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A Finite Element Simulation Model for Cracks in Reinforced Concrete

Modèle de simulation par éléments finis pour des fissures dans le béton armé

Finite Element Simulation von Rissen in Stahlbeton

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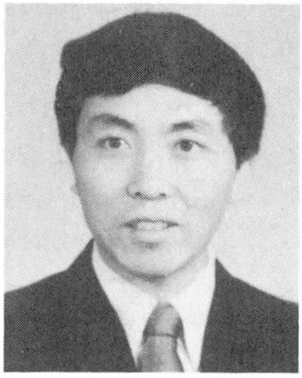


Di Shenglin, born 1951, got his master's degree of engineering at Nanjing Institute of Technology, Nanjing, Jiangsu. Now directed by professors Song Qigen and Shan Bingzi in the Research Division of Structural Mechanics, NIT, he was involved in computational mechanics of RC structures and finite element models for eight years.

SUMMARY

An analytical model has been developed to simulate the crack initiation and propagation. By separating and moving the nodes, the mesh of structure can be changed successively to follow the crack growth. In a computer program the nonlinear behavior of material and bond-slip relationship is taken into account. All the treatments are automatically executed by computer; the rationality of this model has been studied.

RÉSUMÉ

Un modèle analytique simule l'origine d'une fissure et sa propagation. En séparant et déplaçant les noeuds, le réseau de la structure peut être adapté pour suivre l'évolution de la fissure. Un programme d'ordinateur prend en considération le comportement non-linéaire du matériau et la relation adhérence-glissement. Tous les calculs sont exécutés à l'aide de l'ordinateur; le modèle semble rationnel.

ZUSAMMENFASSUNG

Ein numerisches Modell zur Beschreibung der Rissentstehung und deren Fortpflanzung wurde entwickelt. Durch Trennen und Verschieben von Knoten wird das Elementennetz dem Rissfortschritt angepasst. Im Computerprogramm wird nichtlineares Materialverhalten und Verbundverschiebung berücksichtigt. Die Rationalität dieses Modells ist untersucht worden.

1. INTRODUCTION

The crack initiation and propagation is one of the most important characteristics of concrete. In reinforced concrete structure, the cracks cause sudden changes in local stress levels and the local stress will be redistributed. The difference of crack distribution between different structures which are controlled by different strength criteria is obvious, so we must describe the crack initiation and propagation correctly.

There are three different approaches employed for crack modelling: (1) discrete cracking model; (2) smeared cracking model; (3) single crack within an element dealt with by fracture mechanics.

In the first approach, the difficulties are that the location and direction of the cracks are unknown in advance, so Ngo and Scordelis [1] have predicted the diagonal crack in simply supported beam. Based on the discrete cracking model, an approach for computer simulation of crack formation and growth has been developed in this paper. By separating and moving the nodes, the finite element mesh of structure can be changed successively to follow the crack propagation. The crack occurs and grows in accordence with the stress state, it is not necessary to predict the crack location and direction in advance.

2. FINITE ELEMENT SIMULATION MODEL

Consider the boundaries between neighboring element as checking line, if the average stress of two neighboring elements achieve the tensile strength of concrete, f_t', a crack will form along this checking line. According to the crack situation of neighboring boundaries, we can make different treatment, that include, (1) add an extra node, renumber the nodes and equations, separate the corresponding node as shown in Fig.1(a-e); (2) change the direction of cracked boundary to perpendicular to direction of σ_1; (3) calculate the displacement of moved nodes by Lagrangian interpolation formula.

The range in which the nodes can be moved is limited to avoid distorting the element shape exeessively, see Fig.1(f). For simplification, the node can only be moved along one direction, e.g. if the checking line along X coordinate is cracked, the node of this

