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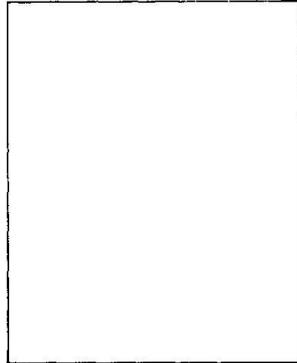
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**Examples of Non-Linear Numerical Analysis with DIANA**  
Exemples d'analyse numérique non-linéaire avec DIANA  
Beispiele nichtlinearer, numerischer Berechnungen mit DIANA

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**SUMMARY**

A number of examples of non-linear numerical analysis with DIANA are made to illustrate the actual possibilities and the progress made with advanced concrete mechanics. The numerical model appears to be a new and powerful instrument especially for specific practical studies and research purposes more generally.

**RÉSUMÉ**

Certains exemples d'analyses non-linéaires numériques avec DIANA permettent d'illustrer les possibilités actuelles et le progrès dans la mécanique des structures en béton armé. Il est évident que le modèle numérique est un instrument nouveau et puissant pour des études pratiques, ainsi que pour la recherche en général.

**ZUSAMMENFASSUNG**

Eine Anzahl von Beispiele nichtlinearer numerischer Berechnungen mit DIANA wurde ausgeführt um zu illustrieren welche Möglichkeiten entstanden sind und welche Fortschritte mit rechnerischer Mechanik von Konstruktionen aus Stahlbeton gemacht wurden. Es stellte sich heraus dass das numerische Modell ein neues und starkes Hilfsmittel für praktische Studien und für Forschung im allgemeinen ist.



## 1. INTRODUCTION

### 1.1 General

The Dutch "Concrete mechanics project" is an on-going co-operative research program. The objective of this project is to analyse the structural behaviour of reinforced concrete members by simulation through an advanced numerical model. In this numerical scheme - which accounts for the geometry of the structure under consideration - advanced material-models are applied and additionally basic-models for effects from cracking of concrete, bond-slip behaviour between reinforcement and concrete and aggregate interlock for stress transfer in cracks. The investigations are carried out by material-scientists and specialists in numerical modelling and applied mechanics.

The Dutch Technical Universities of Delft and Eindhoven, the Rijkswaterstaat - a division of the Ministry of Transport and Public Works - and the Institute for Applied Scientific Research on Building Materials and Structures (TNO-IBBC) are the participants in this co-operative program. The activities are financially supported by CUR - centre for civil engineering research, codes and specifications - and MaTS - marine technological research.

### 1.2 Examples of analysis

Although three numerical models have been developed, the study now concentrates for several years specifically on DIANA, a general purpose finite element code for the analysis of three dimensional structures.

This report concerns a number of examples of non-linear numerical analysis of reinforced concrete structures made with DIANA. The examples are completely explained in Heron 1987 nr. 3: "Examples of non-linear analysis of reinforced concrete structures with DIANA", by dr.ir. J.G.M. van Mier, ir. C.R. Braam, ir. H. Groeneveld, ir. J. F. Marcelis, prof. Chr. Meyer and ir. J.G. Rots.

The examples include the simulation of three structural details (a tooth support of a beam, a corbel and a beam to column connection), three global structures (deep beams in two variants, a tunnel section and a LNG-storage tank subjected to fire) and two dynamic analysis (a beam falling on a shock absorbing element and an explosion in a tunnel). All simulations are two dimensional except for the storage tank, which is treated as an axi-symmetric problem.

### 1.3 Application

In principle the non-linear numerical analysis concerned, fit into a design-procedure like experimental model studies do. This means that the practical application of these analysis will be limited to specific cases in which design has been carried out e.g. by linear analysis or on the basis of equilibrium conditions and non-linear compatibility must be checked properly by specific studies. Of course, this will concern complex combinations of action effects in most cases. Other situations in which additional analysis may be required concern for instance: expected non-linear stress-distributions as in deep beams or redistribution of action-effects specifically at lower load levels or combined with shear. For unlimited use of non-linear analysis in this practical scheme, the simulation of the real behaviour of the structure under consideration must be reliable. In the future this status may be expected in a wide range of application.

