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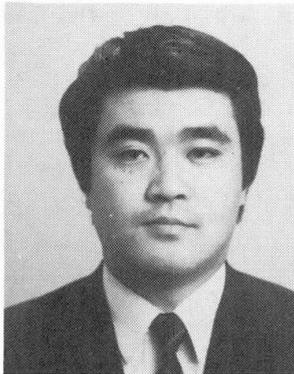
## Application of Finite Element Methods to Design of Reinforced Concrete Structures

Application de méthode des éléments finis au projet des structures en béton armé

Anwendung der Finite Element Methode auf den Entwurf von Stahlbetontragwerken

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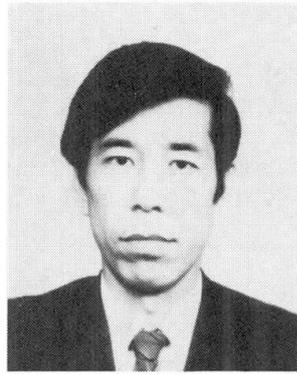
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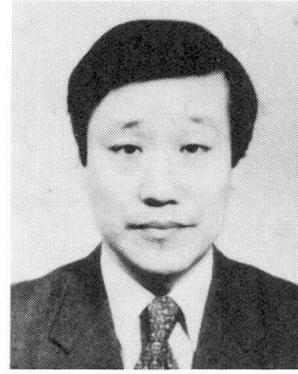
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### SUMMARY

The Japan Concrete Institute had established the Committee on Finite Element Analysis of Reinforced Concrete Structures. One of the final aims of this committee is to propose guidelines for the application of FEM analyses to design. As a first step, the committee circulated a questionnaire on design examples using FEM analyses. Based on results of the questionnaire, this paper outlines the present situation and subjects to be treated in Japan.

### RÉSUMÉ

L'Institut Japonais du Béton a mis sur pied un comité pour l'analyse des structures en béton armé au moyen des éléments finis. Un des objectifs finaux de ce comité est de proposer des directives pour l'application de la méthode des éléments finis au calcul du projet. Dans une première phase, le comité a réalisé un questionnaire sur des exemples de projet à calculer au moyen de la méthode des éléments finis. Sur la base des résultats du questionnaire, l'article explique la situation actuelle et les sujets à étudier au Japon.

### ZUSAMMENFASSUNG

Der Japanische Betonverein hat einen Ausschuss für FE-Berechnungen im Massivbau eingesetzt. Eines der Ziele ist, Richtlinien für die Anwendung von FE-Berechnungen beim Entwurf vorzuschlagen. Als erster Schritt wurde ein Fragebogen verschickt über Beispiele von FE-Entwurfsberechnungen. Auf der Grundlage dieses Fragebogens stellt dieser Beitrag die gegenwärtige Situation und noch ausstehende Fragen in Japan dar.



## 1. INTRODUCTION

Recently according to the widespread adoption and advancement of computers, opportunities for the use of finite element method (FEM) analyses have been rapidly increasing in the fields of both the design and analytical research of reinforced concrete (RC) structures. The following reasons can be indicated as the background of this tendency.

In the field of design, it has always been difficult to apply conventional design procedures to large scale or special structures. Because of the relatively recent occurrence of these structures, there are very few preceding design examples. Therefore, it is not easy to convert them into simple structural members such as beams, columns, plates and frames. Secondly, the number of cases that demand precise examination of local stress conditions in a structure has increased remarkably. Lastly, a method of finite element analysis for thermal cracks in a massive concrete structure has been rapidly developed.

In the field of analytical research, the application of FEM analyses instead of the former experimental approach to derive or refine design equations of RC structures has increased. This is because FEM analyses can accurately compute the flow of internal forces and stress distributions in structures. Fig.1 shows the relationship between FEM analyses and RC structures from the point of view of applied fields.