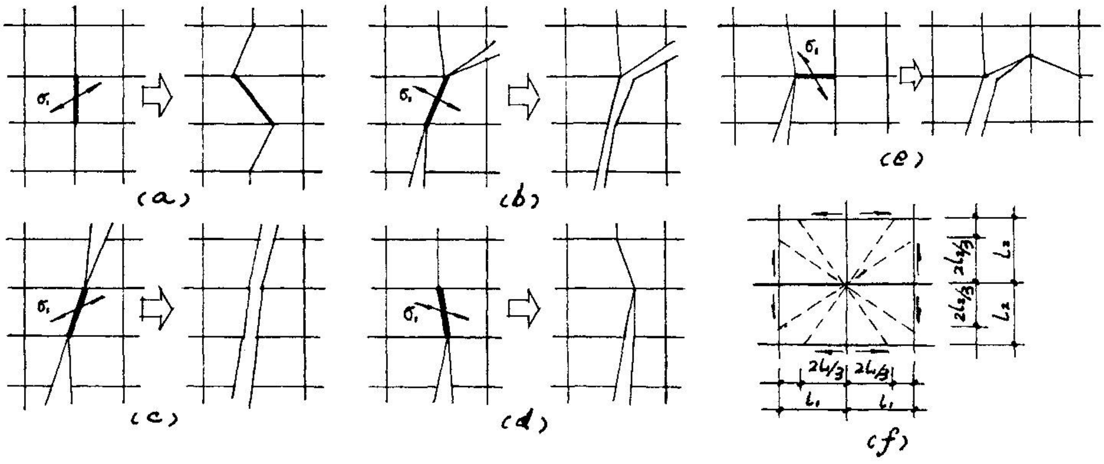


Fig. 1

border can only be moved in the direction of Y coordinate, vice cersa.

If some nodes are disjointed, the program will compute the equivelant node forces of relative elements and release them in next iteration to reflect the stress redistribution occured in actural structures. e.g. assuming node r will be disjointed, we add

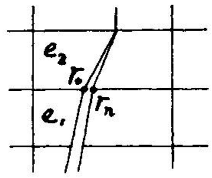


Fig. 2 (m=2)

a new node r_n and call the original node r_o (Fig.2), the equivelant node force of element e_1 respect to node r_o

$$\{P\}_{r_o}^{e_1} = \int_{A_{e_1}} [B]_{r_o}^T \{\sigma\}_{e_1} \cdot t dA \tag{1}$$

where, t,A= the thickness and area of element e_1; $[B]_{r_o}$=node submatrix of strain. Summing up the contribution of all elements

$$\{P\}_{r_o}^{eq} = \sum_{i=1}^{m} \{P\}_{r_o}^{ei} , \quad \{P\}_{r_n}^{eq} = -\{P\}_{r_o}^{eq} \tag{2}$$

is obtained, in which m=number of elements which are linked to node r_o.

According to the experimental results[2], though the bond action at each side of crack is weakened, the range is limited, so if we take the stiffness of linkage element as zero in such a case, a considerable error will be caused. In this program, when the crack crosses the reinforcement and the node is separated, we add an extra linkage element to link the new node of concrete to the same node of reinforcement and then adjust the longitudinal stiffness of these linkage element.

When all the checking lines have been examined, the computer execute a new iteration using the released equivelant node force vector. If in an iteration, there is no new crack occured or propagated and also no element feil, the loading increment of next step will be added.

3. NONLINEAR ANALYSIS

A computer program has been set up for reinforced concrete plane problem. Using the quadratic element for both concrete and reinforcement, one dimension element for stirrup, the program has accepted the tangential stiffness increment method.

The stress-strain relation of concrete under biaxial compression can be expressed as

$$\sigma = \frac{a+b\epsilon}{1+c\epsilon+d\epsilon^2} \tag{3}$$

the constant a,b,c,d can be given by

$$\epsilon=0,\quad \sigma=0,\quad \frac{d\sigma}{d\epsilon}=E_0 \quad ;$$

$$\epsilon=\epsilon_c,\quad \sigma=\frac{\sigma_c}{1-\alpha_i\mu},\quad \frac{d\sigma}{d\epsilon}=0 \quad . \tag{4}$$

Where σ_c, ϵ_c=stress and strain at uniaxial compressive failure;

$$\alpha_1=\sigma_1/\sigma_2\ ,\ \alpha_2=\alpha_1^{-1}\ ;$$

E_0= initial elastic modulus.

then we have

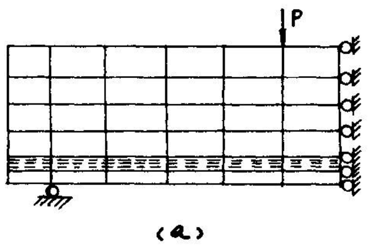
$$E_i=\frac{d\sigma_i}{d\epsilon_i}=\frac{E_0\left[1-\left(\frac{\epsilon_i}{\epsilon_{ic}}\right)^2\right]}{\left[1+\left(\frac{1}{1-\alpha_i\mu}\cdot\frac{E_0}{E_s}-2\right)\cdot\frac{\epsilon_i}{\epsilon_{ic}}+\left(\frac{\epsilon_i}{\epsilon_{ic}}\right)^2\right]} \tag{5}$$

$$(i=1,2)$$

The concrete is considered as an orthotropic body, the modulus of elasticity in compression is calculated from eq.(5), in tension is E_0. The reinforcement is considered as idealized elasto-plastic material. The failure criteria is based upon Kupfer's work[4].