Nowadays, the numerical simulation technique appears to be applicable in certain practical cases already. However, it should be mentioned that the analyses must be carried out by experienced model-engineers, who are familiar with the programs, the underlying material-models and structural behaviour of reinforced concrete structures as well.

Having regard to the state of the art of this moment, the numerical models provide for a new and powerful instrument, especially for research purposes. The use of this numerical analysis may take place in these cases under the required circumstances with guidance and support of specialists or even in combination with experiments. This will result in a better understanding of the behaviour of

reinforced concrete structures under conditions where problems still exist e.g. in certain cases where shear governs the behaviour. The examples to discuss have been chosen in order to demonstrate the current possibilities of a non-linear numerical analysis of reinforced concrete structures with DIANA.

#### 1.4 DIANA

For most of the information about DIANA and for a description of the material-models available in DIANA reference is made to the Heron 1987, nr. 3 mentioned before.

An exception is made for the models related to crack-formation and bond, because these phenomena will be met later in this survey.

In DIANA a smeared crack concept is used. When a non-linear analysis of an unreinforced concrete structure is carried out, the energy release rate, the  $G_f$  concept, is used. The stress-crackstrain diagram is transformed into the bilinear diagram of figure 1.

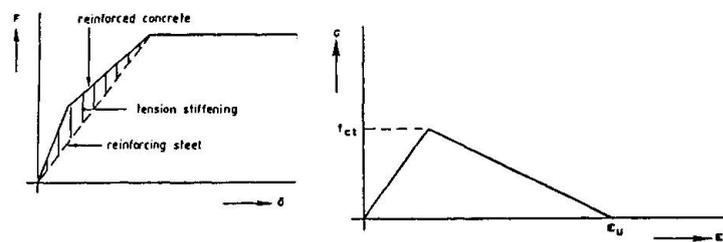
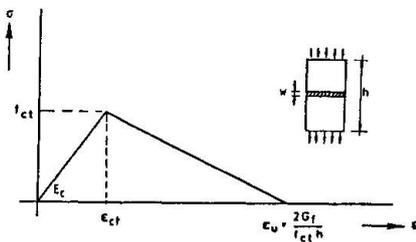


Fig. 1. Linear softening model.

Fig. 2. Tension stiffening concept

When a reinforced concrete structure is analysed, the tension-stiffening concept is used. The principle is explained in figure 2. The tension-stiffening concept is applicable for the analysis of a reinforced concrete structure with uniform distributed reinforcing bars and when cracks are expected to develop perpendicular to the direction of the bars. This is a rather severe restriction: in general cracks do not intersect the reinforcement perpendicularly, but more likely under a certain angle. The assumption of uniformly distributed reinforcing bars generally does not apply also for structures. Note that average stresses are calculated and not the maximum steel stresses that occur in cracks. This is a consequence of the smeared crack approach.

When full bond between steel and concrete is assumed, the reinforcement is modelled by bar elements which are embedded in the concrete. However, when relative displacements between reinforcement and concrete are allowed, special slip-elements (two-dimensional springs) can be applied. The consequence of this approach is that the element-mesh must be adjusted to the reinforcement grid. When numerous reinforcement bars are present, this may lead to an enormous number of elements. Thus, it is very important to decide in an early stage if the application of bond-slip elements is necessary.

The behaviour of bond-slip elements is described by means of two constitutive equations. The first concerns the bond shear stress with the slip along the bar. This relationship is taken as a bi-linear diagram. The second equation describes the relation between the radial component of the bond stress and the radial displacement. The latter becomes important when radial confinement is present.

## 2. THE EXAMPLE-CALCULATIONS

### 2.1 A METRO-beam

This beam is a prefabricated prestressed concrete beam with a total span of 33 m



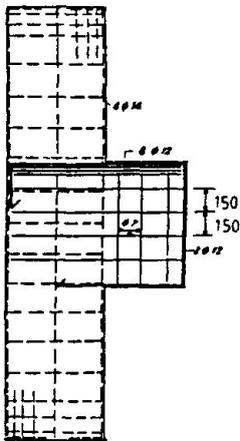
stiffening that has been used. The value of  $\epsilon_{us}$  needed for crack propagation clearly determines the rate at which the diagonal crack in the throat propagates. The dependency of the total structural behaviour on this parameter is such that predictions of the structural response is not very well possible. This parameter has been varied to study the influence of the variation and finally fitted to about the right number.