The present situation and future prospects of FEM in the research field have already been reported by Aoyama and Noguchi[1]. A few practical examples of specific RC structures designed by FEM were presented by Ikeda and Tsubaki[2]. However, there are very few reports that discuss the present situation regarding the application of FEM analyses to design.

The Japan Concrete Institute (JCI) had established the Committee on Finite Element Analysis of Reinforced Concrete Structures (chairman H.Noguchi). One of the aims of this committee is to propose guidelines for the application of finite element analyses to the design of RC structures.

As the first step, a questionnaire was circulated to investigate the actual circumstances of the use of FEM analyses in the practical design of RC structures. General contractors, research institutes and universities were the objects of this investigation. Replies were received from both the fields of

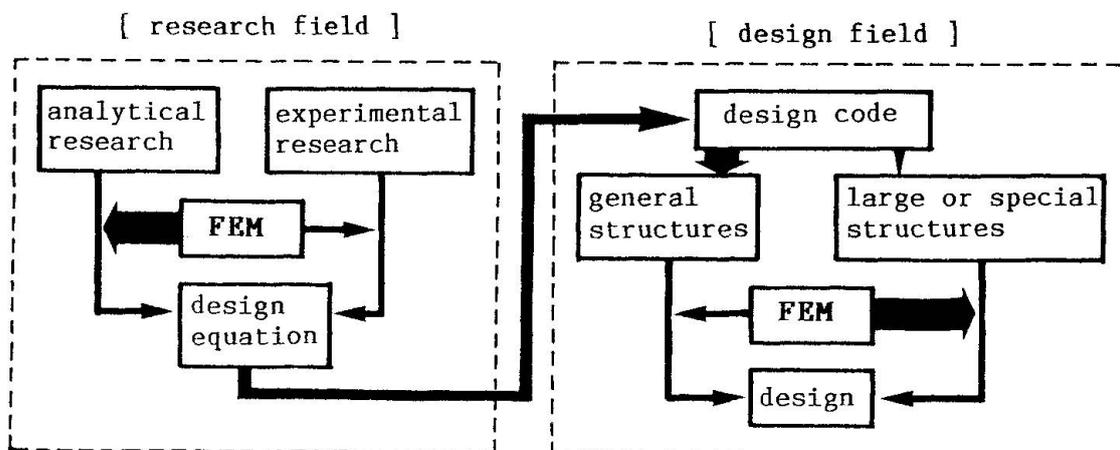


Fig.1 Present situation in the application of FEM to design

civil and architectural engineering. It is considered that the replies realistically represent the tendency of FEM analyses in the design of RC structures in our country.

This paper outlines the present situation and subjects to be solved in the application of FEM analyses to the design of RC structures according to the replies of the questionnaire.

## 2. PRESENT SITUATION IN THE APPLICATION OF FEM TO DESIGN

### 2.1 Summary of Replies of the Questionnaire

Practical design examples using FEM analyses were surveyed in the questionnaire. Questions covered the following:

- (a) practical examples of the application of FEM analyses
- (b) methods of finite element analysis applied to design
- (c) evaluation of analytical results
- (d) methods reflecting analytical results on practical design

Forty-four replies to the questionnaire were collected.

The replies concerning the practical examples of the application of FEM analyses were divided into five groups.

- (1) the analysis of structures whose loading conditions are very complicated, such as floor slabs or base slabs
- (2) the analysis of complicated or hollow structures that consist of many different plates and shells, such as underground tanks storing liquefied natural gas or nuclear reactor plants
- (3) the analysis of specific parts of a structure where it is necessary to examine the internal stress distribution precisely, such as connections, openings or tendon anchorage zones
- (4) the analysis concerning the thermal stress condition or heat conduction in a structure where thermal loads are predominant
- (5) the analysis of thermal cracks in a massive concrete structure

Fig.2 shows an example of the application of an FEM analysis to the design. This is a finite element mesh for a reinforced concrete hollow "Kan-non" (the goddess of mercy) statue that consists of many different shells.

