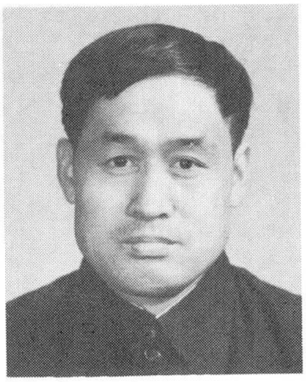
Direct Iteration in Nonlinear Analysis of 3-Dimensional Concrete Structures

Méthode d'itération et analyse non-linéaire de structures tridimensionnelles en béton

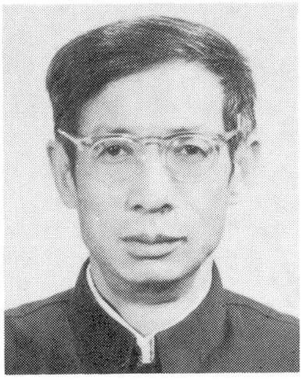
Direkter Iteration bei der nichtlinearen Berechnung von räumlichen Betonkonstruktionen

Jiang JIAN-JING
Tsinghua University
Beijing, China



Jiang Jian-Jing, born in 1938, graduated from Tsinghua University in 1963 and received a Dr. Degree from Chalmers University in 1983. Now he is the director of R.C. structure division at Tsinghua University. He has been investigating mainly in finite element analysis and computer simulation of concrete structures.

Zhu JIN-QUAN
Tsinghua University
Beijing, China



Zhu Jin-Quan, born in 1934, graduated from Tsinghua University in 1957. Now he is the Associate Dean of Civil Engineering Department at Tsinghua University. He has been investigating mainly in reinforced concrete structures.

SUMMARY
In this paper the direct iteration method for nonlinear analysis of three-dimensional reinforced concrete structures is studied. The paper presents a nonlinearity index, which is in term of second invariant stresses deviator tensor. Much computer time can be saved by using this nonlinearity index. One example is included and numerical results are compared with the experimental results. The comparison shows that this method may be recommended for practical use.

RÉSUMÉ
La contribution traite de la méthode d'itération directe pour l'analyse non-linéaire de structures tridimensionnelles en béton armé. Elle présente un index de non-linéarité, qui est un vecteur de tension de second ordre. Un temps d'ordinateur considérable peut être économisé en utilisant cet index de non-linéarité. Quelques exemples sont présentés et les résultats numériques comparés avec les résultats expérimentaux. La comparaison montre que cette méthode peut être recommandée dans la pratique.

ZUSAMMENFASSUNG
In diesem Beitrag wird die direkte Iterationsmethode für nichtlineare Berechnungen von räumlichen Stahlbetonkonstruktionen studiert. Der Beitrag stellt einen Nichtlinearitätsindex vor, der die zweite Invariante des Tensors der Deviatorspannungen verwendet. Mit diesem Index kann viel Rechenzeit gespart werden. Beispiele numerischer Berechnungen werden mit Versuchsresultaten verglichen. Der Vergleich zeigt, dass diese Methode der Praxis empfohlen werden kann.

1. INTRODUCTION

Three dimensional concrete structures are widely used in massive footing of huge machine, offshore platform, concrete reactor vessel, etc. But its detailed behaviour under various stress combination has not been full understood. In this paper the nonlinear finite element techniques are used for analysis of 3-D reinforced concrete structure from beginning of loading to failure of structure. In this paper the reinforcement is regarded as a steel membrance in the concrete, but different constitutive relationships are adopted for two different materials. The nonlinearity and crack growth of concrete, the yield of reinforced bars are considered in the analysis. The direct iteration method is used for solving the nonlinear finite element equation systems, which is quite efficient for the full range of nonlinear analysis. This method is first proposed by Ottosen [1]. Here this method will be extended to analyse three dimensional reinforced concrete structure. In Ottosen's model, the nonlinearity index is defined in term of σ_{3f}/σ_3, in which the interactive calculation is needed to get β. In this paper another nonlinearity index $\sqrt{J_{2f}}/\sqrt{J_2}$ is proposed, which can be directly calculated from the stress state and have evident geometrical means in stress space.