This means that for the right prediction of the behaviour of details like this the bond-slip model is necessary. However, the use of that model causes practical difficulties with regard to the number of required elements.

In spite of the fact that a precise prediction of the structural response of the tooth structure is not very well possible, more insight in the qualitative structural behaviour can be obtained from a numerical simulation by investigation of the influence of some of the model parameters.

2.2 A corbel

This example concerns a single corbel on one side of a column, see figure 6. Corbels like this one are rather widely used, while the analysis and design sometimes may be rather difficult. The concerning corbel was subject of experimental study in 1961.



The analysis of this example was carried out in two variants. At first an analysis was made with perfect bond between reinforcement and concrete. In a second analysis the effect of using bond-slip elements between the main reinforcing bars and the concrete has been considered.

The bearing capacity and the global behaviour of the corbel at failure has been predicted satisfactory with the two dimensional perfect bond model.

The results give a good impression of the way how action effects are distributed within the corbel and the column, see figure 7 and 8.

Fig. 6. reinforcement of the corbel

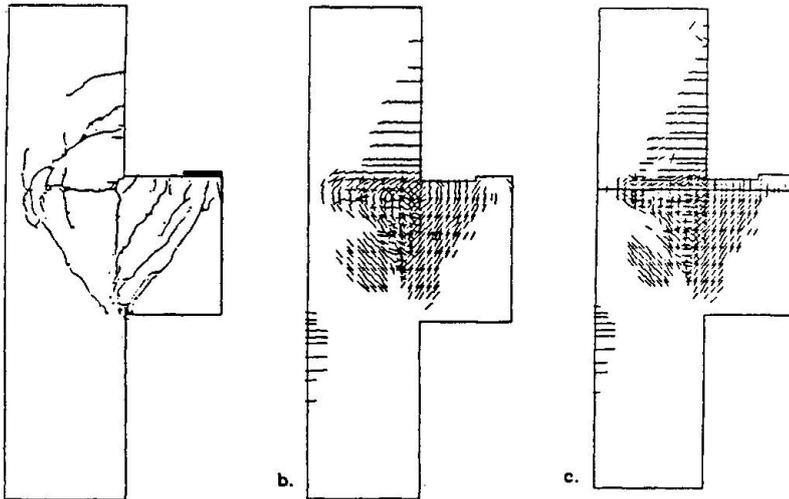


Fig. 7. a. crack pattern at failure  
 b. computed perfect bond  
 c. computed bond-slip

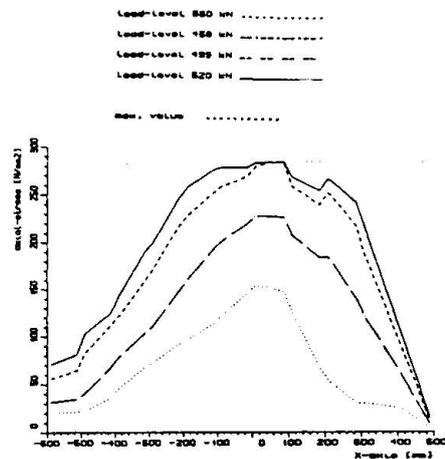


Fig. 8. axial stresses in the bond-slip bar

The current models - the perfect bond and bond-slip elements around the main reinforcement only - the simulation of the localized cracking is not very accurate. Refining of the element mesh and bond-slip elements for all reinforcement may give better results in this respect. However, this creates



difficulties regarding the amount of elements required in this case.

### 2.3 A beam-column connection

In this example a single-bay portal frame is modelled, see figure 9.