These examples can be summarized in three categories.

- (1) cases that involve great difficulties in converting an original structure into an assemblage of simple structural members, such as beams, columns or frames.
- (2) analyses concerning thermal problems
- (3) cases that grasp the mechanical behavior of a structure accurately

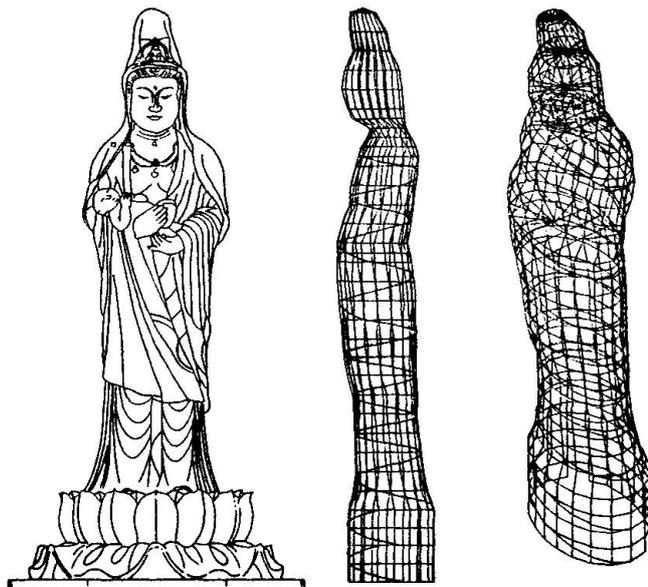
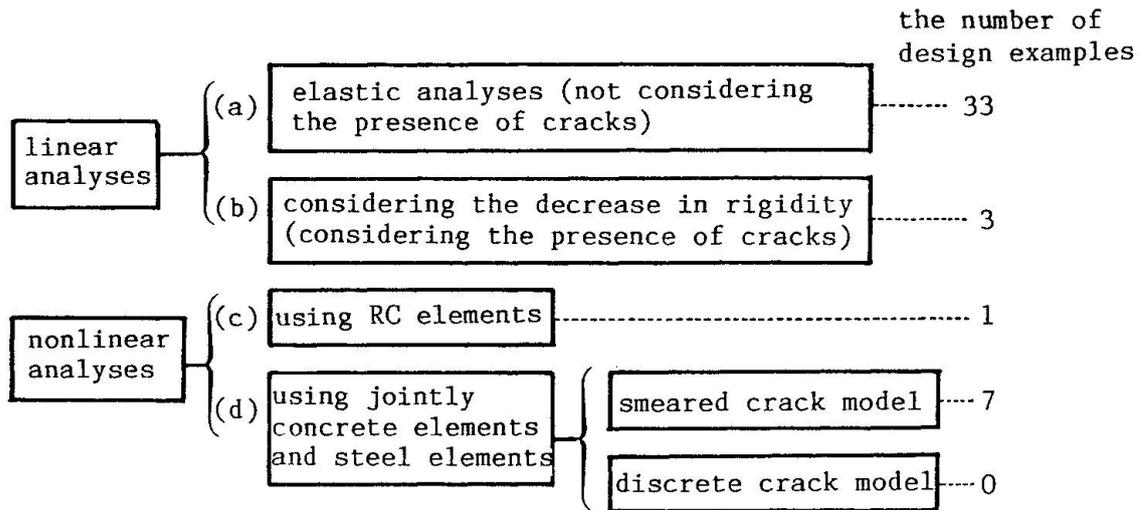


Fig.2 Finite element mesh for the goddess of mercy



**Fig.3** Classification of the methods of finite element analysis and the number of design examples

In general, methods of finite element analysis can be divided as represented in Fig.3. The figure on the right in Fig.3 indicates the number of practical design examples using FEM analyses in the replies.

At first glance it can be seen that most of the analyses applied in practical design are linear analyses. In addition nearly all are perfectly elastic, and don't consider the presence of cracks.