2. FINITE ELEMENT FORMULATION

Taking into consideration the effect of reinforcement. The eight-node isotropic element with reinforcement membrance is used. In this case the strain in the reinforcement is assumed to be the same as the surrounding concrete. Thus two materials are integrated into a single element but have separate stress-strain relations. A detail expla-nation can be found in Reference [2]. Here only the formula which are used in this paper are written as follows.

The stress-strain relation is

$$[\sigma]=[D][\varepsilon]$$

where [D] is the material matrix, which is change with the stress level. The stiffness matrix can be calculated by using standard procedure, i.e.

$$[Kc]=\iiint_v [B]^T [Dc][B]dv$$

where [B] - geometric matrix of solid elements
[Dc] - material matrix of concrete

The contribution of reinforcement membrane to stiffness matrix of element may be calculated as follows

$$[Ks]=t\iint_A [B]^T [L]^T [Ds][L][B]dA$$

where [B] - geometric matrix of solid elements
[L] - matrix of coordinate translation
[Ds] - material matrix of reinforced bar
t - equivalent thickness in reinforced direction.

Then the total stiffness matrix of element [K] can be calculated as

$$[K] = [Kc] + [Ks]$$

3. CONSTITUTIVE RELATION FOR CONCRETE

From the test of concrete under compressive stresses shows that the nonlinear strain is existed at beginning of loading and hasn't evident initial yield surface. On the other hand the stress-strain relation of concrete under triaxial stress condition has not yet been full understood. In this case the Ottosen's nonlinear elastic model is available for monotonously increasing load.

In order to evaluate the modulus of elasticity of concrete at different stress level, three things have been decided upon first, i.e.

(1) The failure criterion of concrete;
(2) The equivalent uniaxial stress-strain formulation of concrete;
(3) The nonlinearity index of concrete.

The failure criterion under triaxial stress state proposed by Ottosen is assumed in this paper. However, some other failure surfaces, such as Mohr-coulomb, Drucker-Prager, W.F.Chen, Willianm-Watnke have been implemented in the program. From the expression of the stress-strain relation under uniaxial loading, the secant modulus of concrete, Ec, can be determined from the uniaxial expression by using the nonlinearity index. Here the following expression proposed by Sargin [4] is adopted:

$$-\frac{\sigma}{f_c'}=\frac{-(Eo/Ep)(\varepsilon/\varepsilon_p)+(D-1)(\varepsilon/\varepsilon_p)^2}{1-((Eo/Ep)-2)(\varepsilon/\varepsilon_p)+D(\varepsilon/\varepsilon_p)^2}$$

in which tensile stress and strain are taken as positive. ε_p is the strain at peak stress f_c', Eo is the initial modulus, and Ep is the secant modulus corresponding to $\varepsilon = \varepsilon_p$. D is a parameter which mainly affects the descending segment of the stress-strain curve (Fig. 1). The nonlinearity index β is defined as the ratio of σ/f_c' . Thus the secant modulus of concrete Ec can be evaluated as

$$Ec=0.5Eo-\beta(0.5Eo-Ep)\pm\sqrt{[0.5Eo-\beta(0.5Eo-Ep)]^2+\beta Ep^2\,[D(1-\beta)-1]}$$

where the positive sign is used for the ascending part and the negative sign is used for descending part of the curves.