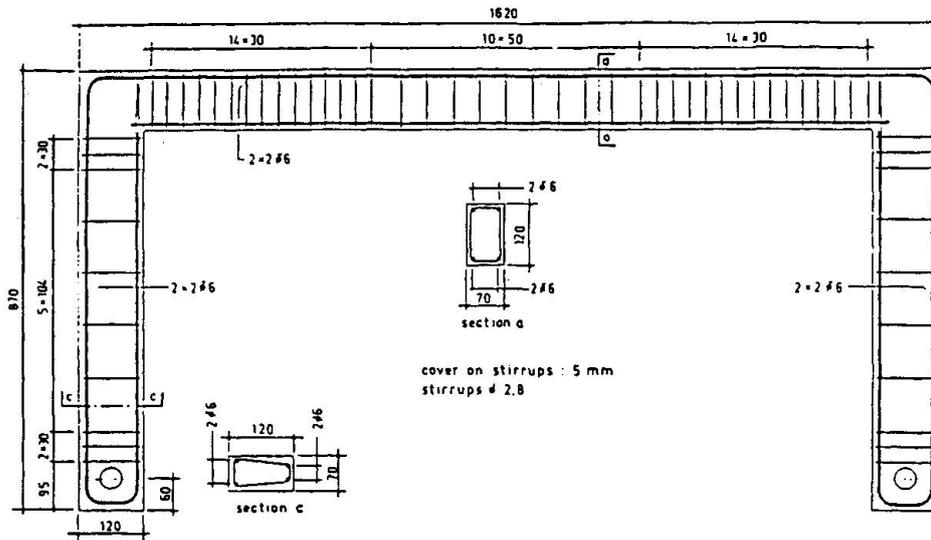


Fig. 9. Portal frame

Special attention has been paid to the structural detailing of the beam to column connection. The behaviour of this joint is interesting because one might easily assume that the joints are as strong as the connected members. However, in certain cases the strength of the joint may be lower.

In this case the beam-column connection is subjected to a negative moment and the behaviour of the joint and the frame has been simulated. Some of the results are compared with findings from experiments.

This numerical simulation gave a good impression about the global behaviour of the frame (see figure 10) and the structural mechanism in the corner-joint.

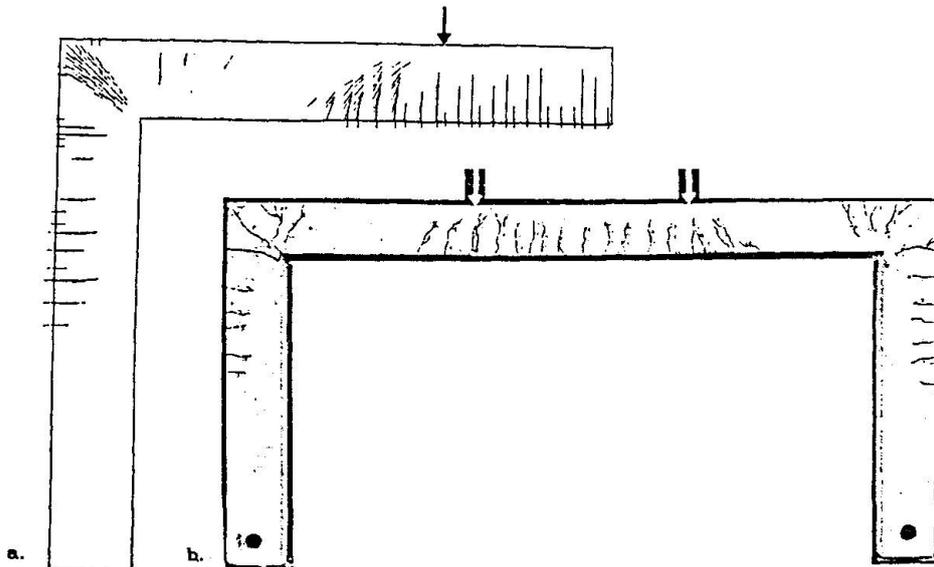


Fig. 10. a. Cracks at failure

b. Experimental crackpattern at failure

After cracking occurred at the corner of the frame, large compressive stresses developed in the diagonal between beam and column. This phenomenon corresponds well with a simple system of equilibrium of internal forces in that region, where the said compression-diagonal transfers the components of the tensile

reinforcement to the compression zones of beam and column at this corner. In the experiments the compression-diagonal caused the splitting tensile stresses at the bend of the tensile reinforcement perpendicular to the plan of the curved bar. This, of course, could not occur in the two-dimensional analysis. For the same reason a localized diagonal corner crack suddenly appeared in the analysis, which developed much more gradual in the experiment.