The methods of finite element analysis that consider the presence of cracks can be classified into three types, as represented in Fig.3.

Item (b) is a linear analysis. It assumes the decrease in rigidity according to the presence of cracks to be uniform throughout the whole of a structure. This method is called "an equivalent elastic analysis", and is generally used in cases where the design load levels are not excessively large and estimation of the overall nonlinearity of a structure is sufficient.

Item (c) is a nonlinear analysis. In this case, reinforced concrete elements are used and the behavior of cracks is not considered directly. Reinforced concrete elements can represent the total behavior of reinforced concrete, and the influence of crack propagation is already contained in the employed tension stiffening model. In this method, nonlinearity that is very near the actual behavior of reinforced concrete must be assumed.

Item (d) is also a nonlinear analysis. In this case, the nonlinearity due to the presence of cracks is considered directly. There are two models, that is "a smeared crack model" and "a discrete crack model" in this method. This method can predict the position, the propagation and the width of cracks.

The replies concerning the evaluation of analytical results and the reflection of them on practical design will be explained in Chapter 3.

## 2.2 Present Situation in the Application of a Linear Analysis

According to replies to the questionnaire, most of the FEM analyses applied to the practical design of RC structures are linear analyses. Nonlinear analyses

considering the presence of cracks and the material nonlinearity are not conducted except for special cases. In most practical design procedures, section forces are obtained by a linear analysis, and stresses of structural members are calculated by a conventional design method stipulated in a design code. A conventional design method ordinarily considers material nonlinearity with a simple form and permits a structure to be decomposed into simple structural members.

In the design of large-scale, complicated structures, such as nuclear reactor plants and underground tanks storing liquefied natural gas, however, the influence of cracks is usually considered. For these structures, the degree of the decrease in rigidity according to cracks is considerably large. To determine the section forces of these structures generated by thermal loads, for instance, the following methods are employed, taking account of the decrease in rigidity.

- (1) the application of a pseudo elasto-plastic analysis
- (2) the use of a uniform decreasing rate of rigidity
- (3) the use of proposed formula predicting the decrease in rigidity

In a pseudo elasto-plastic analysis, a conventional structural analysis is first carried out to obtain section forces. In this structural analysis, the rigidity is determined by assuming that the total area of a section is effective. Secondly, an analysis calculating stresses from the obtained section forces is conducted. As a result of the stress analysis, if it is judged that an arbitrary element contains cracks, the corresponding element is replaced with an anisotropic one whose tensile strength perpendicular to the direction of cracks is zero. Finally the stress analysis is repeated with additional reinforcement arranged in the cracked elements. Though this method is very lucid, it is necessary to execute the analysis twice. Therefore, it demands a certain amount of extra cost for calculation and is troublesome.

In the method using a uniform decreasing rate of rigidity, the elastic rigidity is assumed to be uniform everywhere throughout the whole structure. Usually the uniform decreasing rate is evaluated as one half or one third of the original value. This is due to the presence of cracks and the influence of creep according to long-term thermal loads. Although this method is only tentative, it has been widely adopted in our country because of its ease of application.

In the method using the proposed formula, the decreasing rate of rigidity has to be determined adequately taking account of previous papers, experimental data measured in situ, or other design examples. The method to determine the decreasing rate for combinations of various types of loads is not necessarily proposed, therefore the judgment of the design engineer himself becomes significant. In a linear analysis which does not consider the presence of cracks, section forces and their distributions calculated from a conventional structural analysis are, in general, widely different from actual conditions. Considering the improvement of design, it is advisable to estimate the decrease in rigidity according to cracks. To improve design procedures, it is urgently required that guidelines or standards evaluating the decrease in rigidity be stipulated.