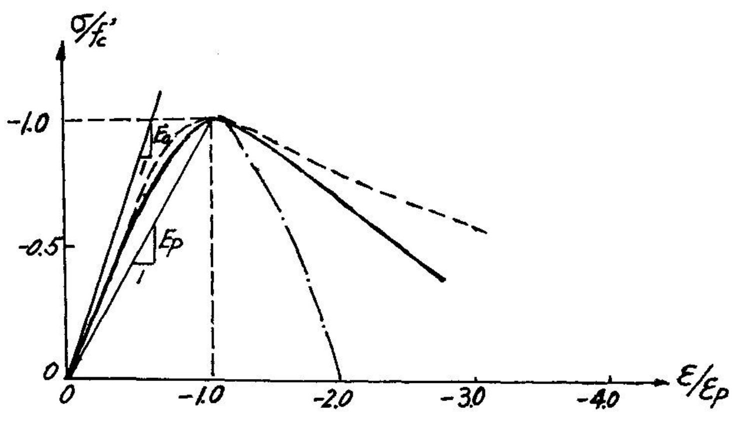
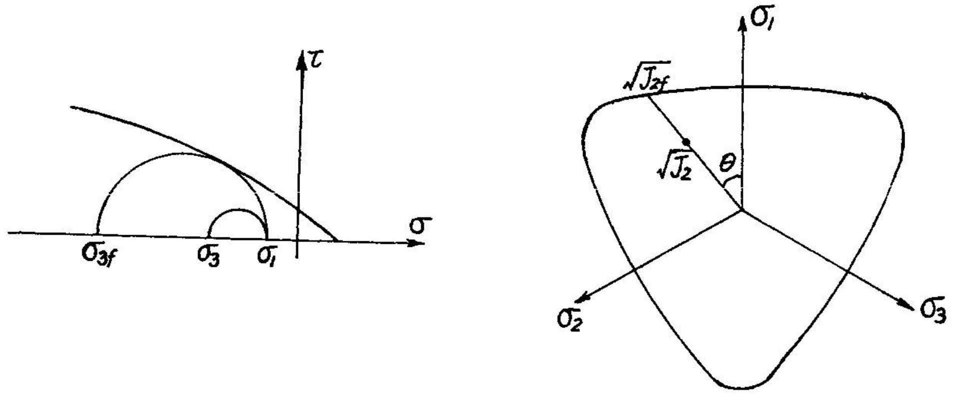


Fig. 1

Under uniaxial loading nonlinearity index β is determined by the scalar stress σ only. How can β be determined under general stress condition? Ottosen suggests

(Fig.2):

$$\beta = \sigma_{3f}/\sigma_3$$



(a) (b)

Fig. 2

where σ_3 is the third principal stress
σ_{3f} is the failure value of σ_3 provided σ_1 and σ_2 are unchanged.

In order to determine the σ_{3f} , the try and error method should be used. In this paper the β value is suggested to be calculated as follows:

$$\beta = \sqrt{J_{2f}} / \sqrt{J_2}$$

where J_2 is the second invariant of stress deviator tensor
J_{2f} is the failure value provided I_1 and θ keep unchanged.

4. DIRECT ITERATION METHOD

The finite element equation

$$[K][U] = [P]$$

is a set of nonlinear equations, in which, [U] the total stiffness matrix changes with the stress level. Here, the "direct iteration method" is developed to solve the nonlinear equations. The iterative steps are as follows:

(1) Evaluate the first approximate displacement $[U_1]$ with the initial stiffness matrix $[K_1]$.

(2) Calculate the strain of each element from the displacement $[U_1]$.

(3) Calculate the stress for each element.

(4) Calculate nonlinearity index β .

(5) Evaluate the secant modulus of concrete, and form the updated material matrix

[Dc].

(6) Check the tension cut-off condition: if $\sigma_1 > f'$, modify [Dc].

(7) Calculate the stress of the reinforcement and check the yielding condition: if $\sigma_s > f_y'$, modify the material matrix [Ds].

(8) Calculate the element stiffness matrix [Kc] and assemble the structural stiffness $[K_2]$.

(9) Evaluate the next approximate displacement $[U_2]$ with $[K_2]$ by

$$[U_2] = [K_2]^{-1} [P]$$

(10) Check the convergence condition: if $\|\delta u\| \leq \varepsilon$, stop the iteration and output the results, where ε is the convergence tolerance; otherwise, replace $[U_1]$ with $[U_2]$, go to step (2), and repeat the procedure.