More detailed research of the behaviour of concerned corner detailing therefore would require a full three-dimensional analysis.

#### 2.4 Analysis of a tunnel-section

The analysis concerns a part of a cross-section through an existing tunnel, see figure 11.

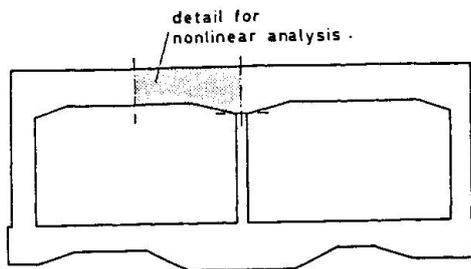


Fig. 11. Cross section of tunnel-structure

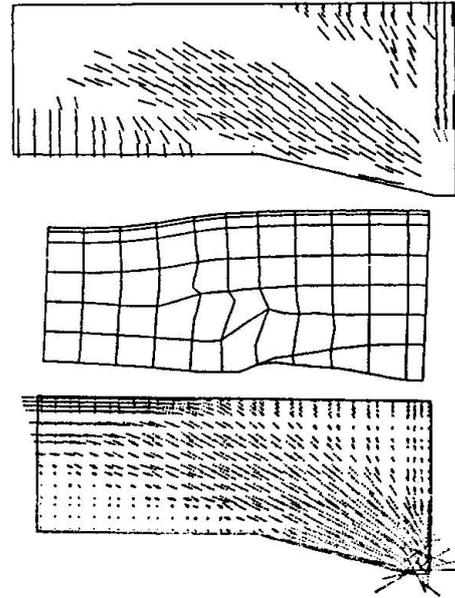


Fig. 12. Crack pattern, incremental deformations and principal stress trajectories at failure

In this case no experimental information is available. The analysis serves a design-check. The main problem is to check the bearing capacity near the middle support. In this part a considerable redistribution of moments is necessary to reach the required bearing capacity. In the part of the section of concern all main reinforcement will yield before the ultimate capacity will be reached. The question was whether this region would have sufficient strength to withstand the belonging shear force while the main reinforcement yields.

The non-linear simulation of a part of the tunnel-section gave very good results. The development of the crack formation, due to flexure and shear, is predicted qualitatively well.

The redistribution of internal stresses, due to crack formation, appears significantly in the analysis. At ultimate an arch-effect develops, which is in equilibrium with the tensile reinforcement in the span, see figure 12. The bond-slip elements appeared to be a feasible model also at the splices. The analysis made use as much as possible of the possibility to limit the section to consider by applying realistic edge-conditions.

#### 2.5 Deep beams

Deep beams are usually analysed and designed on the basis of linear calculation. The behaviour after crack-formation is difficult to estimate. This holds especially when deep beams extend over several supports. Therefore, a non-linear analysis of deep beams forms part of the series of examples considered.

The first example comprises the analysis of a deep beam on two supports, figure 13. In the second example a deep beam extending over three supports is simulated numerically, figure 14.