### 2.3 Present Situation in the Application of Nonlinear Analysis

Procedures for nonlinear analyses with high accuracy have been established in the field of analytical research. Accompanying the recent high-speed proces-



sing of computers, a nonlinear analysis that can predict the actual behavior of RC structures fairly well has become available for practical design. Nevertheless, at present most of the analyses applied to design are linear which consider the equivalent elastic rigidity instead of the actual nonlinearity, as represented in replies to the questionnaire. Even in cases where a nonlinear analysis is carried out, it was replied that the aim is only verification of the results of a linear analysis used in the practical design, or a preliminary analysis for a linear analysis.

Nonlinear analyses have been steadily improved to allow easy and rapid prediction of the actual behavior of RC structures, compared with former days. In spite of the circumstances, a nonlinear analysis still consumes more time and cost for calculation than a linear one. Therefore, in advance of the execution of an analysis for design, a design engineer has to take account of both the merits and demerits of a nonlinear analysis that can adequately predict the actual behavior but will require more time and cost. At that time, a design engineer has naturally accepted as true that the nonlinear behavior of RC structures can be predicted indirectly and almost accurately by a linear analysis corresponding to the level of design loads.

After accepting this situation, the following question was presented in the questionnaire. It represents a case requiring the carrying out of a nonlinear analysis. The replies obtained can be divided into four categories.

- (1) the case where high reliability is necessary for practical design
- (2) the case where rationalization for design and the saving of cost can be realized remarkably by a nonlinear analysis
- (3) the case in which the nonlinearity of a structure is fairly large
- (4) the case in which the results of a linear analysis may become dangerous

High reliability is necessary for the design of important structures, such as nuclear reactor plants. These structures have a great social influence. For these structures, the time and cost for calculation are given no consideration, as a reliable analysis accurately predicting actual behavior is essential. At present, this is the most important reason for the execution of a nonlinear analysis.

A remarkable rationalization of design and saving of cost will be realized, for instance in the case where a redistribution of bending moment can occur in statically indeterminate structures. According to the change from the elastic resistant mechanism of a structure to the inelastic one, section forces obtained by linear analyses will alter considerably. As the limit state design method or the ultimate strength design method becomes predominant, practical design examples corresponding to this case will increase.

The degree of nonlinearity of a structure is naturally influenced by the level of design loads. It is necessary to consider nonlinearity in a structure subject to seismic loads, since the displacement of a structure after yielding must be examined. Besides these structures, nonlinearity due to thermal cracks must be examined for massive concrete structures. In massive concrete, thermal cracks and the change of material properties in the process of concrete hardening have an important effect on analytical results. Under these circumstances, nonlinear analyses for massive concrete have been executed very frequently in our country.

When bending moment of a structural member may increase according to the redistribution or the displacement of a structure after yielding must be checked, it is possible that a linear analysis will provide the results on the dangerous

side.

A nonlinear analysis is indispensable for designs where high reliability and safety are required. However, if a nonlinear analysis is always applied to all types of design, design procedures may become irrational. It is important to adequately determine the adoption of a nonlinear analysis considering the intended structures and limit states, the level of design load and the deformation conditions.

### 3. METHOD REFLECTING ANALYTICAL RESULTS ON DESIGN

The method of reflecting the analytical results of an FEM analysis on practical design is an important matter to design engineers, researchers and developers of computer programs. An established method reflecting analytical results on practical design has not been obtained yet. Therefore, the means of dealing with analytical results differs case by case. However, there are a few almost unified methods according to intended structures or purposes of design.

In a plate bending analysis and a shell analysis, section forces are usually obtained as the output of a calculation. It is possible to use these results directly for a design. In such cases, section forces are rarely calculated by a nonlinear analysis. This is because a linear analysis generally provides conservative results for design. The margin can be incorporated into the degree of the safety for design. Furthermore, the rigorous accuracy of calculation is not necessarily required for a linear analysis.