5. EXAMPLE

The footing structure, Fig. 3, was tested by Nylander. The footing is loaded by a jack, and fixed to the ground by 12 steel bars. Swedish deformed bars (kamstal) of type Ks 60 were used as reinforcement. The actual average yield stress is f_y'=621 MPa. The amount of reinforcement is 17ϕ 8 and 16ϕ 8, see Fig. 3, The load-deflection curve obtained from experiment is shown in Fig. 4. The deflection of the centre obtained from calculation is also shown in Fig. 4 by a dashed line. The strain at the centre of the reinforcement is shown in Fig. 5 in which the solid line shows the experimental result and the dashed line shows the analytical results by this program.

It can be concluded that the calculated load-displacement curve is in reasonable agreement with the experimental data that the analytical stress in the centre reinforcement can reflect the main characteristics of the experimental results. The calculated failure load is about 11% higher than that experimentally obtained. It is reasonable for the concrete structure.

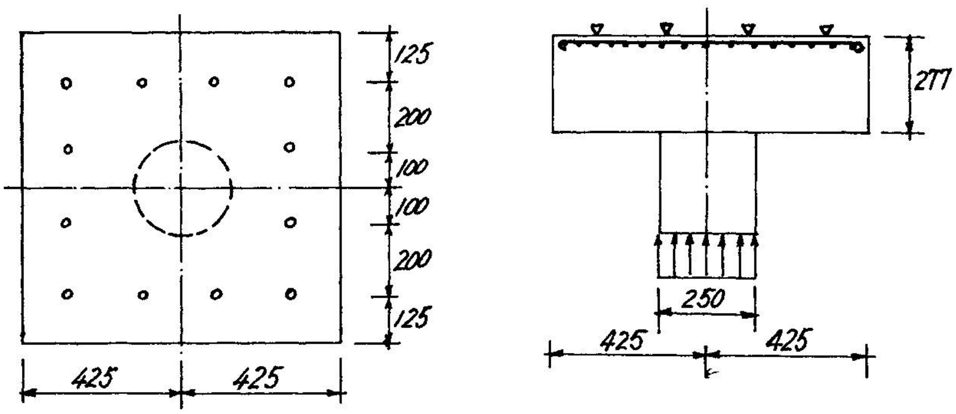


Fig. 3

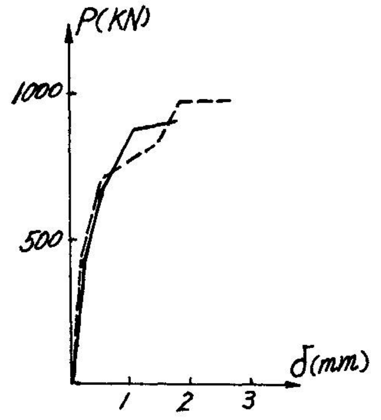


Fig. 4

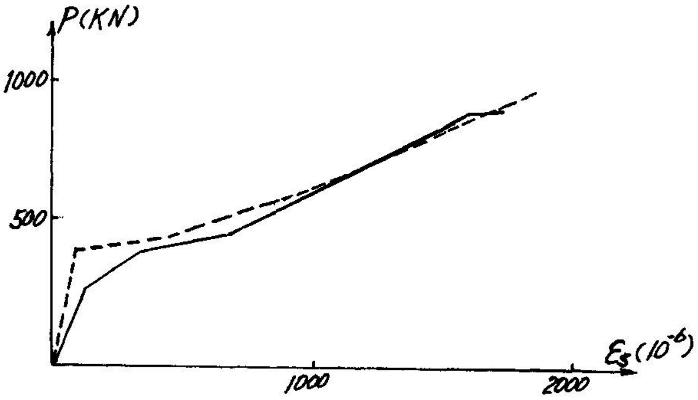


Fig. 5

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