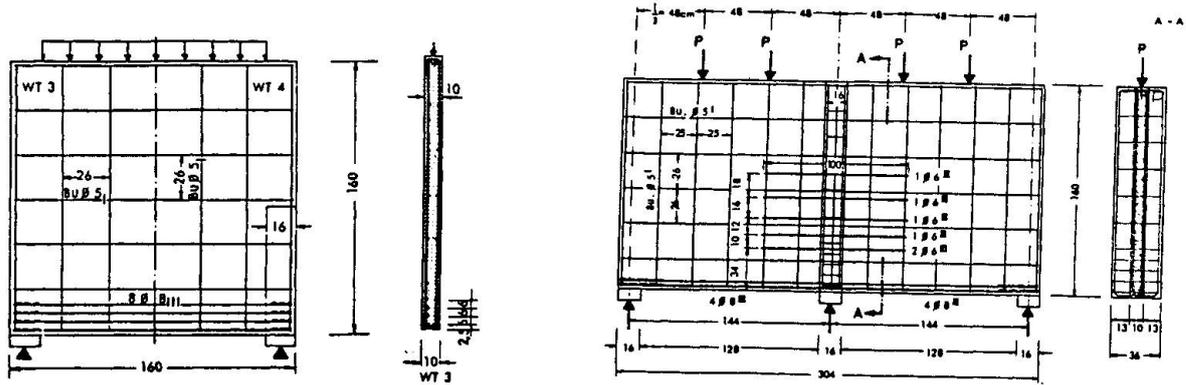


Fig. 13. Deep-beam on two supports. Fig. 14 Deep-beam on three supports

In both cases results of experiments by Leonhardt and Walter are available for comparison.

A realistic simulation of the structural behaviour of reinforced concrete deep beams is possible. The load-deflection curve observed in the experiments and the crack pattern are quite well simulated, figure 15.

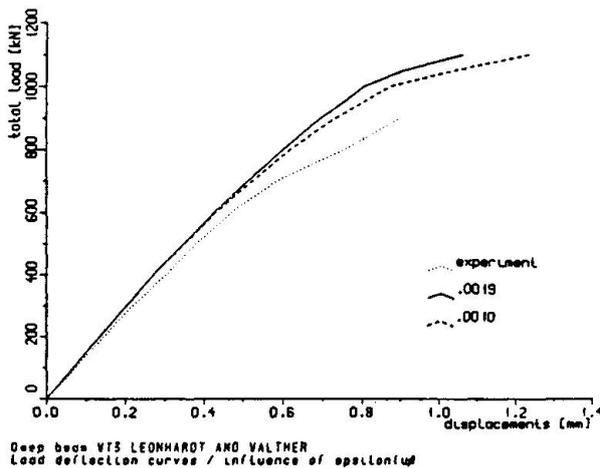


Fig. 15. Comparison between calculated and experimental load-deflection.

However, a specific fit of the tension-stiffening parameter  $\epsilon_{us}$ , was necessary to find satisfactory results. This means a more direct modelling of the bond phenomena would be better, also in these cases.

The behaviour of the deep beam on two supports is, after cracking of the central region, completely determined by the compressive response of a 'column' above the support. The results indicate, that as soon as the central region is separated from the 'column', failure is inevitable. This separation seems to occur through the development of a dominant crack in the beam, just next to the support. This dominant crack was not observed in the experiment.

The behaviour of the deep beam on three supports was determined by steep shear cracks; as soon as this crack develops and the reinforcement in the crack yields, failure is inevitable, figure 16.



Fig. 16. cracking failure in the experiment and calculated strains.

The compressive response of the 'column' above the supports is not important for failure in this case, but seems to influence the stiffness.

### 2.6 A LNG storage tank

The storage tank which is investigated, is analysed and designed by Muller (1965). The tank is shown in figure 17.