To determine section forces in a plane stress analysis and a three-dimensional solid analysis, it is necessary to transform the obtained stresses into section forces. Since the obtained analytical results are influenced firmly by the fineness of divided elements, it is difficult to reflect them directly on design. However, in a plane stress analysis or a three-dimensional solid analysis, the shapes and boundary conditions can be assumed to be very similar to those of the actual structure. It is also possible to execute more rational and accurate analyses in comparison with a plate bending analysis or a shell analysis. Considering the convenience for design, it is desirable to apply obtained analytical results directly to practical design, for instance the determination of a section of a structural member or the arrangement of reinforcement. It is expected that the most suitable design method without transforming obtained stresses into section forces will be proposed.

Besides the reflection of analytical results obtained on practical design, some examples of their use as reference data only are reported in replies to the questionnaire. For the case of decomposing special structures into simple structural members, the results obtained are used as implements to examine whether the decomposition itself is adequate or not. Besides this kind of examination, results obtained are used to grasp the actual mechanical behavior of a structure. For these purposes, a nonlinear analysis is usually carried out to secure the safety of the conventional design method independent of a nonlinear analysis.

The following opinion was predominant in the replies. If section forces obtained by a nonlinear analysis are greater than the values estimated in the conventional design, the amount of reinforcement must be increased. On the other hand, if the section forces obtained are smaller than the estimated values, the amount of reinforcement should not be decreased.

In addition, nonlinear analyses have been executed in the following cases where



the degree of safety must be examined out of the application range of a design code or a standard, the validity of design methods stipulated in a design code must be verified, and the results of the design according to different design codes must be compared with each other.

#### 4. SUBJECTS TO BE SOLVED IN FUTURE

Considering the present situation where the occurrence of large and complicated structures, special structures and new construction methods has been frequent, it is indubitable that the application of an FEM analysis to practical design of RC structures will increase rapidly. In these circumstances, some important subjects still remain to be solved.

- (1) the certification for computer programs
- (2) the preparation of guidelines on the application of FEM analyses for design

Many computer programs for FEM analyses are circulating at present. They can be divided into two types, one being programs developed individually and the other being purchased ones. They are being widely used. In the analysis of complicated structures, it is virtually impossible to verify all analytical results precisely. Design engineers cannot but believe the results to be right. Therefore, it is necessary to certify the validity of a computer program itself, especially for a nonlinear program.

Analytical results are often affected remarkably by the decomposition of an original structure into simple structural members, the division and proportion of finite elements, the assumption of input data and some other causes. Although an FEM analysis can compute stress conditions precisely within an arbitrary limited portion, on the other hand it is possible that an FEM analysis will provide an extremely large output of stresses in some elements. For the cases where the results are changed considerably according to the procedures employed, and extraordinary outputs are provided locally, it is not satisfactory for each design engineer to deal with these problems individually without any restrictions. This matter will bring about a decrease in the reliability of the design. It is urgently required that authorized guidelines for the application of FEM analyses be completed. The preparation of guidelines is intensively desired in the field of design.

#### 5. CONCLUSIONS

Based on the replies to the questionnaire circulated by the JCI Committee, the present situation in the application of an FEM analysis to practical design and subjects to be solved in future were described in the outline. Within the limited replies, it can be adequately predicted that an FEM analysis has been used more often for practical design.

With regard to current tendencies, most of the FEM analyses applied to practical design are linear ones. However, practical examples using nonlinear analyses or linear analyses that indirectly consider nonlinearity are increasing steadily. It has been widely recognized that a nonlinear analysis must be executed in accordance with the intended structure or purpose of the design.

Considering these circumstances, the JCI Committee intends to examine the methods of reflecting analytical results, especially by using a nonlinear analysis, on practical design, and propose guidelines for the application of



FEM analyses to designs.

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I.Shiraishi	Kumagai Gumi Co.	H.Yoshikawa	Hazama-Gumi Co.
Y.Suzuki	Sumitomo Cement Co.	F.Watanabe	Kyoto University
K.Naganuma	Ohbayashi Corporation	Y.Watanabe	Shimizu Construction Co.

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