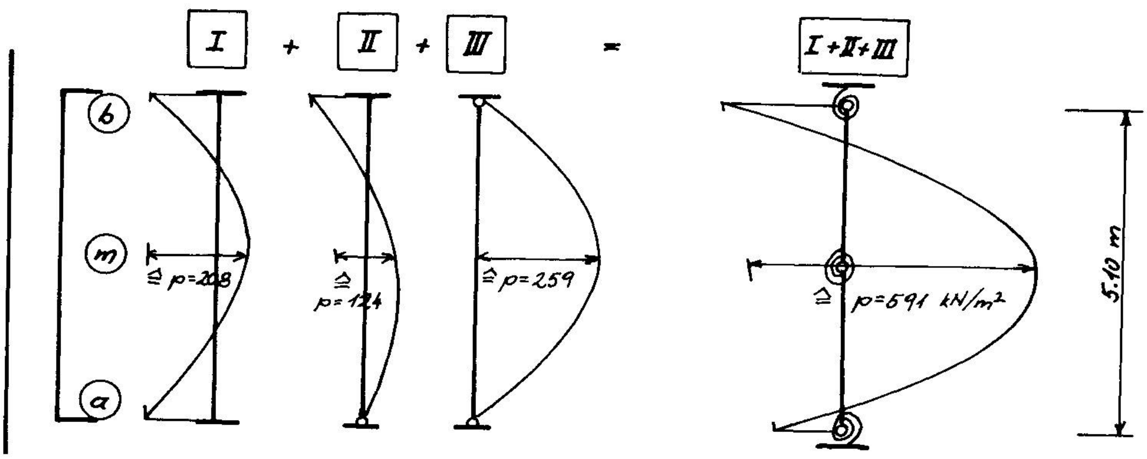



Fig. 9 Comparative calculation based on plastic hinges: system will bending moments and load steps

## 5. SUMMARY

With this example of a single instance of damage due to forced deformation the FE analysis proved to be a powerful tool in understanding the mechanism of the load redistribution. The remaining insecurities are substantially less important than those associated with an estimate based on the occurence of plastic hinges.

This example also shows, that load redistribution is, above all, a consequence of cracking of the concrete in the tensile regions already beginning with low load levels. An obvious rotation as would be expected cannot be observed; the strains stay below permissible values.

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