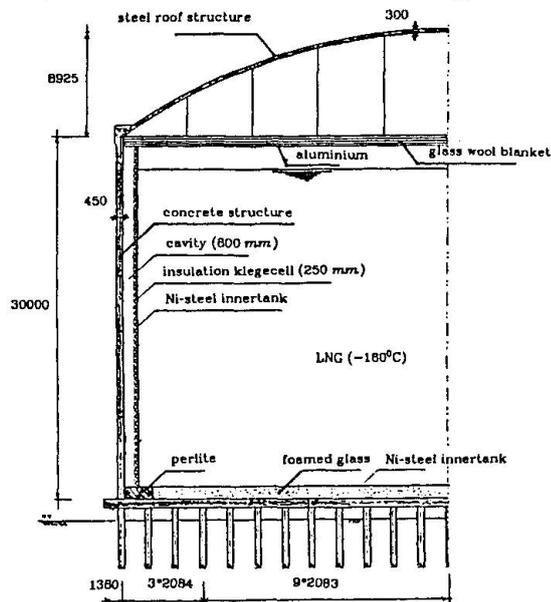


Fig. 17. Cross-section of the LNG tank

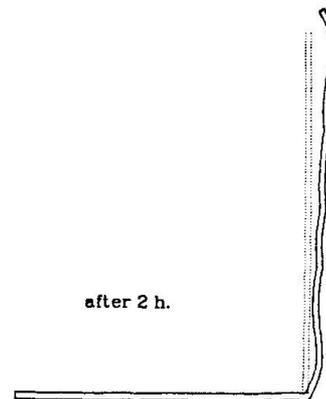


Fig. 18. Deformation of the tank

The roof and the base are monolithically connected to the wall. In this example two analysis were carried out.

The tank subjected to the water test; the behaviour of the tank subjected to static loading was checked.

The tank subjected to fire-load. It is assumed that the heat is caused by a burning adjacent storage tank producing a thermal radiation of  $30 \text{ kW/m}^2$  constant over the height of the tank and constant in time during 40 hours.

The effect of high temperatures on the properties of the concerning materials have been taken into account.

In both cases the analysis is treated as an axi-symmetric problem.

The water test analysis demonstrated good agreement between 'elementary calculations' at one hand and 'numerical simulation' on the other hand. No cracking was observed when the tank was subjected to the water test, which indicates that the tank was designed well.

The result of the fire-load analysis are shown by means of numerically calculated crack patterns and deformations and are in agreement with theoretical considerations, figure 18.

However, one should realise that some simplifications were made in the numerical analysis. The most important ones are that combined plasticity and temperature dependency and 'transient strain'-effects are not taken into account.

### 2.7 Beam falling on a shock absorbing element

This analysis has been performed to show the possibility to make dynamic non-linear calculations with DIANA. The beam considered is 8.15 m long, supported at one end by a hinge, around which the beam can freely rotate in a vertical plane. The other end of the beam is lifted vertically to the desired drop-height. The beam falls on a shock absorbing element and undergoes severe bending within fractions of a second after striking the shock absorber. Figure 19 shows the arrangement as described.

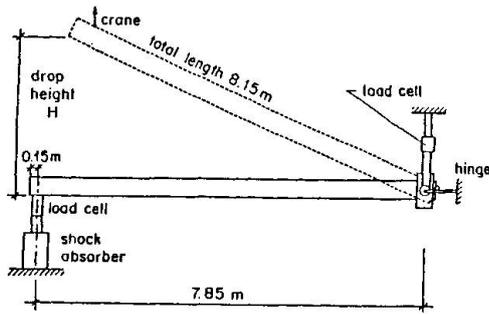


Fig. 19. Test set-up

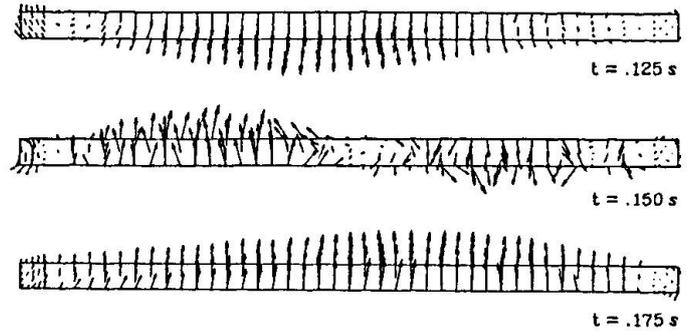


Fig. 20 Velocity field as calculated

It has been shown that a non-linear dynamic finite element analysis is possible. Good results can be obtained, figure 20.

In the analysis the stiffness of the beam was a little overrated, resulting in somewhat smaller deflections and a higher frequency than observed in the experiment. The influence of dynamic loading on the material properties has been overestimated. Further research in this field seems necessary. But in general the numerical results can stand a comparison with the experimental results. Development of a spring element with hysteresis could improve the results of this analysis, because a better model for the shock absorber could be implemented then.