4. NUMERICAL RESULTS

A simply supported beam (Fig.3a) is analysed to study the rationality of this approach. After two loading increments, the cracks developed as shown in Fig.3b. In the second calculation, we load the same two loading increments on such a cracked beam at the same time. Because of the same equilibrium state, the correct results of the second computing must be the same as the first one. In fact, the error are very small(less than 4% in general).



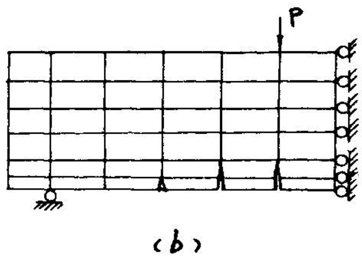


Fig. 3

To examin the whole analytical model and program, a specimen, shown in Fig.4, has been computed. Using 171 concrete elements, we obtained the crack distribution by computer (Fig.5), the crack distribution and direction agree well with the experimental results. The comparison of crack width is not satisfactory, it is obvious that the interlock action in the crack should be included, see Table 1.

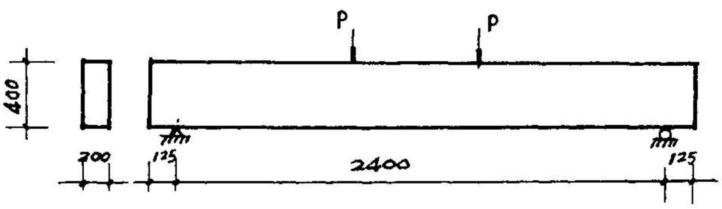


Fig.4 Specimen (mm)

width of crack	load	analysis	experiment
average	4^t	0.090	0.057
	8^t	0.170	0.080
maximum	4^t	0.130	0.060
	8^t	0.300	0.150

Table 1 Comparison of crack width

We have also got a lot of message 3 , such as deflection, stress and the properties of total structure. Combining with the experimental results, the analytical results can help us in study of RC structures.

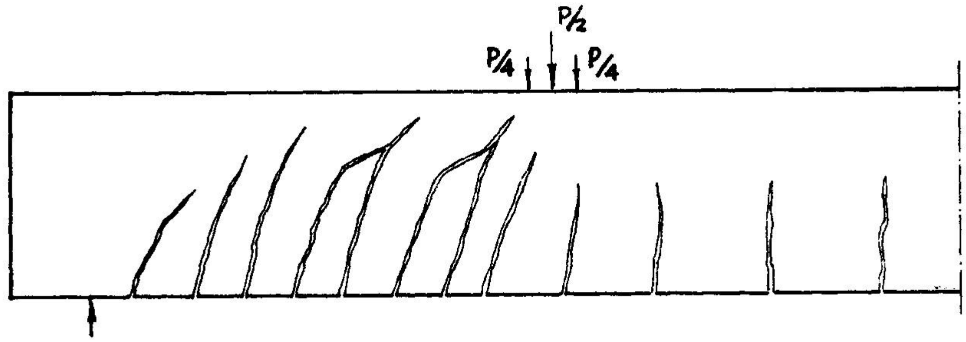


Fig.5 a analytical crack distribution of specimen

REFERENCES

1. Ngo D. and Scordelis A.C., Finite Element Analysis of RC Beam. ACI Journal, March 1967.

2. Song Qigen, Shan Bingzi, Di Shenglin, Study of Reinforced Concrete Bond Properties. International Symposium on Fundamental Theory of RC and PC,Nanjing, PRC, 1986.

3. Di Shenglin, Nonlinear Finite Element Analysis of RC Beams. Journal of Nanjing Institute of Technology, No.2, 1984.

4. Kupfer H.B., Behavior of Concrete Under Biaxial Stress. ACI Journal, August 1969.