Such an improvement can be expected too from a concrete model with even better unloading behaviour than the model that was applied in this analysis.

### 2.8 Dynamic analysis of under water tunnel for gas explosion

Tunnels passing under water ways are normally designed to resist loads associated with dead weight, soil, water and traffic. In the event of an internal gas explosion, the tunnel experiences a load reversal. The question may be whether an accidental gas explosion can cause failure of the tunnel.

Figure 21 shows a typical cross-section which was the subject of non-linear analysis. The main objective of the analysis was to predict the response of the tunnel to the load, associated with a hypothetical internal gas explosion, figure 22.

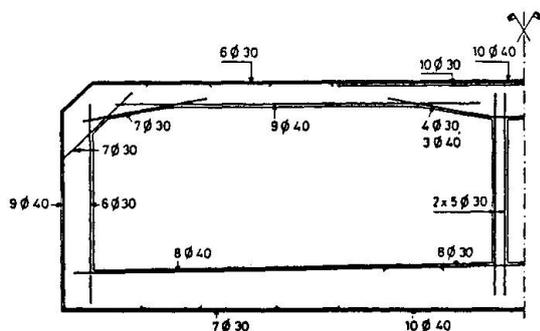


Fig. 21. Tunnel reinforcement for a 1.5 m wide section

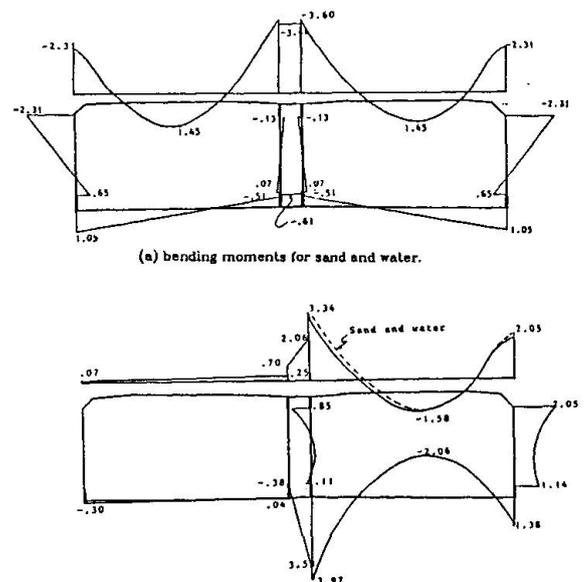


Fig. 22. Bending moments obtained with frame element model

It appears from the analysis, that a conventionally reinforced under water tunnel is unlikely to survive an internal gas explosion.

DIANA appears here a powerful tool to provide rational solutions for complex non-linear dynamic analysis problems.

However, such an analysis is a very difficult task, which may take an experienced analyst at least one full month or more and consumes considerable computer resources. These man-hours and computer expenses are needed primarily for the step-by-step development of the finite element model and its verification.

### 3. CONCLUSION

The application of the non-linear numerical model DIANA as discussed may largely increase the structural insight. It may serve as a simulation tool, a research tool or as a means to improve codes as well. Yet in general an analysis is a difficult task and must be carried out by an experienced model-engineer who is familiar with the adopted material-models and with structural behaviour.

The practical application for design activities of this tool is close to further studies by experiments. That should be beared in mind with regard to the knowledge required and the time and costs involved. However, the use of a finite element package in a simple form may already give considerable insight in structural behaviour.

The performance of the program still seemed rather problem dependent in some cases. This of course puts some restrictions on its predictive power. However, it appears that this weakness mainly follows from the use of simplified models like tension-stiffening where bond-slip elements would have been more appropriate. The use of bond-slip elements has been avoided as much as possible in order to reduce the amount of elements and computation time. In some cases this appears to be too rough for good results.

Nevertheless in some examples the computation time was very much. However, this will decrease with further developments of computer-technology. One should bear in mind, that relative much of this time is spent nearby the ultimate state. For an analysis up to 70% of the bearing capacity roughly only 30% of the total computer time is needed. For the practical application of non-linear analysis, this may be an important fact.

The further development of this powerful non-linear numerical model is worthwhile.

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